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ON THE BEHAVIOUR OF GRAVITY WAVES IN A TURBULENT MEDIUM, WITH APPLICATION TO THE DECAY AND APPARENT PERIOD INCREASE OF SWELL

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Summary

With respect to the decay of gravity waves by turbulence a distinction is made between "internally generated" turbulence, which is generated by the waves themselves, and "externally generated" turbulence, which exists independently of the wave motion. The mechanism of the interaction between "external" turbulence and surface waves is then analyzed and a formula for an eddy viscosity coefficient is derived that shows a dependence of eddy viscosity on the wave-length.

On the basis of Richardson's 4/3-power rule for horizontal eddy exchange processes, the decay and apparent period increase of a swell, having a given initial spectral energy distribution, and suffering no loss of energy by dispersion, is investigated theoretically. The results are compared with observations and a numerical formula for the vertically averaged eddy viscosity, operative in the decay of swell, is found by that comparison.

1. Introduction

In an earlier publication the present author and R. Dorrestein (1950) outlined a theory of the damping effect of turbulence on swell waves, which was based on Richardson's "4/3-power law" of turbulent exchange. K. F. Bowden (1950) has questioned the applicability of this 4/3-power law to the decay of swell waves and developed a theory of decay of swell by turbulence from an entirely different approach. It has already been remarked at an earlier occasion (Groen 1951) that these two different treatments of the problem may be considered as more or less complementary to each other, since they apply to two different kinds of turbulence which may act on waves. Indeed, while Bowden's turbulence is generated by the waves themselves and is, therefore, what we may call internally generated turbulence or, shortly, "internal turbuence", the turbulence which Groen and Dorrestein have considered may be called "external turbulence", since it is supposed to exist independently of the wave motion, which it damps; it may be due to direct wind influences, present or past, or to currents in the sea.

The purpose of the present study is to get a clearer insight into the mechanism of the damping effect of "*external*" turbulence on free water waves. It will be shown in what follows, that this mechanism is such as to make it possible, for a given single wave system and a given horizontally homogeneous field of external turbulence, to define an eddy viscosity coefficient, which has a definite physical meaning. Its value depends on the *scale* of the field of motion in a similar way as, according to Richardson's "law", an eddy diffusivity depends on the scale of the pattern of the diffusing matter. Although it might *apriori* be supposed – as Bowden (l.c.) suggested – that this scale, in the case of water waves, would be determined by the height of the waves as well as by their wave length, it will turn out that, in the case considered here, our eddy viscosity is determined by the wave length alone, if the field of turbulence is given.

Damping by turbulence is not only responsible for the decay but also for an effective period increase of the waves with increasing travel distance, since the selectivity of the damping, acting in favour of the longer components of the wave mixture, makes the wave length of the dominant wave components increase as the damping goes on (Groen and Dorrestein l.c.). For treating the effective period increase theoretically, it is therefore better, physically speaking, to consider the wave *spectrum* than to treat the "significant waves" as if they were single waves having a continually increasing period, as Bowden has done, following Sverdrup and Munk (1947).

2. Qualitative interpretation of the dependence of eddy viscosity on wave length

Experience has shown (see f.i. Richardson 1926, Burke 1946 and Stommel 1949) that the diffusion of a cluster of particles in a turbulent medium goes on in such a way that the effective eddy diffusivity appears to increase with increasing extension of the cluster. This may be understood in the following way. If the cluster is small, only small eddies contribute to the spreading of the cluster, since the larger eddies give rise only to a transport of the cluster, more of less as a whole, which is not called diffusion. If the cluster is large, large eddies too contribute to its spreading, and more effectively so than do the smaller ones, since the larger eddies have greater eddy velocities. Hence, the effective diffusivity is greater in the latter than in the former case.

In a similar way it may be seen how a given field of turbulence, having eddies of various sizes, acts on waves of different wave lengths in different ways. To start with, eddy friction might formally be treated as an eddy diffusion of momentum and a field of wave motion as a field of positive and negative concentrations of, say, horizontal momentum. Thus, it is obvious that the wave length is analogous to the diameter of a diffusing band of matter whereas the wave amplitude determines only the maximum concentrations of momentum, which are analogous to the maximum concentration within the diffusing band of matter, and will, consequently, not have to be expected to enter into the 4/3-power law, if it holds for eddy friction.

We may, however, consider things in a less formal way. It is clear that eddies which are small in comparison to the wave length cause an internal friction and, consequently, a loss of ordered wave energy, or a decay of the waves. Eddies, however, which are large as compared to the wave length, only cause local changes of phase velocity and group velocity. Short waves that have passed through such an eddy may, after leaving its sphere of action, have undergone a refraction, but will not, on the average, have lost energy thereby. To long waves however, eddies of such a size, if small in comparison to the wave length, will again act as turbulence elements that cause an effective internal friction; so, they contribute to the damping of those waves, and more effectively so than do the smaller eddies, because the larger eddies have greater eddy velocities.

