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Dr J. A. BUSINGER

SOME ASPECTS OF THE INFLUENCE OF THE EARTH'S SURFACE ON THE ATMOSPHERE

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STAATSDRUKKERIJ EN UITGEVERIJBEDRIJF

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ERRATA:

age

2 2nd line from bottom should read: a. composition (amongst which *distribution* of moisture).

7 Line 11 — "KM- types" should read: "K- types".

2 20th line from bottom read *errors* instead of mistakes.

0 Equation (2.8.3) should read:

$$k\,\zeta = R\,(1 - \beta\, Sn\, R)^{\frac{1}{4}} \tag{2.8.3}$$

1 Equation (2.8.5) should read:

$$R = \left(\frac{u_*}{u_{*a}}\right)^2 k^2 \zeta^2 Sn + \tfrac{1}{2}\, k\, \zeta\, \frac{u_{*a}}{u_*} \left[1 + \left\{1 + 4\left(\frac{u_*}{u_{*a}}\right)^3 k\, \zeta\, Sn\right\}^{\frac{1}{2}}\right] \tag{2.8.5}$$

2 Line 6 and line 16 — "$k\,\zeta\, Sn \gg 1$" should read: "$k\,\zeta\, Sn \ll 1$".
Equation (2.8.11) should read:

$$U = \frac{1}{k}\, ln\, \zeta - \frac{1}{k}\, ln \left\{ 1 + Sn\, k\,(\zeta - 1) \right\} \tag{2.8.11}$$

4 For equations (2.8.3) and (2.8.5) see errata for page 30 and 31, respectively.

CONTENTS

LIST OF SYMBOLS

Symbol		Meaning	dimension
a	=	thermal diffusivity	m^2/h
a_1, a_2	=	constants (see **3.4**)	
b	=	$\dfrac{\delta(\theta_0-\theta_\infty(0))}{\gamma(\chi_0-\chi_\infty(0))}$ (see **3.5.1**)	dimensionless
c_1, c_2 etc.		constants (see **2.3**)	
c_p, c_v	=	specific heat at constant pressure, volume	kcal/kg °C
c_w	=	specific heat of water	kcal/kg °C
d	=	depth (see **3.2**)	m
D	=	molecular dissipation of turbulent energy (see **2.8**)	kg/h^2m
f	=	gradient of sea surface temperature	°C/m
g	=	acceleration due to gravity	m/h^2
h	=	height	m
j	=	q_e/q_a (see **3.6**)	dimensionless
k	=	v. Kármán's constant	dimensionless
K_h	=	coefficient of eddy conductivity (eddy thermal diffusivity)	m^2/h
K_m	=	coefficient of eddy viscosity	m^2/h
K	=	coefficient of eddy transfer	m^2/h
l	=	mixing length	m
L	=	latent heat of condensation	kcal/kg
m	=	constant	
m_e	=	evaporation of the earth's surface (see **1.4**)	kg/m^2
m_p	=	precipitation (see **1.4**)	kg/m^2
m_s	=	transport of mass from the soil to the surface (see **1.4**)	kg/m^2
m'	=	concentration of water in the air (see **3.2**)	kg/m^3
p	=	pressure	kg/mh^2
q_a	=	flux of sensitive heat of the surface	$kcal/m^2h$
$q_{a,e}$	=	total flux of heat of the surface	$kcal/m^2h$
q_e	=	flux of latent heat of the surface	$kcal/m^2h$
q_p	=	flux of heat on account of precipitation	$kcal/m^2h$
q_r	=	flux of heat on account of radiation	$kcal/m^2h$
q_s	=	flux of heat from the soil to the surface	$kcal/m^2h$
q'_r	=	difference between absorbed and radiated energy	$kcal/m^3h$
q'_e	=	change of latent to sensitive heat in the air	$kcal/m^3h$

		dimension
R	= gas constant	$m^2/h^2\ ^\circ C$
T	= temperature	$^\circ K$
T_m	= mean temperature (see **3.4**)	$^\circ K$
t	= time	h
u	= velocity comp. in x-direction	m/h
u_m	= mean velocity (see **2.2.5**)	m/h
u_*	= friction velocity $= (\tau/\varrho)^{\frac{1}{2}}$	m/h
u_b	= velocity at the upper limit of the surface layer	m/h
v	= velocity comp. in the y-direction	m/h
w	= velocity comp. in the z-direction	m/h
x, y, z	= coordinates	
z_b	= height of the surface layer (see u_b)	m
z_o	= roughness parameter	m
α	= coefficient of heat transfer	$kcal/m^2h\ ^\circ C$
β	= constant	dimensionless
γ	= lapse rate of potential temperature	$^\circ C/m$
δ	= vertical gradient of mixing ratio	l/m
η	= viscosity	kg/mh
θ	= potential temperature	$^\circ K$
θ_o	= potential temperature of the earth's surface	$^\circ K$
θ_∞	= potential temperature at the top of the surface layer	$^\circ K$
$\overline{\theta}_o$	= mean potential temperature of the earth's surface (see **3.5.3**)	$^\circ K$
θ_*	$= \dfrac{q_a}{u_*\, c_p\, \varrho}$	$^\circ K$
$\varkappa$	= coefficient of mass transfer	kg/m^2h
λ	= coefficient of heat conductivity	$kcal/mh\ ^\circ C$
ν	= kinematic viscosity	m^2/h
ϱ	= density	kg/m^3
ϱ_b	= density at the top of the surface layer (see u_b)	kg/m^3
ϱ_w	= density of water	kg/m^3
σ	$= u_*/\theta_*$	$m/h\ ^\circ C$
τ	= shearing stress	kg/mh^2
χ	= mixing ratio	kg/kg
χ_o	= mixing ratio at the surface	kg/kg

		dimension
χ_∞	= mixing ratio at level b (see u_b)	kg/kg
$\chi_* = -\dfrac{q_e}{Lu_*}$	. . . .	kg/kg

Dimensionless groups

$$Gr' = \frac{gl^4 \, \partial \bar{\theta} / \partial z}{K^2 \bar{T}}$$

$Pr = \dfrac{\nu}{a}$ . . . . Prandtl number

$R = \dfrac{K}{u_* z_o}$ . . . . dimensionless coefficient of eddy transfer

Re_x (see **2.2.5**)

$Ri = \dfrac{g \partial \bar{\theta} / \partial z}{\bar{T} (\partial \bar{u} / \partial z)^2}$ . . . . Richardson number

$Sn = \dfrac{g \theta_* z_o}{\bar{T} u_*^2}$ . . . . structure number

$U = \dfrac{\bar{u}}{u_*}$ . . . . dimensionless velocity

$\zeta = \dfrac{z + z_o}{z_o}$ . . . . dimensionless height

$\zeta_1 = -\dfrac{\alpha}{\lambda_s} z$ . . . . dimensionless height (see **3.6**)

$\zeta_b =$ . . . . dimensionless height of level b (see u_b)

$$\eta = \frac{\chi_\infty - \chi_\infty(0)}{\chi_o - \chi_\infty(0)}$$

$\vartheta = \dfrac{\theta_\infty - \theta_\infty(0)}{\theta_o - \theta_\infty(0)}$ . . . . dimensionless temperature

ϑ_1 *and* ϑ_2 (see **3.6**)

$\xi = \dfrac{\gamma}{\theta_o - \theta_\infty(0)} \left(\dfrac{k}{ln \zeta_b} \right)^2 . x$. dimensionless distance (see **3.5.1**)

$\xi_1 = \frac{1}{h}\left(\frac{k}{ln\zeta_b}\right)^2 . x$ . . . . . . dimensionless distance (see **3.6**)

$\tau = \frac{\alpha^2 a}{\lambda^2} t$ (see **3.6**) . . . . . dimensionless time

$\varphi = \frac{k}{ln\zeta_b}$

$\psi = \frac{f}{g}\left(\frac{ln\zeta_b}{k}\right)^2$

ψ' (see **3.5.2**)

Indices

a = adiabatic; K_a, l_a and u_{*a} (not q_a)
e = evaporation; q_e etc.
f = friction part; K_f, l_f etc.
h = at height h; (not K_h)
p = precipitation; (not c_p)
r = radiation; q_r etc.
s = soil; q_s etc.
w = water

CHAPTER 0

INTRODUCTION

A complete description of the meteorological processes necessitates a good insight into the thermodynamics and aerodynamics of the boundary phenomena. Although in recent years meteorological studies have testified to this better insight, the theories have somewhat lagged behind, and, with respect to the boundary phenomena, have not yet taken full account of them. This situation is not so much to be attributed to insufficient data of the boundary area – for compared with the data on higher air layers the former come out favourably – as to the lack of insight into the structure of the boundary phenomena, a complicated structure indeed, which is mainly caused by the great irregularity of the earth's surface and soil condition.

G e r h a r d t and J e h n (1950) arrived at the same conclusion when they state: "Studies of the atmosphere near the ground are certainly facilitated by the mere ability to make measurements, but they are simultaneously hindered manyfold by the complicating factors introduced by the presence of a boundary layer".

Even a good insight into the boundary phenomena will not make it exactly easy to describe atmospheric processes sufficiently well. Our investigation then, must be regarded as an attempt to penetrate the structure of the boundary layer and to indicate ways and means for an improved insight into the transformation of air masses.

In the first chapter processes and properties playing a part near the earth's surface will be discussed so as to secure a rational basis to build upon.

By introducing a classification of the energy balance of the earth's surface it will be possible to classify the whole territory of boundary phenomena and to survey the extent of a complete investigation in this field.

The second chapter will deal with the convective heat transmission of the earth's surface, and it contains a discussion of what way this phenomenon is related with other factors and defines the structure of the atmospheric surface layer. By atmospheric surface layer is understood the lower 25 m of the atmosphere, in contrast to the atmospheric boundary layer which extends to about 1000 m.

In the third, the final chapter, the transformation of the properties of the air as the result of the interaction at the surface will be discussed. This discussion will not be restricted to the surface layer alone, the entire air mass that directly participates in the process will be considered. The equations usually describing this transformation can only be solved graphically. Only in the cold air mass will it be possible to solve some problems appoximately by analysis. Since a complete investigation into this matter would be extremely exhaustive, only a few problems have more or less completely been worked out. An important means to facilitate the calculations of the transformation is the coëfficient of heat transfer which is introduced in the beginning of the third chapter; this calculation could be made because of the special structure found in the atmospheric surface layer when the latter is unstable.

CHAPTER 1

ENERGY AND MASS TRANSFER AT THE EARTH'S SURFACE

1.1 The surface of the earth

When speaking of the earth's surface we mostly have some idea of what we mean. The idea will soon become somewhat vague when a more exact definition is wanted; then a clear cut picture of the earth's surface will be highly necessary.

In the following considerations we have endeavoured to do so. There exists a surface which indicates the transition between the solid mass which belongs to the earth and the air which belongs to the atmosphere. Strictly speaking this surface is the earth's surface. However, it is not easy to study this surface because it is exceptionally freakish and this will only be done when one wishes to consider its fine structure. Especially if the study is limited to macroscopical phenomena one will get another idea of the earth's surface. This transition surface will therefore be called *boundary surface* and by *earth's surface* we understand then the average height where the boundary surface is to be found. The earth's surface will be in most cases an almost horizontal level which cuts through the boundary surface in a freakish way. This definition of the earth's surface blurs each detail of the boundary surface; for this reason it will be important to distinguish the average deviation of the boundary surface from the earth's surface as complemental information for the earth's surface. This average deviation we call *geometric roughness* of the earth's surface. Besides this geometric roughness we have the *aerodynamic roughness* of the surface, a property which in many cases may be deduced from the geometric roughness, especially when the surface shows no vegetation (e.g. sand and water stretches). The aerodynamic roughness is connected with the wind profile and can often be determined from it.

When there is dense vegetation it is necessary to take into consideration a *transition zone* between earth and atmosphere. This transition zone has a great influence on the transport of heat and vapour. The above mentioned definition of the earth's surface can be formally maintained, but is, in fact, only of little significance in this case. Now it is possible to extend the earth's surface to the bare earth and to understand by transition zone the average height of vegetation. The geometric roughness of the surface is here no longer of importance, while the aerodynamic roughness will be determined mainly by the uppermost vegetation limit.

It is furthermore important to talk of *homogeneous* surface when the geometric roughness is constant over the whole considered territory and of *inhomogeneous* surface when this condition does not exist.

Finally we can differentiate between *changeable* and *unchangeable* surfaces in so far as the wind is able to transform the surface or not. Thus seen, changeable surfaces are for example water surfaces, cornfields and, more generally, surfaces where wave phenomena may appear through influence of the wind.

When no further indication is given we shall from now on understand by the surface of the earth a homogeneous unchangeable surface without transition zone. This surface is best suited for an analytical treatment of the physical processes in its environment. All other surfaces include extra complications which will be touched upon only here and there.

1.2 Properties and processes, which play a part in the energy and mass transfer near the earth's surface

The complex of physical phenomena at the earth's surface is determined by a number of properties and processes. These properties and processes can be described by quantities. We shall now trace especially those quantities which play a role in the energy- and mass economy at the surface and differentiate between quantities that can be determined independently, as e.g. temperature and wind velocity, and quantities that can be determined dependently only, as e.g. the heat flux from the earth's surface to the air. In the first case it is not necessary to know other quantities, while in the latter case it is necessary to calculate the whole energy balance. In principle, this difference does not characterize the quantity concerned but shows in how far the technique of measuring is limited, and is therefore of importance to the verifying of theoritical observations. In **2.12** we will come back to this.

The following properties can be determined independently:

Of the earth's surface:

a. geometric roughness
b. absorption coefficient; (absorptivity)
c. contact coefficient.

Of the air:

a. temperature
b. humidity
c. wind velocity
d. pressure
e. lapse rate
f. viscosity.

Of the ground:

a. composition (among which division of moisture)
b. temperature

c. thermal conductivity
d. specific heat
e. density.

Furthermore there are processes at the earth's surface which can be investigated independently:

a. radiation
b. evaporation and condensation (sublimation)
c. precipitation
d. heat flux to and from the soil
e. transfer of momentum as a result of the shearing stress at the surface.

To be determined only dependently are:

a. aerodynamic roughness (see **2.13.2** and **2.13.3**)
b. vertical gradient of wind velocity [1])
c. coefficients of eddy transfer (see **2.5**)
d. mixing length (see **2.6** and **2.8**)
e. heat flux from the earth's surface toward the air (see **1.9**).

Usually only few quantities of this total are measured, namely those that are of direct importance to the comfort of mankind and for agricultural productivity. These are: temperature, humidity, wind velocity, precipitation and sunshine. For the description of the structure of the atmospheric surface layer and the examining of the transformation of air masses, these quantities alone are of relatively little significance and therefore it is essential to take the flux of energy and mass at the earth's surface into consideration. It is not a simple matter to measure all the above mentioned quantities simultaneously. Even if it were possible to succeed in this, it would be difficult to get a deeper insight into the phenomena near the surface of the earth. Therefore it is important to make a thorough study of the mutual coherence of the different quantities, to examine this coherence theoretically and experimentally and as a result to compose a scheme of measuring which yields maximal information with minimal exertion. It is possible that the usual series of observations will fit in with the results of such study, the outcome of the observations would then be advanced. It would supply a want when a survey were given of the instrumental consequences and those of measuring techniques, which are the result of the correct observing of the different quantities.

1.3 The energy balance (heat balance)

Considering that the earth's surface itself does not possess any heat capacity it is possible to give a very simple equation for the energy balance viz. the same

[1]) However, it is conceivable that a measuring instrument will be developed for measuring the vertical gradient of wind velocity.

amount of energy must flow toward the surface as is withdrawn from it. This energy flux can be divided in different parts. The most important are:

1. the energy flux from the soil, q_s
2. the energy flux toward the air $q_{a,e}$, as a result of contact of the air with the surface
3. The net radiation of energy at the surface, q_r.

The balance is now represented by

$$q_s = q_{a,e} + q_r \tag{1.3.1}$$

whereby the energy flux is considered positive in the direction of the positive z-axis. It is clear that 2 of the 3 terms from (1.3.1) determine the third term. We shall make use of this for the classification (see **1.6**).

These 3 energy fluxes can be divided again individually:

q_s exists at a land surface of an energy flux as a result of conduction of heat, of an energy flux as a result of distillation (displacement of moisture) and of a weak energy flux as a result of convection. At a water level we find energy transport mainly as a result of heat conduction and convection, while here an energy flux can also occur as a result of radiation.

$q_{a,e}$ consists of conduction and convection of heat q_a and of energy of evaporation q_e (see remark at **1.9**). As a rule q_a and q_e are balanced separately. This has not been done here due to a main division and a subdivision of the heat balance (see **1.6**).

q_r consists of various components:

1. the direct and indirect irradiation of the sun q_{r1}
2. irradiation of the atmosphere q_{r2}
3. reflection of the earth's surface q_{r3}
4. radiation q_{r4} of the earth's surface
5. absorption by photosynthesis q_{r5}.

Often a separate balance is made of it viz. the balance of radiation (see Albrecht (1933); Geiger (1950) and **1.9**):

$$q_r = q_{r1} + q_{r2} + q_{r3} + q_{r4} + q_{r5} \tag{1.3.2}$$

In the above consideration no account has been taken of the energy flux q_p, which occurs because of precipitation. In fact to the right member of equation (1.3.1) q_p has yet to be added:

$$q_s = q_{a,e} + q_r + q_p \tag{1.3.3}$$

For the sake of clearness this has not been done. Moreover the giving of an energy balance in the form (1.3.3) is only theoretically possible during precipitation as the determination of the separate components is virtually impossible, (see **1.9**). As soon as the intensity of precipitation is of some importance, q_p in the right member of (1.3.3) will predominate with respect to $q_{a,\,e}$ and q_r.

1.4 The mass balance (water balance)

In the same way as the energy balance has been drawn up so, a mass balance of the earth's surface can be given. The earth's surface does not posses any mass, nor has it any heat capacity. Thus it is necessary that just as much mass flows toward the surface as is withdrawn from it.

The mass balance exists of the following components:

1. transport of mass from the soil m_s
2. evaporation of the surface m_e
3. precipitation on the surface m_p.

In equation: (when $m > 0$ is taken in the direction of the positive z-axis):

$$m_s = m_e + m_p \tag{1.4.1}$$

The mass balance is closely connected with the energy balance. q_e will entirely be determined by m_e and for the determining of q_e it is even necessary to make up the mass balance (see **1.9**). A part of q_s corresponds with m_s, and q_p will be determined mainly by m_p.

1.5 Considerations of the heat- and water economy of the earth's surface

So far we have written of energy and mass in order to express ourselves physically correctly. However, it is much more natural to speak of heat and water, when energy and mass are meant. Although it is not quite correct, we shall do so now for clearness sake.

It may be of importance to make up the average values of the different terms of the equations (1.3.3) and (1.4.1). The average of these terms can be made simultaneously about the surface or about the time. In both cases interesting information may be obtained.

The average of the surface is of importance to the transformation of the air mass existing above this surface, when the size of the surface o is of synoptical scale. If the average of the whole earth's surface could be calculated, it would be interesting to consider the values of

$$1/o\int q_s\,do;\ 1/o\int m_s\,do;\ 1/o\int q_{a,e}\,do;\ 1/o\int q_r\,do;\ 1/o\int m_e\,do \text{ and } 1/o\int m_p\,do.$$

If e. g. $$1/o\int q_s\,do < o$$

this would mean that the earth as a whole absorbs more heat than it yields, or if $1/o \int m_e\, do > -1/o \int m_p\, do$ this would mean that the atmosphere as a whole has an increasing content of water vapour. Such temporary changes in the heat and water economy might be of great influence on the synoptic weather picture. For the present, considerations concerning this will as yet be of a speculative character.

The averages of time may also bring about interesting aspects. The peculiarity of a land surface would then present itself, that the horizontal heat flux in regard to the vertical one is negligible. The result is that when at two different moments the enthalpy of the ground is the same, the average value of q_s over the period between those two moments is zero. Thus it is possible to control the applied measuring method. As a rule it will not be possible to neglect the advection in the sea, and therefore it will be also over long periods $\bar{q}_s \neq 0$. The sign of $\bar{q}_s$ denotes whether the sea as an average yields heat to the air or absorbs from it. It does not indicate if the sea as an average is warmer or colder than the air, because the structure of the atmosphere is quite different in both cases, see **2** and **3.1.**

The meaning of the average values of the other terms of the equations (1.3.3) and (1.4.1) is rather obvious. The sign of $\bar{q}_r$ denotes whether the radiation or the irradiation predominates; the sign of $\bar{q}_{a,e}$ whether the air absorbs heat from the earth's surface or yields heat to it. Even then we may not say that the air as an average is colder or warmer than the surface of the earth. The making up of $\bar{q}_p$ gives less direct information and is more suitable to detail study. The sign of $\bar{m}_s$ furthermore denotes whether the evaporation is greater of less than the precipitation.

The equations (1.3.3) and (1.4.1) are not sufficient to describe the entire heat and water economy of the soil and the air, because in that case the heat and water supply has not been taken into consideration. In **3.2** we shall go further into this. Nevertheless the average quantities as outlined above can give us certain characteristics of occuring climates (see **1.8**).

1.6 Classification of the heat balance

As has been remarked in the introduction, it is reasonable to make a classification of the possible types of the heat balance so as to be able to oversee the boundary phenomena at the earth's surface. Here we shall build on the work of Geiger. Geiger (1950) classified the heat balance in two types: the radiation type R and the irradiation type I, which correspond with $q_r > 0$ and $q_r < 0$ resp.. These types were amply treated by him, because they are especially suitable for the study of microclimatological phenomena. Research in microclimatology was engaged in these types especially see e.g. Fransilla (1936) and others.

The typology of G e i g e r can be extended when we take also into consideration $q_{a,e}$ and q_s. Since q_s is determined when $q_{a,e}$ and q_r are known (according to (1.3.1)), it will be sufficient only to classify the direction and the size of q_r and $q_{a,e}$. When $q_{a,e} > 0$ we speak of cold mass K and when $q_{a,e} < 0$ of warm mass W. This definition is not quite in accord with the definition which is used in synoptic meteorology. Here namely is spoken of cold mass when $q_a > 0$ and of warm mass when $q_a < 0$. However seen from an energic angle, it is more convenient to work with our definition, because then K or W denotes whether the enthalpy of the air increases or decreases.

We now come to the following classification:

1. $q_{a,e} > 0$; $q_r > 0$ and $q_{a,e} > q_r$ type KR $\rightarrow q_s > 0$
2. $q_{a,e} > 0$; $q_r > 0$ and $q_r > q_{a,e}$ type RK $\rightarrow q_s > 0$
3. $q_{a,e} > 0$; $q_r < 0$ and $|q_{a,e}| > |q_r|$ type KI $\rightarrow q_s > 0$
4. $q_{a,e} > 0$; $q_r < 0$ and $|q_r| > |q_{a,e}|$ type IK $\rightarrow q_s < 0$
5. $q_{a,e} < 0$; $q_r > 0$ and $|q_{a,e}| > |q_r|$ type WR $\rightarrow q_s < 0$
6. $q_{a,e} < 0$; $q_r > 0$ and $|q_r| > |q_{a,e}|$ type RW $\rightarrow q_s > 0$
7. $q_{a,e} < 0$; $q_r < 0$ and $|q_{a,e}| > |q_r|$ type WI $\rightarrow q_s < 0$
8. $q_{a,e} < 0$; $q_r < 0$ and $|q_r| > |q_{a,e}|$ type IW $\rightarrow q_s < 0$

These 8 types form the main division of the possible energy balances. A survey of this is given by Fig. 1.6.1.

