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Coordinate conversions for presenting and compositing weather radar data

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CONTENTS

ABSTRACT

Weather radars display the position of rain areas with respect to the known topographic position of e.g. an observer. For most practical applications the azimuth and distance of the radar target have to be converted to generally accepted geographical coordinates, or - alternatively - a topographical map has to be converted to the polar radar coordinates.
Because the radar range is not small compared to the earth radius, these conversions have to be carried out with the well-known great-circle navigation formulae.
In many areas of the world the radar coverage is now sufficient to combine many single radar images into composite radar pictures. An example is the experimental COST-73 image that covers a part of Western Europe. For such large-scale pictures the conversion formulae have to account for the non-spherical shape of the earth.
It is advantageous to define a uniform grid for all radars in the composite area, to perform the conversion at each individual radar site, and to use look-up tables for that purpose.
This reports considers the formulae required to display single radar data in polar stereographic projection of a spheroid. This projection is used by the Netherlands weather radars since August 1989. A choice can be made between two sets of formulae, one with very high and the other with acceptable accuracy.

Coordinate conversions for presenting and compositing weather radar data.

1 INTRODUCTION

A weather radar measures the azimuth, distance and other characteristics of a target. Plotting the position on a polar diagram is aequivalent with applying the so-called Postel map projection. If the radar data are used for warnings their position with respect to certain user locations must be known. This can be accomplished by providing a topographic overlay recomputed for the specific radar projection.
This works well for a single radar, but if the data of two or more radars are merged or if the radar data must be compared with mapped meteorological information, the conversion to some common grid is necessary. The monitor presentation of modern radars demands already a local polar-rectangular conversion, so it is advantageous to convert to a universal grid rather than to a local presentation. The necessary accuracy is related with the radar beam width and pulse length and will be of the order of 1 km. Such an accuracy is easy to achieve over ranges of a few 100 km but may require an ellipsoid approximation of the figure of the earth's if the combined radar picture stretches over more than 1000 km.
To avoid the loss of process time by repeating mathematical function computations, it is good practice to perform the grid conversion with a table, which is read in or computed during the system initialisation. The benefits of very accurate conversion formulae can therefore be obtained without extra cost.
In the following sections we will discuss the transformation of radar data to a polar stereographic grid as it is carried out operationally for the Netherlands weather radars. One of the reasons to choose this projection was its acceptance as a standard for the COST-73 composite radar products. The conversion to other projections requires different formulae, but the general procedure will be the same. Special attention will be given to the errors involved in certain approximations. In the appendices accurate computing schemes will be given for the two so-called principal problems of geodesy: computing the coordinates of a target with given range and azimuth, and the inverse problem: the computation of azimuth and distance of a given target.

2 POLAR STEREOGRAPHIC PROJECTION

The simple case of polar stereographic projection on a plane tangent at the north pole N of a spherical earth can be illustrated with a geometrical construction (Fig.1): the projection A' of point A is found by extending S-A. The plotting distance NA' follows from

$$r(\varphi) = \frac{2}{1+\sin\varphi} R \cos\varphi = 2R \tan(45 - \varphi/2) \qquad (1)$$

If dm is a small distance on the sphere, we have as scale factor

$$z(\varphi) = \frac{dr}{dm} = \frac{dr}{d\varphi}\frac{d\varphi}{dm} = \frac{R}{\cos^2(45-\varphi/2)}\frac{1}{R} = \sec^2(45-\varphi/2) \; . \qquad (2)$$

Fig.1.
Polar sterographic projection of a point A on a sphere.

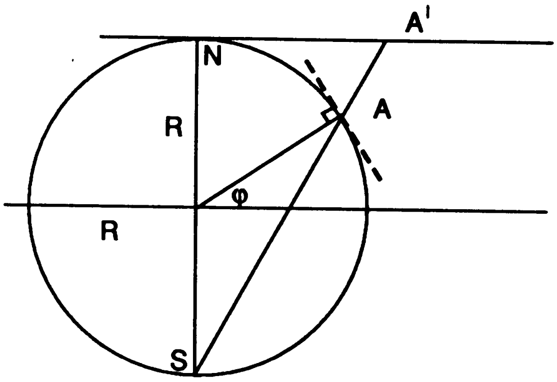
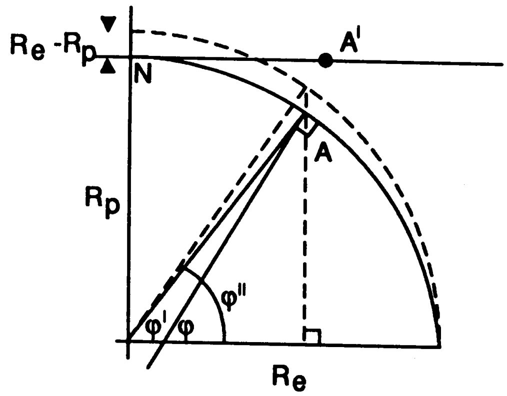


Fig.2.
Geographic φ, geocentric φ' and reduced φ'' latitude of point A on an ellipsoid.



