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THE FORCING OF THE MEAN MERIDIONAL CIRCULATION

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Abstract

With a linear model of the zonal mean circulation an estimate was made of the relative contribution of the various forcing terms to the zonally averaged circulation. The forcing terms considered were the eddy forcing due to the convergence of the horizontal fluxes of heat and momentum, and an idealized diabatic heating (radiation and condensational heating). The eddy forcing was derived from observations (Oort and Rasmussen, 1971). The diabatic heating was prescribed analytically. Both forcings together generate a three cell mean meridional circulation and a zonal mean wind distribution which compare well with observations. The model is not able to make a good simulation of the temperature field. The eddies are responsible for a meridional circulation consisting of three cells, equal in strength. The standing eddy part of the forcing is negligible. The diabatic heating forces a one cell direct circulation. The zonal mean wind and temperature field is mainly generated by the diabatic heating.

I. Introduction

The atmospheric circulation is forced by the differential heating of the sun. The heating of the air at the equator causes rising motion, while the cooling at higher latitudes causes the air to descend. However the mean meridional circulation cannot be described by considering radiation only. This would force a weak direct cell, while observations show a three cell circulation in each hemisphere: a strong direct cell extending from the equator to 30N (the Hadley cell), a weak indirect cell at mid-latitudes (the Ferrel cell) and a very weak direct polar cell (Oort and Rasmussen, 1971; Lorenz, 1967). Radiation however is the triggering mechanism for the mean meridional circulation. The ascending of moist air in the Tropics supplies the required energy in the form of condensational heating to create a direct cell with the correct strength and magnitude (Schneider and Lindzen, 1977; Crawford and Sasamori, 1981; Pfeffer, 1981). The transient eddies generated by baroclinic instability in mid-latitudes and the topograhpically forced stationary eddies are responsible for the indirect cell (Kuo, 1956; Crawford and Sasamori, 1981; Pfeffer, 1981).

Here we will examine the relative importance of both forcings (the diabatic heating due to radiation and condensation and the forcing from the eddies) on the mean meridional circulation as well as on the zonal mean wind and temperature. The forcing terms can be computed from observations or can be prescribed analytically (Kuo, 1956). Various authors have developed simple linear models, to assess the importance of these forcing terms on the mean meridional circulation (Kuo, 1956; Crawford and Sasamori, 1981; Pfeffer, 1981). Schneider and Lindzen (1977) forced their model with radiation, condensational heating (parameterized cumulus convection or simply prescribed) and used a parameterized form of cumulus friction. Transport of heat and momentum by large scale eddies are suppressed in their model. They were able to show that cumulus friction can also force a three cell circulation. The zonally averaged westerly winds were simulated very unrealistically.

Crawford and Sasamori (1981) and Pfeffer (1981) concentrated on the mean meridional circulation. Pfeffer also examined the tendencies of the zonal mean wind and the temperature as computed from the terms involving the meridional circulation. He found a net acceleration of the zonal mean westerly wind due to the eddies poleward of the jet core and due to the diabatic heating equatorward of the jet. He concluded that diabatic heating alone does not maintain the zonally averaged jet in the position we find it. The combined contributions of the eddies and the diabatic heating create the positive acceleration to maintain the jet against dissipation.

We will use a linear steady-state model, similar to Kuo (1956), prescribing the eddies of momentum and heat from observations (from Oort and Rasmussen, 1971) and an idealized diabatic heating. A Newtonian type of dissipation for temperature and a Rayleigh friction is used. Cumulus friction is assumed to be incorporated in the observed eddies. With this simple model it is possible to simulate the mean meridional circulation as well as the mean zonal wind an temperature profile. It appears that while the eddies are responsible for the three cell circulation, the diabatich heating generates the zonal mean wind and temperature.

In Section II the model is described. Also the forcing terms are discussed. In section III the results of the model experiments are shown. First the model is driven by both forcing terms, then the contribution of the separate forcing terms is considered.

II. The model

1. The model equations

We will use a linearized steady state model. The variables are linearized with respect to a zonally averaged basic state:

$$
\begin{array}{lll}
u = \hat{u} & \text{zonal wind} & \\
v = \hat{v} & \text{meridional wind} & \\
\omega = \hat{\omega} & \text{vertical velocity } (\omega = \frac{dp}{dt}) & \quad (1) \\
T = T_n + \hat{T} & \text{temperature} & \\
\Phi = \Phi_n + \hat{\Phi} & \text{geopotential} & \\
\sigma = \sigma_n & \text{static stability} &
\end{array}
$$

The basic state meridional circulation is zero. The geopotential height and temperature are a function of pressure only. Also $U_n = 0$ at the surface, which causes the basic state atmosphere to be at rest. The stability is specified by:

$$\sigma_n = - \frac{\partial T_n}{\partial p} + \frac{RT_n}{pc_p} .$$

When we assume stationarity, the linearized equations become:

$$
\begin{aligned}
-f\hat{v} &= F_X - D_u \\
\frac{\partial \hat{\Phi}}{\partial y} + f\hat{u} &= 0 \\
-\sigma_n \hat{\omega} &= Q + F_T - D_T \qquad (2) \\
\frac{1}{\cos\phi} \frac{\partial}{\partial y}(\hat{v}\cos\phi) + \frac{\partial \hat{\omega}}{\partial p} &= 0 \\
\frac{\partial \hat{\Phi}}{\partial p} &= - \frac{R\hat{T}}{p}
\end{aligned}
$$

Here, R is the gas constant, p the pressure and F_X and F_T the divergence of the poleward eddy fluxes of momentum and heat respectively, consisting of a transient and a standing part:

$$F_X = - [\frac{\partial}{\partial y} \overline{u'v'} - \frac{2\overline{u'v'}\, tg\phi}{a}] - [\frac{\partial}{\partial y} \bar{u}^* \bar{v}^* - \frac{2\bar{u}^*\bar{v}^*\, tg\phi}{a}] \qquad (3)$$

$$F_T = - [\frac{\partial}{\partial y} \overline{v'T'} + \frac{\overline{v'T'}\, tg\phi}{a}] - [\frac{\partial}{\partial y} \bar{v}^*\bar{T}^* + \frac{\bar{v}^*\bar{T}^*\, tg\phi}{a}]$$

The following notation has been used:

$\overline{\quad}$	: time mean
$'$	: deviation from a time mean
$[\quad]$	: zonal mean
$*$	: deviation from a zonal mean.

