KONINKLIJK NEDERLANDS METEOROLOGISCH INSTITUUT

MEDEDELINGEN EN VERHANDELINGEN

73

M. P. H. WEENINK

A THEORY AND METHOD OF CALCULATION OF WIND EFFECTS ON SEA LEVELS IN A PARTLY-ENCLOSED SEA, WITH SPECIAL APPLICATION TO THE SOUTHERN COAST OF THE NORTH SEA

1958

F 8,—

STAATSDRUKKERIJ- EN UITGEVERIJBEDRIJF / 'S-GRAVENHAGE

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ERRATA

Please correct the following errors in the above mentioned publication:

On pages 71 and 72 formulas (21)—(21 g), (22), (23)—(23 g), (25)—(25 g), (26), (27)—(27 g) should be replaced by the formulas printed on the accompanying sheet.

On page 84 formula (1) should read:

$$\vec{\tau}_b = -k\,(z_b)\,\varrho v\,(z_b)\,\vec{v}\,(z_b) \qquad (1)$$

On page 86 formula (9) should read:

$$r = \frac{3\nu/H}{1+3\nu/fkHv_t} \qquad (9)$$

On page 94, in the 12th line from the top, please replace the word "time" by "sine".

On page 105, reference 27, please replace "M. Hunt" by "I.A. Hunt".

following leak effects due to a homogeneous wind field over area N are obtained:

$$h^N_3\,(S) = \left\{-0.47\cos\alpha_N - 0.47\,(0.05+q)\frac{H_N}{H_S}\sin\alpha_N\right\} A_N \qquad (19)$$

$$h^N_3\,(C) = \left\{-0.56\cos\alpha_N - 0.56\,(0.05+q)\frac{H_N}{H_S}\sin\alpha_N\right\} A_N \qquad (19\,a)$$

$$h^N_3\,(V) = \left\{-0.50\cos\alpha_N - 0.50\,(0.05+q)\frac{H_N}{H_S}\sin\alpha_N\right\} A_N \qquad (19\,b)$$

$$h^N_3\,(H) = \left\{-0.44\cos\alpha_N - 0.44\,(0.05+q)\frac{H_N}{H_S}\sin\alpha_N\right\} A_N \qquad (19\,c)$$

$$h^N_3\,(He) = \left\{-0.34\cos\alpha_N - 0.34\,(0.05+q)\frac{H_N}{H_S}\sin\alpha_N\right\} A_N \qquad (19\,d)$$

$$h^N_3\,(F) = \left\{-0.32\cos\alpha_N - 0.32\,(0.05+q)\frac{H_N}{H_S}\sin\alpha_N\right\} A_N \qquad (19\,e)$$

$$h^N_3\,(B) = \left\{-0.24\cos\alpha_N - 0.24\,(0.05+q)\frac{H_N}{H_S}\sin\alpha_N\right\} A_N \qquad (19\,f)$$

$$h^N_3\,(Cu) = \left\{-0.22\cos\alpha_N - 0.22\,(0.05+q)\frac{H_N}{H_S}\sin\alpha_N\right\} A_N \qquad (19\,g)$$

Area S

In the same way as in the foregoing section, by adding the formulas 2.2 (10a), 2.3 (8a) and 2.4 (3a) for h^{*S} is obtained:

$$h^{*S} = (1.13\cos\alpha_S + 0.40\sin\alpha_S)\,B_S, \qquad (20)$$

and, by multiplying this by the leak factors, we find:

$$h^S_3\,(S) = (-0.56\cos\alpha_S - 0.20\sin\alpha_S)\,B_S \qquad (21)$$
$$h^S_3\,(C) = (-0.67\cos\alpha_S - 0.24\sin\alpha_S)\,B_S \qquad (21\,a)$$
$$h^S_3\,(V) = (-0.60\cos\alpha_S - 0.21\sin\alpha_S)\,B_S \qquad (21\,b)$$
$$h^S_3\,(H) = (-0.53\cos\alpha_S - 0.19\sin\alpha_S)\,B_S \qquad (21\,c)$$
$$h^S_3\,(He) = (-0.41\cos\alpha_S - 0.14\sin\alpha_S)\,B_S \qquad (21\,d)$$
$$h^S_3\,(F) = (-0.38\cos\alpha_S - 0.14\sin\alpha_S)\,B_S \qquad (21\,e)$$
$$h^S_3\,(B) = (-0.28\cos\alpha_S - 0.10\sin\alpha_S)\,B_S \qquad (21\,f)$$
$$h^S_3\,(Cu) = (-0.26\cos\alpha_S - 0.09\sin\alpha_S)\,B_S \qquad (21\,g)$$

Area I

By adding 2.2 (12), 2.3 (9a) and 2.4 (4a) we find:

$$h^{*I}\,(S) = (0.64\cos\alpha_I - 0.01\sin\alpha_I)\,B_I, \qquad (22)$$

and for the leak effects:

$$h^{\mathrm{I}}_3\,(\mathrm{S}) = (-0.32 \cos \alpha_{\mathrm{I}} + 0.01 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (23)$$
$$h^{\mathrm{I}}_3\,(\mathrm{C}) = (-0.38 \cos \alpha_{\mathrm{I}} + 0.01 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (23\,\mathrm{a})$$
$$h^{\mathrm{I}}_3\,(\mathrm{V}) = (-0.34 \cos \alpha_{\mathrm{I}} + 0.01 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (23\,\mathrm{b})$$
$$h^{\mathrm{I}}_3\,(\mathrm{H}) = (-0.30 \cos \alpha_{\mathrm{I}} + 0.00 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (23\,\mathrm{c})$$
$$h^{\mathrm{I}}_3\,(\mathrm{He}) = (-0.23 \cos \alpha_{\mathrm{I}} + 0.00 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (23\,\mathrm{d})$$
$$h^{\mathrm{I}}_3\,(\mathrm{F}) = (-0.22 \cos \alpha_{\mathrm{I}} + 0.00 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (23\,\mathrm{e})$$
$$h^{\mathrm{I}}_3\,(\mathrm{B}) = (-0.16 \cos \alpha_{\mathrm{I}} + 0.00 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (23\,\mathrm{f})$$
$$h^{\mathrm{I}}_3\,(\mathrm{Cu}) = (-0.15 \cos \alpha_{\mathrm{I}} + 0.00 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (23\,\mathrm{g})$$