We can now distinguish the rain-type P as soon as $q_p \neq 0$. P corresponds with $q_s > 0$ or with $q_s < 0$ as a result of which the following two types can be denoted:

9. $q_p > 0$ type Kp $\rightarrow q_s > 0$
10. $q_p < 0$ type Wp $\rightarrow q_s < 0$ (see **1.9**)

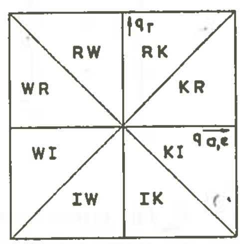


Fig. 1.6.1

A subdivision will be obtained by a closer observation of the components q_a and q_e of $q_{a,e}$. When $q_{a,e} > 0$ (thus with the KM- types) there are two possibilities viz:
1. q_a and $q_e > 0$ and 2. $q_a < 0$ and $q_e > 0$, where $q_e > |q_a|$.
At most surfaces the combination $q_e < 0$ and $q_a > 0$ does not exist because no condensation can occur at a surface that is warmer than the adjacent air, unless that surface is very hygroscopic, e.g. a salt surface. These exceptional cases will not be dealt with here.

When $q_{a,e} < 0$ (as with the W-types) there are likewise two possibilities:

1. q_a and $q_e < 0$ and 2. $q_a < 0$ and $q_e > 0$, where $|q_a| > |q_e|$.

The sign of q_a denotes whether the air is stratified stable or unstable. This will be elaborated in chapter **2**. So we have with the K-types unstable, as well as stable types, while the W-types are always stable.

The sign of q_e denotes whether condensation or evaporation occurs. With the K-types we always find eva- poration, while with the W-types evaporation as well as condensation may occur. The subdivision has now been composed as follows:

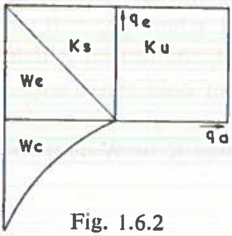


Fig. 1.6.2

unstable cold mass Ku
stable cold mass Ks
warm mass with condensation Wc
warm mass with evaporation We.

Fig. 1.6.2 gives a survey of this subdivision. The quadrant bordered by ($q_e = 0; q_a < 0$) and ($q_a = 0; q_e < 0$) is not filled entirely, because every $q_a < 0$ determines a minimum value of

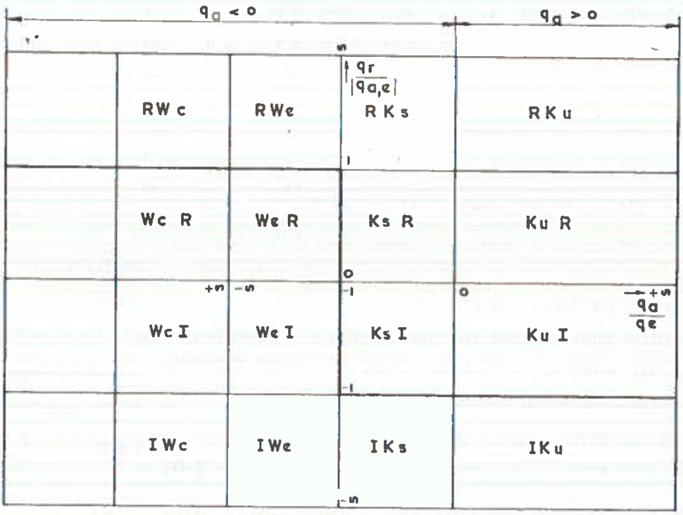


Fig. 1.6.3

$q_e < 0$. The curve indicating these minima is dependent of the air temperature and the wind velocity.

The problems connected with this will not be dealt with.

Fig. 1.6.3 gives us a survey of the whole classification of the heat balance with exception of the precipitation types.

1.7 Discussion on the heat balance types

A complete description of the different types and of the situations connected with it would lead us too far. Therefore we shall go into this only sparingly, especially in connection with the research that has already been done in this field.

Types KuR and KuI. At the earth's surface energy is being withdrawn by convection as well as by evaporation; here the energy flux toward the air is greater than the radiation or the irradiation. These types will occur after a cold wave, where the radiation is suppressed by clouds. Above the sea these types occur frequently in certain places, e.g. the Gulf Stream. When radiation is small in regard to the energy flux toward the air, it will be possible with certain simplifying assumptions to calculate the transformation of temperature and humidity of the air. In chapter **3** this will be dealt with more in detail and special attention will be paid to the investigations of B u r k e (1945) and F r o s t (1949).

The types KsR and KsI are of a quite different character. These types, just as the other Ks types are rather scarce. The temperature of the earth's surface lies between the wet bulb and the dry buld temperature of the air. In that case the heat flux from the air toward the surface can be smaller than the energy flux from the surface toward the air, as a result of evaporation, namely when the earth's surface is sufficiently moist. These types will show up mostly when dry air flows over a water level, e.g. desert air over sea. As far as we know no special investigations have been made about them.

Over land the types RKu and RKs are always transition types toward RWe. Above sea it is possible for these types to maintain themselves for a longer time; then their character is such that the air, as an average, will have the feature of cold mass. So the air is of colder origin.

The radiation types IKu and RWe or RWc occur often; these types predominate when we have clear radiation-wheather above land. Different research workers have studied the phenomena which belong to these types:

1. the occurrence of free convection (see **2.9** and further)
2. the occurrence of ground frost, B r u n t (1932), G r o e n (1947) (theoretical investigations), K e s s l e r and K a e m f e r t (1940) and others (measurements).
3. the occurence of fog.

Investigation of the whole balance of these types has been made by A l b r e c h t (1933), F r a n s i l l a (1936), J e h n and G e r h a r d t (1950), P e e r l k a m p (1944), R i d e r and R o b i n s o n (1951) and others.

The types WcR, WeR, WcI and WeI occur when warm air invades. Here we often find fog and stratiform clouds as a result of which radiation will be of little importance. The convective heat transmission is here smaller as a rule than with the K-types, because of the stable stratification of the atmosphere. The calculation of the transformation of the properties of the air will be exceptionally complicated by this stable stratification (see **3.7**). As far as we know these types have not specially been investigated either.

Above land the types IWc, IWe and IKs are as a rule transition-types toward IKu. However, there are certain circumstances where these types above land can maintain themselves for a longer period, viz. during thawing conditions, when the earth's surface is covered with a layer of snow. Here the well defined condition of the earth's surface can be applied for calculations of transformation. Above sea these types can maintain themselves longer, the same as with the types RKu and RKs. The type IKs will occur only when the air is particularly dry and warm, as eg. with föhn-wheather.

The types Kp and Wp are apart from the 16 types discussed so far and will often cause discontinuities in the course of these types. The type Kp will occur most, because rain usually comes from, and goes through layers of the atmosphere wich have a lower temperature than the surface. However, especially in winter we find decided Wp types, viz. when rain falls on frozen ground.

As an illustration it is interesting to follow the course of the types on a clear day, when the radiation types predominate. As example we have taken the observations in the central part of Finland by F r a n s s i l a (1936). On the 7th–8th of August 1934 we found there an almost cloudless day with practically no wind. The following table of the succession of different heat balance types has been composed from the heat balance measurements by F r a n s s i l a on that day.

TABLE I

Time		Type	Time	Type
7th till 8th	8.00–16.40	IKu		Wc R
of August	16.40–17.30	Ku I		Wc I
	17.30–18.50	Ks I	Aug. 8th 6.10–6.35	We I
		Ks R		I Wc
	18.50–20.20	R Ks		I Ks
		R We		
	20.20– 6.10	R Wc	6.35–8.00	I Ku

This table shows clearly how much the radiation types RWc and IKu predominate, (almost 20 out of the 24 hours), while, and this fact is of more importance, nearly the whole exchange of energy takes place in this period. The rest of the time is divided in 10 types, whereby it is remarkable that the transition from IKu toward RWc lasts almost four hours, whereas the transition from RWc toward IKu takes place in about 25 minutes. So the transition in the evening is much more gradual than the one in the morning. It is noteworthy too, that 12 out of the 18 types occur once in 24 hours.

The great changes in the types of the heat balance make this typology also on account of their intricacy unsuitable for present use. It is, useful however, to make up average values over a shorter or longer time, as was discussed in **1.5.** From this it is possible to make up the character of the air mass in spite of predominating radiation weather (clear weather). Here the average over 24 hours can be of use.

1.8 Consideration of the possible meaning of the given classification to the climate description

It is not intended to try to develop in this place a climatology, but rather to open some perspectives on this matter. The possible importance of it justifies in our opinion a short interruption of our argument.

On account of the above it must be possible to give a climate description on an energetic basis. The making up of averages about the time, as has been discussed in **1.5**, gives us certain predominating types, which are characteristic for the place where the averages have been made. In this case we shall have to investigate especially over which periods must be averaged in order to obtain a differentiation as great as possible. When a very long period is considered, some years for instance, then with land surfaces, as we have seen in section **1.5**, q_s will become equal to zero and the types will then come on the thick-drawn line of Fig. 1.6.3 and the possible differentiation will be limited.

The advantage of such a climate description is that the climate will be brought more in relation with the physical processes at the surface and less with the vegetation. It is conceivable that by doing so the climate may be connected with the heat economy of the troposphere and that in this way at last a better insight in the general circulation will be obtained.

Another aspect of a climatology based on the heat and water economy of the earth's surface, is the possibility that from this climatology we could define the quantity of change of climate which a region will undergo when irrigation, deforestation or planting would be applied extensively. Here it will be necessary to have a good insight also into the microclimatological changes.

1.9 The measuring of the different energy fluxes

The principal reason why so few systematic investigations have been made in the sphere of the energy balance is the fact that it is difficult and intricate to measure the different energy fluxes. Albrecht (1932), (1933), (1950a) has done much work in this field.

The determining of q_s can be done in different ways:

1. Directly by means of a heat flux meter. Albrecht (1932) has developed a so called "Wärmeumsatzmesser", which has been improved by Fransilla. An objection against this heat flux meter is that it changes the structure of the ground, because periodically a heat flux is directed through it at the place of measuring.

An other type of heat flux meter has been developed at the Technical University at Delft by Mulder (1951). This heat flux meter can be used aptly during short periods. During longer periods this instrument has drawbacks,

as the measuring disk obstructs the humidity transport in the ground, on account of which the structure of the ground will be influenced by the presence of the disk.
2. A method which considered from the measuring technique, is simple, but otherwise intricate, has been applied by J e h n and G e r h a r d t (1950). With this method the temperature of the ground at different depths has been registered till such depth where the temperature change with the time is negligible. With the aid of these registrations the enthalpy of the ground can be calculated at different moments, when the specific heat and the density have also been determined.
3. Thanks to the fact that it is now possible to measure the coefficient of thermal conductivity of the ground without structural changes of it with the help of the non stationary method (as has been applied by d e V r i e s (1952)), it is possible to measure q_s also continously in a more or less trustworthy way. However, this way of measuring and the working out are intricate. So this method is suitable to fundamental investigations, but not to routine work.

The determining of q_r can be done well only with the use of a so-called radiation balance meter, which indicates directly the net radiation. Such an instrument has been developed by A l b r e c h t (1933). It is possible to determine the noctunal radiation with the aid of an "actinometer" and to make corrections of the difference of temperature between the meter and the surface and of the absorptivity of the surface; however, even then the possibility of considerable mistakes remains.

$q_{a,e}$ can be determined with the aid of equation (1.3.1) when q_r and q_s have been measured already. Properly speaking in this case $q_{a,e}$ will be made closing entry of the balance. It is more desirable to determine $q_{a,e}$ independently. Doing so would enlarge also the accuracy of the whole balance. In order to determine $q_{a,e}$ independently the components q_a and q_e have to be determined separately.

q_e can be determined with the water balance, which has already been noticed in **1.4**. To do this it is necessary to measure m_s, m_e and m_p. As long as it does not rain the determining of q_e means the measuring of the loss of weight of a ground-sample, which can lose moisture only by evaporation. The surface condition of the sample has to be as good in agreement with the environment as possible.

R i d e r and R o b i n s o n (1951), because of the equality of the coefficients of eddy transfer, (see **2.5**) determined by use of q_e, also q_a, by making a close fit of the two temperature and moisture profiles. The $q_{a,e}$ acquired in this way, was tolerably well in agreement with the $q_{a,e}$ which was obtained as closing entry of the heat balance.

When the structure of the atmospheric surface-layer has been sufficiently investigated at different stabilities, it should be possible to determine q_a and q_e from the profiles of wind velocity, temperature and humidity.

The determining of the q_a is the most difficult of the energy fluxes, discussed in this section. However, in a certain way this quantity is one of the most important too, because it is definitely important to the structure of the atmospheric surface layer, see chapter **2**.

q_p, as has been remarked in **1.3**, is difficult to determine. The reason of it lies in the fact that various measurements cannot be made during precipitation. Thus it is impossible to carry out useful measurements, while from observations on different heights of temperature and moisture, no conclusions can be drawn about heat and moisture transport. It is furthermore very difficult to make up m_e from the water balance, because with weight measurements m_e and m_p cannot be separated. The only quantity that can be measured more or less reliably is q_s (as long as there is no precipitation). From the process of q_s before, during and after a shower of rain, in combination with q_r and $q_{a,e}$ before and after the shower, some conclusions about q_p can be made. In most cases, during rain, it will be allowable to neglect q_r and $q_{a,e}$. So when the vertical temperature and humidity gradient is small, $q_s \approx q_p$.

CHAPTER 2

ANALYSIS OF THE ATMOSPHERIC SURFACE LAYER

2.1 Introduction

In **1.9** the fact was pointed out that a good insight into the structure of the atmospheric surface layer can considerably simplify determining the energy fluxes q_a and q_e. It is these very energy fluxes, which are conclusive for the transformation of air masses. Moreover, it is of importance to have a thorough knowledge of the atmospheric surface layer for many microclimatological investigations. Naturally numerous investigations have been made in this field. A good survey of the most important of these has lately been given by Priestley and Sheppard (1952). It is remarkable that all these investigations have yielded somewhat unsatisfactory results so far. Possibly one of the causes is the fact that most investigations confine themselves too much to detail studies without trying to give a more comprehensive theoretical analysis. Lettau (1949) has developed a more complete theory, however, which will be dealt with further in **2.6** and following.

When investigating atmospheric turbulence one often comes up against the fact that nearly every assumption which is made in order to develop a theory, appears to be untenable during further analysis. Consequently, one is prompted to try and work with ever more fundamental concepts and as a result ever greater difficulties are encountered.

Of course a fundamental analysis is essential in order to build up an entirely satisfactory theory. However, it may be profitable to develop in a less exact, but more tentative manner, a theory, which is useful and capable of practical application. The exact basis of the theory may then be developed afterwards. In working thus, however, it is necessary to proceed with great caution, especially in interpreting possible results. The above mentioned theory by Lettau (1949) is a typical instance of a case in which this caution was insufficiently observed.

Starting from special assumptions we have attempted in this chapter to give a more or less complete model of the atmospheric surface layer. In doing so we have especially built on the mixing length theory of Prandtl (1935) (see **2.4**). The profiles of velocity and temperature and the transfer of momentum and heat, which belongs to it, are especially considered. Once these are known, it is possible to treat the diffusion of water vapour and other mass, without giving rise to considerable new difficulties (see also **2.13.5**).

Various investigators (Batchelor (1950), Inoue (1952) and others) have raised grave objections against the concept of the mixing length and have shown the inadequacy of this concept in certain cases. In its place they have put forward the so-called statistical theory of turbulence. However at present this statistical theory is only applicable to "homogeneous" media such as an adiabatic atmosphere, but not to "inhomogeneous" media in which convective forces occur. It therefore seems to us that the rejection of the concept of the mixing length is at least premature, in particular because this concept has shown its usefulness in adiabatic conditions.

2.2 Definitions and assumptions

2.2.1 *The atmospheric surface layer*

By atmospheric surface layer we understand the part of the atmosphere bordering upon the surface of the earth and in which, approximately, the vertical transfer of heat and momentum may be considered constant with height. According to Ertel (1933) this layer extends on an average for about 25 m. The thickness of the layer depends on the velocity of the wind and the roughness of the earth's surface, see also **2.3.3**. The surface layer forms a part of the atmospheric boundary layer, which extends to a height of approximately 1000 m. Higher up the friction at the surface hardly makes its influence felt.

2.2.2 *Steady States*

In order to keep the equations as simple as possible, only steady states are considered. This means that the various quantities in one and the same place are on an average constant as to time. We now assume that such averages of the various quantities exist and have a real physical significance. A consequence of this assumption is, that the various quantities can be split up into a mean component and a purely turbulent component, which as to time, is zero on an average.

This is expressed as follows; e.g.:

$$u = \bar{u} + u'; \ \varrho = \bar{\varrho} + \varrho' \text{ etc. and } \overline{u'} = \overline{\varrho'} = 0 \text{ etc.}$$

That this splitting up is not obvious, but that an assumption has to be made respecting it, has been pointed out by various authors. For this we refer to the paper by Priestley and Sheppard (1952). Furthermore it is accepted that the mass flow takes place in the x-direction, so that $\overline{\varrho v} = \overline{\varrho w} = 0$ and $\overline{\varrho u} \neq 0$.

2.2.3 *Air is considered as an incompressible medium*

2.2.4 It is supposed that along the whole surface layer *turbulent friction is great with respect to the purely viscous friction,* consequently $K \gg \nu$. Only in the lowest centimeters this will not be quite correct.

2.2.5 It is assumed that the *turbulence is completely developed for the main flow.* This means that $Re_x = \dfrac{\varrho \bar{u}_m x}{\eta} \geqslant 10^9$ in which x is the distance from the edge of the surface where this starts getting homogeneous to the measuring-point and in which $\overline{\varrho u_m}$ is the mass flow. For it we may take e.g. the mean mass flow over the layer of the atmosphere considered so $\overline{\varrho u_m} = 1/h \int_o^h \overline{\varrho u}\, dz$ in which h is the height up to where the layer extends. The value 10^9 of Re_x as boundary for completely developed turbulence will be further explained in **2.13.4.**

2.2.6 The coordination system is chosen in such a way that the mean mass flow takes place in the direction of the x-axis and that the z-axis is at right angles to the earth's surface.

2.3 The basic equations

Generally speaking the equations describing the structure of the atmospheric surface layer are:

The equation of continuity

$$\frac{\partial \varrho}{\partial t} + \nabla (\varrho \vec{v}) = 0, \tag{2.3.1}$$

the equation of state

$$p = \varrho RT, \tag{2.3.2}$$

the Navier-Stokes equations

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}(\text{grad.}\, \vec{v}) = \vec{K} - \frac{1}{\varrho} \nabla p + \nu \triangle \vec{v} + \frac{\nu}{3} \nabla . \nabla \vec{v}, \tag{2.3.3}$$

Fourier equation

$$\frac{\partial (\varrho T)}{\partial t} + \vec{v} \nabla \varrho T = \frac{\lambda}{c_p} \triangle T. \tag{2.3.4}$$

This last equation is a special case of the first law of thermodynamics.

To these may furthermore be added equations for diffusion, which always show the form of (2.3.4) and in which $\lambda/c_p\, \varrho$ must be replaced by the coefficient of diffusivity and T by the concentration of the diffusing matter. As in these diffusion phenomena no convective forces of any significance are met with, except in special cases for water vapour, these may be left out of consideration in the first instance, see **2.13.5.**

The above mentioned equations can be considerably simplified by means of the assumptions from **2.2,** in which also the physical meaning of the various terms is emphasized.

2.3.1 *The equation of continuity*

The equation (2.3.1) may be split up into two equations, one for the mean state and one for the purely turbulent state:

$$\frac{\partial \bar{\varrho}}{\partial t} + div(\bar{\varrho}\,\vec{\bar{v}} + \overline{\varrho'\,\vec{v}'}) = 0 \tag{2.3.5}$$

$$\frac{\partial \varrho'}{\partial t} + div\{\varrho'\,\vec{\bar{v}} + \bar{\varrho}\,\vec{v}' + (\varrho'\,\vec{v}' - \overline{\varrho'\,\vec{v}'})\} \tag{2.3.6}$$

The equation of pure turbulence may be left out of consideration because it is of no importance for our further argument. We shall also do so with the equations (2.3.3) and (2.3.4).

As furthermore only steady states will be considered, in which $\frac{\partial \bar{\varrho}}{\partial t} = 0$ and moreover $\overline{\varrho v} = \overline{\varrho w} = 0$, compare **2.2.2**, equation (2.3.5) becomes

$$\frac{\partial}{\partial x}(\bar{\varrho}\,\bar{u} + \overline{\varrho' u'}) = 0$$

or

$$\bar{\varrho}\,\bar{u} + \overline{\varrho' u'} = f(z) \tag{2.3.7}$$

Moreover, it is of importance to remark that when $\overline{\varrho w} = \bar{\varrho}\,\bar{w} + \overline{\varrho' w'} = 0$ in case a vertical density gradient prevails, $\overline{\varrho' w'} \neq 0$ so that

$$\bar{w} = -\frac{1}{\bar{\varrho}}\overline{\varrho' w'} \tag{2.3.8}$$

So the possibility exists that there is a mean vertical velocity component without mass flow in this direction. $\bar{w}$, however, is not perceptible owing to its small quantity. Even in case of large density differences and strong turbulence we keep $\bar{w} < 1$ cm/sec.

It is easy to see that there must be a correlation between ϱ' and w'. For if we start from the theory that on the whole a vertical motion caused by turbulence takes with it an eddy which is in equilibrium with its surroundings, this eddy on account of the existing mean gradient will show a deviation in density which is proportional to said gradient and to the distance covered by this eddy. This distance again will be proportional to w'. So there is a correlation between w' and ϱ', which may be expressed as:

$$\overline{\varrho' w'} = c_1\,\overline{w'^2} \neq 0 \tag{2.3.9}$$

As various authors (E r t e l (1942, 1944), S c h m i t z (1947)) start from the theory that $\bar{w} = 0$ should be valid, they arrive at the conclusion that $\overline{\varrho' w'} = 0$. Though C a l d e r (1949), points out in above mentioned qualitative manner that $\overline{\varrho' w'} \neq 0$, he opposes these authors but he does not arrive at equation (2.3.8), on the contrary, in continuing his argument he commits the error himself: $\bar{w} = 0$. The way it is dealt with above clearly shows where the incorrect reasoning of E r t e l and S c h m i t z is to be found.

An analogous qualitative observation as given for $\overline{\varrho' w'}$, may also be given for $\overline{\varrho' u'}$, we shall revert to this in **2.3.3**.

From considerations of symmetry we may accept that no correlation exists between ϱ' and v', so that $\overline{\varrho' v'} = 0$.

2.3.2 *The equation of state*

As the density, temperature and pressure variations are small with respect to the mean values (see Calder (1949)), the equation of state (2.3.2) may be written in the form:

$$\frac{p'}{\bar{p}} = \frac{\varrho'}{\bar{\varrho}} + \frac{T'}{\bar{T}} \tag{2.3.10}$$

Calder moreover shows that $\dfrac{p'}{\bar{p}} \ll \dfrac{\varrho'}{\bar{\varrho}}$ and $\dfrac{T'}{\bar{T}}$

So that we have left over in first approximation

$$\frac{\varrho'}{\bar{\varrho}} = -\frac{T'}{\bar{T}} \tag{2.3.11}$$

As moreover in the bottom 25 metres of the atmosphere the difference between $\bar{T}$ and $\bar{\theta}$ and between T' and θ' is very small, henceforth we shall only make use of potential temperature because the gradient of the potential temperature is the gauge of the stability. As a fair approximation we therefore have:

$$\frac{\varrho'}{\bar{\varrho}} = -\frac{\theta'}{\bar{\theta}} \quad \text{for} \quad \frac{\bar{\theta}}{\bar{T}} \approx 1 \tag{2.3.12}$$

2.3.3 *The* Navier-Stokes *equations*

The Navier-Stokes equations may be treated in a manner analogous to that of the equations of continuity. We leave out of consideration the Coriolis force, which is neutralized by a component of the gradient force and take into account only the component of the gradient force in the direction of the main flow.