Q is the diabatic heating. The internal forcing terms F_X, F_T and the external forcing term Q drive the model and will be prescribed.
D_u and D_T are dissipation terms, which we shall choose as $D_u = k_u\hat{u}$ and $D_T = k_T T$. The timescale of the dissipation processes is of the order of 20 days ($k_u = k_T = 5.10^{-7}$).
When now a meridional streamfunction is introduced

$$\hat{v} = -\frac{1}{\cos\phi}\frac{\partial\chi}{\partial p},\quad \hat{\omega} = \frac{1}{\cos\phi}\frac{\partial\chi}{\partial\mu},\quad \mu = \sin\phi \qquad (4)$$

all variables can be expressed in this one variable and the equation for the zonal mean circulation becomes

$$A_1 = \frac{\partial^2\chi}{\partial\mu^2} + A_2\frac{\partial^2\chi}{\partial p^2} = Q' + F' \qquad (5)$$

$$A_1 = \sigma_n K_u \cos\phi$$

$$A_2 = a^2 f^2 K_T p/R$$

$$Q' = -K_u \cos^2\phi\; a\frac{\partial}{\partial\mu}\,(Q + F_T)$$

$$F' = -K_T a^2 p/R\cos\phi\;\frac{\partial}{\partial p}(-fF_X)$$

which is the equation we will use in the model experiments.
Pfeffer (1981), Crawford and Sasamori (1981) and Holton (1972) arrive at the same type of elliptic equation without assuming stationarity. In that case the dissipation coefficients must be taken equal to each other (or zero), to eliminate the tendency terms. Pfeffer (1981) computed these terms after solving the equation in the function. Crawford and Sasamori (1981) assumed stationarity a posteriori.
In our model we can vary the dissipation coefficients. When k_u is a function of height a correction term has to be added to F' in eq. (5).
The model equation is evaluated at 15 equidistant levels in 30 grid points from pole to pole. For further technical detail the reader is referred to the appendix.

2. The forcing

In our experiments the annual mean meridional circulation is simulated by prescribing the corresponding forcing functions F_X, F_T and Q. For the internal forcing F_X and F_T the observed convergence of horizontal eddy momentum and eddy heatflux is used (Oort and Rasmussen, 1971). The vertical transient eddyfluxes are not available in this dataset. In particular the omission of the convergence of the vertical heatflux may be serious in the lower troposphere in mid-latitudes.

In fig. 1a, b the eddy momentum forcing F_X and the eddy heat forcing F_T are shown. They have been made symmetrical with respect to the equator. The momentum flux is directed polewards, transferring momentum to mid-latitudes, and has its maximum at about 200 mb in the jet core. As a consequence F_X is positive polewards of this maximum and negative equatorwards. Thsi can be interpreted as an acceleration resp. deceleration of the zonally averaged westerly wind. In our stationary model the meridional flow and the dissipation have to balance this acceleration exactly (see eq. 2). In the same way the strong northerly fluxes of heat by the eddies at mid-latitudes cause cooling at lower latitudes and heating of the polar region, which has to be balanced by the vertical motion and the Newtonian cooling.

For the diabatic heating (fig. 1c) a simple distribution is used, which corresponds roughly to the observed heating (Newell et al., 1969; Pfeffer, 1981). We will not use observational data because these data, from budget studies, can contain large errors (Hantel and Baader, 1978). We also make a simplification in the heating profile by neglecting the surface heating at mid-latitudes: in the real atmosphere the vertical eddy heatflux, which had to be neglected in F_T is an important mechanism. In this way we avoid the generation of an unstable lower atmosphere which would in reality be modified by the vertical eddies.

The maximum heating in the Tropics is about 0.7 K per day and the cooling at the pole about 0.5 K per day. The global mean heating is zero.