Area II

By adding 2.2 (15a) and 2.3 (10a) we obtain:

$$h^{*\mathrm{II}}\,(\mathrm{S}) = (0.43 \cos \alpha_{\mathrm{II}} + 0.40 \sin \alpha_{\mathrm{II}})\, B_{\mathrm{II}}, \qquad (24)$$

and for the leak effects:

$$h^{\mathrm{II}}_3\,(\mathrm{S}) = (-0.22 \cos \alpha_{\mathrm{II}} - 0.20 \sin \alpha_{\mathrm{II}})\, B_{\mathrm{II}} \qquad (25)$$
$$h^{\mathrm{II}}_3\,(\mathrm{C}) = (-0.25 \cos \alpha_{\mathrm{II}} - 0.24 \sin \alpha_{\mathrm{II}})\, B_{\mathrm{II}} \qquad (25\,\mathrm{a})$$
$$h^{\mathrm{II}}_3\,(\mathrm{V}) = (-0.23 \cos \alpha_{\mathrm{II}} - 0.21 \sin \alpha_{\mathrm{II}})\, B_{\mathrm{II}} \qquad (25\,\mathrm{b})$$
$$h^{\mathrm{II}}_3\,(\mathrm{H}) = (-0.20 \cos \alpha_{\mathrm{II}} - 0.19 \sin \alpha_{\mathrm{II}})\, B_{\mathrm{II}} \qquad (25\,\mathrm{c})$$
$$h^{\mathrm{II}}_3\,(\mathrm{He}) = (-0.16 \cos \alpha_{\mathrm{II}} - 0.14 \sin \alpha_{\mathrm{II}})\, B_{\mathrm{II}} \qquad (25\,\mathrm{d})$$
$$h^{\mathrm{II}}_3\,(\mathrm{F}) = (-0.14 \cos \alpha_{\mathrm{II}} - 0.14 \sin \alpha_{\mathrm{II}})\, B_{\mathrm{II}} \qquad (25\,\mathrm{e})$$
$$h^{\mathrm{II}}_3\,(\mathrm{B}) = (-0.11 \cos \alpha_{\mathrm{II}} - 0.10 \sin \alpha_{\mathrm{II}})\, B_{\mathrm{II}} \qquad (25\,\mathrm{f})$$
$$h^{\mathrm{II}}_3\,(\mathrm{Cu}) = (-0.10 \cos \alpha_{\mathrm{II}} - 0.09 \sin \alpha_{\mathrm{II}})\, B_{\mathrm{II}} \qquad (25\,\mathrm{g})$$

Area III

By adding 2.2 (18a), 2.3 (11a) and 2.4 (5a) we obtain:

$$h^{*\mathrm{III}}\,(\mathrm{S}) = (0.06 \cos \alpha_{\mathrm{III}} + 0.01 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}}, \qquad (26)$$

and for the leak effects:

$$h^{\mathrm{III}}_3\,(\mathrm{S}) = (-0.03 \cos \alpha_{\mathrm{III}} - 0.01 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (27)$$
$$h^{\mathrm{III}}_3\,(\mathrm{C}) = (-0.04 \cos \alpha_{\mathrm{III}} - 0.01 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (27\,\mathrm{a})$$
$$h^{\mathrm{III}}_3\,(\mathrm{V}) = (-0.03 \cos \alpha_{\mathrm{III}} - 0.00 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (27\,\mathrm{b})$$
$$h^{\mathrm{III}}_3\,(\mathrm{H}) = (-0.03 \cos \alpha_{\mathrm{III}} - 0.00 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (27\,\mathrm{c})$$
$$h^{\mathrm{III}}_3\,(\mathrm{He}) = (-0.02 \cos \alpha_{\mathrm{III}} - 0.00 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (27\,\mathrm{d})$$
$$h^{\mathrm{III}}_3\,(\mathrm{F}) = (-0.02 \cos \alpha_{\mathrm{III}} - 0.00 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (27\,\mathrm{e})$$
$$h^{\mathrm{III}}_3\,(\mathrm{B}) = (-0.01 \cos \alpha_{\mathrm{III}} - 0.00 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (27\,\mathrm{f})$$
$$h^{\mathrm{III}}_3\,(\mathrm{Cu}) = (-0.01 \cos \alpha_{\mathrm{III}} - 0.00 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (27\,\mathrm{g})$$

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VOORWOORD

De inhoud van deze publicatie vormt het voornaamste resultaat van een onderzoek dat werd opgezet met het doel te komen tot een veralgemening en verfijning van de bestaande methode van berekening van de windinvloed op de waterstanden voor de Nederlandse kust. Het probleem wordt hier op een semi-theoretische, semi-empirische wijze aangevat, waarbij de voornaamste empirische basis gelegen is in de door SCHALKWIJK gevonden gegevens betreffende stormvloedsverhogingen te Hoek van Holland. Voorlopige resultaten van dit onderzoek zijn reeds eerder gepubliceerd in interim-rapporten van het K.N.M.I. naar aanleiding van de grote stormvloed van 1953. Genoemde voorlopige resultaten worden thans vervangen door de inhoud van deze publicatie. De hier verkregen wiskundige betrekkingen tussen het windveld en de hoogte van het zeeoppervlak worden reeds sinds enige tijd met succes door het K.N.M.I. gebruikt voor de dagelijkse verwachtingen van de afwijkingen van de zeewaterstanden langs de Nederlandse kust. Behalve een practische verwachtingstechniek voor de waterstanden levert deze studie ook een wiskundige theorie van de wind-gedreven horizontale stromingen in de Noordzee.

Deze publicatie werd door de Wis- en Natuurkundige faculteit van de Rijksuniversiteit te Utrecht aanvaard als dissertatie van de auteur ter verkrijging van de doctorstitel. Prof. Dr. P. GROEN, die als leider van het wetenschappelijke onderzoek van de afdeling Oceanografie en Maritieme Meteorologie van het K.N.M.I. leiding aan het onderhavige onderzoek gaf, trad op als promotor.

Bij sommige van de wiskundige bewerkingen ondervond de schrijver hulp van Dr. B. HEIJNA. De heer H. C. BIJVOET hielp hem met de analyses van weerkaarten. De heer J. Moser voerde een groot deel van de numerieke berekeningen uit.