In this way, it is seen, at least qualitatively, how, in the case of waves being damped by external turbulence, an eddy viscosity coefficient may depend on wave length in a similar way as, in the case of diffusion of spreading dots or bands of matter, the effective eddy diffusivity depends on the diameter of such concentrations of matter. It remains to be shown that it is indeed possible to define such an eddy viscosity coefficient, or, shortly, "eddy viscosity", as a physical quantity which, for any given single wave system and a given field of turbulence, has a definite value. This will, for a simple sort of turbulence, be shown in the next two paragraphs.

3. Kinematic analysis

Suppose a field of turbulence is given, which is described by turbulent velocity components $\overset{*}{u}\,(x, y, z, t)$, $\overset{*}{v}\,(x, y, z, t)$ and $\overset{*}{w}\,(x, y, z, t)$ in the x-, y- and z-directions, respectively; the z-axis is supposed to point vertically upwards. This field of external turbulence is supposed to be horizontally homogeneous and stationary, which means that the time-averages of $\overset{*}{u}^2$, $\overset{*}{v}^2$, $\overset{*}{w}^2$ are independent of x and y, whereas the horizontally averaged values $\overline{\overset{*}{u}^2}$, $\overline{\overset{*}{v}^2}$, $\overline{\overset{*}{w}^2}$ are independent of t. Now, suppose further that surface waves have run into this field of turbulence. The total velocity field is now described by the velocity components $U(x, y, z, t)$, $V(x, y, z, t)$, $W(x, y, z, t)$. We suppose that the waves, apart from the perturbations and distortions that are caused by the turbulence, are plane harmonic deep water waves running in the x-direction; this means that

$$\overline{U} = U_0\, e^{kz} \cos(kx - \omega t), \tag{1}$$

$$\overline{W} = U_0\, e^{kz} \sin(kx - \omega t), \tag{2}$$

$$\overline{V} = 0, \tag{3}$$

where the horizontal bar over U, V and W now means averaging over y. In the following we shall continually use such "y-averages", since the undisturbed wave motion velocity field is independent of y; we shall design these averages by a horizontal bar.

For U, V and W we may now write

$$U = \overline{U} + u,\; V = v,\; W = \overline{W} + w, \tag{4}$$

where

$$u = \overset{*}{u} + \grave{u},\; v = \overset{*}{v},\; w = \overset{*}{w} + \grave{w}. \tag{5}$$

It is clear, that u and w are not indentical with the "external" turbulent velocity components $\overset{*}{u}$ and $\overset{*}{w}$, since the latter displace water particles, which, by inertia, take their wave motion velocities with them to their new places, so that a „secondary" velocity perturbation field $\grave{u}$, $\grave{w}$ is induced, which is additional to the "primary" (external) field $\overset{*}{u}$, $\overset{*}{v}$, $\overset{*}{w}$ and which depends on the wave motion velocity field $\overline{U}$, $\overline{W}$; in the y-direction no additional velocity perturbation is induced, since the wave motion velocities have no components in that direction ($\overline{V} = 0$).

The distinction between u, w and $\overset{*}{u}$, $\overset{*}{w}$ becomes most clear, if we consider the case of a primary turbulence which is purely horizontal: $\overset{*}{w} = 0$. In this case $w \neq 0$, nevertheless, because horizontal displacements of water particles change the W-field, so that $W \neq \overline{W}$, or $w \neq 0$; in this case we have: $w = \grave{w}$. We shall start our investigation with this simplest case.

If the distortions of the velocity field by turbulent displacements were only small as compared to the wave length, we might write: $\grave{u} = -\,\xi\, \partial \overline{U}/\partial x$,

$\overset{\circ}{w} = - \xi\, \partial \overline{W}/\partial x$, where $\xi = \xi\,(x, y, z, t)$ is a measure of the displacement in the positive x-direction. More generally, however, we may write:

$$U(x, y, z, t) - \overset{*}{u}(x, y, z, t) = \overline{U}(x - \xi_u, y, z, t),$$

$$W(x, y, z, t) - \overset{*}{w}(x, y, z, t) = \overline{W}(x - \xi_w, y, z, t),$$

whence:

$$\overset{\circ}{u} = \overline{U}(x - \xi_u, y, z, t) - \overline{U}(x, y, z, t), \tag{6}$$

$$\overset{\circ}{w} = \overline{W}(x - \xi_w, y, z, t) - \overline{W}(x, y, z, t). \tag{7}$$

The "free path lengths" ξ_u and ξ_w are now *formally defined* by (6) and (7) and may therefore differ from each other. They are functions of x, y, z and t and are in some way correlated to $\overset{*}{u}\,(x, y, z, t)$. Where $\overset{*}{u}$ is positive, ξ_u and ξ_w will more probably be positive than negative, which means that $\overline{\overset{*}{u}\xi_u}$ and $\overline{\overset{*}{u}\xi_w}$ will be positive.

As a general property of the field of the turbulent velocity components u, w we may state that, since they arise from the cooperative action of the primary turbulence field $\overset{*}{u}$, $\overset{*}{v}$, which is homogeneous with respect to x, and the wave motion field $\overline{U}$, $\overline{W}$, which is periodic in x with wavelength L, any *mean* quantity, characteristic of the field u, w, such as $\overline{u^2}$, $\overline{uw}$ and the like, will also be periodic in x with wavelength L, if not independent of x or *zero*. We shall make use of this general property of the field u, w farther on.