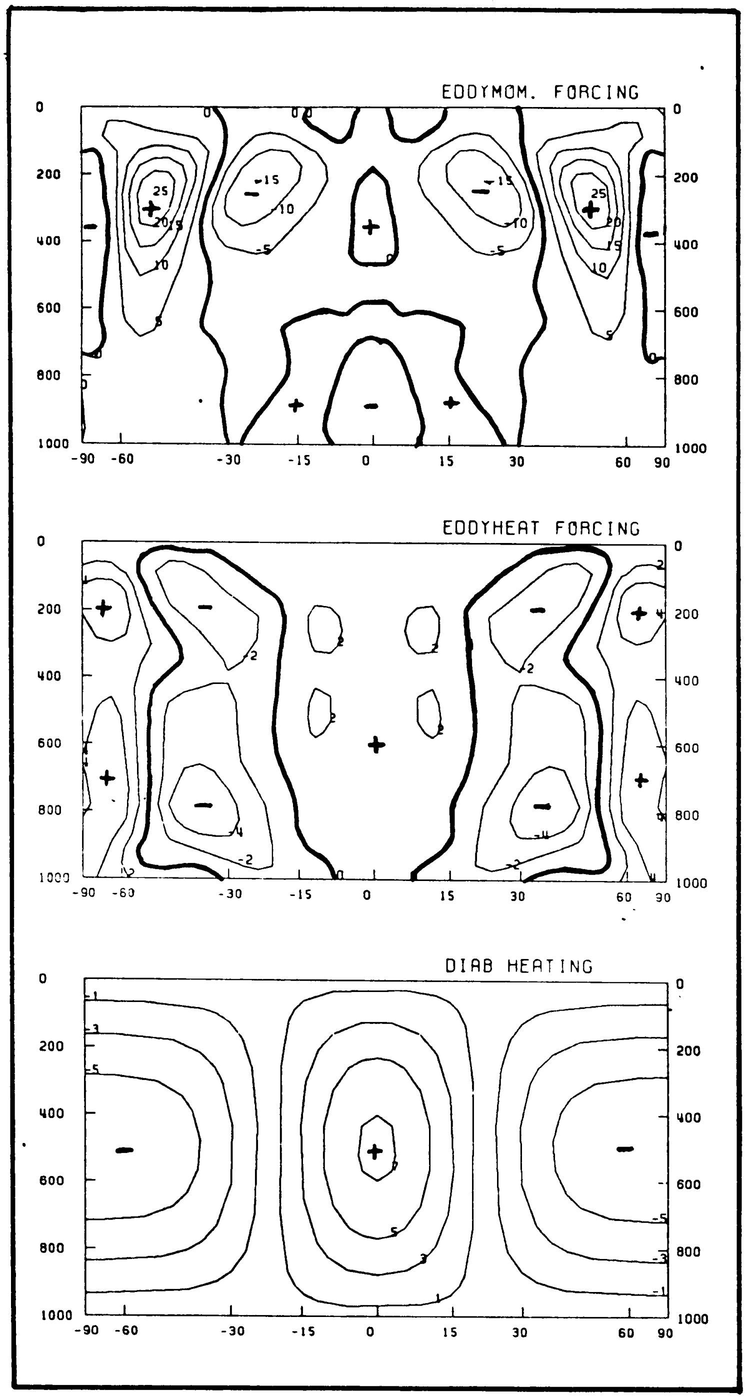

Fig. 1a, b, c Forcings of eddy momentum (10^{-6} m/s^2), eddy heating (10^{-6} K/s) and diabatic heating (10^{-6} K/s).

III. Results

1. Simulation of an annual mean circulation

In a first experiment the three forcing functions as represented in fig. 1 are used to simulate an annual mean circulation. In the following sections we will examine the relative contributions of the different forcings. The dissipation-coefficient for temperature is $k_T = 5\ 10^{-7}s^{-1}$ and for momentum $k_u = 5\ 10^{-7}s^{-1}$ in the upper troposphere. In the lower troposphere k_u increases exponentially to $1.3\ 10^{-6}s^{-1}$ at the lowest level.

In fig. 2 the response of the model is shown of the zonal wind, temperature and the mean meridional circulation. Also the total forcing in the thermodynamic equation, $Q + F_T$, is shown. The zonal wind profile has a maximum at 30^oN at 200 mb of 30 m/s. This as well as the shift with latitude of the jet compares well with observations (fig. 3). The tropical easterlies however in the upper atmosphere are not reproduced. The easterly winds are also too weak (less than 5 m/s). The change of sign occurs at the correct latitude (30^oN). The easterly winds at the poles however are too strong.

The deviation of the temperature from the mean at each level is shown in fig. 2b. The temperature distribution in the tropics is fairly homogeneous, with a sharp transition in the subtropics. The equator to pole contrast is however too small. Also the mid-latitude atmospheric stability is too small. The temperature in the tropical stratosphere is lower than at mid-latitudes, due to the higher stability.

The mean meridional circulation (fig. 2c) shows a 3 cell pattern, which compares well with observations (Oort and Rasmussen, 1971). There is an intense Hadley cell extending from the equator to 30^oNL, and a much weaker Ferrel cell at mid-latitudes (Pfeffer, 1981).