Taking into consideration (2.3.1), the equations of the mean flow become:

$$\frac{\partial}{\partial z}(\overline{\varrho u w}) = -\frac{\partial \bar{p}}{\partial x} + \eta \frac{\partial^2 \bar{u}}{\partial z^2} \tag{2.3.13}$$

All the terms in the y-direction are on an average zero, (2.3.14)

and

$$\frac{\partial}{\partial z}(\overline{\varrho w^2}) = -\frac{\partial \bar{p}}{\partial z} - g\bar{\varrho} + \tfrac{3}{4}\eta \frac{\partial^2 \bar{w}}{\partial z^2} \tag{2.3.15}$$

By means of (2.3.8) we can write for the last term of (2.3.15)

$$\frac{4}{3}\eta \frac{\partial^2 \bar{w}}{\partial z^2} = -\frac{4}{3}\nu \frac{\partial^2}{\partial z^2}\overline{\varrho' w'}$$

Now $\overline{\varrho' w'}$ in the surface layer is practically constant, because this correlation coefficient is a measure for the heat transfer see **2.3.4.** So that $\frac{\partial^2}{\partial z^2}(\overline{\varrho' w'})$ will be negligible. So we retain:

$$\frac{\partial}{\partial z}(\overline{\varrho w^2}) = -\frac{\partial \bar{p}}{\partial z} - g\bar{\varrho} \tag{2.3.16}$$

Let us now consider correlation coefficients $\overline{\varrho u w}$ and $\overline{\varrho w^2}$. These may be written as follows:

$$\overline{\varrho u w} = \overline{(\bar{\varrho}+\varrho')(\bar{u}+u')(\bar{w}+w')} =$$
$$= \bar{\varrho}\,\bar{u}\,\bar{w} + \bar{\varrho}\,\overline{u' w'} + \bar{u}\,\overline{\varrho' w'} + \bar{w}\,\overline{\varrho' u'} + \overline{\varrho' u' w'} =$$

Taking into account (2.3.8) this becomes:

$$= \bar{\varrho}\,\overline{u' w'} + \bar{w}\,\overline{\varrho' u'} + \overline{\varrho' u' w'} \tag{2.3.17}$$

and

$$\overline{\varrho w^2} = \overline{(\bar{\varrho}+\varrho')(\bar{w}+w')^2} = \bar{\varrho}\,\bar{w}^2 + \bar{\varrho}\,\overline{w'^2} + 2\bar{w}\,\overline{\varrho' w'} + \overline{\varrho' w'^2} =$$
$$= \bar{\varrho}\,\overline{w'^2} - \bar{\varrho}\,\bar{w}^2 + \overline{\varrho' w'^2} \tag{2.3.18}$$

Of the terms occuring in (2.3.17) and (2.3.18) $\bar{\varrho}\,\overline{u' w'}$ and $\bar{\varrho}\,\overline{w'^2}$ are great with respect to the other 4 terms. That these 4 terms are small can only easily be grasped from the term $\bar{\varrho}\,\bar{w}^2$, because $\overline{w'^2} \gg \bar{w}^2$.

That the other 3 terms are small may be seen by means of the following qualitative line of argument (see also **2.3.1**).

We shall first observe $\overline{u' w'}$. Because a velocity gradient in vertical direction prevails, an eddy which originally had the mean velocity, will show a deviation with respect to this mean, as soon as it undergoes a vertical motion. On an average a vertical motion will carry with it an eddy having the mean velocity $\bar{u}(z)$. The deviation u' therefore will be proportional to the gradient $\partial\bar{u}/\partial z$ and to the distance covered by the eddy. This distance again will be proportional to w'. So there is a correlation between u' and w', which may be formulated as:

$$\overline{u' w'} = c_2\,\overline{w'^2} \neq 0 \tag{2.3.19}$$

By virtue of such a consideration Prandtl (1932) introduced the mixing length for turbulence as turbulence strives to become isotropic, he supposed that u' as well as w' are proportional to $l\,\partial\bar{u}/\partial z$. In this way he arrived at the following definition for l:

$$\overline{u' w'} = l^2\left(\frac{\partial\bar{u}}{\partial z}\right)^2 \tag{2.3.20}$$

We see that u' as wel as ϱ' correlate with w' so it is obvious to introduce a third relation, analogous to (2.3.9) and (2.3.19):

$$\overline{\varrho' u'} = c_3 \overline{\varrho' w'} = c_4 \overline{w'^2} \neq 0 \tag{2.3.21}$$

The factors c_2 and c_3, unknown for the rest, will be of the order of magnitude one.

For the first two terms of the right-hand side of (2.3.17) we may now write $c_2 \bar{\varrho}\, \overline{w'^2}$ and $-c_3 \bar{\varrho}\, \bar{w}^2$ in doing so it is clear that $c_2 \overline{w'^2} \gg c_3 \bar{w}^2$ from which it appears that $\bar{w}\, \overline{\varrho' u'}$ is small.

The correlation coëfficients $\overline{\varrho' u' w'}$ and $\overline{\varrho' w'^2}$ will both be of the order $\overline{w'^3} \approx 0$. These terms therefore may undoubtedly be neglected.

We now substitute what we have left into the Eqs (2.3.13) and (2.3.16)

$$\frac{\partial}{\partial z}(\bar{\varrho}\, \overline{u'w'}) = -\frac{\partial \bar{p}}{\partial x} + \eta \frac{\partial^2 \bar{u}}{\partial z^2} \tag{2.3.22}$$

and

$$\frac{\partial}{\partial z}(\bar{\varrho}\, \overline{w'^2}) = -\frac{\partial \bar{p}}{\partial z} - g \bar{\varrho} \tag{2.3.23}$$

These equations may be integrated, when we assume that $\dfrac{\partial \bar{p}}{\partial x}$ is constant with height:

$$\overline{u' w'} = -\frac{1}{\varrho} \frac{\partial \bar{p}}{\partial x} z + \nu \frac{\partial \bar{u}}{\partial z} - \nu \left(\frac{\partial \bar{u}}{\partial z}\right)_o \tag{2.3.24}$$

and

$$\bar{\varrho}\, \overline{w'^2} = \bar{p}(0) - \bar{p}(z) - g \int_o^z \bar{\varrho}\, dz \tag{2.3.25}$$

The latter equation expresses, as is known, that the vertical turbulent motion causes a pressure rise with regard to the static pressure in the free flow.

Now the equation (2.3.24) is particularly interesting to us. The term $\nu \left(\dfrac{\partial \bar{u}}{\partial z}\right)_o$ is the shearing stress at the earth's surface τ_o divided by ϱ. The term $\dfrac{1}{\varrho} \dfrac{\partial \bar{p}}{\partial x} . z$ is negligible as to small z. E r t e l (1933) has pointed out that this is the case up to heights of 25 m, so for the very field in observation, see **2.2.1.**

As soon as there is any turbulence the term $\nu \dfrac{\partial \bar{u}}{\partial z}$ is negligibly small with respect to $\overline{u'w'}$. So a near approximation for (2.3.24) in the bottom 25 m of the atmosphere is:

$$-\overline{u'w'} = \frac{\tau_o}{\varrho} = u_*{}^2 \tag{2.3.26}$$

2.3.4 *The* Fourier *equation*

By treating (2.3.4) in the same way as the Navier-Stokes equations the following equation for the mean heat flux is obtained:

$$\frac{\partial}{\partial z}(\overline{\varrho w T}) = \frac{\lambda}{c_p}\frac{\partial^2 \overline{T}}{\partial z^2} \tag{2.3.27}$$

The equation for pure turbulence has again been left out of consideration.

Equation (2.3.27) integrated yields:

$$\overline{\varrho w T} = \frac{\lambda}{c_p}\frac{\partial \overline{T}}{\partial z} - \frac{\lambda}{c_p}\left(\frac{\partial \overline{T}}{\partial z}\right)_o \tag{2.3.28}$$

In this $\lambda(\partial\overline{T}/\partial z)_o = q_a$ is the heat flux at the surface and as the steady state is being considered, this is constant in the layer under observation. Furthermore the term $\frac{\lambda}{c_p}\frac{\partial \overline{T}}{\partial z}$, which indicates the heat flux by pure conduction, is of some importance only quite close to the earth's surface, but for the rest negligible. So we get

$$\overline{\varrho w T} = \frac{q_a}{c_p} \tag{2.3.29}$$

The term $\overline{\varrho w T}$ can be writen as:

$$\overline{\varrho w T} = \overline{\varrho}\,\overline{w}\,\overline{T} + \overline{\varrho}\,\overline{w'T'} + \overline{T}\,\overline{\varrho' w'} + \overline{w}\,\overline{\varrho' T'} + \overline{\varrho' w' T'} =$$

bij means of (2.3.8) and (2.3.12) this is:

$$= \overline{\varrho}\,\overline{w'T'}\left(1 - \frac{\overline{T'^2}}{\overline{T}^2}\right) - \frac{\overline{T}}{\overline{\varrho}}\,\overline{w'\varrho'^2} \tag{2.3.30}$$

In it $\overline{T'^2}/\overline{T}^2$ is very small with respect to 1 and $\overline{w'\varrho'^2}$ is again of the order of $\overline{w'^3}$ and consequently negligible (see **2.3.3**).

We have left over:

$$\overline{\varrho w T} = \overline{\varrho}\,\overline{w'T'} \tag{2.3.31}$$

This substituted in (2.3.29) gives

$$\overline{w'T'} = \frac{q_a}{c_p\,\overline{\varrho}} \tag{2.3.32}$$

On the strength of the equations (2.3.11) and (2.3.12) we may also write for the bottom layer of the atmosphere:

$$\overline{w'\theta'} = \frac{q_a}{c_p\,\overline{\varrho}} \tag{2.3.33}$$

2.4 The velocity profile in an adiabatic atmosphere without density gradient

In the special case that the atmosphere is adiabatic i.e. $\frac{\partial \bar{\theta}}{\partial z} = 0$ equation (2.3.26) can be solved. For H a m e l (1943) has proved what had already been postulated by v. K á r m á n (1930) that

$$\overline{u'w'} = -k^2 \left(\frac{\partial \bar{u}}{\partial z}\right)^4 \Big/ \left(\frac{\partial^2 \bar{u}}{\partial z^2}\right)^2 \tag{2.4.1}$$

Fitted to (2.3.26) this gives:

$$k \left(\frac{\partial \bar{u}}{\partial z}\right)^2 \Big/ \frac{\partial^2 \bar{u}}{\partial z^2} = - u_* \tag{2.4.2}$$

integrated:

$$\frac{\partial \bar{u}}{\partial z} = \frac{1}{\frac{kz}{u_*} + c} \tag{2.4.3}$$

The integration constant c cannot be determined by taking $z = 0$, for then the term $\nu \frac{\partial \bar{u}}{\partial z}$ is no longer negligible. We therefore introduce a distance z_o, which can be determined from observations and which is connected with c as follows: $c = \frac{kz_o}{u_*}$; this substituted into (2.4.3) gives:

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k(z + z_o)} \tag{2.4.5}$$

Now the well-known form of R o s s b y and M o n t g o m e r y (1935) has been found.

Integration of (2.4.5) gives:

$$\frac{\bar{u}}{u_*} = \frac{1}{k} ln(z + z_o) + \text{constant}$$

Also in this case the integration constant should, in fact, not be deduced from the condition $\bar{u} = 0$ if $z = 0$. If we do so after all the error will probably not be great; we then get:

$$\frac{\bar{u}}{u_*} = \frac{1}{k} ln \frac{z + z_o}{z_o} = 5{,}75 \log \frac{z + z_o}{z_o} \tag{2.4.6}$$

The z_o in the numerator and denominator therefore need not be exactly alike. This is not important however, because the z_o of the numerator has only a slight significance, for as a rule we have $z \gg z_o$. The z_o in the denominator on the other hand is of great significance, it being the measure for the roughness of the earth's surface and leaving its mark on the whole velocity profile.

The equation (2.4.6) is in good agreement with the observations. Prandtl (1932) was the first to represent the logarithmic velocity profile for the bottom layers of the atmosphere. Best (1935), Sverdrup (1936), Paeschke (1937), Rossby and Montgomery (1935, 1936) have found a satisfactory agreement with the observations. A good survey has been given by Deacon (1949); more theoretical considerations have been given by Calder (1949b) and Sutton (1949 a and b). Sheppard (1947) has tried to obtain yet another extra confirmation of the theory by a direct measuring of the shearing stress.

Thanks to the relation proved by Hamel (1943) (Eq. (2.4.1)) it was not necessary in the above mentioned derivation to make use of the mixing length introduced by Prandtl and so it was possible to arrive at the equation (2.4.6) by a more exact way. At the same time the concept of the mixing length has thus obtained a thorough basis. This concept, which was drawn up for the atmospheric turbulence by Prandtl on the analogy of Maxwell's model for an ideal gas, in which he arrived at the relation (2.3.20), was greatly supported by the similarity hypothesis of v. Kármán (1930).

v. Kármán came to the relation:

$$l = -k \frac{\partial \bar{u} / \partial z}{\partial^2 \bar{u} / \partial z^2} \tag{2.4.7}$$

By the proof of (2.4.1) these considerations are justified, for when we substitute, (2.4.7) to (2.3.20) we get, as is well known, the equation (2.4.1) (see further remarks **2.13.3**). By working out equation (2.4.7) by means of equation (2.4.5) we get the well-known relation

$$l = k\,(z + z_o) \tag{2.4.8}$$

We can now use this equation, which at first was assumed by Prandtl, as a definition for l in the adiabatic atmosphere.

2.5 The transfer equations

It is much more difficult to calculate the profiles of velocity and temperature when the atmosphere is diabatic, so when the gradient of the potential temperature $\partial\,\bar{\theta}/\partial z \neq 0$. Equation (2.4.1) is no longer valid now and we can no longer solve equation (2.3.26). The cause of it is found in the fact that as soon as an eddy is placed in surroundings of a different temperature a force is exerted on this eddy which asserts its influence on the turbulence present.

As soon as a gradient of the potential temperature is extant eddies will continue to be drawn out of their thermic equilibrium (see **2.3.1**) by which the entire structure of the surface layer is influenced. So the structure is determined not only by equation (2.3.26) but also by equation (2.3.33):

$$\overline{w'u'} = -u_*^2 \tag{2.3.26}$$

$$\overline{w'\theta'} = \frac{q_a}{c_p \varrho} \tag{2.3.33}$$

These equations (the so-called transfer equations), which indicate the flux of momentum and heat respectively, should be considered in their mutual connection. So the problem we have now to face is the solving of the correlation coefficients $\overline{w'u'}$ and $\overline{w'\theta'}$. This will only be possible in an exact way, when the theory of turbulence has been further developed. In doing so, possibly the so-called similarity hypotheses may be used, which have been given by Kolmogoroff, Onsager, Heisenberg and v. Weizsäcker independent of each other. Up to the present this theory has only been useful for fluids in which no convective forces assert themselves and can therefore only be applied for the adiabatic atmosphere. Whether the logarithmic profile has already been derived along these lines, is not known to us.

Considering this state of things we have thought it advisable to follow a less exact way to arrive at a solution. A stimulus for doing so has been given by the theoretic confirmation of the model of the turbulence by Prandtl (1937), see **2.4.** By extending the idea of mixing length and by applying a formal distinction of turbulence into friction turbulence and convective turbulence it proves possible to describe the velocity- and temperature profiles. It should be borne in mind that the mixing length by no means pretends to say anything regarding the precise structure of turbulence, but merely indicates a kind of effective sphere of action.

If we now return to the transfer Eqs (2.3.26) and (2.3.33), we see that we cannot do much with them in this form.

These equations are usually written, (in imitation of Schmidt (1917) and Taylor (1915)) in the following form:

$$K_m \frac{\partial \bar{u}}{\partial z} = u_*^2 \tag{2.5.1}$$

and

$$K_h \frac{\partial \bar{\theta}}{\partial z} = -\frac{q_a}{c_p \varrho} \tag{2.5.2}$$

which gives a formulation analogous to that for the flux of momentum and heat in the laminar flow. (Schmidt has not used K_m and K_h but $A_m = K_m \varrho$ and $A_w = K_h \varrho$).

In doing so the correlation coefficients $\overline{w'u'}$ and $\overline{w'\theta'}$ have been replaced by $K_m\, \partial\bar{u}/\partial z$ and $K_h\, \partial\bar{\theta}/\partial z$. So now the difficulty has been shifted from the correlation coefficients to the coefficients K the so-called coefficients of eddy transfer.

In virtue of the consideration that turbulence was supposed to transfer the various properties of air in a similar way, it was formerly generally accepted that

$$K_h = K_m = K. \tag{2.5.3}$$

Unamity on this point has entirely disappeared during the latter 10 years. Eq. (2.5.3) is now being strongly challenged on theoretical grounds by Ertel (1942, 1944) and Priestly and Swinbank (1947), while Swinbank (1951) and Pasquill (1949) claim to have proved experimentally that $K_h \neq K_m$. On the other hand measurements by Rider and Robinson (1951) have shown that $K_h/K_m = \text{const.}$ Moreover R. and R. argue that this constant can only be equal to one.

Of an entirely different nature is a theory by v. d. Held (1947), which contains an argument in favour of the validity of (2.5.3). Starting from the model of turbulence as given by Prandtl, v. d. Held regards the elements of turbulence as molecules with an infinite number of degrees of freedom. He then follows up the analogy between the molecular viscosity ν and the eddy viscosity K_m and between the molecular thermal diffusivity a and the eddy conductivity K_h, making use of the well-known nondimensional Prandtl number defined by $Pr = \nu/a$. A relation exists, derived from the kinetic theory of gases, between Pr and the number of degrees of freedom of the molecules:

$$Pr = \frac{n+2}{n+4{,}5}$$

where n is the number of degrees of freedom.

It follows that for $n = \infty$, $Pr = 1$ or if we may extend the analogy between molecular and turbulent motions:

$$Pr = \frac{K_m}{K_h} = 1 \text{ or } K_m = K_h = K \tag{2.5.3}$$

From the above it will be obvious that no unanimity exists on this point. In the following Eq. (2.5.3) will be used, bearing in mind that the ratio K_m/K_h, while not necessarily equal to one, will be approximately constant with height for a given profile.

Substituting Eqs (2.5.3) in (2.5.1) and (2.5.2) we obtain:

$$K\frac{\partial \bar{u}}{\partial z} = u_*^2 \tag{2.5.4}$$

$$K\frac{\partial \bar{\theta}}{\partial z} = -\frac{q_a}{c_p\varrho} \tag{2.5.5}$$

These equations contain three unknowns viz. $K, \bar{u}$ and $\bar{\theta}$. Therefore a third relation must be found in order to solve these equations. In the following sections an attempt is made to find such a relation.

2.6 Frictional (mechanical) turbulence and convective turbulence

In an adiabatic atmosphere the turbulence is entirely due to friction at the earth's surface. As soon as a heat flux exists turbulence will either increase or decrease through the liberation or absorption of convective energy, respectively. We may now make a formal distinction between turbulence caused by mechanical friction, the frictional or mechanical turbulence and turbulence caused by convection, the convective turbulence. The problem is to write this formal distinction in an equation. This equation will have the general form:

$$\text{total turbulence} = \text{frictional turbulence} + \text{convective turbulence} \quad (2.6.1)$$

It was Richardson (1920) who for the first time more or less succeeded in achieving this by means of energy considerations. The chief result of Richardson's analysis is the definition of the Richardson number Ri as a measure of the stability of the atmosphere.

$$Ri = \frac{g\,\partial\bar{\theta}/\partial z}{T(\partial\bar{u}/\partial z)^2} \quad (2.6.2)$$

Measurements by Deacon (1949) Pasquill (1949) and others have shown that this number can be used as a stability parameter.

A disadvantage is, however, that Ri varies with height. Moreover, in this form the description of the components of turbulence is still incomplete. Attemps at a more complete description of both components of turbulence have been made by Rossby and Montgomery (1935) and by Lettau (1949).

Though the approach of these authors is quite different yet the supposition which they made may be brought into a comparable form.

The difficulty of putting an equation of the form of (2.6.1) into a formula principally lies in the fact that we have but few points to go by. Practically only the force or the acceleration which is exercised on an eddy, when this eddy has a temperature difference θ' with its surroundings can clearly be stated, viz: $g\,\theta'/T$.

On an average this quantity will have a value, which will be possible to write as follows:

$$\text{mean convective acceleration} = g\,\frac{l\partial\bar{\theta}/\partial z}{\bar{T}} \quad (2.6.3)$$

in which l is the mixing length bearing on the total turbulence. So here we have a quantity indicating in what measure convection contributes to turbulence.

Rossby and Montgomery as well as Lettau built on this point of contact; the first two authors taking a proportional constant, by writing for the mean convective acceleration:

$$\beta g l \frac{\partial \bar{\theta}/\partial z}{\bar{T}} \tag{2.6.4}$$

Besides this convective acceleration we must now attempt to express the total turbulent acceleration and the turbulent acceleration which is caused by friction, to be able to write Eq. (2.6.1) entirely in the dimension of acceleration:

total turbulent acceleration = turbulent acceleration caused by friction + turbulent acceleration caused by convection (2.6.5)

It is obvious to express the total turbulent acceleration in the coefficient of eddy transfer K and the mixing length l. We then obtain:

$$\text{total turbulent acceleration} = K^2/l^3 \tag{2.6.6}$$

This has been done by R. and M. as well as by Lettau. The main difference between these authors is seen in the formulation of the turbulent acceleration caused by friction. R. and M. give for it:

$$\text{turbulent acceleration caused by friction} = K_f^2/l^3 \tag{2.6.7}$$

in which K_f is the transfer coefficient as it would be if only friction were to cause the turbulence. By means of equations (2.4.5) and (2.5.4) this may be written:

$$K_f = k u_* (z + z_o) \tag{2.6.8}$$

In our opinion Eq. (2.6.7) gives only a slightly consistent and therefore only slightly convincing formulation of friction turbulence. So according to Rossby and Montgomery (1935) Eq. (2.6.5) becomes: (Eqs (2.6.4), (2.6.6) and (2.6.7) substituted in (2.6.5))

$$\frac{K^2}{l^3} = \frac{K_f^2}{l^3} - \beta g \frac{l\, \partial \bar{\theta}/\partial z}{\bar{T}} \tag{2.6.9}$$

The proportional constant β has been introduced to express the uncertainty of equation (2.6.9). In analogous equations to be given henceforth such a proportional constant has not been taken along in order to keep the treatment as simple as possible. In possible experiments this should, however, be taken into account.

Lettau's (1949) formulation is more consistent. His reasoning is more or less as follows. In one special synoptic situation a certain friction will arise, that causes turbulence at the earth's surface when there is no heat flux, consequently in adiabatic conditions. The friction velocity belonging to it we shall call u_{*a}, in which index a indicates that the quantity relates to the adiabatic state.

Likewise we have the transfer coefficient $K_a = ku_{*a}(z + z_o)$ according to Eq. (2.6.8) and the mixing length $l_a = k(z + z_o)$ according to Eq. (2.4.8). Now when the situation changes merely because a heat flux sets in, then this change must be entirely ascribed to convective turbulence caused by the heat flux. Eq. (2.5.6) should therefore be written as follows: (Substitute (2.6.4) and (2.6.6) in (2.6.5))

$$\frac{K^2}{l^3} = \frac{K_a^{\,2}}{l_a^{\,3}} - g\,\frac{l\,\partial\bar{\theta}/\partial z}{\bar{T}} \tag{2.6.10}$$

A drawback of the reasoning is that in case of a diabatic atmosphere the friction must then be split up into a pure friction part causing friction turbulence and a part which conduces to convective turbulence. It is more logical to hold friction at all times entirely responsible for friction turbulence. We then come to the following formulation of Eq. (2.6.5):

$$\frac{K^2}{l^3} = \frac{K_f^{\,2}}{l_f^{\,3}} - g\,\frac{l\,\partial\bar{\theta}/\partial z}{\bar{T}} \tag{2.6.11}$$

in which Eq. (2.6.8) applies to K_f and Eq. (2.4.8) applies to l_f as well as to l_a. This last sentence might be considered a new assumption. It appears to us, however, that we had better to define the quantities of frictional turbulence in this way, as otherwise the difference between frictional turbulence and convective turbulence loses its significance.

We are fully aware that Eq. (2.6.11) is based on a weak foundation and that a further motivation would not be superfluous. To do so however, we must in the first place have a better insight into the structure of turbulence.