The projection is 'orthomorphic', which means that the scale is the same in all directions (but not necessarily at all locations), so the values of angles are conserved. Also, circles are projected as circles, because SA' cuts the sphere and the projection plane at equal angles: $90 - \varphi/2$. The scales along both meridians and latitude circles are equal, but of course vary with latitude φ. With Equation (1) the plotting position of A' is known, because the angular position of A' follows directly from the geographical longitude of A.
If a higher precision is required the deviation of the earth's figure from a sphere must be accounted for. For a so-called spheroid the polar and equatorial radius are no longer equal. The shape of such an ellipsoid is usually expressed by its eccentricity

$$e^2 = 1 - (R_p/R_e)^2 \quad . \tag{3}$$

The geocentric latitude φ' and the geographic latitude φ of a point are related by

$$\tan\varphi' = (1-e^2)\tan\varphi. \tag{4}$$

We also have the reduced latitude φ'' (Fig.2) with

$$\tan\varphi'' = \surd(1-e^2)\ \tan\varphi. \tag{5}$$

Orthomorphic projection is obtained with

$$r(\varphi) = 2\frac{R_e^2}{R_p}\tan(45-\varphi/2)\left(\frac{1-e}{1+e}\,\frac{1+e\sin}{1-e\sin\varphi}\right)^{e/2} \tag{6}$$

and the scale (except for φ=90) from

$$z(\varphi) = \frac{dr}{dm} = \frac{dr}{d\varphi}\frac{d\varphi}{dm} = \frac{r(\varphi)(1-e^2)}{(1-e^2\sin^2\varphi)\cos\varphi}\,\frac{(1-e^2\sin^2\varphi)^{3/2}}{R_e(1-e^2)} =$$

$$z(\varphi) = \frac{r(\varphi)\surd(1-e^2\sin^{\varphi})}{R_e\cos\varphi} \quad . \tag{7}$$

The term $dm/d\varphi$ is the local meridional radius of curvature. Again the scale is independent of the direction on the spheroid.
If, however, the geometric construction from the S pole were applied for an ellipsoid, the plotting distance and the scale along latitude circles would be found by using R_e and the reduced latitude φ'' in the equations (1) and (2). The meridional scale, however, is then larger by a factor $\sqrt{(1-e^2\sin^2\varphi)}/\sqrt{(1-e^2)}$. This construction method would therefore not result in an orthomorphic projection as do the Equations (6) and (7).

3 GEOGRAPHICAL COORDINATES AND THE FIGURE OF THE EARTH

Because the actual form of the earth cannot be described by a simple ellipsoid, most countries have selected a certain ellipsoid that offers the best fit over their territory.
The following table compares well-known models of the earth. The rightmost columns illustrate the differences in scale factor for orthomorphic stereographic projection and a typical example of the differences in plotting distance for a 30 deg latitude difference. This demonstrates that the approximation with a sphere leads to an error of about 4 km.

reference	R_e (m)	R_p (m)	e	z(60)	r(30)-r(60)
sphere, same vol.	6371221	6371221	0	1.07179677	3942525
Bessel 1841	6377397	6356079	0.0816965	1.07173221	3937953
Airy	6377563	6356256	0.0816744	1.07173225	3938061
Clarke 1866	6378206.4	6356583.8	0.0822719	1.07173130	3938334
Hayford 1910	6378388	6356912	0.0819918	1.07173174	3938504
IUGG 1967 (IAU 1968)	6378160	6356775	0.0818196	1.07173202	3938399

For meteorologists it is of interest to note that Meteosat software is based on Clarke's ellipsoid with R_e changed to 6378169 m.
Considering that the geographical latitude of a point on the earth is the angle between the ellipsoid axis and the plane tangent to the ellipsoid at the chosen point, it is evident that the values of the geographical coordinates depend on the ellipsoid choice. As an example we may mention the position of KNMI's measuring tower at Cabauw, which is 4.927481 / 51.971255 in the Bessel system of the Netherlands Ordnance Survey maps. However, the IAU-coordinates are 4.926570 / 51.971056. Confusing these reference systems would lead to a position error of 66 m.