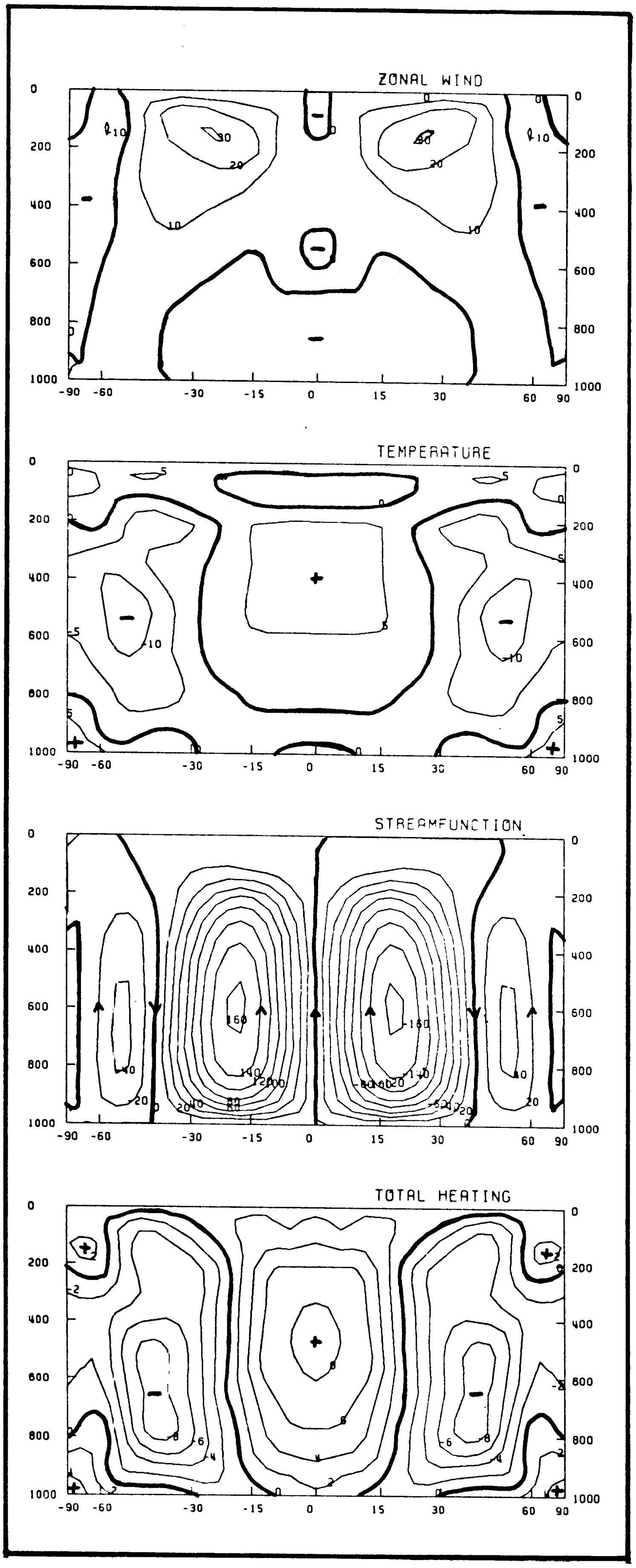

Fig. 2a,b,c,d Response of the model to eddy forcing and diabatic heating in zonal wind (m/s), temperature (10^{-1}K), streamfunction (10^2N/ms) Also the total heating (diabatic + eddies) is shown .

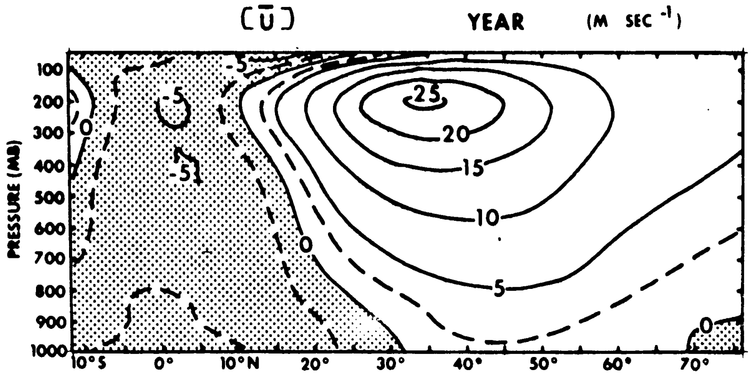

Fig. 3 Observed zonal wind (Oort and Rasmussen, 1971)

2. Forcing with the eddies only

a) Transient and standing eddy forcing

Now only eddy forcings of momentum and heat (cf. fig. 1a, b) are prescribed. When the model is forced with these observational data it generates a three cell circulation (fig. 4c), as was also shown by Kuo (1956) and Pfeffer (1981). Comparing this result with fig. 2 we can conclude that the eddies are mainly responsible for the three cell structure. The Hadley cell and Ferrel cell are now however comparable in magnitude. The resulting circulation can entirely be understood from the momentum flux distribution: in the upper atmosphere there is an approximate balance between the ageostrophic meridional motion and the convergence of eddymomentum $-f\hat{v} = F_X$. This means that convergence of eddy fluxes of momentum (positive F_X) forces equatorward flow and divergence (negative F_X) forces poleward flow in the upper atmosphere.

In the lower atmosphere there exists a balance between dissipation and the meridional motion $f\hat{v} = k_u\hat{u}$. The atmosphere thus gains angular momentum in the subtropics due to the easterlies and looses momentum in mid-latitudes due to the westerly winds (fig. 4a).

The zonal wind and temperature distributions are very irregular. The jet is not present anymore. The easterlies in the polar region can be explained by the forcing from the temperature eddies. The divergence of the eddy heat fluxes at mid-latitudes and the convergence at the poles cause a cooling resp. heating and thus, by the thermal wind relation, reinforce the polar easterly winds in the upper atmosphere.