4. The energy equation and the eddy viscosity

Now we shall develop the energy equation for our velocity-field. We do this in the classical way, following Reynolds (see f.i. Lamb 1932, p. 675), by starting from the equations of motion of an incompressible fluid in the following form:

$$\frac{\partial U}{\partial t} = -\frac{1}{\varrho}\frac{\partial p}{\partial x} - \frac{\partial U^2}{\partial x} - \frac{\partial UV}{\partial y} - \frac{\partial UW}{\partial z},$$

$$\frac{\partial V}{\partial t} = -\frac{1}{\varrho}\frac{\partial p}{\partial y} - \frac{\partial UV}{\partial x} - \frac{\partial V^2}{\partial y} - \frac{\partial VW}{\partial z},$$

$$\frac{\partial W}{\partial t} = -\frac{1}{\varrho}\frac{\partial p}{\partial z} - \frac{\partial UW}{\partial x} - \frac{\partial VW}{\partial y} - \frac{\partial W^2}{\partial z} - g,$$

where $p =$ pressure and $\varrho =$ density, which is assumed to be a constant; molecular viscosity and extraneous forces, other than gravity (g), have been left out of consideration; the fluid is supposed to be incompressible. By substituting (4), averaging and using (3), we find

$$\frac{\partial \bar{U}}{\partial t} = -\frac{1}{\varrho}\frac{\partial \bar{p}}{\partial x} - \frac{\partial \bar{U}^2}{\partial x} - \frac{\partial \overline{u^2}}{\partial x} - \frac{\partial \bar{U}\bar{W}}{\partial z} - \frac{\partial \overline{uw}}{\partial z}, \tag{8}$$

$$\frac{\partial \bar{W}}{\partial t} = -\frac{1}{\varrho}\frac{\partial \bar{p}}{\partial z} - \frac{\partial \bar{U}\bar{W}}{\partial x} - \frac{\partial \overline{uw}}{\partial x} - \frac{\partial \bar{W}^2}{\partial z} - \frac{\partial \overline{w^2}}{\partial z} - g. \tag{9}$$

In deriving (8) and (9) we have used the fact that, according to the adopted wave model and the horizontal homogeneity of the primary turbulence field, all averages are independent of y.

Using the continuity equation for the U, W-field,

$$\frac{\partial \bar{U}}{\partial x} + \frac{\partial \bar{W}}{\partial z} = 0,$$

we transform (8), (9) into

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x} + \bar{W}\frac{\partial}{\partial z}\right)\bar{U} = -\frac{1}{\varrho}\frac{\partial \bar{p}}{\partial x} - \frac{\partial \overline{u^2}}{\partial x} - \frac{\partial \overline{uw}}{\partial z}, \tag{10}$$

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x} + \bar{W}\frac{\partial}{\partial z}\right)\bar{W} = -g - \frac{1}{\varrho}\frac{\partial \bar{p}}{\partial z} - \frac{\partial \overline{uw}}{\partial x} - \frac{\partial \overline{w^2}}{\partial z}. \tag{11}$$

Multiplying (10) by $\bar{U}$, (11) by $\bar{W}$ and adding, we obtain the energy equation

$$\frac{1}{2}\frac{d}{dt}(\bar{U}^2 + \bar{W}^2) = -g\bar{W} - \frac{1}{\varrho}\left(\bar{U}\frac{\partial \bar{p}}{\partial x} + \bar{W}\frac{\partial \bar{p}}{\partial z}\right) + \\ - \bar{U}\left(\frac{\partial \overline{u^2}}{\partial x} + \frac{\partial \overline{uw}}{\partial z}\right) - \bar{W}\left(\frac{\partial \overline{uw}}{\partial x} + \frac{\partial \overline{w^2}}{\partial z}\right), \tag{12}$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x} + \bar{W}\frac{\partial}{\partial z}.$$

The first two terms of the right hand side of (12) represent the work done by gravity and by pressure forces per unit of time, per unit of mass. In small amplitude deep-water waves, as described by (1), (2), (3), these terms cancel each other at any point, as the theory of such waves shows. (In shallow water waves, they do so at any level, when integrated over one wave length in the x-direction). So, we are left with

$$\frac{1}{\varrho}\frac{dE'_k}{dt} = -\bar{U}\left(\frac{d\overline{u^2}}{\partial x} + \frac{\partial \overline{uw}}{\partial z}\right) - \bar{W}\left(\frac{\partial \overline{uw}}{\partial x} + \frac{\partial \overline{w^2}}{\partial z}\right),$$

where $E'_k = \frac{1}{2}\varrho\,(\bar{U}^2 + \bar{W}^2)$ is the *kinetic* wave energy *per unit volume*. We integrate both sides of this equation horizontally over one wave-length, from $x = x_0$ to $x = x_0 + L = x_0 + 2\pi/k$, and vertically from $z = -\infty$ to the