De Hoofddirecteur van het
Koninklijk Nederlands Meteorologisch Instituut

C. J. WARNERS

PREFACE

The contents of this publication form the main outcome of an investigation which was started with the object of generalizing and refining the existing method of forecasting wind effects on sea levels off the Netherlands coast. The present approach to the problem is a semi-theoretical, semi-empirical one, the empirical basis being mainly given by part of SCHALKWIJK's results on storm surges at Hoek van Holland. Preliminary results have already been published earlier in interim reports of the KNMI concerning the great North Sea storm surge of 1953. Those preliminary results are now replaced by the contents of this publication. The wind effect formulas arrived at here have already been used successfully for some time by the KNMI for its daily forecasts of sea level heights. Besides resulting in a practical forecasting technique for sea level disturbances the present investigation gives also a mathematical theory of wind-driven horizontal currents in the North Sea.

This publication was accepted as a doctor's thesis by the Philosophical Faculty of the University of Utrecht. Professor Dr. P. GROEN, who, as research leader of the Division of Oceanography and Maritime Meteorology of this institute, gave guidance to the investigation, acted as promotor.

In part of the mathematical work the author was assisted by Dr. B. HEIJNA. Mr. H. C. BIJVOET helped with weather map analyses. Mr. J. MOSER performed much of the computational work.

The Director in Chief
Royal Netherlands Meteorological Institute

C. J. WARNERS

CONTENTS

CHAPTER 0

HISTORICAL SURVEY

The present study may be considered an extension and a refinement of SCHALKWIJK's work [1] on the relation between the disturbance of the sea level at Hook of Holland and the wind field over the North Sea and the Channel, including storm surges. In that publication an extensive survey of the work on this subject published before 1947 has been made. The present survey is confined to investigations on the theoretical and empirical aspects of atmospherically induced disturbances of sea level that have been published after 1946 up to present day (middle of 1958) as far as they concern disturbances in enclosed or partly enclosed seas, leaving unconsidered the sea level disturbances of the ocean or the continental shelf area caused by e.g. moving cyclones. The countries round the North Sea whose coasts have, often in their history, been endangered by storm surges have given much attention to the study of wind-induced disturbances of the sea level. Especially Great Britain, Germany and the Netherlands have suffered several floodings and it is therefore self-evident that in these countries research on storm surges has received much attention. In these countries research proceeded along different lines and led to different methods of forecasting. Below these methods are reviewed in some detail. The disastrous storm surge of February 1st, 1953 has again given great impetus to the research, especially in the countries mentioned, but also in other countries bordering on the North Sea. In America special attention has been given to the wind-induced disturbances (of the type considered here) in the Great Lakes and in Lake Okeechobee for purposes of coastal protection (most of the work on surges in America concerns the type excluded here).

In the Netherlands three bodies have tackled the problem of the relations between a wind field over a sea area and the corresponding sea level disturbances, viz. the "Koninklijk Nederlands Meteorologisch Instituut" (K.N.M.I.) the "Mathematisch Centrum" and the "Rijkswaterstaat" (Department of dykes, roads and waterways).

The latter two carried out research on the probabilities of high water levels in order to have a sound basis for the computation of the required height of the dykes and on the mathematics of the mechanism of storm surges.

The K.N.M.I. has also made contributions to the first mentioned subject viz. studies of the probabilities of certain types of depressions crossing the North Sea, and of high wind speeds along the Dutch coast, but moreover it has been undertaken here to extend and refine SCHALKWIJK's method of forecasting sea level disturbances from the daily weather maps, the results being given in the present publication. Therefore SCHALKWIJK's work will be reviewed here in detail.

SCHALKWIJK's study on storm surges consists of two parts, a theoretical and an empirical, which are almost independent of each other. The theoretical part rests on the equation for the volume transport, i.e. the vertical integral of the horizontal velocity component from surface to bottom, taking into account the Coriolis force and the bottom friction, but neglecting the non-linear inertia terms. The density was taken constant. As to the bottom friction SCHALKWIJK has made two computations, one in which the velocity at the bottom has been assumed to be zero, the vertical eddy viscosity being a function of the height above the bottom, and one in which the eddy viscosity is supposed to be constant, implying that some slipping of the water over the bottom (i.e. over the thin boundary layer where a considerable shear is observed) must be assumed. In the latter computation SCHALKWIJK used a linear dependency of the bottom stress on the velocity just above the laminar boundary layer. For both cases he computed the velocity profile from the still more simplified equation of motion and from this the volume transport was derived for each case, assuming that the wind stress and the slope of the sea surface are known. Equating these computed transports SCHALKWIJK obtained an expression for the friction parameter connecting the bottom stress to the depth-mean velocity. For the eddy viscosity in the first case he assumed the power law established by FJELDSTADT to be valid, implying zero viscosity at the bottom and a viscosity increasing with height according to a power law.

SCHALKWIJK has shown that in the case of zero transport the slope of the sea surface is in first approximation inversely proportional to the depth and that the direction of the slope is deflected to the left over a small angle with respect to the direction of the wind. The factor of proportionality between the wind stress and the integral of the horizontal gradient force over the vertical from surface to bottom was shown to depend on the ratio of the actual depth to the depth of frictional influence as defined by EKMAN, except in the case of a viscosity increasing linearly from bottom to surface. In the latter case the factor of proportionality is equal to 1; moreover the angle of deflection becomes zero in this case. In the other limiting case, viz. of a constant viscosity, the factor of proportionality becomes 1.5, at least if the actual depth is small as compared with the frictional depth. For conditions as observed in the North Sea during storm surges the factor of proportionality would amount to about 1.1 according to SCHALKWIJK.

SCHALKWIJK solved the problem of finding the slope of the sea surface in an enclosed sea if the bottom depth is constant and the wind field is homogeneous. He also made some tentative computations for a rectangular bay of constant depth. In this case the volume transport will not generally be zero everywhere, even if the wind field is homogeneous. SCHALKWIJK developed the differential equation for the stream function of the volume transport in the general case of a sloping bottom and an inhomogeneous wind field and applied it to the cases of a bay with constant depth and with a uniformly sloping bottom.

It has been shown (VELTKAMP [2]) that there is some inconsistency in the way in which SCHALKWIJK has treated the influence of the open connection with the ocean. His assumption, that the influence of the bay on the current pattern in the ocean is only significant in an area that is comparable to the dimensions of the bay, so that the values of the stream function of the transport at the border of such an area, where the influence of the bay is no longer felt, may be considered to be known, contradicts his assumption that at the "corners" of the bay the disturbance of the sea level is zero; the stream function being given at the border of the bay and the neighbouring ocean area, the disturbance at the corners may no longer be arbitrarily chosen, but can be computed from the stream function.

If one assumes that at the corners of the bay the disturbance vanishes, one should replace SCHALKWIJK's assumption concerning the stream function by the condition that at great distance from the bay the stream pattern should approach the stream pattern, already known, of an ocean without a bay.