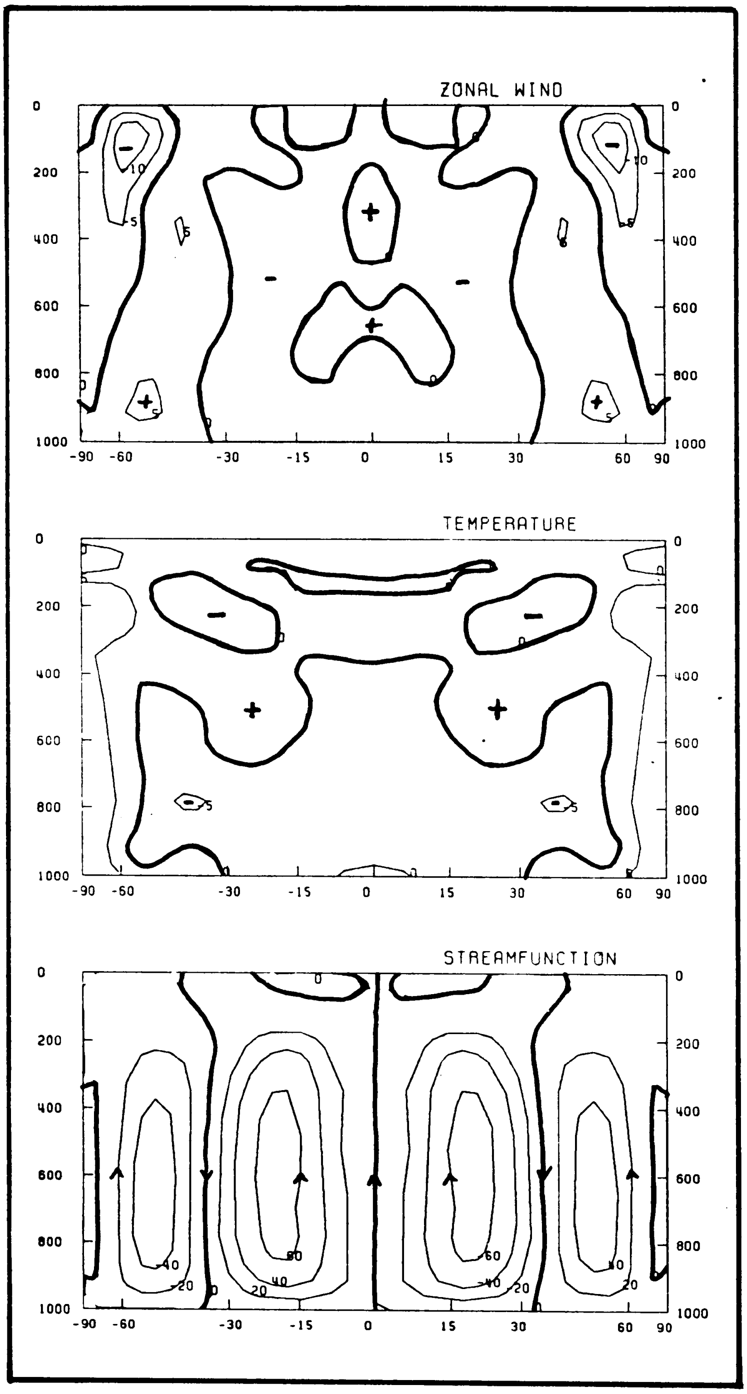

Fig. 4a,b,c Response of the model to eddy forcing only (units as in fig. 2)

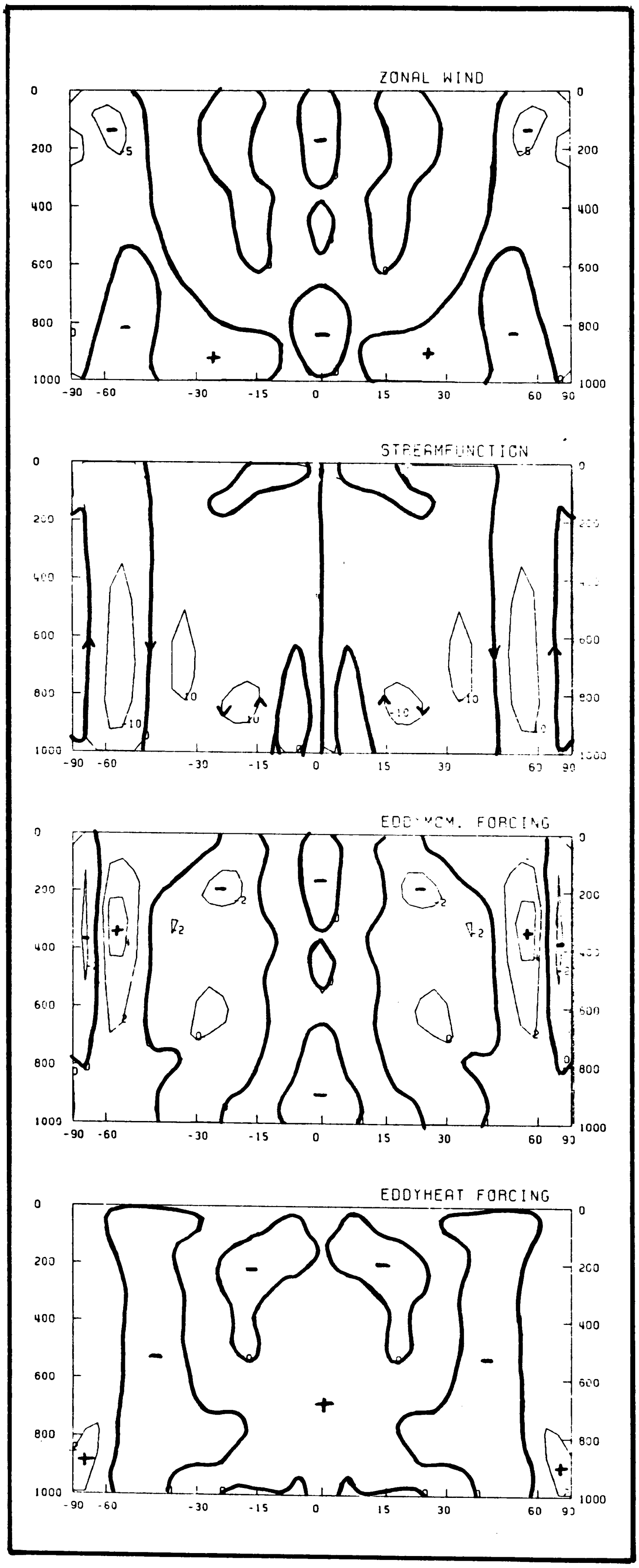

Fig. 5a,b,c,d Response due to standing eddies. Also the standing eddy forcing is shown (Units as in fig. 1 and 2)

b) Standing eddies

In this experiment we force the model with the standing part of the eddies only. The response and the forcing are shown in fig. 5. The standing eddy forcing generates a noisy meridional circulation. Also the contribution to the zonal wind and temperature profile is very small (fig. 5a, b).