So far all formulations of the general Eq. (2.6.1) have been given in the dimension of acceleration, because the convective term is easiest written in this form. No further arguments, however, can be put forward why it is this very dimension in which the best formulation could be given. With as good a reason and in an analogous way we are able to give a formulation of (2.6.1) in the dimension of energy or energy per unit of mass. Such an equation would appear as follows:

$$\frac{K^2}{l^2} = \frac{K_f^{\,2}}{l_f^{\,2}} - g\,\frac{l^2\,\partial\bar{\theta}/\partial z}{\bar{T}} \tag{2.6.12}$$

Some consequences of the various formulations of (2.6.1) will be treated in **2.8.**

Although we have now drawn nearer to a solution of the problem, a new variable l has been introduced and therefore a fourth relation is required beside the relations (2.5.4), (2.5.5) and one of the formulations of (2.6.1). Before deriving such a relation considerations of similarity are introduced in order to reduce the number of parameters and to define a suitable stability parameter.

2.7 Similarity and stability

From the well known logarithmic law (Eq. (2.4.6)) a non-dimensional velocity and a non-dimensional height may at once be derived, viz. $U = \bar{u}/u_*$ and $\zeta = \dfrac{z + z_o}{z_o}$.

Substituting these expressions in Eq. (2.5.4) and introducing another non-dimensional parameter $R = K/u_* z_o$ one gets:

$$\frac{\partial U}{\partial \zeta} = \frac{1}{R} \tag{2.7.1}$$

The similarity between Eqs (2.5.5) and (2.5.4) is brought out by writing (2.5.5) as follows:

$$\frac{\partial \vartheta}{\partial \zeta} = \frac{1}{R} \tag{2.7.2}$$

where $\vartheta = \dfrac{\bar{\theta} - \theta_o}{\theta_*}$ and $\theta_* = \dfrac{-q_a}{u_* c_p \varrho}$

The meaning of θ_* may be clarified by identifying the velocity profile and the temperature profile. This can be achieved by using u_* as the unit of velocity and θ_* as the unit of potential temperature. The ratio of the scales of both profiles is given by $u_*/\theta_* = \sigma$. From this ratio and u_*, θ_* can be at once determined:

$$\theta_* = u_*/\sigma \tag{2.7.3}$$

The problem is now to determine R as a function of height and stability, so for instance

$$R = R\,(\zeta, Ri) \tag{2.7.4}$$

It has been shown experimentally that Ri can be used as a stability parameter, see (2.6.2). A disadvantage is however, that Ri is an unknown function of height. Therefore it is desirable to find another stability parameter which is independent of height. That such a quantity must exist may be deduced from the fact that if Ri is known at one height, while u_* and θ_* are also known, the velocity and temperature profiles and consequently the rate of change of Ri with height are entirely determined. One may now try to split Ri in two parts one of which is independent of height. The easiest way to achieve this is to express Ri in the quantities defined above. We then find:

$$Ri = g\frac{\partial\bar{\theta}/\partial z}{T(\partial\bar{u}/\partial z)^2} = g\frac{\theta_* z_o}{T u_*^2} \cdot R \tag{2.7.5}$$

This is really what we looked for: a term $g\dfrac{\theta_* z_o}{T u_*^2}$ which is constant with height and a known quantity R which varies with height. Writing:

$$-\frac{g\theta_* z_o}{u_*^2 \bar{T}} = Sn \tag{2.7.6}$$

it follows that instead of (2.7.4) one may write:

$$R = R(\zeta, Sn) \tag{2.7.7}$$

Sn is now used as stability parameter instead of Ri.

The practical significance of this equation is, that if the profiles have once been determined for one given set of conditions involving a given value of Sn, then they are also determined for any other set of conditions which yields the same value of Sn. Furthermore, the number Sn enables us to draw up a survey of all states of the atmospheric surface layer in which the assumptions of **2.2** are valid.

However, the stability parameter thus defined can only assume its full importance when the above reasoning has been confirmed experimentally.

2.8 The relation $R(\zeta, Sn)$

We have now progressed so far that an attempt can be made to define relation $R(\zeta, Sn)$. Once this relation is known the profiles of wind velocity and temperature can easily be calculated by integrating the Eqs. (2.7.1) and (2.7.2). As has been pointed out at the end of **2.6** we need for the calculation of $R(\zeta, Sn)$, besides the equation (2.5.4), (2.5.5) and a formulation of (2.6.1) yet another equation which shows a connection between l on the one side and K, $\bar{u}$ and $\bar{\theta}$ on the other side. It is necessary to make yet another assumption. Rossby and Montgomery (1935) made the obvious assumption that equation (2.3.20) holds good even when the atmosphere is diabatic. In combination with Eq. (2.5.4) this may be written:

$$K = l^2 \frac{\partial \bar{u}}{\partial z} \tag{2.8.1}$$

By means of equations (2.6.8) and (2.5.4) this assumption can be written in the following simple form:

$$\frac{K}{K_f} = \frac{l}{l_f} \tag{2.8.2}$$

By combining (2.5.4), (2.5.5), (2.6.9) and (2.8.2) we now find:

$$R = k\,\zeta \left\{\tfrac{1}{2} + \tfrac{1}{2}(1 - \beta\,k\,\zeta\,Sn)^{\frac{1}{2}}\right\}^{-\frac{1}{2}} \tag{2.8.3}$$

An unsatisfactory solution because for positive values of $Sn > \dfrac{1}{\beta k \zeta}$ the the right-hand side of (2.8.3) becomes complex. Qualitatively there is not a single indication why such a critical value should appear in the unstable field.

Neither do the profiles which can be calculated by means of (2.8.3) show a good agreement with the observations.

Holzman (1943) has applied an approximation in the working out, by which a form was found which could be handled somewhat better. He has, however, introduced no alteration in the assumptions.

Lettau (1949) started from the assumption that the vertical turbulent wind component w' changes on an average proportionally to l, owing to which K becomes proportional not to l, but to l^2. Thus he comes to the equation:

$$\frac{K}{K_a} = \left(\frac{l}{l_a}\right)^2 \tag{2.8.4}$$

in which the index a refers again to the adiabatic state. By means of Lettau's assumptions we find, by combining the Eqs. (2.5.4), (2.5.5), (2.6.10) and (2.8.4):

$$R = \frac{u_*}{u_{*a}} k^2 \zeta^2 Sn + \tfrac{1}{2} k \zeta \frac{u_{*a}}{u_*} \left\{ 1 + \left(1 + 4 \frac{u_*}{u_{*a}} k \zeta Sn\right)^{\frac{1}{2}} \right\} \tag{2.8.5}$$

In it occurs a new parameter viz. u_*/u_{*a}. In order to define this parameter it is necessary to consider the entire boundary layer, because a change of u_* by the appearance of a heat flux is linked up with a structure change of the boundary layer.

A problem is introduced by it which is beyond the scope of this research; a problem that in our opinion need not be introduced (see **2.6**). At the same time a practical objection to Lettau's theory now makes its appearance viz. that in testing this theory it is necessary that the synoptic situation remains constant for a long time, so that u_{*a} can at least be determined. Lettau finds a satisfactory agreement of his theory with the observations. This agreement, however, is by no means convincing, because the observations used by him are all of them incomplete, so that it was always possible to fit in the missing quantities as favourably as possible. At most we may say of this theory that the observations do not exclude it.

Building on Lettau's starting point for the equation (2.8.4) we would formulate this assumption as follows:

$$\frac{K}{K_f} = \left(\frac{l}{l_f}\right)^2 \tag{2.8.6}$$

By means of the equations (2.6.8) and (2.5.4) a form may be given for it analogous to (2.8.1): $K_f = l^2 \, \partial\bar{u}/\partial z$.

Now by combining the equations (2.5.4), (2.5.5) and (2.6.11) with (2.8.6) we find: (see figure 2.11.1)

$$R = k^2 \zeta^2 Sn + \tfrac{1}{2} k \zeta \left\{ 1 + (1 + 4 k \zeta Sn)^{\frac{1}{2}} \right\} \tag{2.8.7}$$

An equation showing much resemblance to (2.8.5) and giving about the same form of the profiles. By substituting this equation in (2.7.1) and by integrating we get: (see figure 2.11.3)

$$U = \frac{1}{k}\left\{ln \frac{[(1+4k\,\zeta\,Sn)^{\frac{1}{2}} - 1][(1+4k\,Sn)^{\frac{1}{2}}+1]}{[(1+4k\,\zeta\,Sn)^{\frac{1}{2}}+1][(1+4k\,Sn)^{\frac{1}{2}}-1]} + \right.$$
$$\left. + \frac{2\,(1+4k\,\zeta\,Sn)^{\frac{1}{2}}}{[1+4k\,\zeta\,Sn)^{\frac{1}{2}}]^2} - \frac{2\,(1+4k\,Sn)^{\frac{1}{2}})}{[1+(1+4k\,Sn)^{\frac{1}{2}}]^2}\right\} \tag{2.8.8}$$

As long as $k\,\zeta\,Sn \gg 1$ equation (2.8.8) may be approximated by:

$$U = \frac{1}{k}\,ln\,\zeta - 2\,Sn\,(\zeta - 1) \tag{2.8.9}$$

in which the well known logarithmic profile has been retrieved amplified by a correction term.

A particularly simple relation $R(\zeta, Sn)$ is obtained when the assumptions (2.6.12) and (2.8.6) are combined with the equations (2.5.4) and (2.5.5): (see figure 2.11.2)

$$R = k\,\zeta + Sn\,(k\,\zeta)^2 \tag{2.8.10}$$

This equation substituted in (2.7.1) and integrated yields: (see figure 2.11.4)

$$U = \frac{1}{k}\,ln\,\zeta - \frac{1}{k}\left\{1 + Sn\,k\,(\zeta - 1)\right\} \tag{2.8.11}$$

This equation too can be approximated as long as $Sn\,k\,\zeta \gg 1$ now, however, by means of

$$U = \frac{1}{k}\,ln\,\zeta - Sn(\zeta - 1) \tag{2.8.12}$$

So we find here another correction term than in equation (2.8.9). Of the solutions (2.8.8) and (2.8.11) it may be said that the observations so far do not preclude that they are in agreement with them.

An advantage of these solutions over that of Lettau is that they can be verified more easily and so possibly have greater applicability as a provisional theory.

From what precedes it clearly appears that there is a great number of possibilities to arrive at a relation $R(\zeta, Sn)$ and it seems that by no means all forms of this relation that have a chance of succeeding, have as yet been given.

Yet without further comment it may be seen from various combinations that they cannot suffice. Thus e.g. the assumption of Rossby and Montgomery (2.8.2) in combination with (2.6.11) states, that K and l would decrease in case of increasing instability, which is contrary to observation, whereas the same assumption combined with (2.6.12) gives no solution whatever. Furthermore it is possible to more or less confine the field where the correct solution is to be found by considering the equation for the energy of the turbulence.

Calder[1]) (1949) has further elaborated the energy consideration by Richardson and derived the following equation:

$$\bar{\varrho} K_m \left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \frac{g q_a}{c_p \bar{\bar{T}}} - D -$$

$$- \frac{\partial}{\partial z} \{ \overline{w'(p' + \tfrac{1}{2}\bar{\varrho}\, u'^2)} + \overline{w'(p' + \tfrac{1}{2}\bar{\varrho}\, v'^2)} + \overline{w'(p' + \tfrac{1}{2}\bar{\varrho}\, w'^2)} \} = 0 \qquad (2.8.13)$$

The last three terms express a diffusion of the turbulent kinetic and potential energy. Probably these terms are small with respect to the others. If we neglect these terms we see that the terms of Richardson (the first 2) need only be amplified by a term for the molecular dissipation D, which may also be seen qualitatively. We have then left:

$$\bar{\varrho}\, K_m \left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \frac{g\, q_a}{c_p \bar{\bar{T}}} = D \qquad (2.8.14)$$

In the special case that $q_a = 0$ we can write for D, by means of the equations (2.5.4) and (2.6.8):

$$D = \frac{\bar{\varrho}\, u_*^{\,3}}{k\,(z + z_0)} \qquad (2.8.15)$$

It is now possible by means of the equations (2.5.4) and (2.5.5) to come to a relation $R\,(\zeta\, Sn)$, by assuming a certain course of D as a function of stability.

In doing so, no use is made of the assumptions given previously and therefore not of the mixing length either. Reversed it is possible to calculate the course of D by means of the solutions (2.8.8) and (2.8.11).

The simplest assumption therefore to be made respecting D is, that equation (2.8.15) holds, independent of stability. Working out equation (2.8.14) by means of it we find:

$$R = \frac{k\,\zeta}{1 - k\zeta\, Sn} \qquad (2.8.16)$$

integrated this gives:

$$U = \frac{1}{k}\, ln \zeta - Sn\,(\zeta - 1) \qquad (2.8.12)$$

by which equation (2.8.12) has been recovered.

Presumably D will on the whole increase in case of increasing instability, so with increasing Sn, and D = constant will only occur as a first approximation. As a matter of fact equation (2.8.10) satisfies in this respect. Equation (2.8.7)

[1]) Kano (1950) derived more or less the same equation indepedent of Calder.

TABLE II.

Authors	1st assumption	2nd assumption	$R\,(\zeta, Sn)$
Rossby and Montgomery	$\dfrac{K^2}{l^3} = \dfrac{K_f^2}{l^3} - \beta g \dfrac{l\, \partial\bar{\theta}/\partial z}{\bar{T}}$	$\dfrac{K}{K_f} = \dfrac{l}{l_f}$	$R = k\zeta \left\{\frac{1}{2} + \frac{1}{2}\,(1 - \beta k\zeta\, Sn)^{\frac{1}{2}}\right\}^{-\frac{1}{2}}$
Eq.	(2.6.9)	(2.8.2)	(2.8.3)
Lettau	$\dfrac{K^2}{l^3} = \dfrac{K_a^2}{l_a^3} - g \dfrac{l\, \partial\bar{\theta}/\partial z}{\bar{T}}$	$\dfrac{K}{K_a} = \dfrac{l^2}{l_a^2}$	$R = \dfrac{u_*}{u_{*a}} k^2 \zeta^2\, Sn + \frac{1}{2} k\, \zeta \dfrac{u_{*a}}{u_*} \left\{1 + \left(1 + 4 \dfrac{u_*}{u_{*a}}\, k\, \zeta\, Sn\right)^{\frac{1}{2}}\right\}$
Eq.	(2.6.10)	(2.8.4)	(2.8.5)
Businger	$\dfrac{K^2}{l^3} = \dfrac{K_f^2}{l_f^3} - g \dfrac{l\, \partial\bar{\theta}/\partial z}{\bar{T}}$	$\dfrac{K}{K_f} = \dfrac{l^2}{l_f^2}$	$R = k^2\zeta^2\, Sn + \frac{1}{2}\, k\zeta \left\{1 + (1 + 4\, k\zeta\, Sn)^{\frac{1}{2}}\right\}$
Eq.	(2.6.11)	(2.8.6)	(2.8.7)
Businger	$\dfrac{K^2}{l^2} = \dfrac{K_f^2}{l_f^2} - g \dfrac{l^2\, \partial\bar{\theta}/\partial z}{\bar{T}}$	$\dfrac{K}{K_f} = \dfrac{l^2}{l_f^2}$	$R = k\zeta + k^2\zeta^2\, Sn$
Eq.	(2.6.12)	(2.8.6)	(2.8.10)
General	total turb. = frictional turb. + convective turb.	$\dfrac{K}{K_f} = f\left(\dfrac{l}{l_f}\right)$	$R = R\,(\zeta, Sn)$
Eq.	(2.6.1)		(2.7.7)

shows at first a decrease of D for small Sn and D does not increase until we have a greater Sn. So it seems that for small values of $|Sn|$, equation (2.8.11) gives a better description of the profiles than equation (2.8.8). In **2.9** and **2.10** we shall consider the equations (2.8.7) and (2.8.10) also for greater values of $|Sn|$.

A synopsis of the various theories mentioned above and the corresponding relations $R(\zeta, Sn)$ is given in Table II.

The relations $R(\zeta, Sn)$ given here do not hold for all conditions in the atmospheric surface layer. Thus for $Sn > 0$, there is a field where convection turbulence prevails to such an extent that free or natural convection establishes itself, (see **2.9**) and for $Sn < 0$ there is a field where turbulence is entirely subsided and where only laminar flow still occurs, see **2.10.**

In the following sections we shall observe more closely in connection with both these transitions the equations (2.8.7) and (2.8.10), worked out in the figures 2.11.1 and 2.11.2 respectively.

2.9 Free and forced convection

2.9.1 *Definitions*

A good deal of confusion prevails in the various branches of physics as to the conception of convection. The terms used up till now in which the word convection occured such as convective force and convective turbulence have been sufficiently explained when quoted, see **2.6.** As soon, however, as free and forced convection is spoken of, or convection without further comment, a more precise definition is certainly desirable. For our purpose we have thought it best to link up with the terminology in use in the field of heating technique.

Thus we understand by *forced convection* the heat transmission in a flowing medium at right angles to the mean flow direction, the force of friction being greater than the convective force, and by *free convection* the vertical heat transmission, the convective force being greater than the frictional force. Free and forced convection may appear in a laminar as well as in a turbulent flow. So we can distinguish: *laminar and turbulent forced convection* and *laminar* and *turbulent free con- vection* respectively.

Now it is possible to translate the terms which are used in meteorology into the above defined conceptions:

Laminar forced convection hardly occurs in meteorology, therefore no denomination is given for it.

Turbulent forced convection corresponds with *turbulent heat flux.*

Laminar free convection (e.g. cells of Bénard) is often expressed by *steady state convection.*

For turbulent free convection various conceptions are used. Often one simply speaks of *convection.* When the flow form is especially considered (e.g. for glider pilots) we speak of *thermic.*

When turbulent free convection is very evident, so that it gives rise to cumulus clouds the term *penetrating convection* is often used. Thus the conceptions forced and free convection have been sufficiently circumscribed for our further argument.

2.9.2 *The transition between free and forced convection*

By means of a rather rough qualitative reasoning Taylor has succeeded in expressing the transition between free and forced convection His reasoning more or less comes down to this:

We again observe an eddy, which, with respect to its surroundings has an other potential temperature. When this eddy furthermore has a characteristic length l, a vertical force will be exercised upon it (see also **2.6**) for which we shall be able to write something like:

$$-\varrho g \frac{\theta'}{T} l'^3 \qquad (2.9.1)$$

As soon as the eddy possesses a certain velocity w', a frictional force will be exercised upon it, which is proportional to the velocity gradient w'/l' at the edge of the eddy, to its surface l'^2 and to the intensity of the turbulence, which latter quantity is characterized by the coefficient of eddy transfer. For the frictional force we therefore get a relation, looking more or less like this:

$$\varrho K l'^2 \frac{w'}{l'} = \varrho K l' w' \qquad (2.9.2)$$

We now have various possibilities, viz. the convective force may be greater, equal to or smaller than the frictional force. In the first case the vertical motion is accelerated and we can speak of free convection. It is possible to maintain the motion if there is a sufficiently great negative gradient of the potential temperature, so that the convective force, the frictional force, and the attendant turbulent exchange continues to prevail. In the second case there is a balance between the two forces so that the eddy will have a constant motion. There we have the transition between free and forced convection. In the last case the frictional force prevails, by which the eddy will be checked and will further disintegrate by the turbulent exchange. Then we speak of forced convection.

The relation of the two forces is therefore a criterion for the form which the convection will assume. There is little sense, however, in making out this relation for one single eddy, because we are especially interested in the mean state.

Now by replacing the momentary quantities in the equations (2.9.1) and (2.9.2) by means and subsequently dividing the two equations by each other, we get a parameter, which looks like this:

$$g \frac{l^4 \, \partial\bar{\theta}/\partial z}{K^2 \bar{T}} = Gr' \qquad (2.9.3)$$

This relation closely resembles the number of G r a s h o f from the theory of heat transmission. We have therefore called this number Gr'. In virtue of what precedes we may now expect that there will be a definite critical value of this number, indicating the transition from free to forced convection.

We now observe that the left-hand side of equation (2.9.3) can also be obtained from equation (2.6.11) as well as from equation (2.6.12) bij dividing the second term of the right-hand side by the left-hand side.

If we assume that in the transition between free- and forced convection the frictional turbulence is equal to the convective turbulence, than according to equation (2.6.11):

$$\frac{K_f^2}{l_f^3} = -g\frac{l\,\partial\bar{\theta}/\partial z}{\bar{T}} \tag{2.9.4}$$

and according to equation (2.6.12):

$$\frac{K_f^2}{l_f^2} = -g\frac{l^2\,\partial\bar{\theta}/\partial z}{\bar{T}} \tag{2.9.5}$$

In both cases we find:

$$Gr' = -\tfrac{1}{2} \tag{2.9.6}$$

At the same time it is possible to give a critical value of Ri. By combining (2.9.4) with (2.6.11), (2.8.6), (2.8.7) and (2.7.5) we find: $Ri = -8$.
and bij combining (2.9.5) with (2.6.12), (2.8.6), (2.8.10) and (2.7.5) we find: $Ri = -2$.

In the following we shall investigate whether these results have any consequences for the assumptions (2.6.11) and (2.6.12) and hence for the equations (2.8.7) and (2.8.10).

2.9.3 *Extension of the relation $R(\zeta, Sn)$ to the field of free convection*

T a y l o r (1931) succeeded in deriving a relation for the field of free convection, with which, by means of the foregoing, the form of the velocity profile may be deduced. For he observes that as soon as an eddy can accelerate its motion on account of a prevailing of the convective force, the frictional force will increase together with the velocity, owing to which the two forces will again approach each other. The convective flow will establish its own balance, as this flow causes the turbulence to increase to such a degree that the convective force is about equal to the frictional force. This means, (see **2.9.2**) that in the field of free convection will hold:

$$Gr' = \text{constant} \approx -\tfrac{1}{2} \tag{2.9.7}$$

In working this out by means of equations (2.9.3), (2.7.6) and (2.7.2) we find:

$$R^3 = 2\,Sn\left(\frac{l}{z_o}\right)^4 \tag{2.9.8}$$

From observations by Johnson and Heywood (1938) (see Sutton (1948)) it has been derived that

$$K = \text{constant } z^{1.75}$$

or

$$R\,(:)\,\zeta^{1.75} \tag{2.9.9}$$

When we try to fit this relation closely to equation (2.8.7), we see that this can be done for a value of $Ri = -8$, in which way the same critical value of Ri is found along a quite different line as is obtained by means of equation (2.9.4) for the transition from forced to free convection. This fit brings out that for the field of free convection holds:

$$R = 0.47\, Sn^{0.75}\, \zeta^{1.75} \tag{2.9.10}$$

and after integration of equation (2.7.1) and fitting to (2.8.8)

$$U = -5.75 \log Sn + 1 - 2.8\,(Sn\,\zeta)^{-0.75} \tag{2.9.11}$$

A slightly less favourable result is obtained by fitting equation (2.9.9) to (2.8.10). This yields $Ri \approx -10$, whereas $Ri = -2$ had been found with equation (2.9.5). It is, however, possible that the transition from forced to free convection takes place gradually and that in the process a stretch of $Ri = -2$ to $Ri = -10$ is passed through. This adaptation gives for the field of free convection the relation:

$$R = 0.35\, Sn^{0.75}\, \zeta^{1.75} \tag{2.9.12}$$

after integration of (2.7.1) and fitting to (2.8.11)

$$U = -5.75 \log 2k\, Sn + 1.9 - 3.8\,(Sn\,\zeta)^{-0.75} \tag{2.9.13}$$

So it appears to be possible by means of the assumptions (2.6.11), (2.8.6) and (2.9.4) as well as by the assumptions (2.6.12), (2.8.6) and (2.9.5) to come to a complete notion of the unstable surface layer, so when $Sn > 0$.

By means of the first series of assumptions the transition from free to forced convection is more elegantly described than by means of the second series, which is a consequence of the assumption, that $Gr' > -\frac{1}{2}$, see (2.9.6). If we alter this assumption slightly by taking $Gr' = -\frac{3}{4}$ the second series of assumptions gives a more elegant adaptation. Careful observations will be able to give considerable indications in what direction the solution must be sought.

