Anomalous Scaling of Cumulus Cloud Boundaries

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The geometrical properties of cumulus clouds modeled by a numerical technique called Large-Eddy-Simulation are investigated. Surface-Volume analyses reconfirm previous scaling results based on satellite data. This technique allows for the first time a direct analysis of the scaling behaviour of cloud boundaries of individual clouds.

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Over the last decade, numerical simulation has become an increasingly more important tool to study atmospheric turbulence [1]. Due to improving computer resources it is now feasible to resolve a spatial scale range of 2 to 3 orders of magnitude, i.e. $(L/l_0) \simeq 10^2 \sim 10^3$, where l_0 is the grid spacing and L the domain size. Atmospheric dynamics however ranges over 10 orders of magnitude, from the smallest dissipative Kolmogorov scale $\eta \sim 1$ mm up to largest synoptic disturbances at scales of several thousand kilometers. So depending on the scale range of interest one has to choose the appropriate resolution. On the large scale side of this spectrum we have general circulation models (GCM's), such as numerical weather prediction models that use a resolution of $l_0 \simeq 50$ km. On the small scale side of the spectrum one finds Direct Numerical Simulation (DNS) models that operate at a resolution of the dissipative Kolmogorov scale η. In this case the Navier-Stokes equations are integrated directly without having to make any subgrid scale assumptions, but for atmospheric turbulence the domain size would be $L \simeq 10$ cm. Interesting scaling results from both model types have been reported recently, that supply a useful complement to laboratory and field experiments [2,3].

In the present study we are interested in the geometry of cumulus clouds. Clearly, we cannot utilize the above mentioned simulation techniques since the linear size of cumulus clouds ($0.1 \sim 10$ km) is well *above* the domain size L of any DNS model and well *below* the resolution l_0 of any GCM in the foreseeable future. We will therefore employ another numerical technique, which is Large-Eddy-Simulation (LES) [4]. The resolution of an LES model is such that only the largest and most energetic turbulent eddies of a three-dimensional turbulent field are explicitely resolved, whereas the smaller subgrid eddies are described by a diffusive subgrid filter that is chosen such as to obey the well known scaling behaviour in the inertial subrange. This approach, pioneered by Deardorff [5] in the early seventies, was originally applied to the clear convective atmospheric boundary layer, but can be used as well to simulate cloudy boundary layers [6–12]. Comparisons with field experiments have shown that present state of the art LES models are by now well capable of numerically resolving the dynamics of atmospheric turbulence in the presence of cumulus clouds [10–12]. Therefore LES models provide a powerful numerical laboratory generating three-dimensional fields that are now widely used as testbeds for conceptual models of cloud dynamics.

No effort has been made however to do a critical test (or even to formulate one) for the geometrical properties of these numerically produced synthetic clouds. Fortunately, Nature does provide an excellent and critical test, since one of the most striking examples of statistical self-similarity is the fractal scaling of cloud boundaries as first reported in Refs. [13,14]. Further empirical evidence can be found in a number of papers on area-perimeter analyses of the geometry of satellite- and radar-determined cloud and rain patterns [15–18]. These analyses suggest a fractal scaling of the cloud perimeter in the horizontal plane with a fractal dimension D_p close to 4/3 over a spectrum of four orders of magnitude in size, ranging from small fair weather cumuli ($\sim 10^{-1}$ km) up to huge stratusfields ($\sim 10^3$ km), although occasional scale breaks have also been reported [16,17]. For the dimension of the cloud surface D_s this observed scaling implies, assuming isotropy, $D_s = D_p + 1 \simeq 7/3$. This scaling exponent has been observed for interfaces of several classical turbulent flows and Reynolds number similarity arguments have been used to explain this behaviour [19,20].

The main purpose of this Letter is twofold: Firstly we will use this strong observational scaling fingerprint of cloud geometries as a critical test for the synthetic clouds such as simulated by LES. Secondly we will discuss the consequences of these results for turbulent mixing across the cloud boundaries. This process, called entrainment, is an important process in the atmospheres hydrological cycle, since cumulus clouds form an important transport mechanism of heat and moisture from the Earth's surface into the free atmosphere. Yet, the exact intensity of this mixing process and its parameterization in large scale weather and climate models is still an active field of research [21–23]