This means that the largest contribution to the response due to the eddies is given by the transient disturbances in the atmosphere. The topographically forced eddies have only a small influence on the mean meridional circulation.

3. Forcing with an idealized diabatic heating

Now the diabatic heating as shown in fig. 1 is prescribed only. This simple heating forces a simple circulation (fig. 6). In the Tropics the warmer air will rise, while at higher latitudes the cooling of the air causes descending motion, resulting in a one cell circulation. The balance in the thermodynamic equation is mainly between the vertical motion and the diabatic forcing: $-\sigma_n\hat{\omega} = Q$, while the stability σ_n is prescribed, so positive Q will cause rising motion etc. The continuity equation then requires meridional motion: poleward in the upper atmosphere and equatorward in the lower atmosphere. The balance in the zonal momentum equation is exactly between the meridional flow and the dissipation: $f\hat{v} = k_u\hat{u}$. The meridional flow is deflected in the zonal direction and loses momentum by dissipation. Also the system loses energy by Newtonian cooling. In this way a simple one cell circulation is generated (fig. 6). The zonal wind and the temperature are in thermal balance resulting in the largest increase in the zonal wind at the subtropics, where the horizontal temperature gradient is largest. The vertical asymmetry in the zonal wind, resulting in a jet in the upper atmosphere and weaker easterly winds at the surface is caused by the large dissipation at the surface (cf. section II). When the dissipation coefficient of momentum k_u is chosen uniform through the whole atmosphere $k_u = 5.10^{-7}s^{-1}$ the response is as shown in fig. 7. The surface easterlies have increased.

The vertically integrated zonal wind is zero because the vertically integrated meridional flow is zero, which follows from the zonal momentum equation. The zonal wind profile and the mean meridional circulation is not exactly vertically symmetric, because the static stability (prescribed) is higher in the stratosphere.

It is clear that the atmospheric circulation as forced by the diabatic heating, which represents only half the forcing of the atmosphere, cannot fulfil the requirement of the balance of angular momentum. Also the resulting atmosphere is definitely baroclinically stable: all instability has been removed by putting the eddy forcing F_X and F_T equal to zero.

Comparing the responses of fig. 6, 7 with fig. 2 we can conclude that the diabatic heating is responsible for the position of the jet. The dissipation coefficient determines the strength of the jet.

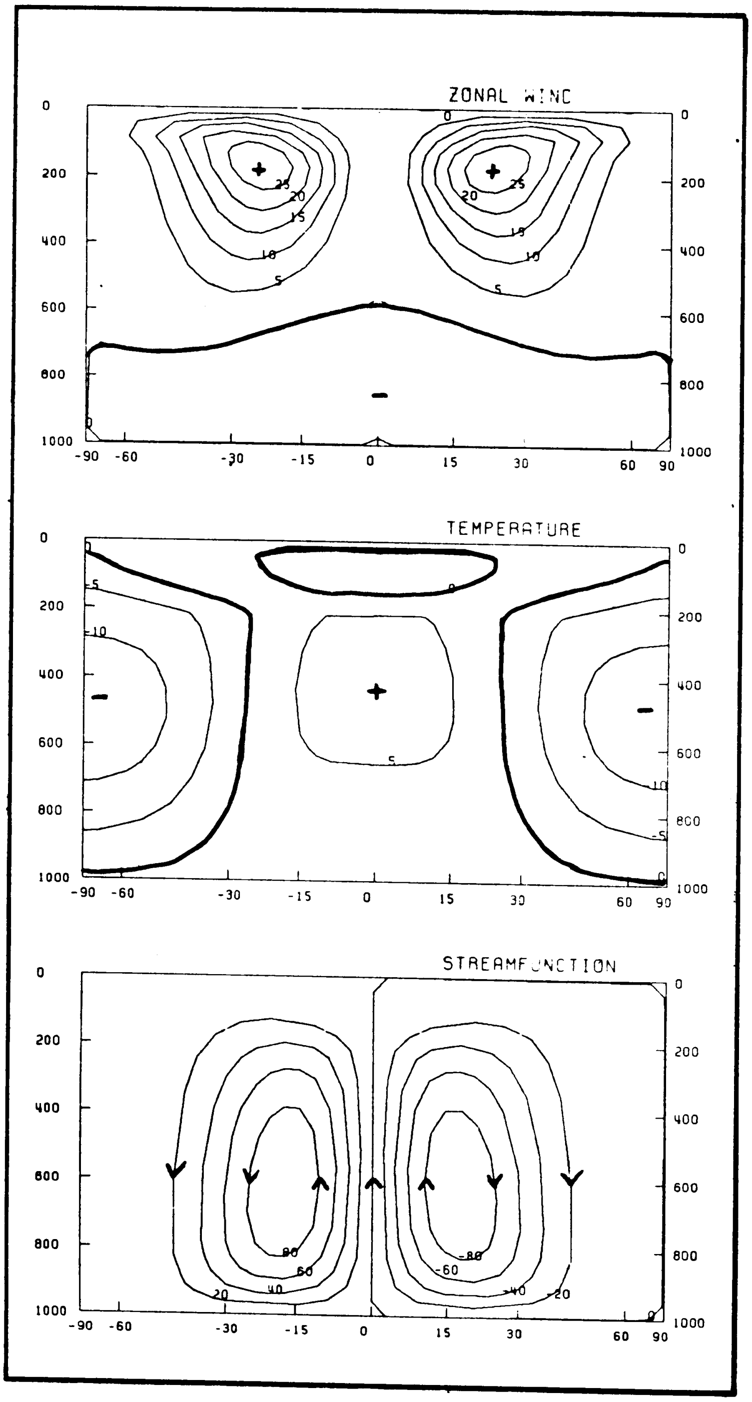

Fig. 6 Response due to an idealized diabatic heating (the Rayleigh friction constant is $k_u = 5.10^{-7}s^{-1}$ in the middle and upper atmosphere, increasing to $1.3\ 10^{-6}s^{-1}$ at the lowest level).