For a rectangular bay and ocean both having the same constant depth SCHALKWIJK arrives at the result that at the point halfway of that coast of the bay that is farthest from the ocean the wind effect only depends on the component of the wind that is parallel to the longitudinal axis of the bay. He has shown that the stream function in the bay may be thought to be composed of the stream function that would be present if the sea were closed and the stream function that represents a stream penetrating the bay from the ocean. For a rectangular bay with a bottom sloping towards the ocean, having dimensions and a slope corresponding to that of the North Sea, he found that in the case of a homogeneous wind the wind effect at the middle of the coast lying opposite the opening, has its maximum if the wind direction is about 15 degrees west of the direction of the axis of the sea. As to the influence of the Straits of Dover and the Channel on the sea level disturbances at Hook of Holland SCHALKWIJK assumes that this body of water may be approximately modelled by an infinitely long channel, for which he shows that the disturbances along the coasts of the Channel mainly depend on the component of the wind stress parallel to the coast. He shows however that for Hook of Holland this contribution will usually be small as compared with the North Sea effect. Moreover he states that the Straits of Dover act as a leak having a lowering influence on the disturbance height at Hook of Holland with respect to the disturbance in the case of the Straits being supposed to be closed. SCHALKWIJK estimates that this influence may amount to some 20 per cent, but he states at the same time that "these estimates are very arbitrary".

As to the effect of an inhomogeneous wind field SCHALKWIJK made some calculations only for the case of a kind of stationary depression situated at different places over the North Sea and its surroundings. The result showed that for cases where a depression lies over the North Sea the effect at Hook of Holland may be computed by nearly the same formula as was derived for a homogeneous wind field except that a correction term has to be applied which is always negative.

As to the non-stationary state SCHALKWIJK has first treated the case of a one-dimensional sea (no Coriolis force) over which suddenly a wind of constant force starts to blow. It was shown that a damped oscillation is set up which decays gradually. The period of oscillation amounts to four times the time a long wave requires to propagate from the open end of the sea to the closed end. Moreover it was shown that the damping is mainly due to the shallow part of the sea. These results were generalized to the cases of gradually varying wind fields by approximating a gradually varying wind by a step function. Applying this generalization to a wind field varying as a sine function of time, SCHALKWIJK distinguishes three possibilities according to whether the period of the wind is relatively long, relatively short or about equal to the characteristic period of the sea. In the first case the sea level can easily adjust itself to the external forces, so that in this case the disturbance corresponds to the equilibrium disturbance. In the second case the sea has no time to follow the short period oscillations of the wind field so that the sea surface is little affected by it. In the third case resonance occurs involving a much larger amplitude of the disturbance than corresponds with the equilibrium value. Moreover there appears a phase shift of about 90 degrees in this case. In practice most cases lie between the first and third cases, which involves that mostly an overshooting of the equilibrium maximum still occurs and also a damped oscillation after the storm. Moreover a phase shift is present, a time lag, of less than 90 degrees.

In all these theoretical computations SCHALKWIJK had in mind the practical applicability of his results and thus often used physical reasoning where strictly mathematical derivations would be too complicated.

In the empirical part of SCHALKWIJK's investigation use was made of the theoretical result that the equation of motion may be linearized in first approximation, which implies that the principle of superposition of solutions may be applied. Thus in the first place the astronomical tide may be separated from the atmospheric effect, whereas the atmospheric effect may be split up into the contributions of the northern and southern halves of the North Sea and of the Channel. In this way SCHALKWIJK has found empirically the relation between the wind-induced sea level disturbance at Hook of Holland and the corresponding three wind vectors over the areas mentioned. It was shown that the wind effects could be approximated with fair accuracy by the equilibrium wind effects, which would occur if the wind blowing at a certain moment had been blowing constantly before. Further it appeared that the effect of the wind over the southern part of the North Sea was by far the most important for Hook of

Holland. The atmospheric pressure effect was shown to be much smaller than the wind effect, which appeared to depend quadratically on the wind speed. SCHALKWIJK has shown that the dependency on the direction of the wind is asymmetrical, at least for Hook of Holland. Theoretically one might expect a sine function of the wind direction. SCHALKWIJK ascribed the asymmetry to the influences of the thermal stability of the atmosphere and of the length of fetch on the wind stress at the sea surface, for a given wind velocity or gradient wind velocity. From present knowledge the second effect seems to be of minor importance or even negligible.

External effects originating from the wind or pressure field over the ocean did not appear to be of importance for the Netherlands' coast, and for the continental North Sea coast in general. As to the effects of the non-stationariness of the wind field SCHALKWIJK has shown empirically that these consist mainly of a time lag, of an "overshooting" of the maximum equilibrium wind effect and of a damped oscillation of the sea surface height around the equilibrium height. He has shown that the time lag, the retardation of the actual wind effect with respect to the wind field, amounts to about 2.5 hours at Hook of Holland for winds over the southern half of the North Sea. For winds over the northern half of the North Sea SCHALKWIJK has also used the time lag of 2.5 hours which surely is much too small. It would be better to use different time lags for different areas. According to CORKAN a time lag of 8 or 9 hours has to be applied for wind effects at Southend caused by wind over the northern half of the North Sea. As to the effect of overshooting SCHALKWIJK has established a rule which states that this effect is proportional to the maximum rate of rise in the ascending branch of the equilibrium wind effect curve. A routine practice of more than 10 years has shown that SCHALKWIJK's method of forecasting sea level disturbances for Hook of Holland gives reliable results in most cases.