The LES model used in this study is standard and extensively described in the literature [10,11]. The governing set of equations are the filtered Navier-Stokes equations within the Boussinesq approximation for the velocity field, and transport equations for heat and moisture. For these quantities a monotone upwind advection scheme is used in order to deal with the strong gradients around the cloud boundaries [24]. A longwave radiation model based on the "grey-body" approximation [25] and a shortwave "two-stream" radiation model [26] is used. Subgrid-scale turbulent fluxes are determined using a 1.5 order closure scheme for which an additional equation of the turbulent kinetic energy is solved. Local pressure fluctuations are calculated solving a Poisson equation. Condensation effects are taken into account whenever oversaturation effects are encountered. The equations are solved on a computional domain of roughly 5 km^2 in the horizontal and 3 km in the vertical which is divided in $128 \times 128 \times 75$ grid points with a grid size length $l_0 = 40$m. The lateral boundary conditions are cyclic and the surface fluxes for heat and moisture at the bottom are prescribed.

The numerical simulation is based on the Barbados Oceanographic and Meteorological Experiment (BOMEX), during which nonprecipitating shallow cumulus clouds, typical for the whole trade wind region, were the only type of cumuli that were observed under steady-state conditions. A time integration of 10 hours has been made during which a realistic steady state was achieved in agreement with observations. For a detailed discussion on the the resulting dynamics for this case we refer to the literature [11]. Our main focus here is on the cloud geometry.

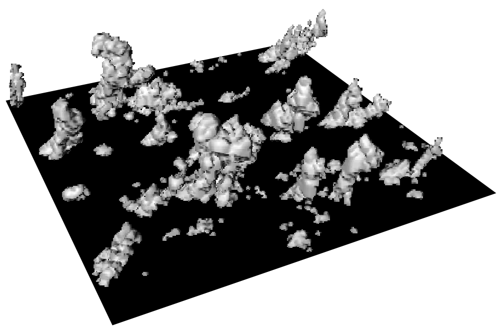

FIG. 1. Snapshot of a three-dimensional cloud field generated by the LES model.

In Fig. 1 we show a snapshot of a typical cloud field after a simulation time of 4 hours. In analogy with the aforementioned area-perimeter analyses we apply a volume-surface analysis for the cumulus field shown in Fig. 1 by measuring the surface area A and the volume V of each cloud. A linear size of each cloud is then defined as $L \equiv V^{1/3}$. For smooth shapes (balls, cubes etc.) the surface area should scale as $A \sim L^{D_s}$ with a surface dimension of $D_s = 2$.

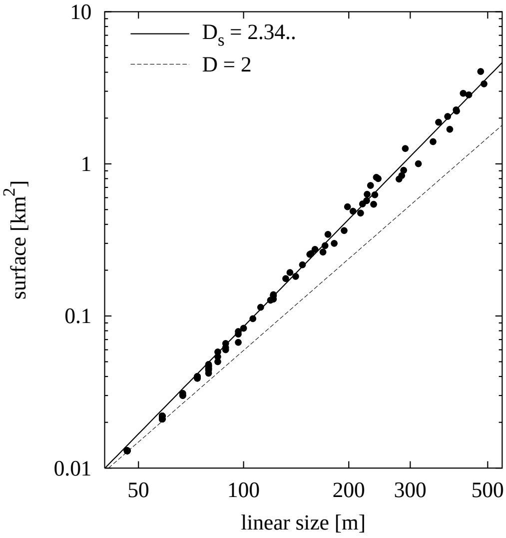

FIG. 2. Log-log plot of the surface area A versus its linear size $L = V^{1/3}$ of all clouds displayed in Fig. 1 along with a linear fit and a line corresponding to the Euclidean case $D_s = 2$

Figure 2 shows a scatter plot of A versus L of all the clouds in logarithmic coordinates. All the points corresponding to clouds with a linear size $l > 4l_0$ are fitted to a straight line. To improve statistics, this procedure is repeated each 5 minutes over a period of 3 hours, thereby building up a database of over 6000 clouds (Fig. 3). This demonstrates that cloud surfaces show a strong and significant non-Euclidian scaling behaviour indicating a surface dimension close to $D_s \simeq 7/3$. Let us stress the fact that this result is highly non-trivial since it is hard to define a dynamics that creates non-Euclidean shapes. Arbitrary random shapes (lattice animals etc.) all fall on the Euclidean ($D_s = 2$) line.