2.10 The transition from turbulent to laminar flow

It is widely known that in the stable atmosphere, turbulence may entirely subside. Richardson's original research was especially aimed at investigating under what circumstances turbulence disappears owing to stable conditions. During this research he came to the formulation of the parameter Ri

(see equation (2.7.5)). On theoretical grounds he gave a critical value $Ri_{cr} = 1$ for the disappearance of turbulence.

Many subsequent investigators have occupied themselves with this problem and arrived at different values for Ri_{cr}, varying from 0,04 tot 1 (see Sutton (1949b)). Without bringing into further discussion the various investigations, we will yet bring forward a single aspect of the transition from turbulent to laminar flow.

As soon as turbulence begins to subside, there will be a moment when the molecular viscosity can no longer be neglected with respect to the coefficients of eddy transfer. The last part of equation (2.7.5) is then no longer correct. Instead of it we can now write:

$$Ri = -Sn \frac{K+\nu}{K+a} \cdot \frac{K+\nu}{u_* z_o} \tag{2.10.1}$$

When turbulence has entirely disappeared, we have $K = 0$ and this equation passes on into:

$$Ri = -Sn \frac{\nu}{a} \cdot \frac{\nu}{u_* z_o} = -Sn. Pr \frac{\nu}{u_* z_o} \tag{2.10.2}$$

by which a final value of Ri has been found. For a stationary state this final value is constant with height and may consequently be derived from the profiles, for then the temperature as well as the velocity in the laminar field increases linearly with height.

It is easy to see that this final value of Ri under various circumstances may be quite different, owing to which it certainly is not a critical value. The fact that Ri is dependent on the extent to which turbulence is subsided, is the cause that this number cannot be easily handled as a criterion for the transition from turbulent to laminar flow. It seems to us that the criterion for this transition can be best expressed by $k \zeta Sn$ by means of which quantity Ri can always be defined in a single integration according to the equations (2.8.7) and (2.8.10). As long as $K \gg \nu$ this quantity is not dependent on whether the turbulence subsides or not. If we now consider the equations (2.8.7) and (2.8.10) for $Sn < 0$ we see that in both equations R at first increases with height until a maximum value is attained. For equation (2.8.7) this maximum is found on the line $k \zeta Sn = -{}^3/_{16}$ and for equation (2.8.10) on the line $k \zeta Sn = -\frac{1}{4}$. Beyond this maximum R decreases until for equation (2.8.7) when $k \zeta Sn = -\frac{1}{4}$ this number passes from real to complex. In equation (2.8.10) we find a gradual decrease of R until in case of $k \zeta Sn = -1$ the value zero is reached.

The transition obtained by equation (2.8.10) is physically more acceptable than that of equation (2.8.7). Especially the transition to a complex value of R cannot be attributed to a physical phenomenon, but is solely a mathematical consequence of the equations (2.6.11) and (2.8.6).

Here too, however, the final word rests with observation.

2.11 The ζ-Sn diagram

In the three preceding sections we did give a more or less simplistic, but yet a complete notion of the structure of the surface layer in view of the equations (2.8.7) and (2.8.10). In the figures 2.11.1 and 2.11.2 these equations have been worked out.

By choosing $k\,\zeta$ as abscis and Sn as ordinate a clear insight in the states occuring in the surface layer may be given. The line $Sn = 0$ runs through the middle of the figure and divides them into two fields viz. te stable field ($Sn < 0$) and the unstable field ($Sn > 0$).

Furthermore lines of constant $k\,\zeta\,Sn$ have been drawn, lines in agreement with a constant Ri, as long as (2.7.5) holds. Certain critical values of $k\,\zeta\,Sn$ indicate in the stable field, where transition from turbulent to laminar flow will be possible and where R is maximal, whereas in the unstable field a critical value of $k\,\zeta\,Sn$ shows the transition from forced to free convection. By means of the lines of constant $k\,\zeta\,Sn$ lines of constant R may easily be constructed. For the field of free convection use has been made of the equations (2.9.10) and (2.9.12) for the figures 2.11.1 and 2.11.2 respectively. It is also possible to draw lines of constant U in these figures by means of the equations (2.8.8) and (2.8.11). For the sake of surveyability this has been omitted and instead of them two new figures have been composed, in which $ln\,\zeta$ has been taken as abscis and U as ordinate. From these figures 2.11.3 and 2.11.4 the form of the profiles clearly stands out.

2.12 Experimental verification

There is a great need of observations, which are so complete that the theories can be tested. So far no observations have come to the knowledge of the author, which are sufficiently complete for this purpose. The experimental confirmations, which Lettau claims to have found for this theory, cannot in fact be accepted as such, since the observations which he used could be made to fit the theory because they were incomplete (see **2.8**).In order to confirm a theory it is necessary that all quantities are measured and that no unknown quantities are inferred from the theory.

On the basis of the preceding considerations it is possible to formulate an observational programme which is sufficiently complete. Starting from the principle that Sn and the shape of the velocity and temperature profiles must be measured independently and further that conditions of section **2.2** must be fulfilled as fully as possible, it appears that the theory can be tested if the following points are satisfied:

1. It is important to have the disposal of two very large fields with different z_o, the surfaces of which must be as uniform as possible in order to ensure a

Fig. 2.11.1

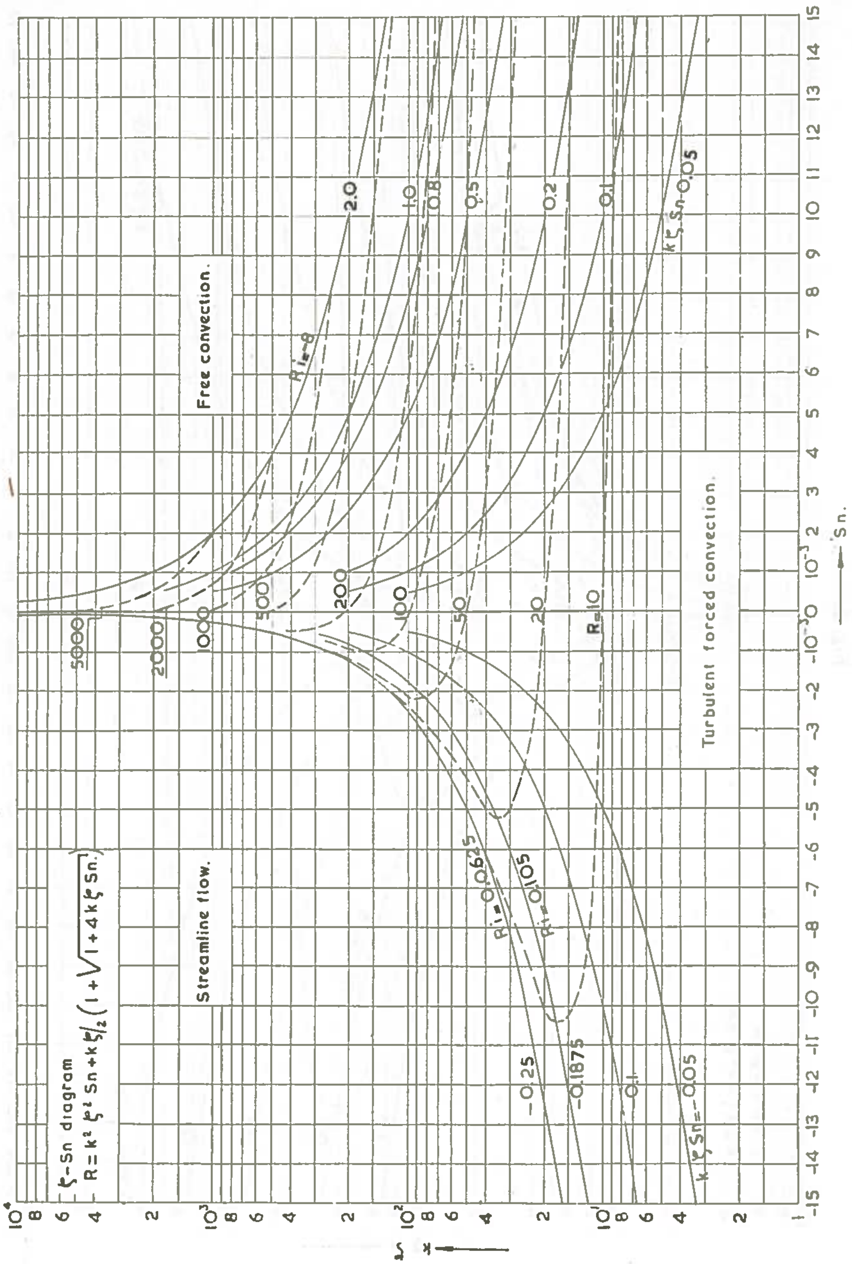
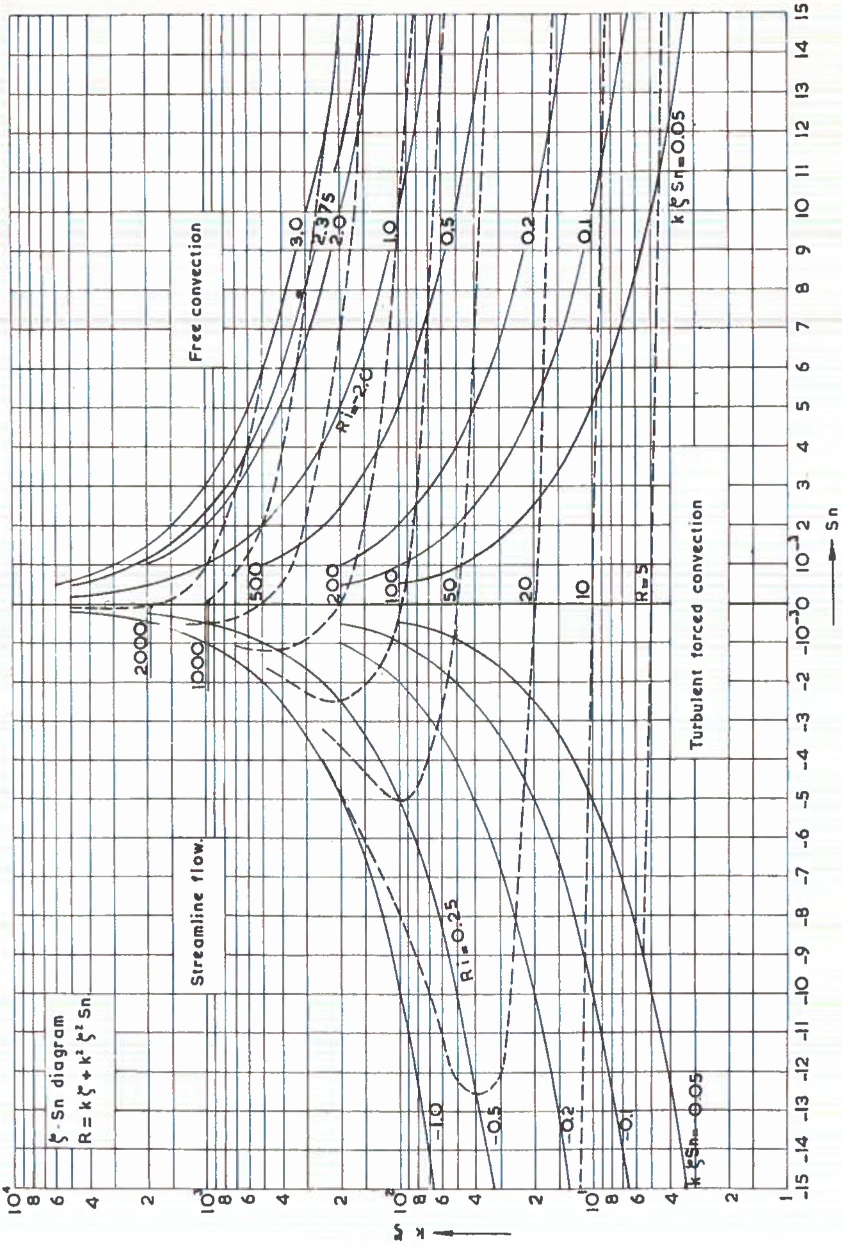

ξ-Sn diagram
R = k² ξ² Sn + kξ/2 (1 + √(1 + 4kξ Sn))
Free convection.
Streamline flow.
Turbulent forced convection.
Sn.
k ξ Sn = 0.05
k ξ Sn = -0.05
5000
2000
1000
500
200
100
50
20
R = 10
2.0
1.0
0.8
0.5
0.2
0.1
-0.25
-0.1875
-0.1
Ri = 0.0625
Ri = 0.105

Fig. 2.11.2


ζ - Sn diagram
R = kζ + k²ζ² Sn
Free convection
Streamline flow.
Turbulent forced convection
Ri = -2.0
Ri = 0.25
k ζ Sn = 0.05
k ζ Sn = -0.05
R = 5
Sn
k ζ

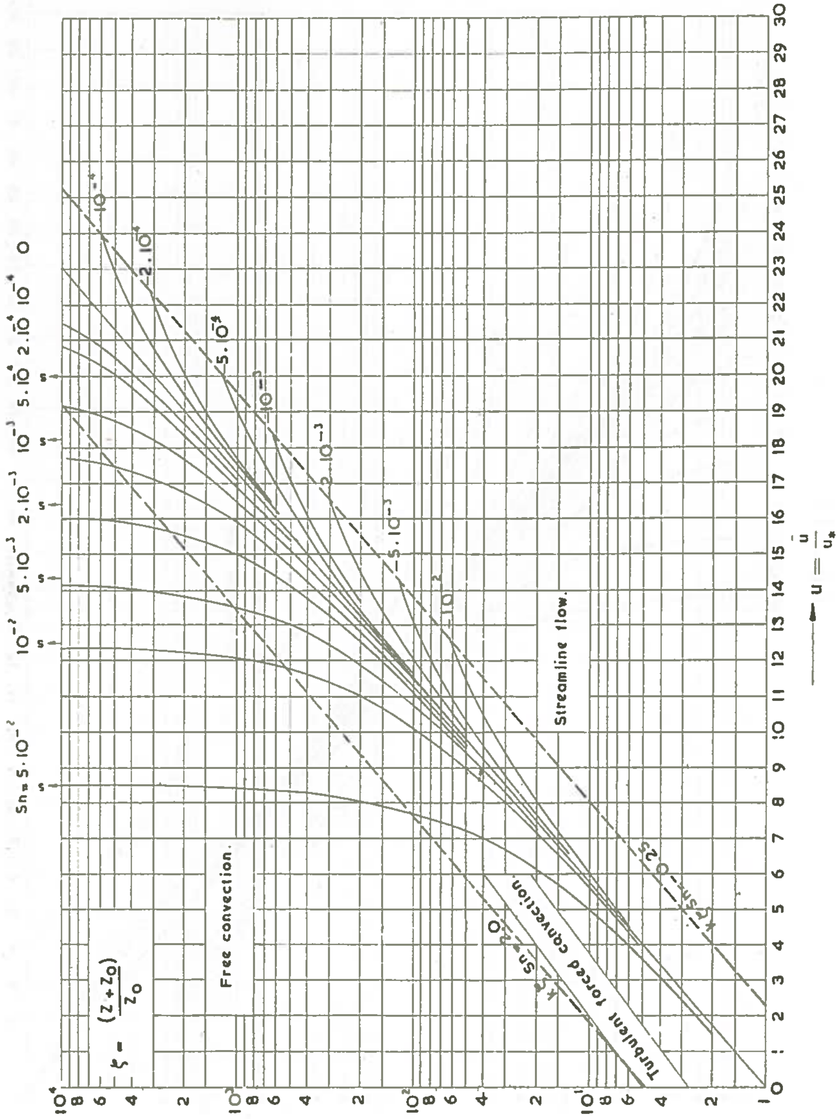

$\zeta = \frac{(Z + Z_0)}{Z_0}$
Free convection.
Streamline flow.
Turbulent forced convection
$Sn = 5 \cdot 10^{-2}$
10^{-2}
$5 \cdot 10^{-3}$
$2 \cdot 10^{-3}$
10^{-3}
$5 \cdot 10^{-4}$
$2 \cdot 10^{-4}$
10^{-4}
O
$u = \frac{\bar{u}}{u_*}$
ζ

Fig. 2.11.3

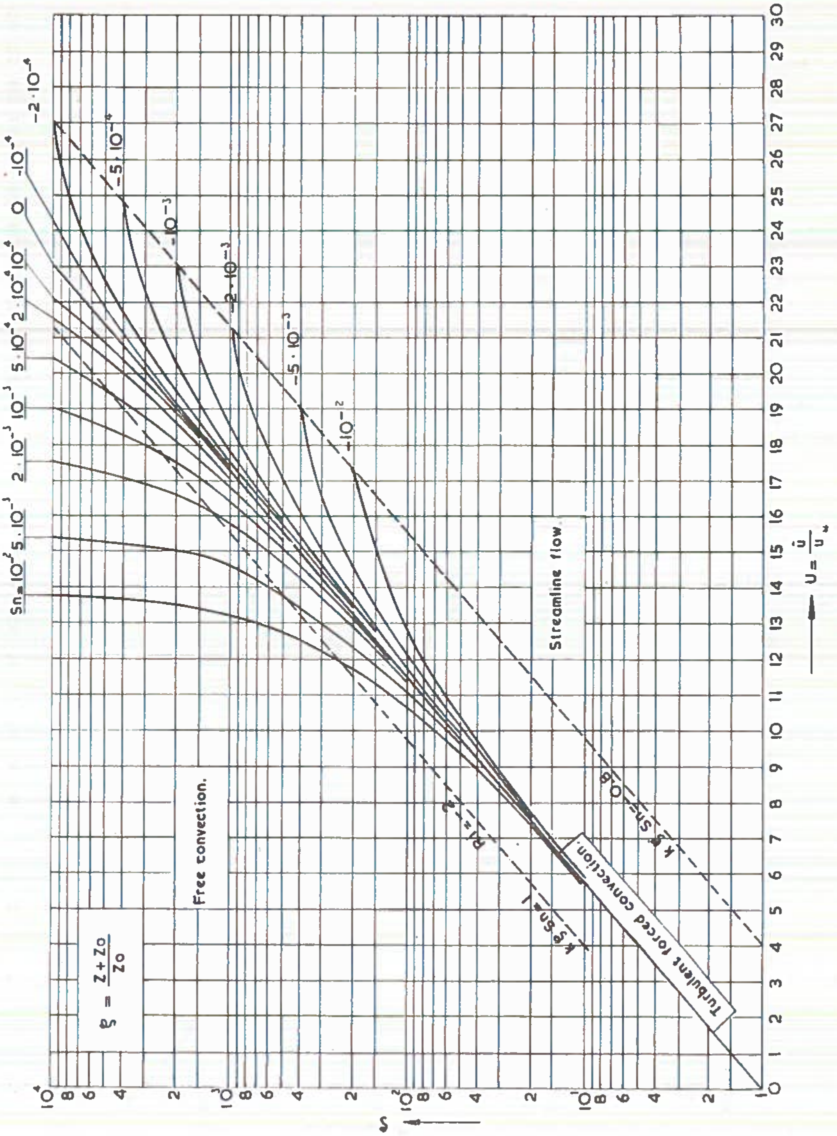

Free convection.
Streamline flow.
Turbulent forced convection.
ς = (Z + Zo)/Zo
U = ū/u*
Ri = ...
k ς Sn = -0.8
k ς Sn = 1
Sn = 10⁻²
5·10⁻³
2·10⁻³
10⁻³
5·10⁻⁴
2·10⁻⁴
10⁻⁴
0
-10⁻⁴
-2·10⁻⁴
-5·10⁻⁴
-10⁻³
-2·10⁻³
-5·10⁻³
-10⁻²

Fig. 2.11.4

constant value of z_o. Moreover these fields should change as little as possible in the course of time so that the observations can be reproduced. The minimum extent of the fields is determined by Re_x. There are reasons to suppose that Re_x must be $\geqslant 10^9$ (see **2.13**) for the profiles to be fully developed. This value of Re_x corresponds with a distance of 2–5 km. There are not many places on earth where uniform areas of such extent are found, which are indifferent to change in time. However, it is possible that the salt-deserts in the U.S.A. satisfy these requirements.
2. The surface drag must be measured directly and in such a manner that the surface is not disturbed, for an accurate determination of u_* is of fundamental importance for testing the theory. No doubt considerable experimental difficulties will be encountered, but in our opinion these should not be insurmountable.
3. Complete measurements of the heat- and water budgets are required for independent determination of q_a, a quantity which likewise is of fundamental importance for testing the theory. The experimental difficulties associated with these measurements have been largely overcome.
4. Finally it is necessary that measurements of wind velocity, temperature and humidity are made at different heights, e.g. at 0.10, 0.20, 0.50, 1.00, 2.00, 5.00, 10.00 and 20.00 m. Here also the experimental difficulties have been largely overcome.

2.13 Some remarks

2.13.1 The analysis presented above by no means claims to be a complete description of the atmospheric surface layer, but it contains a first fundamental step from which further progress can be made. There are many problems bordering on the problem discussed, which are just as fundamental and which may have even more practical value, once they have been solved. Some of these problems are:

a. in which manner do the velocity- and temperature profiles change during transition from one field to another with a different value of z_o.

b. what is the influence of the surface roughness on the flux of heat.

c. what are the critical values of $k\ \zeta\ Sn$.

d. in which manner do the profiles vary as a function of the heat flux, other factors being equal.

e. what is the structure of the atmosphere throughout the surface layer.

Although many authors have studied these and similar problems, no really satisfactory results have been obtained so far.

2.13.2 *The roughness parameter*

The roughness of the earth's surface forms a problem in itself. In the above the roughness parameter z_o has not been further discussed. In fact the analysis presented is only valid for land surfaces not covered by vegetation. As soon as the surface is covered by vegetation the roughness parameter ceases to be the sole determining surface parameter and a second parameter is required, which might be called the zero point displacement. The surface of the earth will now no longer coincide with the level where $\bar{u} = 0$ from Eq. (2.4.6). The difference between both levels is generally denoted by d.

$$\zeta = \frac{z + z_o}{z_o} \quad \text{must then be replaced by} \quad \zeta = \frac{z - d + z_o}{z_o}$$

Another problem arising from the surface roughness is the dependence of z_o on the wind velocity. When the surface is covered by high vegetation the wind velocity will exercise a considerable influence on the surface. The same applies to a water level.

2.13.3 *Rough and smooth surfaces*

In deriving equation (2.4.6) the situation close to the surface has been systematically blurred by introducing the roughness parameter z_o. We have not gone further into the matter because the earth's surface is practically always aerodynamically rough, which has been proved by Calder (1949).

For aerodynamically smooth surfaces, however, a slightly better detailed solution can be given for the velocity profile:

$$\frac{\bar{u}}{u_*} = \frac{1}{k} \ln \frac{u_* z}{\nu} + 5.5$$

Extensive observations on rough and smooth surfaces have been made by Schlichting (1937) in which he arrived at the following criteria:

$$\text{rough flow} \quad \frac{u_* z_o}{\nu} > 2.5$$

$$\text{transitional flow} \quad 2.5 > \frac{u_* z_o}{\nu} > 0.13$$

$$\text{smooth flow} \quad \frac{u_* z_o}{\nu} < 0.13$$

2.13.4 Re_x

It is not easy to make out at what value of Re_x the turbulent flow along the surface has been fully developed. A first essential indeed is that the boundary layer really extends over the bottom 25 m in the atmosphere. Schlichting (1951) calculates the thickness of the boundary layer of a smooth level plate with parallel flow, starting from the equation:

$$\frac{\bar{u}}{u_*} = 2.545\, ln\left(1 + 8.93\frac{u_*\, z}{\nu}\right)$$

when in doing so we replace $\left(1 + 8.93\frac{u_*\, z}{\nu}\right)$ by $\frac{z + z_o}{z_o} = \zeta$ we nearly recover equation (2.4.6).

This adaptation very probably is admissible. For $z_o \approx 1$ cm the boundary layer, therefore, must extend to $\zeta \approx 2500$ or $\frac{u_*\, z_o}{\nu} \approx 280$. Then Schlichting (1951 p. 399) finds:

$$Re_x \approx 7.5.\ 10^8$$

It is, however, probable that in case of a rough surface the boundary layer will be developed faster. So presumably the requirement that

$$Re_x \geqslant 10^9$$

if the turbulence in the surface layer must be fully developed, will be on the safe side in most cases.