4 APPLICATION TO THE REDUCTION OF SINGLE RADAR DATA

A polar stereographic grid is defined by
- the sheroid parameters, e.g. R_p and e,
- the grid orientation, which is parallel to a certain meridian λ_0, with eastward longitude assumed positive, and
- the grid scale, usually valid at latitude 60 N.

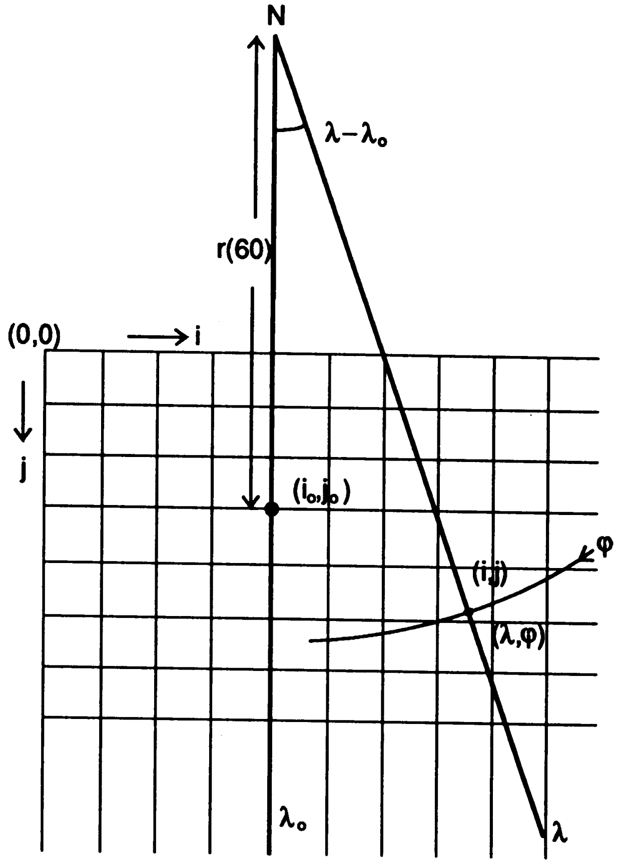
In the projection plane a grid with square pixels of size p (m) can be defined (Fig. 3) and pixel coordinates (i,j) can be used with the origin at the upper left corner of the grid.
The value of j is increasing southward to conform with computer transmission and presentation practice.

For true scale at the earth surface for latitude 60 N, the distance increments in the projection plane become p*z(60). The geographical coordinates (λ,φ) of the upper left corner of an arbitrary pixel (i,j) follow from two equations

$$j-j_0 = (r(\varphi)*\cos(\lambda-\lambda_m) - r(60)) / (p*z(60)) \quad (8a)$$

$$i-i_0 = [(j-j_0) + r(60)/(p*z(60))] *\tan(\lambda-\lambda_0) \quad (8b)$$

Fig.3.
Polar stereographig grid aligned at latitude λ_0.



Here the pixel coordinates (i_0, j_0) of the point (λ_0 E, 60 N) are used as a reference to allow an offset of the grid with respect to this point. If necessary, (i_0,j_0) can be found by substituting (i=0,j=0) and applying the Equations (8) for the longitude and latitude of the true origin of the grid. These coordinates i_0 and j_0 are not necessarily integer values.
Solving Equation (8a) for φ requires iteration, because of the complex form of r in Equation (6). These Equations are hardly suitable for on-line data processing, although they might be speeded up by tabulating the eccentricity -dependent term in Equation (6) as a function of φ. A better procedure is to use the Equations (8) to generate a look-up table for e.g. the polar-rectangular conversion of single radar data. Such a table contains an azimuth and distance (with respect to the radar) for each pixel (i,j), so that the grid can be filled with the appropriate polar radar data.

The coordinate transformation table may be produced in three steps:

a The grid coordinates (i_m, j_m) of the radar have to be computed from the geographical coordinates (λ_m, φ_m) with the help of Equations (8). Of course the projection of the radar is generally not at a pixel corner, but at non-integer coordinates (i_m, j_m). It would be optimal to use coordinates valid for the same spheroid as chosen for the projection. It was noted in Section 3 that differences between spheroids could lead to position errors of the order of 100 meters, which seems acceptable. Using a sphere, however, could cause errors up to 4 km.

b Then establish the range of integer values $(i_1, j_1), (i_2, j_2)$ of the rectangular pixel frame that has to be filled with data from the radar concerned. It may be considered to off-centre the radar in the picture frame because of radar coverage characteristics or forecasting demands.