$$-\int_0^L\int_{-\infty}^0 \frac{\overline{dE_k'}}{dt}\,dxdz = -\int_0^L\int_{-\infty}^0 \varrho \left\{ \overline{\mathring{u}(\cos k\xi - 1)}\,\frac{\partial \overline{U}^{2}}{\partial x} + \overline{\mathring{u}(\cos k\xi - 1)}\,\frac{\partial \overline{W}^{2}}{\partial x} + \right.$$

$$\left. - 2\overline{\mathring{u}k^{-1}\sin k\xi}\left(\frac{\partial \overline{U}}{\partial x}\right)^2 - 2\overline{\mathring{u}k^{-1}\sin k\xi}\left(\frac{\partial \overline{W}}{\partial x}\right)^2 \right\} dxdz =$$

$$= \int_0^L\int_{-\infty}^0 \varrho\,\overline{\mathring{u}k^{-1}\sin k\xi}\left\{ 2\left(\frac{\partial \overline{U}}{\partial x}\right)^2 + 2\left(\frac{\partial \overline{W}}{\partial x}\right)^2 \right\} dxdz =$$

$$= \int_0^L\int_{-\infty}^0 \varrho\,K_h(z)\left\{ 2\left(\frac{\partial \overline{U}}{\partial x}\right)^2 + 2\left(\frac{\partial \overline{W}}{\partial x}\right)^2 \right\} dxdz, \tag{18}$$

where we have taken the averages of the forms with $\mathring{u}$ and ξ_u to be equal to the corresponding averages with $\mathring{u}$ and ξ_w and to depend on z only, since they are supposed to be wholly determined by the primary turbulence field.

If we compare (18) with the classical formula for the internal energy loss of a two-dimensional field of motion by molecular viscosity (see f.i. Lamb, l.c., p. 580):

$$-\iint \frac{\overline{dE_k'}}{dt}\,dxdz = \iint \varrho\nu \left\{ 2\left(\frac{\partial \overline{U}}{\partial x}\right)^2 + 2\left(\frac{\partial \overline{W}}{\partial z}\right)^2 + \left(\frac{\partial \overline{U}}{\partial z} + \frac{\partial \overline{W}}{\partial x}\right)^2 \right\} dxdz, \tag{19}$$

we see that

$$K_h = \overline{\mathring{u}k^{-1}\sin k\xi} \tag{20}$$

plays the role of an effective kinematic viscosity coefficient, the „eddy viscosity"; the index h reminds us of the fact, that the eddies of the primary turbulence field that gives rise to this eddy viscosity are essentially *horizontal*. The same fact causes a term corresponding to $2\,(\partial W/\partial z)^2$ of (19) to be absent in (18), while only part of the term $(\partial U/\partial z + \partial W/\partial x)^2$ of (19) is present in (18), where we have:

$$2\left(\frac{\partial \overline{W}}{\partial x}\right)^2 = \frac{\partial \overline{W}}{\partial x}\left(\frac{\partial \overline{U}}{\partial z} + \frac{\partial \overline{W}}{\partial x}\right).$$

Considering formula (20), we see that, if ξ would everywhere be sufficiently small as compared to $L/2\pi = 1/k$, the expression for K_h would become

$$K_h = \overline{\mathring{u}\xi}, \tag{21}$$

which is a well known formula for the eddy viscosity in a velocity field where the gradients of the mean velocity are sufficiently constant over a space interval of the order of magnitude of the "mixing length". Since, however, $k\xi$ needs not to be small in our case, we shall have to accept formula (20).

From the above analysis we see that, for the sort of turbulence considered here, an eddy viscosity coefficient exists for any simple harmonic wave system and that this "eddy viscosity" depends, within any given level (z), on the wave length only (if the field of turbulence is given, as we have supposed).

Since, for deep water waves, the integrand of (18) does not depend on x and neither does E'_k, we may drop the integration over x in (18). Furthermore, we define:

$$\frac{\overline{dE}}{dt} = \int_{-\infty}^{0} \frac{\overline{dE'_k}}{dt}\, dz,$$

where E may be called the mean *total* wave energy *per unit area* of the sea surface (since any loss of wave energy is supposed to originate as a loss of *kinetic* wave energy). We have:

$$-\frac{dE}{dt} = \varrho \int_{-\infty}^{0} K_h \left\{ 2\left(\frac{\partial \overline{U}}{\partial x}\right)^2 + 2\left(\frac{\partial \overline{W}}{\partial x}\right)^2 \right\} dz. \tag{22}$$

The bar over d/dt has been dropped now, although the meaning of this differential operator has remained the same as it was.