2.13.5 *Convective forces owing to humidity differences*

Because water vapour has a different density from air, convective forces may arise too, owing to humidity differences in the air. As a rule these convective forces are small with respect to the convective forces caused by temperature differences in the air. An estimation of the two convective forces can be made as follows: If ϱ_1 is the density of dry air and ϱ_2 the density of water vapour, the density of moist air is

$$\varrho = \varrho_1 + \chi\,(\varrho_1 - \varrho_2)$$

in which χ is the mixing ratio. Now when an eddy has a humidity deviating with respect to its surroundings e.g. $\bar{\chi} + \chi'$ this eddy will have a deviation in density:

$$\varrho' = \chi'\,(\varrho_2 - \varrho_1)$$

Admittedly humidity differences and temperature differences, cannot be fully compared. A point of contact, however, is obtained by observing the enthalpy of the air. When 2 eddies have the same deviation in enthalpy with respect to the surroundings, and one of the eddies has this deviation on account of a deviation in humidity content, and the other eddy on account of a temperature deviation, the convective forces will be in the following proportion:

$$\frac{\chi'\,(\varrho_2 - \varrho_1)\,\theta}{-\,\varrho\,\theta'} \approx 0.045$$

So cases are certainly conceivable, in which convective forces owing to humidity differences may not be neglected.

CHAPTER 3

SOME CONSIDERATIONS ON THE TRANSFORMATION OF AIR MASSES

3.1 Introduction

The influence of the earth's surface on the atmosphere is mainly restricted to the troposphere. A l b r e c h t (1950) shows in a paper on the heat balance of the troposphere that the troposphere may be considered as a part by itself of the atmosphere, which practically has no transfer of heat and water vapour to the higher layers of air.

In the troposphere dynamic phenomena occur owing to the transfer of energy at the earth's surface. The course of these phenomena, as it appears, can be understood for the greater part without taking into account the stearing action emanating from the earth's surface. It appears that various atmospheric processes have such a great inertness and experience so slight a transformation that their dynamics for a long time are in agreement with those of an energetically closed system. The dynamic meteorology has mainly occupied itself up to the present with the investigation of the processes in this form. However, it is clear that to obtain a good insight, the gradual transformation, owing to contact with the earth's surface, must also be taken into account. In the dynamic meteorology a link is missing as it were. In the last few years there has been a growing consciousness of this situation and there is a tendency to more general investigations of the atmospheric problems. In this direction some qualitative considerations have been given by B l e e k e r (1949, 1950). The changes taking place in an air mass are of two kinds. In the first place by direct contact with the earth's surface and in the second place by physical processes in the air. Excepting the radiation of the sun outside the troposphere, all quantities, playing a part in transformation, stand in mutual interaction. This renders the theoretic treatment exceedingly complicated, owing to which we are more less compelled to simplify matters, so as not to get into a labyrinth.

These simplifications may be introduced by two methods. In the first place we can start from mean states and by them design a stationary image of the atmosphere. This has been done by A l b r e c h t (1950) which has yielded interesting results. Detail studies have been made by W e x l e r (1944) and others.

In the second place the problem may be approached from the non stationary angle by starting from a simplified physical image. This always refers to detail problems. The significance of this approach is that in this direction the missing link of the dynamic meteorology must be formed to obtain a better insight eventually into the complicated momentary state of the weather. Only few investigations have been made in these directions, especially by Burke and Frost. In this chapter also attention is mainly directed to this aspect of transformation. In chapter 2 it was emphasized that the transfer between atmosphere and the surface of the earth is considerably more intensive in unstable conditions than in stable ones. Because the periods of these conditions are on the average nearly equal, a consequence of it is, that the earth's surface as a whole cedes more sensible and latent heat to, than it absorbs from the atmosphere. By continuous radiation of especially the topmost layer of the troposphere this heat again yields to space. The influence of evaporation in this process is twofold, in the first place the water vapour content of the air is increased by it and concequently the rate of radiation and in the second place a great quantity of condensation heat is eventually liberated by it, when condensation sets in, whereas the cloud formation, which then asserts itself, again greatly influences radiation.

3.2 The heat- and water balance of the troposphere

To be able to observe more in detail the significance of the phenomena at the earth's surface with respect to the atmosphere, it is desirable to give a formulation of the heat- and water balance of the troposphere.

In a rather general form we can draw up the following equations: firstly, an equation of the sensible heat

$$q_a + q_p = \int_o^h \left\{ u \frac{\partial (c\varrho\theta)}{\partial x} + v \frac{\partial (c\varrho\theta)}{\partial y} + \frac{\partial (c\varrho\theta)}{\partial t} - q_r' - q_e' \right\} dz \qquad (3.2.1)$$

in it h is the height where the stratosphere begins, q_r' is the more absorbed than liberated radiation energy per unit of volume, likewise q_e' is the liberated condensation heat per unit of volume. This equation renders the water budget of a vertical column of air with a section which is the unit of surface and which extends from the earth's surface to the stratosphere. Secondly, an analogous equation may be made out for the latent heat q_e:

$$q_e = \int_o^h \left\{ L \left(u \frac{\partial \chi}{\partial x} + v \frac{\partial \chi}{\partial y} + \frac{\partial \chi}{\partial t} \right) + q_e' \right\} dz \qquad (3.2.2)$$

By the side of above mentioned equations a third equation may be given rendering the water balance.

$$\frac{dm_p}{dt} = -\int_o^h \left(u \frac{\partial m'}{\partial x} + v \frac{\partial m'}{\partial y} + \frac{\partial m'}{\partial t} - \frac{q_e'}{L} \right) dz \qquad (3.2.3)$$

here m' is the quantity of water per unit of volume contained by the air.

These equations should be considered in connection with the equations (1.3.1) and (1.4.1).

For the sake of completeness the equations applying to the soil may be added. Thus

$$q_s = \int_{-d}^{o} \frac{\partial (c_s \varrho_s T)}{\partial t} dz \tag{3.2.4}$$

for a land surface holds, in which d is a level where temperature may be considered constant.

For a water level holds,

$$q_s = \int_{-d}^{o} \left(u \frac{\partial (c\varrho T)}{\partial x} + v \frac{\partial (c\varrho T)}{\partial y} + \frac{\partial (c\varrho T)}{\partial t} - q_r' \right) dz \tag{3.2.5}$$

in which q_r' is in this case the difference between radiated and absorbed energy. Whilst eventually another equation may be given for the water transfer from the soil to a land surface:

$$\frac{dm_s}{dt} = -\int_{-d}^{o} \left(\frac{\partial (um')}{\partial x} + \frac{\partial (vm')}{\partial y} - \frac{\partial m'}{\partial t} \right) dz \tag{3.2.6}$$

For a water level this equation has no practical value.

The influence which the transition from water to ice exercises, has not been taken into account in the above mentioned six equations. In principle this can be formulated in an analogous way. The whole, however, gets more complicated and less surveyable so that it has been dropped.

It is obvious that working out the equations given above is not possible analytically on account of their generality, whilst graphic elaboration is extremely cumbrous and requires an exceedingly large quantity of observation material. As these equations, however, are of fundamental significance for the physics of the troposphere it may yet be very useful to make a systematic investigation into their elaboration.

In what follows we shall confine ourselves only to the direct transfer of heat and mass from the earth's surface to the atmosphere, whilst the processes in the atmosphere which are rendered by the quantities q_e' and q_r' from equation (3.2.1) are left out of consideration. Special attention will be paid to the unstable condition, to the K-types therefore of chapter **1**, because in them the structure is much simpler than in stable conditions.

3.3 The coefficient of heat transfer and the coefficient of mass transfer

When the atmospheric surface layer is unstable, this layer at the same time is a transition layer. By it we wish to express that this layer forms practically the whole resistance to heat and mass transfer. Above this layer the air has a homogeneous character owing to the intensive mixing, i.e. along the entire field where the mixing extends, practically a constant potential temperature and a constant mixing ratio prevails. As the surface layer has a slight heat capacity with respect to the layer above, it may be neglected and we shall only take into account the heat resistance. In the field of heat transmission, see e.g. Mc. Adams (1951) and others, this is a very usual procedure. Here it is assumed that the heat flux from a surface to the medium bound to the surface is proportional to the temperature difference between surface and medium. In our case this assumption amounts to the following:

$$q_a = \alpha (\theta_o - \theta_\infty) \tag{3.3.1}$$

Here the proportional constant α is the so-called coefficient of heat transfer. In order to investigate how far α is really constant, we must eliminate $\theta_\infty - \theta_o$ and q_a from (3.3.1) by means of the equations which can be determined by means of section **2.7**:

$$q_a = - u_*^2 c_p \varrho \frac{\theta_*}{u_*} \tag{3.3.2}$$

$$\theta_\infty - \theta_o = \theta_* f(Sn) \tag{3.3.3}$$

Because θ_∞ has a final value only in case of unstable condition, we had rather start from the equations for free convection in determining $f(Sn)$. Here, by taking $\zeta = \infty$, $f(Sn)$ according to equation (2.9.11), is:

$$f(Sn) = - \frac{1}{k} \ln Sn + 1 \tag{3.3.4}$$

It is also possible to determine $f(Sn)$ by means of equation (2.9.13), but we have omitted this for the sake of surveyability and because we have slightly more confidence in equation (2.9.11) than in equation (2.9.13). By substituting equations (3.3.3) and (3.3.2) into (3.3.1) we now get as a result a general formulation:

$$\alpha = \frac{u_* c_p \varrho}{f(Sn)} \tag{3.3.5}$$

or by substituting (3.3.4) and (3.3.2) to (3.3.1):

$$\alpha = \frac{u_* c_p \varrho}{1 - \frac{1}{k} \ln Sn} \tag{3.3.6}$$

This shows that α is not constant, as Sn as well as u_* in case of a uniform mainflow and z_o, are dependent on q_a. Now this dependence will be of slight significance for a small Sn and a strong main flow. This will especially be the case when the logarithmic profile extends for about the entire surface layer. Above the surface layer the similarity between temperature and velocity profile disappears. This is a consequence of the fact, that the term $-\frac{1}{\varrho}\frac{\partial \bar{p}}{\partial x} z$ from (2.3.24) can no longer be neglected. A consequence of this is, that the potential temperature approaches a final value, whereas the velocity continues increasing over a large field. (The influence of the Coriolis force and the direction change of the wind has in this case been left out of consideration; the consequence of this simplification is not essential for the argument given here). That the potential temperature is practically constant with the height above the surface layer, as soon as the atmosphere is unstable, is a fact of experience given by many observations (see Johnson and Heywood (1938)). Now the question is what value will $f(Sn)$ assume as a limit in case of a very slight unstable condition (so in case of very small positive values of $Sn (Sn \leqslant 5.10^{-4})$, when the roughness of the earth's surface is given.

It appears to us that a reasonable estimation of it is obtained by taking:

$$f(Sn) = \frac{1}{k} \ln \zeta_b \tag{3.3.7}$$

in which $\zeta_b = \frac{z + z_o}{z_o}$; in it z_b corresponds with the height of the surface layer. In this way a constant value of $f(Sn)$ has been obtained. Furthermore for such small values of Sn, u_* may also be considered as independent of q_a, so that under these circumstances α is indeed a constant, which is proportional to u_* and therefore to the wind velocity. Now when $\bar{u}_b$ is the velocity on level z_b, we may write without committing too great an error:

$$\alpha = \frac{u_b \varrho_b k^2 c_p}{(\ln \zeta_b)^2} \tag{3.3.8}$$

From this it appears that α is dependent not only on the velocity u_b but also on the roughness, as might be expected. An important part is played by α in the calculations in the following sections.

In a way analogous to the one in which the coefficient of heat transfer has been defined, a coefficient of mass transfer may be introduced. The equation for the mass transfer may be written in a form analogous to (2.5.5):

$$K\frac{d\chi}{dz} = -\frac{q_e}{\varrho L} \tag{3.3.9}$$

When in doing so we call

$$\frac{-q_e}{\varrho L u_*} = \chi_* \tag{3.3.10}$$

we can write for (3.3.9) analogous to (2.7.1):

$$\frac{d\chi/\chi_*}{d\zeta} = \frac{1}{R} \tag{3.3.11}$$

Furthermore we can give for the mass transfer an equation analogous to equation (3.3.1):

$$\frac{q_e}{L} = \varkappa(\chi_o - \chi_\infty) \tag{3.3.12}$$

where $\varkappa$ is the coefficient of mass transfer.

By combining the equations (3.3.11) with (3.3.12), equations analogous to (3.3.5), (3.3.6) and (3.3.8) respectively may be drawn up:

$$\varkappa = \frac{\varrho_b u_*}{f(Sn)} \tag{3.3.13}$$

$$\varkappa = \frac{\varrho_b u_*}{1 - \frac{1}{k} ln\, Sn} \tag{3.3.14}$$

$$\varkappa = \frac{\varrho_b u_b k^2}{(ln\, \zeta_b)^2} \tag{3.3.15}$$

3.4 Temperature transformation and moisture transformation

An air mass is principally characterized by its temperature and moisture content. The transformation of an air mass amounts to the transformation of its temperature and moisture. In general temperature and moisture transformation can only be treated in their interaction, which, on the whole, is a particularly complicated problem. In the first instance the temperature difference between the air and the earth's surface determines the velocity of the processes (heat transfer as well as evaporation and with them temperature and moisture change of the air). However because heat is withdrawn from the earth's surface by evaporation, the enthalpy of the soil changes and then again the temperature difference between air and earth's surface is influenced. Owing to this the moisture content of the air therefore exerts an indirect influence on the velocity of the processes. Moreover, there is still another way in which the moisture content of the air causes an indirect effect, viz. as soon as condensation in the air sets in. This changes the structure of the atmosphere in such a way, that this too influences the velocity of the processes. Now in the special case, that the enthalpy of the soil is so great that the transfer of heat to the air does not effect the surface temperature in first approximation and that moreover, no condensation in the air sets in, the temperature transformation and the moisture transformation may be treated independently of each other. This has been done in **3.5**. Under certain circumstances there appears to exist a complete similarity between temperature and moisture content, which considerably simplifies the calculation. This can only be so when the surface temperature is a constant in time as well as in place.

As soon, however, as differences or changes occur in the surface temperature, similarity is only possible in approximation, viz. as long as the mixing ratio at the surface is a linear function of the temperature:

$$\chi(z=0) = a_1 \{T(z=0) - a_2\} \tag{3.4.1}$$

By looking for a suitable adaptation of the constants a_1 and a_2 in the temperature stretch in observation, equation (3.4.1) over a stretch of ca 10°C is serviceable with a maximum error in χ of ca 5 %.

As we have continually to deal with differences of temperature and moisture content the constant a_2 may be eliminated:

$$\triangle \chi = a_1 \triangle T \approx a_1 \triangle \theta \tag{3.4.2}$$

According to the equation of Clapeyron (see Holmboe, Forsyte, Gustin (1945)) the constant a_1, is

$$a_1 = \frac{0.622 \, L \, \varrho_m \, \chi_m}{p \, T_m} \tag{3.4.3}$$

in which χ_m is the maximum mixing ratio at the temperature T_m for which latter quantity must be taken somewhere about the mean temperature of the stretch in observation.

When condensation in the air mass sets in a drastic change in the structure of the atmosphere takes place, owing to which the transformation calculation becomes very difficult.

3.5 Transformation of the cold air mass above the sea

3.5.1 *Transformation of the cold air mass with a constant temperature of the sea level*

We have now progressed so far that an attempt may be made to calculate the change of temperature and the mixing ratio of an air mass which flows over a warmer sea level. The sea has been especially chosen for it because sea water has such a great heat capacity compared with air, that the temperature of the sea level may in first approximation be considered constant with time. A second advantage of the sea is that we have a homogeneous surface.

The problem of the temperature change is determined by the equations (1.3.3), (3.2.1) and (3.3.1), which are supplemented by boundary conditions viz. the original condition of the layers of the air mass and the surface temperature distribution of the sea. Simple, but yet acceptable boundary conditions are:

a. the original air mass has a constant lapse rate.

b. the state of the air mass on arrival at $x = 0$ (i.e. the coastline) is constant with time.

c. the surface temperature of the sea is the same for every x.

The solution of the problem can be made exceedingly simple by further assuming that:

1st. the mean wind direction is constant with the height in the x-direction.
2nd $q_r' = q_e' = q_p = 0$ up to that height h, in which the influence of q_a is felt.

By the assumption that the temperature of the sea water is constant with time, equation (1.3.3) may be left out of consideration and there remains (by combining (3.2.1) with (3.3.1)):

$$\alpha\,(\theta_o - \theta_\infty) = \int_o^h \left\{ u \frac{\partial\,(c_p\,\varrho\,\theta_\infty)}{\partial x} + \frac{\partial\,(c_p\,\varrho\,\theta_\infty)}{\partial t} \right\} dz \tag{3.5.1}$$

of this equation the term $\frac{\partial\,(c_p\,\varrho\,\theta_\infty)}{\partial t}$ may be neglected, whilst the following simplification is finally made:

$$\alpha\,(\theta_o - \theta_\infty) = c_p\,\overline{\varrho u}\,h\,\frac{\partial \theta_\infty}{\partial x} \tag{3.5.2}$$

The height h up to which the air rises is dependent on the original lapse rate of the air mass. If we call the difference between this lapse rate and the adiabatic lapse rate γ, (γ therefore is the lapse rate of the potential temperature), we can write down the following equation for h:

$$\theta_\infty(x) - \theta_\infty(0) = \gamma\,h\,(x) \tag{3.5.3}$$

By substituting this into (3.5.2) and by eliminating α by means of equation (3.3.8), we get:

$$\frac{k^2\,\gamma}{(ln\,\zeta_b)^2} \cdot \frac{\varrho_b\,u_b}{\overline{\varrho\,u}}(\theta_o - \theta_\infty) = (\theta_\infty - \theta_\infty(0))\,\frac{\partial \theta_\infty}{\partial x} \tag{3.5.4}$$

We now further assume that

$$\overline{\varrho u} = \varrho_b\,u_b \tag{3.5.5}$$

which means that mass flow is constant with height. This assumption probably gives a better approximation than the assumption, that velocity is constant with height. According to (3.5.5) velocity therefore must vary with height inversely proportional with the density. This gives a velocity profile of the following shape (see H o l m b o e and others 1945):

$$u = u_b \frac{z}{(c_p - c_v)\,\theta_\infty \left\{ 1 - \left(1 - \frac{z}{c_p\,\theta_\infty}\right)^{c_p/c_p - c_v} \right\}} \tag{3.5.6}$$

By means of assumption (3.5.5) equation (3.5.4) can be easily integrated. If in doing so we introduce the following quantities:

$$\vartheta = \frac{\theta_\infty - \theta_\infty(0)}{\theta_o - \theta_\infty(0)} \text{ and } \xi = \frac{\gamma}{\theta_o - \theta_\infty(0)} \left(\frac{k}{ln\,\zeta_b}\right)^2 x \tag{3.5.7}$$

As a result we then find:

$$ln(1 - \vartheta) + \vartheta + \xi = 0 \tag{3.5.8}$$

In figure 3.5.1 the course of equation (3.5.8) has been represented.

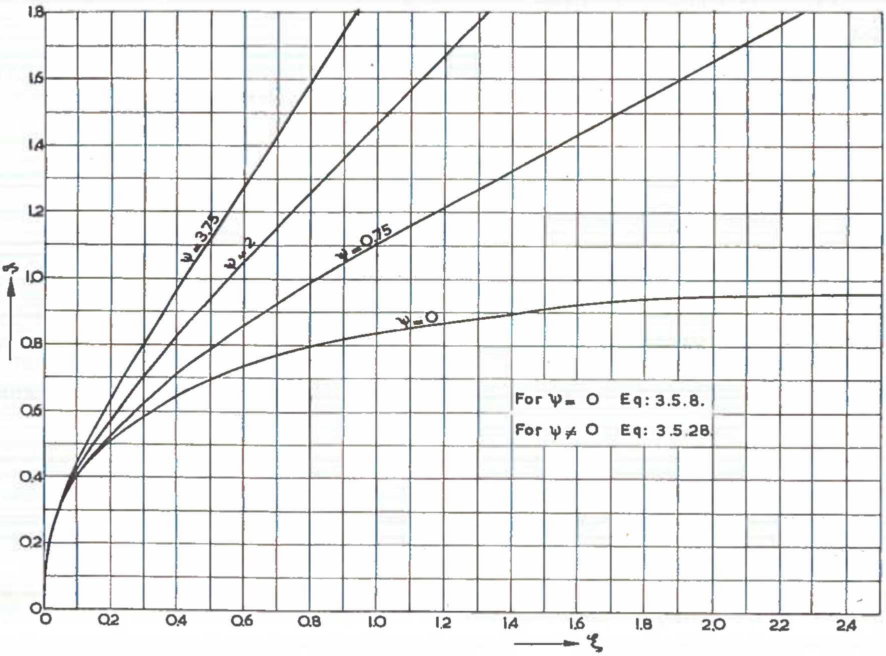


Fig. 3.5.1

The change of the moisture content (i.e. mixing ratio) is determined by (3.5.8) and an equation analogous to (3.5.2):

$$\varkappa(\chi_o - \chi_\infty) = \overline{\varrho u}\, h \frac{\partial x_\infty}{\partial x} + \overline{\varrho u}(\chi_\infty - \chi_h(0)) \frac{\partial h}{\partial x} \tag{3.5.9}$$

in which the last term indicates the discontinuity in mixing ratio between the original air and the air already warmed up. An analogous term is missing in (3.5.2), because the air rises until the original potential temperature is equal to that of the warmed up air, a circumstance by which height h is determined, see equation (3.5.3). For the latter equation we can also write:

$$h = \frac{(\theta_o - \theta_\infty(0))\,\vartheta}{\gamma} \tag{3.5.10}$$

if we now further introduce $\dfrac{\chi_\infty - \chi_\infty(0)}{\chi_o - \chi_\infty(0)} = \eta$, a quantity analogous to ϑ, we can write for (3.5.9) by means of (3.5.10), if we assume that $\chi_h(0)$ is constant with height:

$$1 - \eta = \vartheta \frac{d\eta}{d\xi} + \eta \frac{d\vartheta}{d\xi} \tag{3.5.11}$$

This equation together with (3.5.8) determines the humidity transformation.

By eliminating ξ we get:

$$\frac{d\eta}{d\vartheta} = \frac{1-\eta}{1-\vartheta} - \frac{\eta}{\vartheta} = \frac{\vartheta - \eta}{\vartheta(1-\vartheta)} \tag{3.5.12}$$

with the boundary condition

$$\eta = 0 \text{ when } \vartheta = 0 \tag{3.5.13}$$

Bij application of a development in a series: $\eta = a_o + a_1\vartheta + a_2\vartheta^2 + \ldots.$ (3.5.12) can be solved:

$$\eta = \frac{1-\vartheta}{\vartheta}\, ln(1-\vartheta) + 1 \tag{3.5.14}$$

By means of (3.5.8) therefore the relation between η and ξ has been fixed, which can rendered by way of a diagram in a simple manner. Generally speaking $\chi_h(0)$ will not be constant with height. The problem may be extended rather easily by taking instead of $\chi_h(0)$ is constant:

$$\chi_h(0) = \chi_\infty(0) + \delta h \tag{3.5.15}$$

We then find instead of equation (3.5.12):

$$\frac{d\eta}{d\vartheta} = \frac{\vartheta - \eta}{\vartheta(1-\vartheta)} + b \tag{3.5.16}$$

in which

$$b = \frac{\delta(\theta_o - \theta_\infty(0))}{\gamma(\chi_o - \chi_\infty(0))} \tag{3.5.17}$$

This equation may be solved in a way analogous to (3.5.12). This with boundary condition (3.5.13):

$$\eta = (1-b)\left\{\frac{1-\vartheta}{\vartheta}\, ln(1-\vartheta) + 1\right\} + b\vartheta \tag{3.5.18}$$

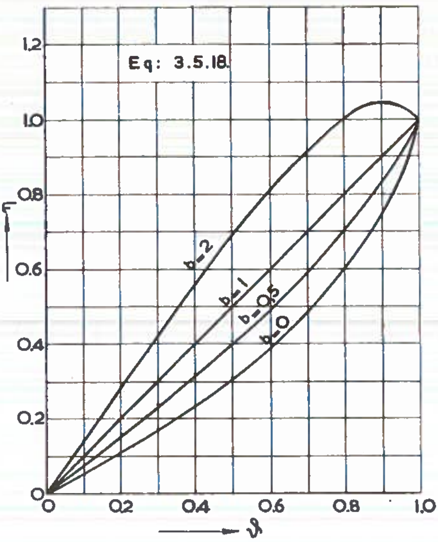


Fig. 3.5.2

This equation has been rendered in Fig. 3.5.2 for some values of b.