c For all pixels in the frame chosen at step b the azimuth A (measured clockwise from North) and the distance D with respect to the radar position have to be found. As the radar data have to be allocated to the pixel centers, the grid data (i,j) used in the following are $(i_1+0.5,\ j_1+0.5)$, $(i_1+1.5,\ j_1+0.5)$, ... etc. For this stage two methods are available:

c1 First all pixel centres (i,j) have to be converted to geographical coordinates (λ, φ). The next step is the problem of determining the bearing and distance from point (λ_m, φ_m) to each pixel (λ, φ) on a spheroid, which is the 'inverse principal problem' of geodesy. A solution is given in the Appendix. It should be noted that the use of 'great circle formulae' for a sphere would cause errors up to 2 km at a radar range of 250 km. This error is comparable with the pixel size and also large compared to other uncertainties like the variation of the radio refractive index.

c2 An alternative to the accurate, but complicated solution c1 is to profit from the orthomorphic properties of the projection. The azimuth and distance between the points (i,j) and (i_m, j_m) in the projection plane can be computed with plane trigonometry. The projection of a radar ray (up to 250 km range) is then approximated by a straight line. The azimuth so found, has to be rotated over an angle $\lambda_m - \lambda_0$. As a first approximation

$$D= \sqrt{((i-i_m)^2 +(j-j_m)^2)} *p*z(60)/z(\varphi_m) \quad , \qquad (9a)$$

$$A= \arctan(-(i-i_m)/(j-j_m)) +(\lambda_m-\lambda_0) \quad , \qquad (9b)$$

where 180 deg. has to be added to A for pixels with $j>j_m$. At far radar ranges (250 km) these approximations lead to errors up to 3 km, respectively 0.5 deg. From comparisons (at latitudes between 30 and 70 deg) with the exact solutions obtained above under c1 the following corrections were obtained

$$D= D \ / \ (1 +0.040*\tan(45-\varphi_m/2)*\cos A) \quad , \qquad (10a)$$

$$A= A + 4.49*10^{-6}*D*\tan(45-\varphi_m/2)*\sin A \quad . \qquad (10b)$$

The errors that remain after these corrections are restricted to 100 m and 0.01 deg at the 250 km range, which is sufficiently smaller than other radar location errors.

5 APPLICATION TO THE COMPOSITING OF RADAR DATA

If the same grid is used for different radars the combination of the pictures is relatively easy. If the grids use a different pixel size, the merging algorithms become slightly more complicated. Of course the most optimal situation is, were the composite pixel size is a multiple of the contributing pixel sizes.
The advantages of using a uniform grid can hardly be overestimated. Most evident is the saving of computing time during the merging process. The resulting composite will become more valuable for the contributing service, because it arrives more timely and also because it may more easily be compared (or actually be merged at the far ranges) with the more detailed local picture. Another advantage is the possibility to exchange topographical overlays based on the common grid.
The choice of a reference ellipsoid for the projection is not critical, considering the other uncertainties of radar location. However, the use of a spherical approximation for an area as large as Europe, would cause errors of about 4 km, which is comparable to the pixel size. Also, the radar positions (λ_m, φ_m) are usually known with reference to an ellipsoid and would then have to be converted to positions on a spherical earth. These problems are easily avoided by choosing an ellipsoid and applying the formulae (6)-(10).

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APPENDIX

RELATIONS BETWEEN AZIMUTH AND RANGE AND POSITIONS ON A SPHEROID

The solutions of the principal problems of geodesy in this appendix are based on work by Helmert dating from 1880 (Bodemüller, 1955). Auxiliary quantities are the reduced lengths l_m, l and the reduced arc lengths s_m, s with respect to the most northern point (latitude φ^*) of the ellips that forms the intersection between the spheroid and the plane through the points (λ_m,φ_m), (λ,φ) and the spheroid centre.

Bodemüller presents an example that can be used for checking a computer program based on the following formulae. For Hayford's ellipsoid and starting point 10 E, 50 N he reaches for azimuth 140 deg and distance 15000 km the point 105.093973 E, 62.950890 S. The formulae and examples of Collins (1980) were also used for checking. It proved, however, that Collins is not using the actual azimuth but its projection from the earth's center on a concentric sphere with radius R_p.

a. Azimuth and range from target coordinates (λ,φ).