It should be noted however that this simple volume-surface analysis is not a complete numerical demonstration of fractal scaling of the individual cloud surfaces. The possibility of some perverted self-affine scaling cannot be excluded. For instance, if clouds were made of cylinders with radius r and a height $z(r)$ that scale with the radius as $z \sim r^{5/2}$, a volume-surface analysis would also give $D_s = 7/3$ even though the individual clouds are ordinary Euclidean objects. To exclude this possibility we performed two additional tests. Firstly, in order to mimick the observational satellite analysis method as much as possible, we show in Fig. 3 also the results of an area-perimeter analysis based on a two dimensional projection of the three-dimensional cloud fields. The method is less accurate than the volume-surface analysis since it clearly suffers more from discretisation effects. Nevertheless, a linear fit over the same size range gives a slope close to $D_p \simeq 4/3$ in agreement with observations. Apparently, the isotropy assumption $D_s = D_p + 1$ is justified.

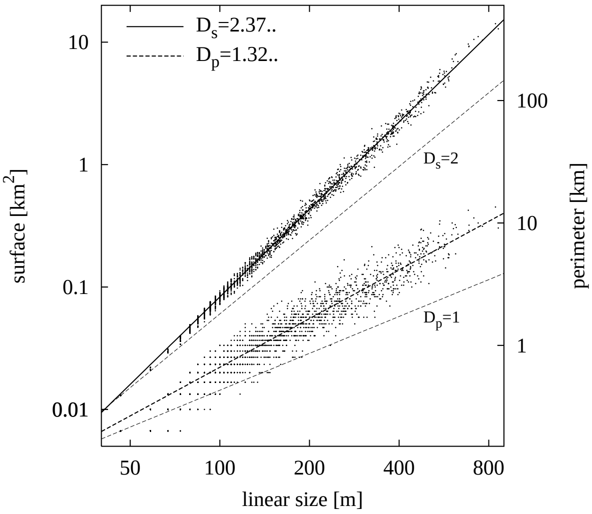

FIG. 3. The same analysis as in Fig. 2, but now using a database of 6000 clouds. In the same plot we also show the results of an area-perimeter analysis along with a fit and a line corresponding to the Euclidean case $D_p = 1$.

Secondly, we performed a direct correlation dimension analysis based on individual clouds. This is done by using the correlation integral $C(l)$ [27] which measures the probability of finding two points $\vec{x}_i$ and $\vec{x}_j$ of the cloud boundary within a cell of linear size l

$$C(l) = \sum_{i,j} \theta \left(l - |\vec{x}_i - \vec{x}_j|\right) \sim l^{D_s} \tag{1}$$

where $\theta(x)$ is the Heaviside step function, and where the summation is taken over all possible pairs of cloud boundary points.

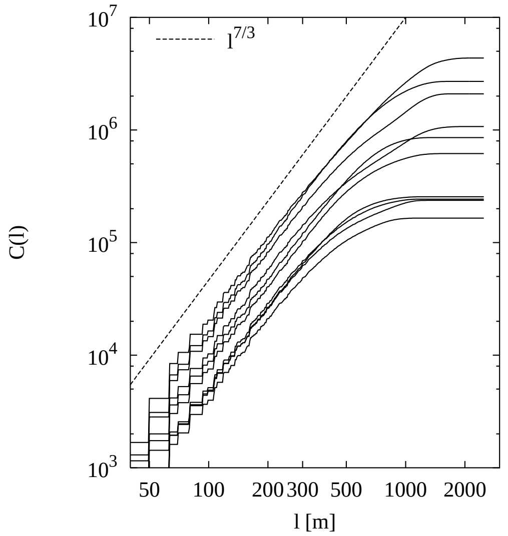

FIG. 4. Log-log plot of the correlation integral Eq. 1 for ten individual clouds. Note that the scaling behaviour of the individual cloud surface areas is identical to the surface-volume scaling displayed in Fig. 3