In the special case that $b = 1$ we get $\eta = \vartheta$, that means that then, there is a complete analogy between temperature transformation and humidity transformation. In this case therefore $\chi_h\,(0)$ must comply with the relation (which follows from (3.5.15) and (3.5.17)):

$$\chi_h(0) = \chi_\infty(0) + \frac{\chi_o - \chi_\infty(0)}{\theta_o - \theta_\infty(0)} \cdot \gamma\, h \tag{3.5.19}$$

The calculation of the temperature and humidity change above the sea becomes very simple when the original condition of the air is already adiabatic to a height h, whilst a distinct inversion is found at this height. We may then assume that h is all but constant, in doing which equation (3.5.2) can be at once integrated by means of the assumption (3.5.5). Taking into account equation (3.3.8) this gives:

$$\theta_\infty - \theta_o = (\theta_\infty(0) - \theta_o)\, exp\left\{-\frac{x}{h}\frac{k^2}{(ln\,\zeta_b)^2}\right\} \tag{3.5.20}$$

With an adiabatic atmosphere the mixing ratio will generally be constant with height, so that we also have:

$$\chi_\infty - \chi_o = (\chi_\infty(0) - \chi_o)\, exp\left\{-\frac{x}{h}\frac{k^2}{(ln\,\zeta_b)^2}\right\} \tag{3.5.21}$$

3.5.2 *Transformation of the cold air mass with a sea level temperature which increases linearly with x*

In this problem only the boundary condition c from **3.5.1** is slightly altered, viz, the temperature of the sea level is:

$$\theta_o = \theta_o(0) + f\,.\,x \tag{3.5.22}$$

By substituting this boundary condition into (3.5.2) and by moreover eliminating h with (3.5.3) we get the equation:

$$\theta_o(0) - \theta_\infty + f\,.\,x = \frac{c_p\,\overline{\varrho\, u}}{\alpha\,\gamma}(\theta_\infty - \theta_\infty(0))\frac{\partial\,\theta_\infty}{\partial x} \tag{3.5.23}$$

Now by introducing by the side of ϑ and ξ (see equation (3.5.7)):

$$\psi = \frac{f}{\gamma}\left(\frac{ln\ \zeta_b}{k}\right)^2 \tag{3.5.24}$$

we get (see equation (3.3.8)):

$$\frac{d\vartheta}{d\xi} = \frac{\psi\xi}{\vartheta} + \frac{1}{\vartheta} - 1 \tag{3.5.25}$$

The boundary condition in it is $\vartheta = 0$ if $\xi = 0$ and ψ is a parameter. This equation may be solved by means of the following substitution:

$$\xi = X - \frac{1}{\psi} \text{ and } \vartheta = X\,Y \tag{3.5.26}$$

This gives:

$$ln\,X = \int \frac{Y\,d\,Y}{\psi - Y - Y^2} + C$$

When $f > 0$, so when the surface temperature increases by an increasing x, then we have $\psi > 0$ therefore also $\psi + \frac{1}{4} > 0$. The solution may then be worked out to:

$$ln\,X = -\tfrac{1}{2}\,ln(-Y^2 - Y + \psi) - \frac{1}{4(\frac{1}{4}+\psi)^{\frac{1}{2}}}\,ln\,\frac{Y + \frac{1}{2} + (\frac{1}{4}+\psi)^{\frac{1}{2}}}{-Y - \frac{1}{2} + (\frac{1}{4}+\psi)^{\frac{1}{2}}} + C \tag{3.5.27}$$

Substituting the boundary condition, gives for C:

$$C = -\tfrac{1}{2}\,ln\,\psi + \frac{1}{2(1+4\,\psi)^{\frac{1}{2}}}\,ln\,\frac{(1+4\,\psi)^{\frac{1}{2}}+1}{(1+4\,\psi)^{\frac{1}{2}}-1}$$

By transforming back and slightly remodelling (3.5.27) we get as a solution:

$$ln\{(\xi\,\psi+1)^2 - \vartheta\,(\xi\,\psi+1) - \vartheta^2\,\psi\} + \frac{1}{(1+4\,\psi)^{\frac{1}{2}}}\,ln\,\frac{2(\xi\,\psi+1)+\vartheta\{(1+4\,\psi)^{\frac{1}{2}}-1\}}{2(\xi\,\psi+1)-\vartheta\,\{(1+4\,\psi)^{\frac{1}{2}}+1\}} = 0 \tag{3.5.28}$$

In Fig. 3.5.1 some curves for $\psi =$ constant are drawn.

When $\psi = 0$, equation (3.5.25) may be directly integrated to the already well-known result:

$$ln(1-\vartheta) + \vartheta + \xi = 0 \tag{3.5.8}$$

This equation may easily be derived from (3.5.28) if we first divide by ψ.

As has been stated in **3.4** the change of humidity content can only be given in first approximation when the surface temperature increases linearly with x, because χ_o does not increase linearly with temperature. We now make use of the approximation formula (3.4.2). This combined with (3.5.22) therefore becomes the boundary condition for the moisture content.

$$\chi_o - \chi_o(0) = a_1 f x \tag{3.5.29}$$

in it a_1 may be considered slightly dependent on the stretch in observation. Substituting (3.5.29) into (3.5.9) and then eliminating h by means of (3.5.10) and $\chi_h(0)$ by means of (3.5.15) yields:

$$\vartheta \frac{d\eta}{d\xi} + (\eta - b\vartheta)\frac{d\vartheta}{d\xi} - \psi'\xi + \eta - 1 = 0 \tag{3.5.30}$$

in which

$$\psi' = \frac{a_1(\theta_o(0) - \theta_\infty(0)) f u_b}{(\chi_o(0) - \chi_\infty(0)) \varkappa \gamma}$$

In the special case that $b = 1$, $\vartheta = \eta$ is valid again and so we recover equation (3.5.25).

In case we have $b \neq 1$ then the equation (3.5.30) and (3.5.28) together with the boundary condition $\vartheta = \eta = 0$ if $\xi = 0$, states the problem. This system is difficult to solve.

3.5.3 *Comparison with the calculations by* Burke *and* Frost

Burke (1945) as well as Frost (1949) made a calculation for the transformation of air above a relatively warm sea level with the same boundary conditions as those given in **3.5.1** and **3.5.2.**

Burke starts from practically the same shape of the atmosphere as is given in the beginning of **3.3**, in which he makes use of Rossby and Montgomery's (1936) equation:

$$\tau = \varrho\, \varphi^2 u_b{}^2 \tag{3.5.31}$$

where $\varphi^2 = 2.6 \,.\, 10^{-3}$, if u_b is measured at about 15 m. In doing so he comes to the following coefficient of heat transfer:

$$\alpha = c_p\, \varrho\, u_b\, \varphi^2 \tag{3.5.32}$$

if we compare this with (3.3.8) then we must write:

$$\varphi = \frac{k}{ln\, \zeta_b} \tag{3.5.33}$$

Thus the above value of φ corresponds with a z_o lying between 0.7 and 0,8 cm, which is a plausible value for moderate winds. For the rest Burke's calculation reads slightly different from ours because he makes somewhat different assumptions, which give a less simple solution.

Frost starts from quite another shape of the atmosphere. He does not make use of a coefficient of heat transfer, but starts from the conjugate power law:

$$\frac{\bar{u}}{\bar{u}_1} = \left(\frac{z}{z_1}\right)^m \text{ and } K = mz^{1-m} z_o^{2m} \bar{u}_1 z_1^{-m} \qquad (3.5.34)$$

in which he takes for $z_o = 1$ cm and $m = {}^1/_7$.

It is self-evident that the subsequent calculation is more complicated than those given in **3.5.1** and **3.5.2**. Two serious drawbacks adhere to Frost's method, viz. the similarity between the velocity profile and the temperature profile is extended over too large a field and the original lapse rate of the air is not taken into account. A consequence is, that this method cannot be used for distances larger than 550 km. Furthermore Frost extends his calculation by starting from a temperature which increases linearly with x, instead of starting from a constant surface temperature. So its solution must be compared with equation (3.5.28).

By means of Burke's observations, which Frost also used for his calculations, it is possible to compare the various results with each other. These observations are summarized in Tables III and IV. In working out the equations (3.5.8) and (3.5.28) the quantity $\varphi^2 = (k/ln\ \zeta_b)^2$ could reasonably be fitted to observation. A reasonable fit was obtained by taking $\varphi^2 = 2 \,.\, 10^{-3}$. This value is smaller than the value which Burke takes for it (see equation (3.5.33)) which is a consequence of the fact that Burke refers the measured wind velocity to the geostropical wind velocity, whereas here equation (3.5.6) has been made use of. It is also of importance to draw attention to the mean sea level temperature, from which Burke starts viz:

$$\bar{\theta}_o = \frac{\theta_o(0) + 3\theta_o(x)}{4} \qquad (3.5.35)$$

which temperature is used for distances smaller than 650 km. For distances larger than 650 km Burke uses the ultimate temperature as the mean sea level temperature. Thanks to the fact, that the boundary temperatures of the sea level are given in Table III, we can here compare equation (3.5.28) and Frost's method with equation (3.5.8) and Burke's method. In order to avoid making the table too large, only the deviations with respect to the observed temperature change of the various methods are compared with each other.

In Table IV only the mean sea level temperature according to Burke is given, so that only equation (3.5.8) can be applied here.

The following conclusions may now be drawn from the tables:

1. By means of equation (3.5.8) and the mean sea level temperature better results are obtained than with equation (3.5.28) and a constant gradient of

TABLE III

Nr	Length of over water trajectory $x \cdot 10^{-5}$	Average lapse rate of potential temp. $\gamma \cdot 10^{2}$	Average temp. gradient of sea surf. temp. $f \cdot 10^{5}$	$\theta_0(0)-\theta_\infty(0)$	$\overline{\theta_0}-\theta_\infty(0)$	$\theta_0(x)-\theta_\infty(0)$	Observed temp. change
1	3.55	0.53	0.70	7.0	8.0		3.0
2	3.70	1.00	2.16	16.5	22.5		14.0
3	4.20	0.64	1.67	9.5	14.5		8.5
4	4.30	0.69	1.63	8.5	13.5		7.5
5	4.40	0.71	0.68	13.0	15.0		7.5
6	4.90	0.73	2.04	15.0	22.5		11.5
7	5.00	0.87	0.60	15.5	17.5		10.5
8	5.00	0.75	1.80	18.0	25.0		14.0
9	5.20	0.72	1.25	14.0	19.0		12.0
10	5.50	0.70	1.27	14.0	19.0		11.5
11	5.55	0.86	2.25	16.5	26.0		19.0
12	5.55	0.94	1.44	13.5	19.5		13.5
13	5.60	0.50	0.18	7.0	8.0		7.0
14	5.60	1.18	1.96	17.0	25.0		18.0
15	5.90	0.74	1.36	12.0	18.0		12.0
16	6.20	1.05	1.77	15.0	23.0		16.0
17	6.30	0.90	0.71	17.0	20.5		14.5
18	6.80	0.64	1.69	11.0	19.5	22.5	15.0
19	7.80	1.42	1.60	16.5	26.0	29.0	21.0
20	7.80	0.67	1.80	11.5	22.0	26.0	18.5
21	7.80	0.88	1.54	13.5	22.5	25.5	17.5
22	8.00	0.77	1.43	14.0	22.5	25.5	17.5
23	8.00	0.91	1.43	13.5	22.0	25.0	17.0
24	9.00	0.60	1.55	18.0	16.5	22.0	13.5
25	9.60	1.10	1.15	15.0	23.0	26.0	21.0
26	11.10	1.18	1.22	16.0	26.0	29.5	24.0
27	12.20	1.29	0.98	17.0	26.0	29.0	24.0
28	12.40	0.68	1.01	10.0	19.5	22.5	16.5

the sea level temperature. This indicates that the assumption of a constant surface temperature gives a better approximation than that of a constant temperature gradient.

2. With Frost's methods slightly better results are obtained than with Burke's method and equation (3.5.8), which is a consequence of the better fit Frost can make owing to the smaller field which he observes.

3. For distances larger than 650 km it is better to work with the ultimate temperature $\theta_o(x)$ than with $\overline{\theta_o}$.

4. With equation (3.5.8) practically the same results are obtained as with Burke's method. From this it appears that by means of the method developed in this chapter it is not so much an improvement which is obtained as

Deviation from equation (3.5.28)	Deviation from Eq. (3.5.8) based on $\bar{\theta}_0$	Deviation from Eq. (3.5.8) based on $\theta_0(x)$	Deviation from Burke's method based on $\bar{\theta}_0$	Deviation from Burke's method based on $\theta_0(x)$	Deviation from Frost's method; constant gradient of sea level temp.
2.5	2.5		2.0		1.5
- 1.0	- 0.5		- 1.0		- 1.5
- 0.5	0.5		0.0		- 0.5
1.5	1.5		1.5		0.5
2.0	2.0		2.0		1.0
1.5	2.0		1.0		1.5
1.5	1.5		0.5		- 0.5
0.5	1.0		0.0		0.0
0.0	0.0		0.0		- 1.5
0.5	1.0		1.0		- 0.5
- 3.5	0.0		- 2.5		
- 0.5	0.5		- 0.5		
- 1.0	- 1.0		- 1.0		
- 1.0	0.0		0.0		
0.0	0.5		0.0		
0.0	0.5		0.5		
0.0	0.0		0.0		
- 3.0	- 2.0	- 0.5	- 2.0	- 0.5	
- 2.5	0.0	1.5	0.5	1.5	
- 4.0	- 3.0	0.0	- 3.0	- 2.0	
- 1.5	- 1.0	- 0.5	- 1.0	0.5	
- 2.0	- 1.5	- 0.5	- 1.5	0.0	
- 1.0	- 0.5	1.0	- 1.0	0.5	
- 1.0	- 1.0	1.5	- 1.5	2.0	
- 3.0	- 2.0	- 0.5	- 2.0	0.0	
- 3.5	- 2.0	0.0	- 2.5	- 1.0	
- 2.5	- 1.5	0.5	- 1.5	1.5	
- 2.0	- 0.5	1.0	- 0.5	1.0	

considerable simplification. This is clearly illustrated by the fact, that the line $\psi = 0$ from Fig. 3.5.1 is equivalent to the 6 figures given by Burke at the end of his paper. Another advantage is that the method given here is entirely analytic, whereas Burke has to make use of graphic integration to arrive at a solution.

Remarks

1. The observations 23, 24, 25, 32, 47, 52, 53 and 54 from the table IV have not been stated quite correctly by Burke, so that they cannot be safely compared. The observation 60 has been incorrectly worked out by Frost.

2. It is conceivable that the method of calculation may be improved by also

TABLE IV

Nr	Length of over water trajectory $x \cdot 10^{-5}$	Average lapse rate of potential temp. $\gamma \cdot 10^2$	$\bar{\theta}_0-\theta_\infty(0)$ or $\theta_0(x)-\theta_\infty(0)$	Observed temp. change	Deviation from equation (3.5.8)	Deviation from Burke's method	Deviation from Frost's method
1	9.25	0.80	21	17	−1	−1	
2	14.80	0.60	31	23	0	0	
3	18.50	0.53	22	17	2	1	
4	10.20	0.20	11	4	3	4	
5	18.50	0.80	20	16	3	2	
6	10.20	0.40	8	6	1	0	
7	10.20	0.87	24	18	0	1	
8	9.25	1.27	27	23	−1	0	
9	12.90	0.47	23	19	−3	−1	
10	12.90	0.67	22	20	−3	−2	
11	10.20	1.00	26	22	−1	−1	
12	7.90	0.73	14	12	−1	−1	
13	12.50	0.87	16	14	0	0	
14	6.50	1.13	17	12	2	1	
15	7.40	0.73	17	12	1	1	
16	12.90	0.47	24	20	−3	−2	
17	8.00	0.75	9	8	0	0	
18	9.25	0.47	5	5	0	0	
19	9.25	1.27	18	18	−2	−2	
20	9.25	0.87	12	11	0	0	
21	11.10	0.80	11	11	−1	−1	
22	9.25	0.60	23	13	3	3	
23	9.25	0.40	29	17	−1	−4	
24	7.40	0.60	16	15	−3	−4	
25	7.40	0.67	18	17	−4	−4	
26	11.10	0.93	27	20	1	1	
27	8.70	0.27	14	7	1	2	
28	9.25	0.60	20	16	−2	−2	
29	10.20	0.93	5	7	−2	−2	
30	8.30	0.40	15	8	2	2	
31	25.90	1.13	32	28	2	2	
32	20.40	0.67	24	23	−2	−1	
33	8.30	0.67	5	4	1	1	
34	15.70	0.93	15	14	0	0	
35	10.20	0.60	13	8	3	3	
36	12.90	0.74	12	9	2	2	
37	8.30	0.67	5	4	1	1	
38	7.40	0.67	5	3	2	2	
39	8.30	0.53	2	1	1	1	
40	11.10	0.47	3	2	1	1	
41	8.30	0.80	7	5	2	2	
42	12.90	0.53	12	11	−1	−1	
43	12.90	0.93	13	10	2	2	
44	18.50	0.53	16	13	1	1	
45	12.00	0.53	18	12	2	2	
46	11.10	0.33	19	11	1	1	
47	4.25	0.87	4	1	3	3	1.0

TABLE IV *(continued)*

Nr	Length of over water trajectory $x \cdot 10^{-5}$	Average lapse rate of potential temp. $\gamma \cdot 10^2$	$\bar{\theta}_0 - \theta_\infty(0)$ or $\theta_0(x) - \theta_\infty(0)$	Observed temp. change	Deviation from equation (3.5.8)	Deviation from Burke's method	Deviation from Frost's method
48	11.60	0.87	13	10	2	2	
49	20.70	0.53	19	17	−1	0	
50	3.30	0.53	14	8	0	0	−1.0
51	13.35	0.60	8	5	1	−1	
52	11.20	0.80	24	17	2	2	
53	15.35	0.27	15	15	−4	−4	
54	11.75	0.33	26	18	−3	0	
55	9.65	0.93	37	29	−4	−3	
56	10.90	0.93	17	15	0	0	
57	15.05	1.20	34	27	2	2	
58	16.40	0.80	31	27	−1	−1	
59	8.30	0.73	28	24	−5	−5	
60	2.40	0.27	6	4	−1	0	−0.5
61	6.80	0.47	19	10	1	2	
62	8.30	0.80	24	18	−1	−1	
63	16.40	0.47	16	10	3	4	
64	5.05	0.47	6	4	1	1	−1.0
65	8.20	0.50	22	17	−3	−1	
66	3.20	0.47	3	3	0	0	−1.5
67	15.30	0.67	18	14	1	1	
68	4.00	0.60	5	4	0	0	−1.5

taking into account the mean wind velocity, because z_o is dependent on it.
3. The reasonable agreement of equation (3.5.8) with the observations enables us to state the value of the coefficient of heat transfer α. Because we have accepted $\varphi^2 = 2 \cdot 10^{-3}$, we can write for equation (3.5.32):

$$\alpha \approx 2.15\, u_b$$

in which u_b in m/sec must be taken. In Table V some values of α in case of various values of u_b have been given for orientation.

TABLE V

u_b (m/sec)	$\alpha (kcal/m^2 h^o C)$
1	2.15
2	4.3
5	10.8
10	21.5
20	43

3.6 Transformation of the cold air mass above land

The transformation of the cold air mass above land is far more difficult to calculate than that above the sea. This is due to the fact that the surface temperature can no longer be considered as constant with time. Now equation (2.3.3) begins to play a part and together with it also (3.2.4). As a rule the capriciousness of the land surface will render it impossible to make a calculation yet it seems sensible to us to give a calculation, which indeed is based on a considerable simplification, but which under a few special circumstances may certainly be fitted in and by means of which useful data could be derived from the earth's surface (e.g. α and $\varkappa$). As to the temperature transformation we take for granted that the following conditions have been fulfilled:

a. at the time that $t = 0$ the air as well as the soil have the same potential temperature for all $x > 0$ and all $z < h$: $\theta (t = 0) = T(t = 0)$, whilst the air for $x < 0$ has the temperature $\theta_\infty (0)$.
b. the atmosphere is adiabatic to a height h, at which height an inversion is found (the same condition as the one underlying equation (3.5.23)).
c. the earth's surface has a constant roughness.
d. the soil has a constant composition i.e. λ_s and $(c\varrho)_s$ are constant.
e. $q_r = q_p = q_e = 0$, so that (1.3.3) becomes $q_s = q_a$.
f. the pressure at the surface is 1000 mb, which makes

$$T(z = 0) = \theta (z = 0)$$

Furthermore it is supposed that the equations (3.3.8) and (3.5.5) are valid.

Now the equations constituting the problem are:

$$q_a = \alpha (\theta_\infty - \theta_o) = c_p \varrho_b u_b h \frac{\partial \theta}{\partial x} + c_p \varrho_b h \frac{\partial \theta}{\partial t} \tag{3.6.1}$$

in which $\theta_o = T(z = 0)$

for the surface:
$$q_s = q_a \tag{3.6.2}$$

and for the soil:
$$\frac{\partial T}{\partial t} = a_s \frac{\partial^2 T}{\partial z^2} \tag{3.6.3}$$

with boundary condition:
$$\lambda_s \frac{\partial T}{\partial z} (z = 0) = - q_s \tag{3.6.4}$$

Then we also have the boundary conditions and the initial conditions which are given under a:

for the air:
$$\theta_\infty (x = 0; t) = \theta_\infty (0) \tag{3.6.5}$$

for the soil:
$$T\left(x; z; t = \frac{x}{u_b}\right) = T_o \tag{3.6.6}$$

and
$$T(x; -\infty; t) = T_o \tag{3.6.7}$$

We now introduce a new time variable: $t' = t - \dfrac{x}{u_b}$ which has been chosen thus, that the front passes at $t' = 0$ at every place x. The equations (3.6.2), (3.6.3), (3.6.4), (3.6.5) and (3.6.7) now also hold with t' instead of t, because in the entire T— problem x occurs only as a parameter. Because we move along with the air, owing to which $t' = $ constant, equation (3.6.1) now becomes:

$$-\alpha\{\theta_\infty(x, t') - T(x, 0, t')\} = c_p\, \varrho_b\, u_b\, h\, \frac{\partial\theta(x, t')}{\partial x} \tag{3.6.8}$$

and equation (3.6.6) becomes:

$$T(x, z, 0) = T_o \tag{3.6.9}$$

By means of the equations (3.6.2), (3.6.3), (3.6.4), (3.6.5), (3.6.7), (3.6.8) and (3.6.9) the problem has now been entirely stated.