Up till now we have only considered horizontal turbulence, giving rise only to horizontal exchange of momentum. If there is also vertical turbulent exchange, characterized by a "vertical" eddy viscosity K_z, the energy equation would be:

$$-\frac{dE}{dt} = \varrho \int_{-\infty}^{0} \left[K_h \left\{ 2\left(\frac{\partial \overline{U}}{\partial x}\right)^2 + \frac{\partial \overline{W}}{\partial x}\left(\frac{\partial \overline{U}}{\partial z} + \frac{\partial \overline{W}}{\partial x}\right) \right\} + \right.$$

$$\left. + K_z \left\{ 2\left(\frac{\partial \overline{W}}{\partial z}\right)^2 + \frac{\partial \overline{U}}{\partial z}\left(\frac{\partial \overline{U}}{\partial z} + \frac{\partial \overline{W}}{\partial x}\right) \right\} \right] dz =$$

$$= \varrho \int_{-\infty}^{0} (K_h + K_z) \left\{ 2\left(\frac{\partial \overline{U}}{\partial x}\right)^2 + 2\left(\frac{\partial \overline{W}}{\partial x}\right)^2 \right\} dz =$$

$$= n\varrho \int_{-\infty}^{0} K_h \left\{ 2\left(\frac{\partial \overline{U}}{\partial x}\right)^2 + 2\left(\frac{\partial \overline{W}}{\partial x}\right)^2 \right\} dz, \tag{23}$$

where, for sea waves,

$$1 < n < 2, \tag{24}$$

since, in general, $K_z < K_h$ in the sea.

5. Dependence of the eddy viscosity on wave length

We shall now consider, for a moment, formula (20) for K_h. For any given value of the wave number $k = 2\,\pi/L$ the contributions to the average of $\mathring{u} k^{-1} \sin k\xi$ will mainly come from combinations of $\mathring{u}$ and ξ having $|\xi|$ not much larger than $\pi/k = L/2$, since contributions from larger ξ-values, where $\sin k\xi$ oscillates between -1 and $+1$, will tend to cancel each other to a large extent. In this qualitative way we may see that K_h will have larger values for large wave lengths than for small wave lengths. A more quantitative evaluation of the dependence of K_h on L can only be made if we know more about the turbulence spectrum. We shall confine ourselves here to a few further qualitative remarks. If we put

$$\overline{\mathring{u}k^{-1}\sin k\xi} = \mathring{u}_l\, l,$$

where l denotes a sort of mean free path or mixing length, we may, on account of what has been said above and on dimensional grounds, take l to be proportional to L. Now, if we suppose a statistical equilibrium to prevail between the eddies of various sizes, we have, according to theory (see f.i. von Weizsäcker 1948): $\mathring{u}_l \sim l^{1/3}$, so that $K_h \sim l^{4/3} \sim L^{4/3}$, in our case.

For the surface layer of the sea the validity of Richardson's 4/3-power rule for horizontal turbulence has more or less been confirmed by observations on eddy diffusion (Richardson and Stommel 1948, Stommel 1949, Inoue 1950). We shall therefore assume it to hold for our eddy viscosity too, in the surface layer. It is, however, not probable that it holds also at subsurface levels, unless the field of horizontal turbulence be threedimensionally homogeneous, since the horizontal eddies at subsurface levels have probably a stronger coupling to the eddies of overlying water layers than to each other, so that a statistical equilibrium should not be expected to prevail within each separate level. In fact, we shall assume that the dependence of K_h on z may be expressed in the following form:

$$K_h = L^{4/3}\,\varphi(2\pi z/L), \tag{25}$$

$\varphi(2\pi z/L)$ being a monotonously increasing function of z. Formula (25) is based on the idea that the larger eddies, which are operative for the longer waves, diminish less rapidly with increasing depth than the smaller eddies, which are operative for the shorter waves. As we shall see presently, assuming this form is aequivalent to assuming that the effective, *vertically averaged* horizontal eddy viscosity is proportional to $L^{4/3}$.

6. Evaluation of the energy loss and apparent period increase of deep water waves on the basis of a supposed validity of the 4/3-power rule

Substituting (1) and (2) in (23) and using (25), we find:

$$-\frac{dE}{dt} = 2n\varrho\left(\frac{2\pi}{L}\right)^2 U_0^2 \int_{-\infty}^{0} K_h\, e^{2kz}\, dz = 8\,\pi^2 L^{-2}\, n\varrho\, U_0^2 L^{4/3} \int_{-\infty}^{0} \varphi(kz)\, e^{2kz}\, dz =$$

$$= 8\,\pi^2 L^{-2}\, nf L^{4/3} E = 8\,\pi^2 L^{-2/3}\, nf E, \tag{26}$$

where f is a weighted average of $\varphi\,(kz)$, defined by

$$\int_{-\infty}^{0} \varphi(kz)\, e^{2kz}\, dz = f \int_{-\infty}^{0} e^{2kz}\, dz,$$

and where E denotes the mean total wave energy (= twice the mean kinetic energy) per unit area of the sea surface. Using the above formula means using a vertically averaged value of $K_h(L,z)$, which we shall call $K^*(L)$ and which is equal to $f L^{4/3}$. In order to get an idea of the relation between K^* and the surface value $K_h(L, 0) = \varphi(0)\, L^{4/3}$, we might assume, as a working hypothesis:

$$\varphi(2\,\pi z/L) = \varphi(0)\, e^{m\,.\,2\pi z/L}, (m > 0)$$

which would imply that

$$f = \frac{2\,\varphi(0)}{2+m}, \text{ or: } K_h(L, 0) = \left(1 + \frac{m}{2}\right) f L^{4/3} > f L^{4/3} = K^*(L). \qquad (27)$$