In England a totally different empirical method of forecasting storm surges is used. This method has been developed by CORKAN [3] and improved by J. and M. DARBYSHIRE [4], and is partly based on the theoretical work of PROUDMAN and DOODSON [5]. CORKAN assumed that the wind effect at Southend is partly due to the "local" wind, i.e. the wind over the southern part of the North Sea between England and Holland, the "North Sea effect", due to the wind over the northern and middle parts of the North Sea, and occasionally an external effect due to the wind or the pressure pattern over the ocean. Moreover CORKAN takes into account the non-astronomical disturbance of the sea level at Dunbar and he assumes that it consists mainly of a "local effect" due to the wind over the northern and middle parts of the North Sea and also of the external effect, if any. The external effect is thought to propagate southward along the east coast of Britain without changing its magnitude and with a speed that is about equal to that of the semi-diurnal tidal wave. It is, as CORKAN showed, probably closely related to an atmospheric pressure drop at the Faroes, the time lag between this drop and the effect at Dunbar being about 15 hours. The "North Sea effect" at Southend has a time lag of about 9 hours which is exactly the time interval required for a long wave to be propagated from Dunbar to Southend. The local effect on the contrary is assumed to have no time lag. CORKAN defines a "meteorological tide" at Southend as the difference between the perturbation at Southend and that of Dunbar 9 hours before. For this "meteorological tide" he has derived an empirical formula, consisting of a "local effect" and a "distant effect". To this "meteorological tide" the perturbation of Dunbar 9 hours before (consisting of a local Dunbar effect and an external effect, if any) is added in order to obtain the total atmospheric effect at Southend. As to the external effect CORKAN himself has stated that "there is no final evidence for the existance of these effects". For, due to the rather strong correlation of the wind fields over the North Sea and over its surroundings, a pressure drop at the Faroes is mostly followed by an atmospheric disturbance over the North Sea, so that it is difficult to decide whether the disturbance at Dunbar is of "local" or oceanic origin. The formula given by CORKAN seems to be applicable with fair success provided the wind field is not too inhomogeneous. CORKAN [6] applied his method e.g. to the storm surge of January 8th, 1949 and found that the predicted disturbances at Southend agreed well with the observed ones.

CORKAN has also constructed "co-disturbance lines" on the chart of the North Sea for a number of times during this surge, from which a still better insight can be obtained than from the disturbances at the coast as to how the sea was filled up and emptied and how, due to the Coriolis force, the incoming water was forced towards the east coast of Britain and the returning, i.e. north-going water towards the continental coast. Of the surge of February 1st, 1953, both GROEN [7] and ROSSITER [8] have drawn sets of co-disturbance lines on the chart of the North Sea, the patterns of which appear to agree in the principal features. For this case too ROSSITER could show that fair predictions were obtained from CORKAN's forecasting formula for Southend.

For strongly inhomogeneous wind fields however CORKAN's formula fails completely, as CORKAN [3] himself has shown for the surge of April, 15–17, 1928, when a strong easterly wind was blowing over the southern part of the North Sea and no appreciable wind over the northern part, so that there was no local effect at Dunbar. This failure proves that CORKAN's splitting up of the effect at Southend is rather arbitrary, underestimating the influence of the nearby winds and exaggerating the effect of distant wind fields. For a normal wind field this does not matter much because of the correlation between the different contributions to the total effect. If the correlation were very strong, the whole effect at Southend could even be predicted from the wind over the northern part only.

Another shortcoming of CORKAN's method is that it does not correlate wind effects with the components of the "squared wind vector" as should be done, since the wind stress, which is the driving force, depends quadratically on the wind speed, but with the squares of the wind components, thus giving different weights to different wind stress components.

J. and M. DARBYSHIRE [4] found a better correlation by taking the wind effect to be a linear function of the wind components themselves. This does not imply however that still better results could not have been obtained if the components of the "squared wind vector" had been used. J. and M. DARBYSHIRE have empirically derived a formula for the computation of the "meteorological tide" at Lowestoft with respect to that of Aberdeen, similar to Corkan's formula for Southend. Moreover they have shown that the rate of change of the atmospheric pressure difference between Wick and Bergen was closely related to the disturbance at Aberdeen and that these changes may possible be the origin of the external effects.

The work of CORKAN, ROSSITER and J. and M. DARBYSHIRE shows that for the east coast of Britain the travelling wave is an essential feature in surge phenomena. It is therefore self-evident that in England theoretical research on surges has tried to account for this feature.

PROUDMAN [9] has shown that approximate formulas for the development and decline of the disturbance of the sea surface may be obtained without considering the vertical distribution of the current. Furthermore, in applying this result he has given explicit solutions for the propagation, deformation and reflection of a given sea level disturbance in a rectangular closed basin and a semi-infinite channel, assuming a linear law for the bottom-friction stress and a uniform depth, but neglecting the Coriolis force and the non-linear terms in the equations of motion. In another paper PROUDMAN [10] took into account the Coriolis force as well as the bottom friction and for this case too he could show that in first approximation the equations for the disturbance of the sea level only depend on the depth-mean values of the current but not on the vertical distribution of the current. The solution PROUDMAN obtained of the homogeneous equations for the case of no external forces represent damped Kelvin- and damped Poincaré-waves.

GOLDSBROUGH [11] has theoretically tackled the problem of a surge penetrating the North Sea. He neglected the Coriolis force and non-linear terms in the equations of motion and treated the North Sea as a relatively narrow channel, thus reducing the problem to a onedimensional one. GOLDSBROUGH solved the problem for a given disturbance of a special form at the entrance of the channel and found that the amplitude of the wave entering the channel increases during the inward propagation. Whereas in GOLDSBROUGH's "explanation" of the external surge phenomenon the Coriolis force is left out of consideration it is of predominant importance in a treatment by CREASE.

CREASE [12] considers a system of plane waves approaching a semi-infinite barrier on a rotating earth. The waves have transverse accelerations to balance the Coriolis force. The effect of the barrier is to form a shadow zone on its lee side but the transverse accelerations remain on the edge of this zone and cause Kelvin- waves to be propagated at right angles with the direction of the original waves, into the region behind the barrier.

The amplitude of these Kelvin-waves depends on the ratio of the original wave period to the length of the pendulum day. Thus, if storms coming in from the Atlantic can be considered as generating these waves and if the British Isles are regarded as the semi-infinite barrier, then this mechanism might account for external surges.

As stated before, CORKAN's method of forecasting storm surges at Southend uses the conception of a Kelvin-wave travelling southward along the east coast of Britain.

LEPPIK [13] has developed a method of forecasting surges along the German North Sea coast which is more or less comparable to CORKAN's method. He assumes that the sea level disturbances propagate eastward along the southern coast of the North Sea together with the tidal wave, in the mean time being changed by the effect of the wind over the coastal area. He uses the wind effect at Flushing as a basis of forecasting wind effects in the German Bight and applies corrections to it in order to account for the additional effect of the wind superposed on this basic wind effect while the disturbance travels along the coast. LEPPIK asserts that this method has the advantage that it is less dependent on an accurate wind forecast than the other methods in use on the continent, since he supposes that the wind effect at Flushing already contains most of the effect of the wind on the German North Sea coast. Moreover, according to LEPPIK, other methods suppose rather homogeneous, quasi-stationary wind fields, which is irrelevant for LEPPIK's method. As to the corrections to be applied to the wind effect at Flushing LEPPIK assumes that they consist of a part that is proportional to the difference in the wind components perpendicular to the coast at the place considered in the German Bight and at Flushing, and of a part that accounts for the local effect, which is supposed to be proportional to the component of the wind perpendicular to the coast at the place considered. The factors of proportionality have to be found empirically.