Before proceeding to solve it, we make the equations dimensionless by the substitutions:

$$t' = \frac{\lambda_s^2}{\alpha^2 a}\tau\,;\; x = \frac{c_p\, \varrho_b\, u_b\, h}{\alpha}\xi_1\,;\; z = \frac{-\lambda_s}{\alpha}\zeta_1$$

$$\theta_\infty = T_o + \vartheta_1(\theta_\infty(0) - T_o) \text{ and } T = T_o + \vartheta_2(\theta_\infty(0) - T_o)$$

We then get after elimination of q_s and q_a:

for (3.6.3):

$$\frac{\partial\vartheta_2}{\partial\tau} = \frac{\partial^2\vartheta}{\partial{\zeta_1}^2} \tag{3.6.10}$$

for (3.6.4) and (3.6.8):

$$\frac{\partial\vartheta_2(\xi_1, 0, \tau)}{\partial\,\zeta_1} = \frac{\partial\zeta_1(\xi_1, \tau)}{\partial\xi_1} = \vartheta_2(\xi_1, 0, \tau) - \vartheta_1(\xi_1, \tau) \qquad \begin{cases}(3.6.11)\\(3.6.12)\end{cases}$$

for (3.6.5), (3.6.7) and (3.6.9) respectively:

$$\vartheta_1(0, \tau) = 1 \tag{3.6.13}$$

$$\vartheta_2(\xi_1, \infty, \tau) = 0 \tag{3.6.14}$$

$$\vartheta_2(\xi_1, \zeta, 0) = 0 \tag{3.6.15}$$

We then apply a Laplace transformation $\tau \rightarrow s$, in which

$$\mathcal{L}\{\vartheta_1, (\xi_1, \zeta_1, \tau)\} = \bar{\vartheta}_1(\xi_1, \zeta_1, s)$$

and

$$\mathcal{L}\{\vartheta_2(\xi_1, \tau)\} = \bar{\vartheta}_2(\xi_1, s)$$

Thus we obtain:

$$s\bar{\vartheta}_2 = \frac{\partial^2\bar{\vartheta}}{\partial\,{\zeta_1}^2} \text{ (because } \vartheta_2(\xi_1, \zeta_1, 0) = 0) \tag{3.6.16}$$

$$\frac{\partial\bar{\vartheta}_2(\xi_1, 0, s)}{\partial\zeta_1} = \frac{\partial\bar{\vartheta}_1(\xi_1, s)}{\partial\xi_1} = \bar{\vartheta}_2(\xi_1, 0, s) - \bar{\vartheta}_1(\xi_1, s) \qquad \begin{cases}(3.6.17)\\(3.6.18)\end{cases}$$

$$\bar{\vartheta}_1(0, s) = \frac{1}{s} \tag{3.6.19}$$

$$\bar{\vartheta}_2(\xi_1, \infty, s) = 0 \tag{3.6.20}$$

This system can be solved. The solution of (3.6.16) which fulfills (3.6.20) is:

$$\vartheta_2 = A\, exp\,(-\zeta_1 \sqrt{s})$$

equation (3.6.17) gives:

$$-A\sqrt{s} = \frac{\partial \bar{\vartheta}_1}{\partial \xi_1}$$

so for $\zeta_1 = 0$ we get $\vartheta_2(\xi_1, 0, s) = -\dfrac{1}{\sqrt{s}} \dfrac{\partial \bar{\vartheta}_1}{\partial \xi}$

This, substituted in (3.6.18) gives:

$$\left(1 + \frac{1}{\sqrt{s}}\right) \frac{\partial \bar{\vartheta}_1}{\partial \xi_1} + \bar{\vartheta}_1 = 0 \qquad (3.6.21)$$

The solution of this, complying with (3.6.19) is:

$$\bar{\vartheta}_1(\xi_1, s) = \frac{1}{s}\, exp\left(-\frac{\xi_1 \sqrt{s}}{1+\sqrt{s}}\right) \qquad (3.6.22)$$

Now we must transform back this function. This is only possible by developing $\bar{\vartheta}_1$ in a series and then transforming back term after term:

$$\bar{\vartheta}_1(\xi_1, s) = \frac{1}{s} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!}\, \xi_1{}^n \frac{(\sqrt{s})^{n-2}}{(1+\sqrt{s})^n} \qquad (3.6.23)$$

Now

$$\frac{(\sqrt{s})^{n-2}}{(1+\sqrt{s})^n} = \frac{\{(\sqrt{s}+\frac{1}{2}) - \frac{1}{2}\}^{n-1}}{\{(\sqrt{s}+\frac{1}{2}) + \frac{1}{2}\}^n} \cdot \frac{1}{(\sqrt{s}+\frac{1}{2}) - \frac{1}{2}} \qquad (3.6.24)$$

We first try to find the back transformation of

$$\frac{(s-\frac{1}{2})^{n-1}}{(s+\frac{1}{2})^n} \cdot \frac{1}{s-\frac{1}{2}}$$

In the tables by D o e t s c h (1947) we find:

$$\mathcal{L}^{-1}\left\{\frac{(s-\frac{1}{2})^{n-2}}{(s+\frac{1}{2})^n}\right\} = e^{-\frac{1}{2}t} L_{n-1}(t) \quad \text{for} \quad n \geqslant 1$$

in wich L_{n-1} are the polynomes of L a g u e r r e

Furthermore $$\mathcal{L}^{-1}\left\{\frac{1}{s-\frac{1}{2}}\right\} = e^{\frac{1}{2}t}$$

We now get a convolution:

$$\mathcal{L}^{-1}\left\{\frac{(s-\frac{1}{2})^{n-1}}{(s+\frac{1}{2})^n} \cdot \frac{1}{s-\frac{1}{2}}\right\} = e^{\frac{1}{2}t} * \{e^{-\frac{1}{2}t} L_{n-1}(t)\} =$$

$$= \int_0^t e^{\frac{1}{2}(t-u)}\, e^{-\frac{1}{2}u} L_{n-1}(u)\, du \qquad (3.6.25)$$

Moreover we find in the transformation rules that if

$$\mathcal{L}^{-1}\{\bar{f}(s)\} = f(t) \qquad \text{then}$$

$$\mathcal{L}^{-1}\{\bar{f}(\sqrt{s})\} = \int_0^\infty \frac{v}{2\sqrt{\pi}\, t^{3/2}} \cdot exp\left(-\frac{v^2}{4t}\right) f(v)\, dv$$

and
$$\mathcal{L}^{-1}\{\bar{f}(s+\tfrac{1}{2})\} = e^{-\frac{1}{2}t} f(t)$$

Consequently

$$\mathcal{L}^{-1}\{\bar{f}(\sqrt{s}+\tfrac{1}{2})\} = \int_0^\infty \frac{v}{2\sqrt{\pi}\, t^{3/2}}\, exp\left(-\frac{v^2}{4t} - \tfrac{1}{2}v\right) f(v)\, dv$$

By applying this to the right-hand side of (3.6.24) we find, thanks to (3.6.25):

$$\mathcal{L}^{-1}\left\{\frac{(\sqrt{s})^{n-2}}{(1+\sqrt{s})^n}\right\} = \int_0^\infty \frac{v}{2\sqrt{\pi}\, t^{3/2}}\, exp\left(-\frac{v^2}{4t} - \tfrac{1}{2}v\right) \int_0^v e^{\frac{1}{2}(v-u)}\, e^{-\frac{1}{2}u}\, L_{n-1}(u)\, du =$$

In it we have written τ instead of t, in agreement with the original transformation

$$= \frac{1}{2\sqrt{\pi}\,\tau^{3/2}} \int_0^\infty v\, exp\left(-\frac{v^2}{4\tau}\right) dv \int_0^v e^{-u} L_{n-1}(u)\, du =$$

$$= -\frac{1}{(\pi\tau)^{\frac{1}{2}}} \int_0^\infty \frac{d}{dv}\left\{exp\left(-\frac{v^2}{4\tau}\right)\right\} dv \int_0^v e^{-u} L_{n-1}(u)\, du =$$

by integrating partially we find:

$$= \frac{1}{(\pi\tau)^{\frac{1}{2}}} \int_0^\infty exp\left(-\frac{v^2}{4\tau} - v\right) L_{n-1}(v)\, dv$$

substitution $w = \dfrac{v}{2\sqrt{\tau}} + \sqrt{\tau}$ yields:

$$= \frac{2}{\pi} e^{\tau} \int_{\sqrt{\tau}}^\infty exp(-w^2)\, L_{n-1}\{2\sqrt{\tau}(w-\sqrt{\tau})\}\, dw$$

So now we get for the transformed series of (3.6.23):

$$\vartheta_1(\xi_1,\tau) = 1 + \sum_{n=1}^\infty \frac{(-1)^n}{n!}\, \xi_1^{\,n} \frac{2}{\sqrt{\pi}} e^{\tau} \int_{\sqrt{\tau}}^\infty exp(-w^2)\, L_{n-1}\{2\sqrt{\tau}(w-\sqrt{\tau})\}\, dw \tag{3.6.26}$$

as the back transformed of $\dfrac{1}{s} \to 1$.

This series represents the course of the temperature of the invading cold air as a function of time and place. The first terms of the series appear as follows

(The L a g u e r r e polynomes are rational functions, so we can connect the integrals with the error integral):

$$\vartheta_1(\xi_1, \tau) = 1 - \xi_1\, e^{\tau}\, erfc\sqrt{\tau} + \frac{\xi_1}{2!}\left\{(2\tau + 1)\, e^{\tau}\, erfc\sqrt{\tau} - \left(\frac{4\tau}{\pi}\right)^{\frac{1}{2}}\right\}$$

$$-\frac{\xi^3}{3!}\left\{(2\,\tau + 5\,\tau + 1)\, e^{\tau}\, erfc\sqrt{\tau} - (\tau + 2)\left(\frac{4\tau}{\pi}\right)^{\frac{1}{2}}\right\} + \ldots \qquad (3.6.27)$$

This series converges swiftly and is especially serviceable for calculations in the first period after an invasion of cold. Equation (3.6.27) is expressed in Fig. 3.6.1 and 3.6.2. Now in Fig. 3.6.1, the retarded time t' has again been reduced to t for the following example.

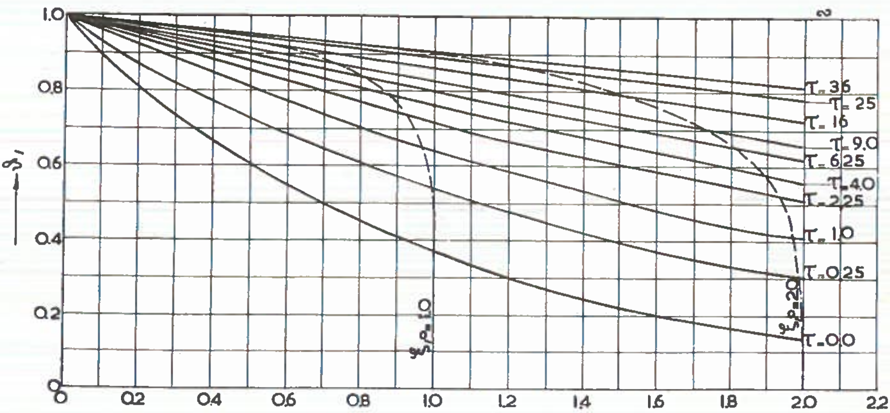


Fig. 3.6.1

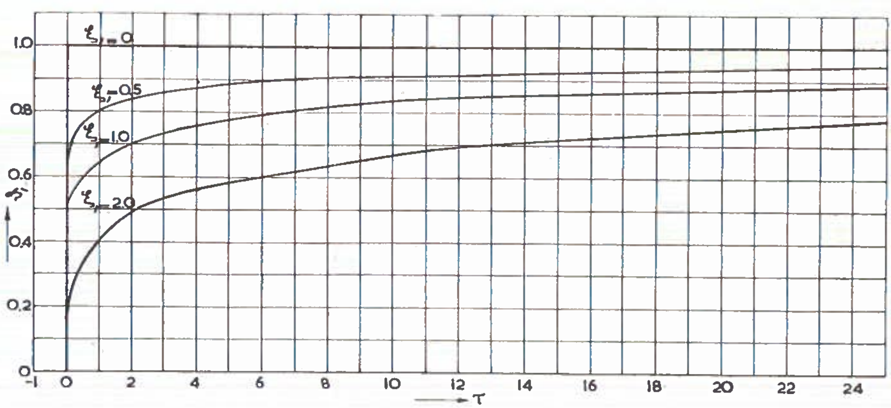


Fig. 3.6.2

Example: Suppose $\lambda = 0.3$ kcal/m h °C; $\alpha = 10$ kcal/m² h °C
$a = 0.001$ m²/h then $0.9\,\tau = t'$
so if $\tau = 1$ then $t' = 54$ minutes
If we further suppose: $c_p\,\varrho_b = 0.3$ kcal/m³ °C; $h = 1000$ m
$u_b = 36000$ m/h
then $1.08 \,.\, 10^6\,\xi_1 = x$
so if $\xi_1 = 1$ then $x = 1080$ km.

Now $t' = t - \dfrac{x}{u_b}$ so when the front has arrived at $\xi_{10} = 1$, then $\tau_o = 33.3$ on the place $\xi_1 = 0$

The course of the temperature as a function of the place at one and the same moment is represented by:

$$\tau = 33.3\,(\xi_{10} - \xi_1)$$

These curves have been drawn for $\xi_{10} = 1$ and $\xi_{10} = 2$ in Fig. 3.6.1.

The transformation of the moisture content under the conditions given by us in the beginning of this section is not interesting because we have assumed that $q_e = 0$ and consequently no change of the moisture content sets in.

As soon as $q_e \neq 0$, the whole preceding calculation is modified because we must then use instead of equation (3.6.2):

$$q_s = q_a + q_e \tag{3.6.28}$$

When equation (3.4.2) may be applied the fraction $q_e/q_a = j$ is practically constant and the preceding calculation holds, if we define the dimensionless time and depth as follows:

$$\tau = \frac{\alpha^2(1+j)a}{\lambda_s^2}\,t' \text{ and } \zeta = \frac{\alpha(1+j)\,z}{-\lambda_s} \tag{3.6.29}$$

Now there is a complete analogy between temperature change and humidity change. When the mixing ratio is constant with height, which is quite plausible for an adiabatic atmosphere, we may then write:

$$\frac{\chi_\infty - \chi_o}{\chi_\infty(0) - \chi_o} = \frac{\theta_\infty - \theta_o}{\theta_\infty(0) - \theta_o} = \vartheta_1 \tag{3.6.30}$$

and therefore also use (3.6.27) for the calculation of the moisture transformation.

3.7 Transformation of the warm air mass

The transformation of the warm air mass is much harder to calculate than of the cold air mass. The stable atmosphere makes it impossible to distinguish a transitional layer, in which the resistance to transfer of heat and mass is principally localized.

According to **2.10** as soon as $Sn < 0$ from a specified height where $k\,\zeta\,Sn$ attains a critical value, turbulence must subside and consequently the heat resistance must greatly increase. Now this only holds for the surface layer and not above it. Owing to the gradual direction change of the wind above the surface layer, the turbulence, however, will subside less swiftly, as might be expected according to the equations (2.8.7) or (2.8.10).

Furthermore it is of importance to remark that, the higher the level at which $k\,\zeta\,Sn$ attains the critical value the slower the subsidence of the turbulence will take place. A consequence is that a turbulence originated already before, may still make itself felt for a long time in the field where $k\,\zeta\,Sn < (k\,\zeta\,Sn)_{cr}$ owing to which the interaction in this field becomes entirely vague, at any rate not to be treated with the expedients developed during this investigation. From the above it appears how difficult an analytic treatment of the transformation of the warm air mass is.

A consolation, however, is that the total transfer in the warm air mass is of much less importance than that of the cold air mass. Observations, moreover, give rise to the supposition that, if only $|Sn|$ is small enough ($|Sn| \leqslant 10^{-4}$) the calculation can be made in the same way as for the cold air mass. The transfer will then be of some importance only in case of a strong wind.

3.8 Some remarks to conclude with

It need not be pointed out that the preceding considerations about transformation are anything but complete. We have imposed restrictions upon ourselves by taking for granted special boundary conditions and initial conditions as well as by introducing simplifying suppositions, such as $\alpha =$ constant; $z_o =$ constant; stationary flux, etc. Extensions of the initial and boundary conditions, as have been given in the calculations, practically only bring new mathematical difficulties, but, apart from the results, offer few new physical perspectives. Nevertheless it will be of importance for practical purposes, if an investigation could be made into all calculable combinations, in order to see in how far changes in frequently occuring situations can be calculated quantitatively with a fair approximation. It is of importance besides looking for situations in the atmosphere in which the simple initial and boundary conditions, as given in the preceding calculations, have reasonably been complied with. So that it is possible by means of observations to collect data of the poorly known physical quantities α, $\varkappa$, λ_s and a. Finally we will discuss a few physical phenomena, which assert themselves in case of transformation.

3.8.1 *The influence of the increased heat and mass transfer near the coast line*

In the sections **3.5** and **3.6** we have continually made use of equation (3.3.8), which means that $f(Sn) =$ constant, see equation (3.3.7). When cold continental air passes the coastline and suddenly comes into contact with a warm sea level, the lowest layers of this air mass will become particularly unstable and Sn will assume a relatively great value. Owing to this α, as might be expected, will become greater than is in agreement with equation (3.3.8). A consequence of this again is, that especially for small values of x the calculations of **3.5** will not be correct. The error, however, does not become so serious as might be expected from the deviation of α with equation (3.3.8), because the sea level temperature does not remain quite constant owing to the increased heat transfer. For there will also be a certain heat resistance from the surface to the deeper layers of the water and it will continue to play an ever greater part according as the resistance from the surface to the air becomes smaller.

3.8.2 *Change of the pressure field on account of the transformation of air masses*

The most important aspect of the transformation of air masses appears to be the change of the pressure field caused by it. Bleeker (1950) has devoted some qualitative considerations to this subject. His considerations are based on a differential heat liberation from the earth's surface to the air. Considering from this point of view e.g. the cold wave above the sea, it appears that, if the isobars originally were straight and at right angles to the coastline, a curve of these isobars sets in, which is anticyclonal at the earth's surface whereas at a greater height cyclonally curved isobars arise.

The quantitative calculation of this phenomenon is exceedingly complicated, because there apparently exists an interaction between heat absorption, wind velocity and wind direction of the air mass. So we should not start from the assumption that the original air flow remains unchanged during the transformation, a supposition which has always been tacitly made use of in the calculations.

3.8.3 *The appearance of inversion layers during the transformation of the cold air mass*

In the calculations of **3.5** has been taken for granted without going further into the matter, that the warmed air rises to a level where the original air has the same potential temperature. Actually the air will rise somewhat more, because the warmed air will posses a certain climbing velocity and therefore a certain kinetic energy, which at the specified level has not disappeared without more ado. As the warmed air will continue to rise somewhat above this level, this air will get into surroundings where it is relatively cold itself. A consequence of this again is that an inversion is formed.

The appearance of an inversion in the cold air mass can easily give rise to a misunderstanding in connection with localizing at greater heights the cold front preceding the cold wave.

Fig. 3.8.1 gives a schematic representation of a vertical section of the atmosphere as it will appear when the cold air has been warmed up in the lowest

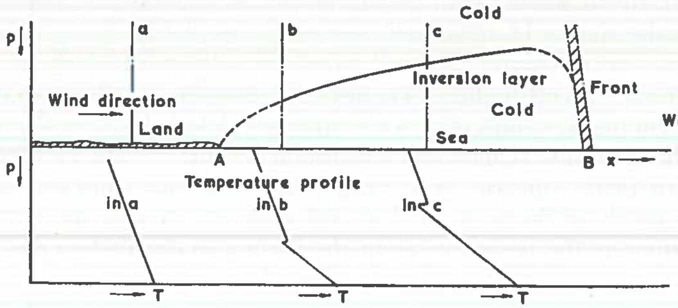


Fig. 3.8.1

layers behind the front along a certain stretch AB. The movement of the front is supposed in the x-direction.

The apearance of an inversion will have no influence worth mentioning on the calculations of **3.5**.

SUMMARY

This thesis gives an account of the transfer of energy and mass at the surface of the earth and of some of its consequences to the atmosphere. A preliminary analysis concerning the phenomena at the surface supplies a classification of the different types of heat balances. This classification may be of some importance in describing the climate. Beside the surface the atmospheric surface layer extending till ca 25 m is of fundamental importance to the transfer of heat and mass. Chapter **2** also discusses in which way the structure of the surface layer is connected with the transfer of heat and momentum at the surface and with the roughness of the surface. An analysis is given of the connection between the basic equations and the so-called equations of eddy transfer whereby it appears to be possible to deduce in a more or less exact manner the latter equations from the former. The different states of the surface layer are summarized in a diagram, in which a dimensionless number, the so-called structure number is used. This structure number is deduced from the stability analysis of Richardson with the aid of a similarity consideration. Further by introducing a modification of Lettau's theory of turbulence, a more or less complete description of the surface layer has been obtained. Finality, however, has not yet been reached in this field. Particularly the transition from turbulent forced convection on the one hand to free convection in the unstable area and on the other to laminar flow in the stable area requires closer investigation. Especially the unstable atmosphere is suited to an extension of the investigation, because in this case nearly the whole resistance to heat and mass transfer is localized in the surface layer. In the unstable atmosphere it is possible to introduce a coefficient of heat transfer and a coefficient of mass transfer in order to facilitate the calculation of the transformation of air masses. So especially the outbreak of cold air is suited to an analytical treatment. The transformation of the cold air mass above the sea is the simplest calculation, because the sea has a homogeneous level and an extremely large heat capacity, so that as a first approximation the sea level temperature may be considered as a constant. The transformation of the cold air mass above land is much more difficult to calculate. A treatment of it is given with some simplifying assumptions. No calculations are given for the warm air mass because in this case the surface layer may no longer be considered as a transition layer. It is necessary therefore to make an analysis of the whole boundary layer, which extends till ca 1000 m. Finally some consequences of the applied simplifications are discussed.

SAMENVATTING

In dit proefschrift worden de energieuitwisseling aan het aardoppervlak en enige van de consequenties, die dit met zich brengt voor de atmosfeer, behandeld. Een inleidende analyse betreffende de verschijnselen aan het aardoppervlak zelf levert een classificatie van de verschillende warmtebalanstypen op. Deze typologie kan mogelijk van betekenis zijn bij de klimaatbeschrijving. Naast het oppervlak is de atmosferische grenslaag welke zich uitstrekt tot ca 25 m hoogte van fundamentele betekenis voor de warmte en stofuitwisseling. Besproken wordt op welke wijze de structuur van deze grenslaag samenhangt met de uitwisseling van warmte en impuls aan het oppervlak en met de ruwheid van het oppervlak. Hierbij wordt een analyse gegeven van de samenhang tussen de grondvergelijkingen en de zgn. uitwisselingsvergelijkingen en hoe deze laatste min of meer exact uit de eerste afgeleid kunnen worden. Het blijkt mogelijk te zijn om de verschillende toestanden die in de grenslaag voor kunnen komen samen te vatten in een diagram, waarbij gebruik gemaakt wordt van een dimensieloos getal, het zgn. structuurgetal. Dit structuurgetal is voortbouwend op de stabiliteits-analyse van Richardson, afgeleid met behulp van een gelijkvormigheidsbeschouwing. Door voorts een modificatie van Lettau's turbulentie theorie te geven, blijkt het mogelijk tot een min of meer volledige beschrijving van de grenslaag te komen, hoewel op dit punt het laatste woord nog niet gezegd is. De overgangen van turbulente gedwongen convectie enerzijds naar vrije convectie in het instabiele gebied en anderzijds naar laminaire stroming in het stabiele gebied vereisen in het bijzonder een nader onderzoek. De instabiel gelaagde atmosfeer leent zich speciaal voor verder onderzoek, omdat hierbij vrijwel de gehele overgangsweerstand voor warmte- en stoftransport zich in de grenslaag bevindt. Het blijkt mogelijk om bij de instabiele atmosfeer gebruik te maken van een warmteovergangscoefficient en een stofovergangscoefficient voor de berekening van de transformatie van luchtsoorten. De koude inval leent zich dus in het bijzonder voor een analytische behandeling. Het eenvoudigst te behandelen is de koude inval boven de zee, omdat de zee een homogeen oppervlak en een bijzonder grote warmtecapaciteit heeft, waardoor als eerste benadering de oppervlakte temperatuur constant beschouwd mag worden. Belangrijk gecompliceerder is de koude inval boven land, waarvoor een berekening gegeven is met behulp van enige vereenvoudigende aannamen. Van de transformatie in de warme massa zijn geen berekeningen gegeven, omdat hierbij de grenslaag niet meer als overgangslaag beschouwd mag worden, waardoor een analyse van de gehele wrijvingslaag welke zich tot ca 1000 m uitstrekt vereist is. Tenslotte worden nog enige consequenties van de gemaakte vereenvoudigingen besproken.

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