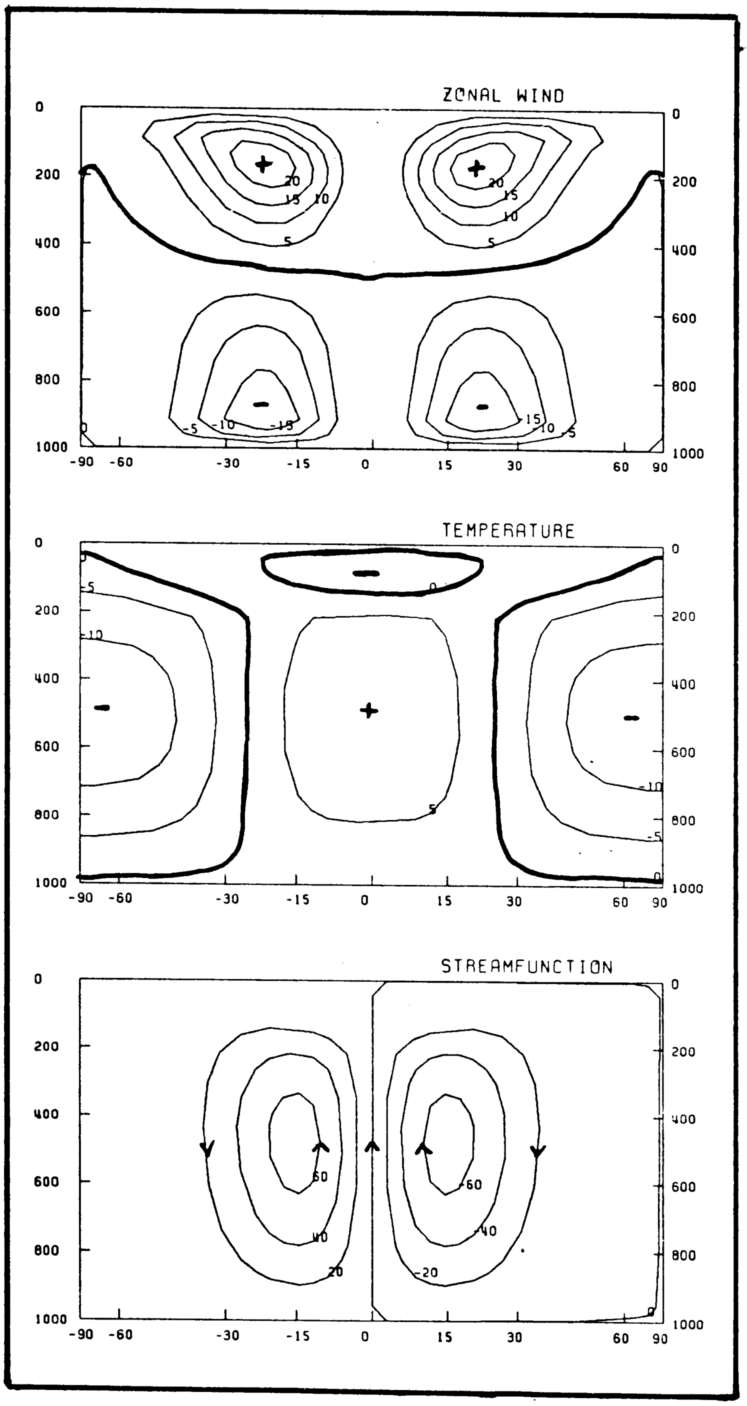

Fig. 7 Response due to an idealized diabatic heating with a different choice of friction parameters (the Rayleigh friction constant is $k_u = 5.10^{-7}s^{-1}$ in the whole atmosphere).

IV Conclusion

With a simple linear model it was possible to estimate the relative contribution of the eddies (momentum and temperature) and the diabatic heating (radiation and condensational heating) to the forcing of the zonal mean meridional circulation. The eddy forcing was given by observations (Oort and Rasmussen, 1971). The diabatic heating was prescribed analytically.

It appeared that the eddies are responsible for a 3 cell meridional circulation. The cells are equal in strength (cf. Kuo, 1956, Pfeffer, 1981). The diabatic heating is important for the zonal wind and temperature profile. This heating reinforces the direct Hadley cell as generated by the eddies, making the Hadley cell about four times stronger than the Ferrel cell. Also the choice of the value of the momentum dissipation constant k_u is important for the strength of the subtropical jet in the zonal mean wind distribution (fig. 6, 7).

The contribution of the standing eddy forcing seems to be negligible. There are some deficiencies in the model and the forcing. The easterly winds in the tropics are too small. In the polar region they are too large, which is caused by the gradient in the eddy heat forcing at mid-latitudes (fig. 4). The equator to pole contrast of the temperature distribution is too small (fig. 1, 6, 7). In the lower atmosphere in mid-latitudes the stability decreases significantly, despite the fact that the surface diabatic heating in this region as observed was neglected. The conclusion is that the divergence of vertical heat fluxes by the eddies is important and that a careful assessment of these terms must be made compared with the diabatic heating.