LEPPIK's method is no longer used in the German surge forecasting service, because it is thought that the underlying principle of a travelling wave does not agree with the actual mechanism of storm surges along the continental coast, though Leppik's method may give satisfactory results in many cases.

At present a method, worked out by Tomczak, is used in surge forecasting for the German North Sea coast.

Tomczak [14, 15] assumes that most of the wind effect on the German North Sea coast originates from the wind field over the German Bight. He has empirically developed a method for the computation of the sea level disturbance in Cuxhaven and some other places along the German Bight. This method is based on the correlation between the difference in height of the observed high water at Cuxhaven and the astronomical high water, which need not be simultaneous (contrary to the wind effect, which has been defined as the difference between simultaneous observed and astronomical tide levels), and the mean wind over the German Bight as derived from lightship observations three hours earlier.

In cases where the wind effect computed by this method begins the diverge appreciably from the actual wind effect the wind field farther out on the North Sea is taken into account by means of the atmospheric pressure difference between Aberdeen and Skudesnäs. Cuxhaven has been chosen as a reference station.

For the other places along the German Bight Tomczak has compiled empirical tables giving the additional effect with respect to Cuxhaven for different wind conditions. The shallow wadden areas off the German coast appear to be very important for the wind effect on the coast, as has been shown by Tomczak [16] for a few cases by means of tide gauges laid out in the German Bight for that special purpose. This becomes especially evident from the difference between the wind effect at high water and that at low water under the same wind conditions; a correction of about +50 per cent has to be applied to the high water wind effect in order to obtain the low water wind effect, because of the different effective depths in both cases (personal communication). External effects, i.e. effects of oceanic origin are not taken into account since they appear to be negligible for the German Bight. For the period of a longitudinal oscillation of the North Sea Tomczak empirically found about 37 hours.

No corrections for non-stationary effects are applied. In the empirical graphs however the climatological mean inertia effects are included. Tomczak assumes that these effects are negligibly small with regard to the total effect, at least for the German North Sea coast.

Weenink [17], in a case study, has shown that non-stationary effects can be of importance on the continental North Sea coast, at least on the southern part of it. He succeeded in describing the wind effects of a twin surge at Hook of Holland formally as the response of a damped harmonic oscillator to a time-dependent external "force" which corresponds to the equilibrium wind effect. For the case considered resonance appears to have occurred. Moreover it was shown that due to heavy damping the maximum wind effect of the second surge of a series of equal storms following each other with equal time intervals is about the maximum disturbance ever to occur in the sequence of surges brought about by these storms.

Thus far only investigations related to forecasting methods concerning *North Sea* surges have been reviewed here. A comparable recent study of another sea area is that by MILLER [18].

MILLER has investigated empirically the wind effects on the coastal waters of New England. He found that the wind-induced sea level disturbances may be thought to be composed of a part that depends on the wind direction as a sine function and a part that depends on the very local topography and the fetch lengths to the place under consideration, which part is therefore an "irregular" function of wind direction. The first part refers to the wind-induced sea level disturbances along the outer (ocean) coast. In order to eliminate the astronomical tide and other non-predictable quasi-periodic motions MILLER used a numerical filtering procedure. He too considered the problem as being a quasi-stationary one.

The following four papers to be reviewed here deal with monthly or annual mean sea level heights in relation to monthly or annual mean winds.

WYRTKI [19] has empirically investigated the relation between the monthly mean wind over the North Sea and the monthly mean sea level of the North Sea. It appeared that the deviations of the monthly mean values of the sea level heights from the corresponding annual mean values have the same sign all over the North Sea.

WYRTKI found that the annual variation of the mean monthly disturbance of the sea level can be ascribed to the effects of the variation of density of sea water, of the atmospheric pressure variation and of the variation of the wind. The first is small as compared to the other two. The last is the most important one. WYRTKI has made some estimates of the relative importance of the northern and of the southern outlet of the North Sea, as dependent on the mean wind vector over the North Sea, with respect to the water budget of the North Sea. He found that on the average only winds between WNW and SSW result in a filling up of the North Sea.

THIEL [20], like SCHALKWIJK and TOMCZAK, has found empirically that for the Baltic Sea the wind effect depends quadratically on the atmospheric pressure gradient. THIEL has shown that the meteorological factors are the predominant causes of the monthly-mean sea level variations. It appears that on the average the barometric effect on the water level is proportional to the deviation of the local pressure from the mean pressure at the same place.

The factor of proportionality appeared for some places to be greater than would be expected according to hydrostatic equilibrium. The cause of this is probably the correlation between the atmospheric pressure deviation and the wind direction, implying that part of the apparent "barometric" effect in reality consists of wind effect.

DIETRICH [21], investigating the secular rise of the sea level with respect to the land at Esbjerg, had to reduce the scatter in the monthly mean sea level values. This was accomplished by, amongst other things, subtracting the effect of the wind on the sea level.

In order to be able to compute this effect separately he established a relation between the deviation of the monthly mean wind vector from the annual mean vector for the same year and the difference between the monthly mean value of the sea level and the annual mean for the same year. It turned out that this relation could be described by a linear dependency for each direction with a fair approximation. The scatter would probably be reduced still more if DIETRICH had used root-mean-square wind vectors in stead of mean wind vectors.

BERGSTEN [22] has studied, for a number of places on the Swedish coast, the relation between the deviation of the annual mean sea levels from the calm weather levels and the local winds, both corrected for air pressure and an assumed land uplift. He had separately computed from the data over a number of years, the linear regression coefficients between this deviation and the annual sums of the squares of the wind velocities for the different wind directions. In this way BERGSTEN could derive the dependency of the wind effect on the wind direction. By means of the relation thus found he could compute the deviations for a long period and decide whether the assumed value of the land uplift was correct or not. If not, he repeated the procedure with another value of the uplift until the smoothed curve of the computed deviations was in agreement with the smoothed curve of the observed deviations.

Since the relation between wind stress and wind effect is known from theory and the relation between wind and wind effect can be found empirically it is evident that the dependency of wind stress on wind speed can be derived by comparing these two relations. A great many investigations have dealt with wind effects for this special purpose. Below two recent studies of this kind are reviewed.