For the solution of the inverse principal problem the values l_m and l are iteratively found from the Equations (A1)-(A5):

$$\frac{l+l_m}{2} = -\arctan\left(\frac{\sin(\varphi''-\varphi_m'')}{\sin(\varphi''+\varphi_m'')\tan(l/2-l_m/2)}\right) , \qquad \text{(A1)}$$

where the first estimate of $l-l_m$ is $\lambda-\lambda_m$. Of course l and l_m are

$$l = \frac{l+l_m}{2} + \frac{l-l_m}{2} \qquad \text{and} \qquad l_m = \frac{l+l_m}{2} - \frac{l-l_m}{2} .$$

The position of the intersecting plane is found from

$$\tan\varphi^* = \tan\varphi_m'' / \cos l_m , \qquad (\varphi^* = 90 \text{ deg if } l = l_m) . \qquad \text{(A2)}$$

We have another auxiliary quantity

$$k = \tan^2([\arctan(\frac{e}{\sqrt{(1-e^2)}}\sin\varphi^*)]/2) . \qquad \text{(A3)}$$

The reduced arc lengths are determined from

$$\cos s_m = \sin\varphi_m'' / \sin\varphi^*, \text{ with } s_m < 0 \text{ for } l_m > 0 , \text{ and} \qquad \text{(A4a)}$$

$$\cos s = \sin\varphi'' / \sin\varphi^*, \text{ with } s < 0 \text{ for } l > 0 . \qquad \text{(A4b)}$$

Finaly we use the rather accurate approximation

$$l - l_m = \lambda - \lambda_m \pm e^2\cos\varphi^* \, [(\frac{R_e}{R_e+R_p} - \frac{k}{4} - \frac{k^2}{8})(s-s_m) +$$

$$- \frac{k}{4}\cos(s+s_m)\sin(s-s_m) + \frac{k^2}{16}\cos(2(s+s_m))\sin(2(s-s_m))] . \qquad \text{(A5)}$$

The - sign for the $\pm$ term is used if $s > s_m$ or $l > l_m$, but not if both conditions apply. The iteration (A1)-(A5) proceeds until φ^* remains constant.

Then the azimuth A is determined from

$$\tan A = 1/(\tan \varphi^* \sin s_m) \text{ , but 0 or 180 deg for } l = l_m. \qquad (A6)$$

Here 180 deg. has to be subtracted from or added to A, if $\lambda - \lambda_m$ and d_m have the same sign (or in the rare case that $\varphi > -\varphi_m$). Add 360 deg if A is negative. The distance D follows with a very high precision (about 1 m over the entire globe) from

$$D = \frac{R_p\,(1+k^2/4)}{(1-k)} \left[\, |s-s_m| + k\left(1-\frac{3k^2}{8}\right)\cos(s+s_m)\sin(|s-s_m|) + \frac{k^2}{8}\cos(2(s+s_m))\sin(2(|s-s_m|)) + \frac{k^3}{24}\cos(3(s+s_m))\sin(3(|s-s_m|))\right]. \qquad (A7)$$

b.**Target coordinates from azimuth and range.**

The computation of the principal problem proceeds without iteration. The position of the intersecting ellipse follows from

$$\cos \varphi^* = \sin A \;\cos \varphi_m'' \quad , \qquad (A8)$$

and the auxilliary coordinate of the starting point from

$$\tan s_m = -\cos A \,/\, \tan \varphi_m'' \quad . \qquad (A9)$$

By means of the constant k (Equation A3) we know the angular distance d of the target

$$d = D/R_p \;(1-k)/(1+k^2/4) \quad . \qquad (A10)$$

Twice the auxiliary angular distance of a reference point midway between starting point and target is approximated by

$$s' = 2\, s_m + d + k*\sin(2\, s_m) - (k^2/8)*\sin(4\, s_m) \; . \qquad (A11)$$

The auxiliary angular distance of the target point follows from

$$s = s_m + d - k(1-9k^2/16)\cos(s')\sin(d) + +5k^2/8\,\cos(2s')\sin(2d) - 29k^3/48\,\cos(3s')\sin(3d), \qquad (A12)$$

neglecting higher order terms. The latitude φ'' follows with Equation (A4b) and the geographical latitude from (5).
Apart from the trivial cases A= 0 or A=180 the auxiliary longitude of the starting point is found from

$$\tan l_m = \tan s_m \,/\, \cos \varphi^* \quad , \qquad (A13)$$

and the longitude difference $\lambda - \lambda_m$ can be solved from (A5). The result is added to λ_m to obtain λ.

For a spherical approximation of the earth the above formulae are greatly simplified, mainly because $\varphi = \varphi''$, k=0 and e=0. Of course the resulting equations will be aequivalent to the well-known explicit formulae for great-circle navigation.