The model is capable of making a good estimate of the contribution of various forcing terms. Here we used a limited dataset, which contained no vertical eddy fluxes of heat and momentum (Oort and Rasmussen, 1971). The diabatic heating was chosen independently, which caused some inconsistencies. The model can however also be used to diagnose the model output of a General Circulation Model. By doing this we can gain insight in the role the various forcings play in the model and the atmosphere.

Appendix

I The modelequations

1. The linearized equations of motion of the zonally symmetric circulation have the form:

a. $-f\hat{v} + k_u\hat{u} = F_X$

b. $f\hat{u} + \frac{\partial\hat{\phi}}{\partial y} = 0$

c. $-\sigma_n\hat{\omega} + K_T\hat{T} = \hat{Q} + F_T$

d. $\frac{1}{\cos\phi}\frac{\partial}{\partial y}(\hat{v}\cos\phi) + \frac{\partial\hat{\omega}}{\partial p} = 0$

e. $\frac{\partial\Phi}{\partial p} = -\frac{R\hat{T}}{p}$

with:

dy	$= ad\phi$
a	: radius of the earth
ϕ	: latitude
$\hat{u}$, $\hat{v}$	: perturbation horizontal wind
$\hat{\omega} \equiv \frac{dp}{dt}$	: vertical velocity
$\hat{T}$	: perturbation temperature
$\hat{\Phi}$	: perturbation geopotential
R	: gasconstant
f	: Coriolisparameter
F_X, F_T	: eddy forcing
Q	: diabatic heating
σ_n	: static stability $(= -\frac{T_n}{p} + \frac{RT_n}{c_p p})$
T_n	: basic state temperature
c_p	: specific heat

2. Eliminating $\hat{u}$ from 1a and 1b, $\hat{T}$ from 1c and 1e gives us 3 equations in $\hat{v}$, $\hat{\omega}$, and $\hat{\Phi}$.
Further elimination of $\hat{\Phi}$, by making use of the thermal wind relation, and the introduction of a meridional streamfunction results in 1 equation in 1 variable:

$$A_1 \frac{\partial^2 \chi}{\partial \mu^2} + A_2 \frac{\partial^2 \chi}{\partial p^2} = Q' + F'$$

with:

$\mu = \sin\phi$

$\hat{v} = - \frac{1}{\cos\phi} \frac{\partial x}{a \partial \mu}$

$\hat{\omega} = \frac{\partial \chi}{a \partial \mu}$

χ = meridional streamfunction

$A_1 = K_u \sigma_n \cos^2\phi$

$A_2 = K_T p / R f^2 a^2$

$Q' = - \cos\phi \; K_u a \frac{\partial}{\partial \mu}(Q + F_T)$

$F' = - K_T p / R a^2 \frac{\partial F_x}{\partial p}$

II Numerical description

1. The equation in χ was solved at 15 equidistant levels (0-1000mb) in 30 gridpoints from $\mu = -1$ (SP) to $\mu = +1$ (NP).
   The boundary condition in χ was $\chi = 0$ at all boundaries: there is no flux across the boundaries. When χ was found (a method as described by Lindzen and Kuo, 1969, was used) the other variables could be evaluated on the staggered grid as described in fig. (X1).

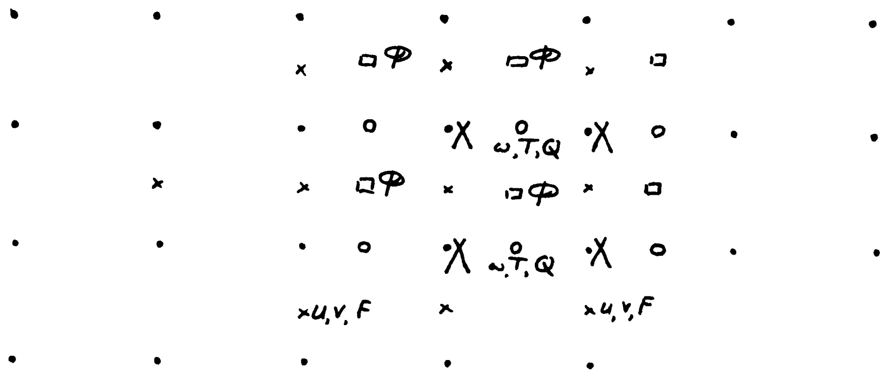

fig. (X1)

2. The computations occur in the following order.
   a) χ is evaluated at the gridpoints (denoted by •)
   b) v and ω are computed from the streamfunction in the intermediate points

$$\hat{v} = \frac{-1}{\sqrt{1-\mu^2}} \frac{\partial \chi}{\partial p} \qquad \text{(in the x-points)}$$

$$\hat{\omega} = \frac{1}{a} \frac{\partial \chi}{\partial \mu} \qquad \text{(in the 0-points)}$$

   c) From the thermodynamic eq. follows the temperature

$$\hat{T} = (Q + F_T + \sigma_n \hat{\omega})/K_T \qquad \text{(in the 0-points)}$$

   d) From the zonal momentum equation the zonal wind

$$\hat{u} = (F_x + f\hat{v})/K_u \qquad \text{(in the x-points)}$$

e) For the remaining variable, the geopotential height Φ, we need a boundary condition. We cannot prescribe $\hat{\Phi} = 0$ at the lowest Φ-level (in the -points) because this would conflict with the $\hat{u}$, we already found. So we fix the geopotential in one point and then integrate horizontally. We have chosen $\hat{\Phi} = 0$ at the equator at the lowest Φ-level. Then we can integrate horizontally and vertically making use of the hydrostatic balance:

$$\Phi = \int \frac{RT}{p} \, dp$$

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