We shall now apply formula (26) to the energy loss of free, monochromatic surface waves traveling over deep water with the group velocity $\frac{1}{2}\, C = \frac{1}{2}(L\, g/2\pi)^{1/2}$. Then we have, according to (26),

$$\frac{dE_L}{dx} = \frac{2}{C}\,\frac{dE_L}{dt} = -\frac{4(2\,\pi)^{5/2}\, n f E_L}{g^{1/2}\, L^{7/6}},$$

so that

$$E_L(x) = E_L(0)\, e^{-\alpha\, x\, L^{-7/6}}, \qquad (28)$$

where $\alpha = 4\,(2\pi)^{5/2}\, n f g^{-1/2}$; the subscript L denotes that the energy of waves with wave-length L is meant. The energy is taken per unit area.

From here on we follow the same reasoning as used by Groen and Dorrestein (1950). We take $L^{7/6} = \lambda$ as the independent variable and describe a continuous wave energy spectrum by means of an energy function $E\,(\lambda, x)$, such that $E\,(\lambda, x)\, d\lambda$ is the wave energy belonging to the λ-interval λ to $\lambda + d\lambda$. As an analytical representation of the initial wave spectrum we assume the following form:

$$E(\lambda, 0) = A\, \lambda^{-a}\, e^{-b/\lambda}, \qquad (29)$$

where a and b are positive constants, as yet unknown. According to (28), then:

$$E(\lambda,x) = A\, \lambda^{-a}\, e^{-(b+\alpha x)/\lambda}. \qquad (30)$$

We shall assume the square of the "significant height" of the waves to be proportional to the total energy of the spectrum. The significant wave length or period, however, needs not to correspond to the value of λ where $E\,(\lambda, x)$ has its maximum. According to results of statistical investigations (see Pierson *et al.* 1953), we may take the significant period to be $\sqrt{1/2}$ times the period corresponding to the wave frequency at which the energy spectrum, expressed *as a function of frequency*, has its maximum.

Using $\omega = 2\pi/T \sim \lambda^{-3/7}$, instead of the frequency $(1/T)$ itself, we have:

$$E(\lambda(\omega), x)\, d\lambda = E(\lambda(\omega), x)\frac{d\lambda}{d\omega}\, d\omega = \text{const.}\, E(\lambda, x)\, \lambda^{10/7}\, d\omega = E^*(\omega, x)\, d\omega.$$

Consequently, the maximum of the spectrum function $E^*(\omega, x)$ as a function of frequency corresponds to the maximum of $E(\lambda, x)\,\lambda^{10/7}$. According to (30), this maximum is found by solving the equation

$$(-a + 10/7)\,\lambda^{-1} + (b + \alpha x)\,\lambda^{-2} = 0$$

for λ, which yields:

$$\lambda = \frac{b + \alpha x}{a - 10/7}.$$

The significant period T_x becomes now:

$$T_x = \left(\frac{\pi}{g}\right)^{1/2} \cdot \left(\frac{b + \alpha x}{a - 10/7}\right)^{3/7} = T_0\left(\frac{b + \alpha x}{b}\right)^{3/7}, \tag{31}$$

where

$$T_0 = \left(\frac{\pi}{g}\right)^{1/2} \left(\frac{b}{a - 10/7}\right)^{3/7} \tag{32}$$

is the initial significant period.

It should be borne in mind that a tacit assumption underlying these deductions is, that at the beginning of the decay distance a steady state prevails, for otherwise, if (29) were an initial spectrum function at one moment, the function (30) would not describe a spectrum at one moment, since the various components forming (29) have different travel times over the distance x. In other words: *we have assumed that no dispersion is operative.*

By integration of (30) the total energy is now found to be

$$E_{\text{tot}} = E_0\left(\frac{b + \alpha x}{b}\right)^{1-a}, \tag{33}$$

where E_0 is the initial total energy:

$$E_0 = A \int_0^\infty \lambda^{-a} e^{-b/\lambda}\, d\lambda = A\, b^{1-a}\,\Gamma(a - 1).$$

Accordingly,

$$\frac{H_x}{H_0} = \left(\frac{b + \alpha x}{b}\right)^{\frac{1-a}{2}}, \tag{34}$$

H_0 being the initial significant height. Combining (34) and (31), we find:

$$\frac{H_x}{H_0} = \left(\frac{T_x}{T_0}\right)^{\frac{7}{6}(1-a)} \tag{35}$$

7. Tentative comparison with observations

Although we know that not all of the turbulence, affecting swell waves, is of the "externally generated" type, we shall assume an eddy viscosity proportional to $L^{4/3}$ to be applicable more or less generally and shall therefore try to compare the results of the preceding section with observations.

Sverdrup and Munk (1947) have, on a semi-empirical basis, deduced that $H_x/H_0 = (T_x/T_0)^{-2.65}$, which by comparison with (35), would yield: $a = 3.3$. Later observations, however, (see f.i. Bretschneider 1952) have shown that the period increase goes on less rapidly than according to the graphs of Sverdrup and Munk.