KEULEGAN [23] has studied the wind effects on lake Erie caused by gales. From his results he derived a relation between wind stress and wind speed which suggests a somewhat smaller power dependency than the quadratic one.

HELLSTRÖM [24] has studied empirically the relation between wind and wind effect on Ringköbing Fjord in Denmark. Assuming that the wind over the Fjord is homogeneous and that the maximum wind effect of each surge considered represents the equilibrium wind effect corresponding to the maximum wind speed during that surge HELLSTRÖM finds that the wind effect is proportional to the 1.8th power of the wind speed. In a former study concerning wind effects on Lake Okeechobee in Florida he even found a 1.6th power.

It should be borne in mind however that the wind speeds used in HELLSTRÖM's investigations are not measured wind speeds, but have been derived from Beaufort estimates by using a conversion scale. Since from HELLSTRÖM's publication it is not clear which conversion scale has been used it cannot be concluded that HELLSTRÖM's results are in contradiction with a quadratic relationship, the more so since more recent studies by SAVILLE, HUNT and KIVISILD on wind effects in Lake Okeechobee appear to be in favour of, or at least not in contradiction with, the quadratic relation. These latter studies are reviewed below.

SAVILLE [25] described some surges in Lake Okeechobee by means of the most simple formula for the relation between wind and sea level disturbances, assuming a quasi-stationary development. The parameter relating the wind stress to the square of the wind speed was fixed so as to give the best fit of the theoretical curve to the observed wind effect curve. This value of the parameter was found to be almost identical with that of the so-called "Zuiderzee formula" [26]. SAVILLE correlated the "set-up" (as he calls the wind effect) in two ways with the wind field, viz. with the root-mean-square wind speed as derived from the wind along the fetch and with the root-mean-square wind speed over the whole lake. The second method appeared to yield somewhat better results.

HUNT [27] has also developed a technique for forecasting wind tides in shallow lakes and applied it to Lake Okeechobee. His method is also a quasi-stationary one, as is mostly a fair approximation to actual conditions in shallow waters of sufficiently limited extent. No Coriolis force is taken into account, as is permissible in such lakes.

The difference between SAVILLE's and HUNT's treatment is, that HUNT uses curved fetches instead of straight ones, as SAVILLE did. The lake is divided into a number of "cells" formed by a set of wind fetches and a number of their orthogonal trajectories. The number of cells to be chosen must be such that in each cell fairly homogeneous conditions prevail as to depth and wind force. In each cell the slope of the lake surface is computed by assuming equilibrium. If now a line of zero disturbance is assumed the "set-up" along each fetch strip can be determined. Lateral differences in elevation, thus found, have to be smoothed out, e.g. by a method such as developed by VOLKER [28] in his theoretical investigation on the wind effect over a uniform channel having a depth profile consisting of parts of constant but different depths. In this way the lake level disturbances can be computed. In a closed sea or lake the total volume of water may be assumed to be constant and thus the mean disturbance height over the whole surface must be zero. If this appears not to be the case, a different line of zero disturbance must be assumed and the computation must be repeated until indeed the mean disturbance vanishes.

In his study HUNT has also given an elaborate account and survey of the dependence of the wind stress on wind speed and air mass stability.

KIVISILD [29] has developed a step-by-step technique for computing non-stationary sea level disturbances that are caused by varying winds or atmospheric pressure. This method is based on a numerical integration of the equations of motion and of continuity, yielding the sea level disturbance height and the volume transport, i.e. the vertical integral from top to bottom of the current velocity. The procedure runs as follows. From a given transport field and a given sea level disturbance field the transport field a short time later can be computed by means of the equations of motion; from the transport field thus found the sea level changes can be determined by means of the equation of continuity; etc. This procedure can easily be performed with graphs KIVISILD has worked out his method for a network consisting of equilateral triangles covering the whole sea considered; the disturbance heights are computed in the apices of the triangles, whereas the mass transport vectors refer to the centres of gravity of the triangles. The Coriolis force is neglected and a quadratic bottom friction is taken into account. KIVISILD has applied his method to a number of storm surges on Lake Okeechobee.

Numerical or graphical methods of computing sea level disturbances and mass transports, such as the methods mentioned just now, are, if great accuracy is required, very lengthy. HANSEN and WELANDER have recently developed methods of computation making use of electronic computers.

HANSEN [30], like KIVISILD, uses difference equations which, in the usual way, replace the differential equations (of motion and of continuity) and thus replace the problem to the problem of solving a set of linear algebraic equations. The solution yields the currents and the sea level disturbance heights at grid points of a network laid over the sea. As for the bottom friction HANSEN assumes a quadratic dependency on the depth-mean velocity. The computations run as follows. From the given velocities and wind effects at certain times t_0 and $t_0+\triangle t$ the velocities at $t_0+2\triangle t$ are computed by means of the equations of motion. By means of the equation of continuity these velocities together with the wind effects at time $t_0+\triangle t$ yield the wind effects at time $t_0+3\triangle t$, etc. In order to ascertain the convergence of this iterative procedure, the time interval $\triangle t$ has to be chosen so that the ratio $\triangle t/l$, where l represents the mesh width of the network, is sufficiently small. It appeared that the method was not yet suitable to be applied to oscillations of periods of about 12 hours because of convergence difficulties. HANSEN published the results of an application to the storm surge of February 1st, 1953. There is a fair agreement between the computed and the observed wind effects, although deviations of 0.5–1 meter are not rare. The mesh width of the network was about 50 km. It is not clear from HANSEN's paper what boundary conditions he assumed for the northern boundary of the North Sea.

WELANDER [31] has extended EKMAN's classical results so as to make them applicable to a shallow sea. He arrives at the result that the divergence of the wind stress field may be as important as the vorticity. This result will however only apply to laminar flow since WELANDER assumed the viscosity to be constant from surface to bottom and the velocity at the bottom nevertheless to be zero. In more natural conditions, such as those described above (in the discussion of SCHALKWIJK's work) the effect of the divergence of the wind field will be of little importance on the wind effects and the effect of wind stress vorticity will be of more importance than according to the theory developed by WELANDER.

WELANDER has laid special emphasis on the well-known fact that in the absence of lateral stresses the velocity profile is uniquely determined by the local time-histories of the wind stress and of the surface slope. To find the explicit expression of the velocity profile, he starts from two special profiles, representing the response to a sudden increase of unit strength of the wind stress and of the surface slope, respectively. Using these elementary profiles WELANDER derived, by means of the equation of motion, an integro-differential equation for the sea level disturbance, which can be solved numerically for a given basin and a given wind stress field by using a step-by-step technique. Applying this technique to the North Sea WELANDER determined the response curves, for some places along the coasts, corresponding to a homogeneous wind over each of the cells into which he has divided the North Sea. This procedure, which is still being developed, promises to become a suitable forecasting technique if use is made of electronic computers.