In Fig. 4 we show a plot of $\log l$ versus $\log C(l)$ for the 10 largest clouds from the above mentioned cloud database along with a line with a slope of 7/3 as a guide to the eye. As an estimate for the surface dimension we use the average of all the slopes of the lines in Fig. 4 for which we find $D_s = 2.32 \pm 0.01$ where the error is based on the standard deviation. This demonstrates that individual clouds do all scale with the same dimension D_s as obtained by the volume-surface method. Note that this is direct numerical evidence for the fact that upscaled small clouds are statistically indistinghuisable from coarse grained large clouds. In the present case the lower cut-off is due to the model resolution l_0. In this respect the LES model can be regarded as an experimental device that can "measure" the surface only up to an accuracy of $l = l_0$. The real physical cut-off is set by the Kolmogorov scale $l = \eta$, i.e. the scale where molecular diffusion smoothes the fields. The variation of the surface area $S(l)$ with the resolution l in the inertial subrange can in general be written as [18,20]

$$S(l) = S_L \left(\frac{l}{L}\right)^{2-D_s} \qquad \eta < l < L \tag{2}$$

where $S_L \simeq L^2$ is the normalizing area that would be obtained if the surface area was measured with resolution equal to the the outer scale L. From Eq. 2 two important conclusions can be drawn. Firstly a naive Euclidean estimate S_L underestimates the "true" cloud surface area $S(l = \eta)$ by a factor of $(\eta/L)^{2-D_s}$ which can be as large as a factor of 100. Secondly, even using a model resolution of 40 m underestimates the real cloud surface area by a factor of 5.

This result raises serious doubts whether LES can provide realistic estimates for turbulent transport across such a cloud boundary. On the other hand LES results are reported indicating that this entrainment process is reasonably resolution independent [22]. To resolve this apparent paradox, let us write the diffusive transport T of a scalar with concentration c across a fractal scaling boundary at a model resolution l_0 as a product of the area of the interface $S(l_0)$ and the diffusive flux across the interface $F(l_0) = -K\dfrac{\partial c}{\partial x}$

$$T(l_0) = S(l_0)\, F(l_0) \simeq S(l_0)\, K(l_0)\, \frac{\Delta c}{l_0} \tag{3}$$

where Δc is the typical concentration jump across the interface. For the eddy diffusivity K in the LES model the usual turbulent kinetic energy closure is used [4]

$$K(l_0) \simeq l_0\, |\delta \vec{u}(l_0)| \tag{4}$$

where $|\delta \vec{u}(l_0)|$ is a typical velocity difference over a length l_0. Invoking the usual Kolmogorov scaling

$$|\delta \vec{u}(l_0)| \simeq |\delta \vec{u}(L)| \left(\frac{l_0}{L}\right)^{1/3} \tag{5}$$

illustrates that the closure given by Eq. 4 obeys the Richardson 4/3 law and allows us, using Eq. 2, to write Eq. 3 explicitly as a function of the model resolution l_0

$$T(l_0) \simeq \Delta c\, S_L\, |\delta\vec{u}(L)| \left(\frac{l_0}{L}\right)^{7/3-D_s} . \quad (6)$$

This result implies that the turbulent transport $T(l_0)$ becomes resolution independent only if $D_s = 7/3$, very close to the value that we obtained. From this result the following physical picture emerges: At the resolved scales $l_0 < l < L$ an enlongated fractal scaling interface is created under the action of turbulent stretching and folding. Then at the resolution scale of the model l_0, diffusion does the actual mixing *across* the twisted interface. As one can read off from Eq. 2, the length of the cloudy boundary is underestimated by a factor $(\eta/l_0)^{2-D_s}$ due to the model resolution. On the other hand, due to the subgrid closure Eq. 4, the diffusion across the interface is enhanced according to Richardsons law in precisely such a way that the resulting transport across the interface becomes resolution independent.

It is interesting to remark that this result is closely related to an heuristic proof of Sreenivasan et al. [18] showing that the surface dimension of interfaces in turbulent flows *has to be* $D_s = 7/3$. Repeating the arguments that led to Eq. 6 at the Kolmogorov scale $l = \eta$ gives

$$T(\eta) \simeq \Delta c\, S_L\, |\delta\vec{u}(L)| \left(\frac{\eta}{L}\right)^{7/3-D_s} \sim \mathrm{Re}^{-3/4(7/3-D_s)} \quad (7)$$

where in the last step we used that η/L scales with the Reynolds number as $\mathrm{Re}^{-3/4}$, whereas the other terms are independent of the Reynolds number. Subjecting the boundary flux $T(\eta)$ to Reynolds number similarity, which states that the flux should be Reynolds number independent, the dimension of the surface must be $D_s = 7/3$.

In conclusion, the anomalous scaling of cloud boundaries such as observed by satellites is strongly reproduced by LES models thereby providing numerical evidence that such models generate a dynamics that reproduce the correct cloud geometry. This allows more careful research on the origin of this scaling since we now have a numerical laboratory where we can study the cloud dynamics and cloud geometry in more detail.

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