Bretschneider (*l.c.*) has tried to fit the decay data into a 3-parameters-representation, according to which the decay and period increase are not determined by x and T_0 alone (as according to Sverdrup and Munk 1947), but depends also on x/F, F being the fetch length of the generating area from which the waves originate. The observations seem not, however, to fit that representation very much better than the simpler two-parameters-representation, where H_x/H_0 and T_x/T_0 are functions of the dimensionless quantity $2\pi x/gT_0^2 = x/L_0$. For a given value of x/T_0^2, the observed values of H_x/H_0 and T_x/T_0 show considerable scattering, it is true. Moreover, if we take average values of H/H_0 and T/T_0 for any fixed value of x/T_0^2 and plot corresponding pairs of average values $\overline{H_x/H_0}$ and $\overline{T_x/T_0}$, thus found, against each other, we donot find a relation of the general form $\overline{H_x/H_0} = (\overline{T_x/T_0})^r$ with constant r; this can most easily be seen on a double-logarithmic diagram, where it appears that log $(\overline{H_x/H_0})$ is not a linear function of log $(\overline{T_x/T_0})$, as it would be according to (35), if our parameter a is to be a true constant. But here we should remember that many of the observational data have been influenced by dispersion and angular spreading, which we have left out of consideration. Since dispersion and angular spreading tend to lower the significant wave height, whereas, at some fixed place, at some time, the observed significant period can, by dispersion, as well be smaller as larger than it would be there without dispersion, we might, in order to compare our result with observations, proceed as follows. If we plot the observed values of T_x/T_0 against x/T_0^2, we can draw a sort of mean curve for T_x/T_0, representing a function of x/T_0^2, which we denote by $\overline{T_x/T_0}$. If we now plot the observed values of H_x/H_0 against the values of $\overline{T_x/T_0}$ corresponding to the (x/T_0^2)-values concerned, we can draw a smooth line that forms a sort of upper limiting curve to the observed values of H_x/H_0, apart from very few occasional exceptions. The points of this curve are likely to correspond to those cases of swell where dispersion and angular spreading have had the least influence on the decay. For any value of $\overline{T_x/T_0}$ the curve gives a value of H_x/H_0 which we shall denote by $(H_x/H_0)^*$ and now we may compare $(H_x/H_0)^*$ as a function of $\overline{T_x/T_0}$ with equation (35). A first, preliminary (and rather rough) analysis of the data collected by Bretschneider (*l.c.*) shows that, if H_x/H_0 and $\overline{T_x/T_0}$ are plotted on a double-logarithmic diagram, a limiting line can be drawn that is fairly close to a straight line and may (at least for not too small values of H_x/H_0) be roughly represented by the old formula of Sverdrup and Munk, referred to at the beginning of this section. Therefore we shall, for the present, retain the value of a that was derived there: $a = 3.3$.

It follows now from (32) that

$$b = c\,T_0^{7/3}, \tag{36}$$

where $c = (a - 10/7)(g/\pi)^{7/6} = 7\ \mathrm{m}^{7/6}\ \mathrm{sec}^{-7/3}$.

By formula (36) the equations (31) and (34) have now become representations of T_x/T_0 and H_x/H_0 as functions of x and T_0. These representations bear a strong formal resemblence to the representations which Sverdrup and Munk arrived at from an entirely different approach. Indeed, equation (31), for instance, may be written as follows:

$$\frac{T_x}{T_0} = \left(1 + \frac{\alpha}{c}\,\frac{x}{T_0^{7/3}}\right)^{3/7}, \tag{37}$$

whereas Sverdrup and Munk found:

$$\frac{T_x}{T_0} = \left(1 + \beta\,\frac{2\pi}{g}\,\frac{x}{T_0^{2}}\right)^{1/2}, \tag{38}$$

α being a geophysical quantity, proportional to the turbulence parameter f, and β being a numerical constant. For not too large values of $(T_x/T_0) - 1$ the formal difference between (37) and (38) is even smaller yet, since (37) then becomes:

$$\frac{T_x}{T_0} = 1 + \alpha\,\frac{3}{7c}\,\frac{x}{T_0^{7/3}} \tag{37a}$$

and (38):

$$\frac{T_x}{T_0} = 1 + \beta\,\frac{\pi}{g}\,\frac{x}{T_0^{2}}. \tag{38a}$$

Sverdrup and Munk found β to be 0.9×10^{-4}, but later observational data, already referred to above, show that β must have about half that value, *if* the empirical relation between T_x/T_0 and x and T_0 is to be represented in the form of (38) or (38a).

By plotting and averaging the observed values of T_x/T_0 against $x/T_0^{7/3}$, it is possible to find, by comparison with (37) or (37a), an average value of α and thereby of the turbulence parameter f. We found $\alpha = 5.5 \times 10^{-4}\,\mathrm{m}^{1/6}$ to give a reasonable fit with the T_x/T_0-averages for not too large values of $x/T_0^{7/3}$. This yields:

$$nf = \frac{g^{1/2}\,\alpha}{4(2\pi)^{5/2}} = 4.5 \times 10^{-6}\,\mathrm{m}^{2/3}\,\mathrm{sec}^{-1},$$

$$(1 < n < 2)$$

so that we have now

$$nK^* = 4.5 \times 10^{-6}\,(\mathrm{m}^{2/3}\,\mathrm{sec}^{-1})\,L^{4/3}, \quad (1 < n < 2) \tag{39}$$

where K^* denotes the vertically averaged horizontal eddy viscosity operative in the decay of waves of wave length L. It should be kept in mind here that these values of f have been computed by discarding the effect of air resistence on the decay of swell and by treating the turbulence that is effective in the decay process as if it were all of the "external turbulence" type, *i.e.* as if the turbulence-as-a-whole followed the 4/3-power rule; which is, perhaps, not too bad a supposition, after all (see the next paragraph).

8. Comparison with eddy diffusion

Richardson (1926) has shown that eddy diffusion may conveniently be described by means of a concept denoted by $q(l)$, where l denotes the separation between two diffusing particles and q means that the number of pairs of particles having separations of between l and $l + dl$ is $q(l)\,dl$. In close analogy to the well-known Fickian equation of diffusion (in terms of concentration), he proposed the following equation for describing the process of eddy diffusion in terms of $q(l)$:

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial l}\left[F(l)\,\frac{\partial q}{\partial l}\right],$$

where $F(l)$ is analogous to the diffusivity K of the Fickian equation. Finally he showed (see Richardson 1952, where a correction to the 1926 paper is given) that, if $F(l)$ follows the 4/3-power rule:

$$F(l) = \varepsilon\, l^{4/3}, \tag{40}$$

then the ordinary "eddy diffusivity" K, although it cannot describe the process of eddy diffusion in a physically adequate way, may be put equal to $F(\sigma)/2.4$, where σ means the standard deviation of the spreading cluster or band of matter; in the latter mentioned case, the spreading process and the standard deviation are considered in one dimension, perpendicular to the axis of the band. (*N.B.*: Richardson's 1926 paper had: $F(\sigma) = 3.03\,K$; it should, however, be: $F(\sigma) = 2.4\,K$, see his 1952 paper).

Consequently: $$K = \varkappa\,\sigma^{4/3}, \text{ with } \varkappa = \varepsilon/2.4. \tag{41}$$

Now, for horizontal eddy diffusion at the surface of the sea, Stommel (1949) has found empirical values of F that follow Richardson's 4/3-power rule fairly well, for each series of observations, the value of ε in formula (40) not being the same in all the series, but ranging from about 6×10^{-4} to about 20×10^{-4} m$^{2/3}$ sec^{-1}. From this it would follow that the cofactor $\varkappa$ of (41) ranged about from 2.5×10^{-4} to 8×10^{-4} m$^{2/3}$ sec^{-1}.

Values of K have also, by various investigators (see f.i. R. Witting 1933, C. J. Burke 1946, G. F. Mc Ewen 1950, M. Hanzawa 1953), been computed directly, by means of the Fickian equation of diffusion, from observations on spreading patches or bands of dye or other matter in the sea. The values thus found appear to fit in the scheme of formula (41) fairly well and yield values of $\varkappa$ that are of the same order of magnitude as are those mentioned above. It has been shown (see E. Inoue 1950) that the applicability of the 4/3-power rule extends to very large scale lengths (up to 10^7 m). Inoue (*l.c.*) has proposed a value 0.01 cm$^{2/3}$ sec^{-1} = 4.6×10^{-4} m$^{2/3}$ sec^{-1} of the cofactor $\varkappa$.

In order to find a way, now, towards comparing the eddy diffusivity (41), as a function of σ, with the eddy viscosity (39), as a function of L, we shall look upon the wave system described by (1)-(3) as a system of parallel bands of positive and negative concentrations of, say, horizontal momentum and treat them as if they ware bands of diffusing matter, having concentrations $\overline{U}$ and a distribution of concentrations as described by equation (1). The standard deviation of one band (lying between $x = -L/4$ and $x = L/4$) from its axis ($x = 0$) is then easily found to be $\sigma = 0.11\,L$ or $L = 9\,\sigma$. Applying this to (39), we find:

$$n\,K^* = 0.8 \times 10^{-4}\,(\text{m}^{2/3}\,\text{sec}^{-1})\,\sigma^{4/3}$$
$$(1 < n < 2)$$

We conclude that the vertically averaged horizontal eddy viscosity K^* that is effective in the observed decay of swell waves seems to be essentially smaller than the horizontal eddy diffusivity K observed at the surface of the sea. This difference has at least two causes: first, as has been said, K^* is a weighted average over the vertical, which should be expected to be smaller than the surface value (see eq. (27)); secondly, the decay data of swell apply to atmospheric conditions, prevailing over the decay area, that are rather more quiet than are the average atmospheric conditions over the ocean (with exception of the horse latitudes and the doldrums), since such cases of decay under "quiet" conditions have even deliberately been selected, in order to avoid large secondary wind effects in the decay graphs, for which they served as a basis.

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