As to the influence of the Straits of Dover on the sea levels in the North Sea a few investigations have been made.

LAUWERIER [32] has, in a theoretical treatment, studied the behaviour of the sea level in a funnel-shaped sea under the influence of a certain type of wind field.

BOWDEN [33] has empirically derived a linear relation between the strength of the current through the Straits of Dover, the slope of the sea surface and the square of the wind speed over the neighbouring sea area. The transport through the Straits was derived from measurements of the voltage difference, induced by the current, between boths ends of a telephone cable over the bottom of the Straits of Dover.

WYRTKI [34] has studied the relation between the monthly mean values of the component of the current through the Straits of Dover (as observed on board the lightship "Varne") parallel to the predominant current direction and the SW component of the wind over the southern North Sea as derived from the pressure data of Tynemouth, Emden and Paris. He found a linear relation. Moreover WYRTKI has constructed a polar diagram from which for a given wind vector the corresponding speed and direction of the current can be read.

This historical survey will be concluded with a number of theoretical papers dealing with forced and free oscillations of sea level, with the non-linear problem of the interaction between astronomical tide and wind effect and with the problem of finding the relation between special types of wind fields over seas having specified coastal and bottom configurations.

LEE HARRIS [35], investigating the problem of wind tides and seiches in the Great Lakes of the United States [1]), distinguished between disturbances of local character and those which involve the whole lake. He derived a partial differential equation for the disturbance of the lake surface and concluded that the problem is an eigenvalue problem. The solutions of the homogeneous differential equation, representing the case of no external forces acting, represent the different modes of seiches. The solutions of the inhomogeneous equation (external forces present) can be represented by a linear combination of these modes. LEE HARRIS showed that the effect of the Coriolis force is an increase of the frequencies of the free oscillations with respect to the case of zero Coriolis force, the relative increase being a function of the ratio of the free period to that of the pendulum day. As to the one-dimensional case he has given an explicit solution and applied it to a few cases of special practical interest. He finally proved that the general solution in the one-dimensional case can be written as the sum of a series of eigen-functions, the coefficients of which appear to satisfy the equation of a harmonic oscillator.

SAITO [36] has given an explicit solution of the problem of wind tides caused by a homogeneous wind field over a channel of uniform depth and width. He neglected the Coriolis force but took into account a bottom friction proportional to the depth-mean velocity.

REID [37] too has investigated forced seiches, as well as free seiches, using a numerical approach. His method of characteristics, which is applicable to relatively deep narrow channels of variable cross-section where bottom friction may be ignored, is based upon the linearized one-dimensional equation of motion. REID has applied and verified his method for the case of a seiche in a closed basin with linearly increasing depth and width, for the case of a free progressive wave in a channel of variable cross-section open at one end, and for the case of a forced surge in such a channel.

DOODSON and PROUDMAN have both tackled the problem of the interaction of the astronomical tide with the storm surge, which is in fact equivalent to the

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[1]) It has already been said that storm surges of the open ocean are left out of consideration in this survey.

problem of solving the non-linear equations of motion. DOODSON [38] investigated this interaction numerically for a long gulf of uniform depth and width by a step-by-step technique after having approximated the differential equations by difference equations. Applying his method to four different cases, corresponding to whether the time of maximum surge occurs at the time of astronomical high water, of low water or of the half-tides, DOODSON could show that for the average of these four cases no obvious direct tidal interaction was found, but he derived that the apparent surge, defined as the difference between the simultaneous observed and predicted astronomical sea level, depends upon the coefficient of bottom friction and that the magnitude of this coefficient is greatly affected by the tides.

PROUDMAN [39] has investigated the non-linear problem of the combined propagation of tide and surge in an estuary. He showed that for a single progressive wave the height of a surge whose maximum occurs near the time of tidal high water is less than that of a surge whose maximum occurs near the time of tidal low water. This effect appears to be due to friction. PROUDMAN treated this non-linear problem of interaction between tide and surge in an estuary of variable cross-section by first solving the usual linear equations which are found by omitting the non-linear terms, including the quadratic bottom stress and then substituting this solution in the non-linear terms of the complete equations; by solving these equations second order approximations of the height of the sea level disturbance and of the mean value of the current over a cross-section are found. These again can be substituted in the non-linear terms of the equation and thus a third order approximation can be obtained, etc.

SCHÖNFELD [40] too has treated the problem of the non-linear terms in the equations of motion. In an elaborate study he has developed the method of "characteristics" as a useful tool in computations concerning the propagation of tides and other long waves in rivers and estuaries taking into account the non-linearity. In another paper dealing with tides and storm surges in a shallow sea SCHÖNFELD [41] uses an iterative procedure by which the non-linear equations are solved in successive approximations. SCHÖNFELD [42] has also shown that it is permissible, with a high degree of accuracy, to use the depth-mean values of the currents in studying sea level disturbances. He found that in channels with parallel shores as solutions of the homogeneous equations of motion, referring to the case of no external forces, Kelvin- and Poincaré-waves are found. Applying his theory to a rectangular sea which along one or two sides is in open connection with an ocean, SCHÖNFELD arrives at the problem of finding Fourier expansions of the wind effect and of the stream function with terms having a prescribed phase.

LAUWERIER [43] and VELTKAMP [44], who came to the last mentioned problem from a somewhat different direction, have solved it. VELTKAMP [2, 45] also solved, at least in principle, the problem of calculating the wind effects and currents caused by a stationary, but otherwise arbitrary, wind field over a sea of arbitrary shape, having a constant depth or consisting of parts having constant but different depths.

The solution was obtained by transforming the equations of motion for a sea of constant depth into the well-known equations of Cauchy-Riemann, which may be solved by the methods of the classical theory of complex functions. For a sea consisting of parts having different constant depths the problem was reduced to solving a singular integral equation, which can in principle be done by the methods developed by Poincaré. VELTKAMP applied his general results to some special cases, e.g. a half-plane sea of constant depth, and a half-plane sea with a rectangular marginal sea, both having constant but different depths. Of the latter example one result is of special importance, viz. that the wind effect at the border between the sea and the ocean may be neglected with a fair approximation if the depth of the ocean is great as compared with the depth of the sea.

Since an arbitrary depth configuration can be approximated by a depth configuration that consists of parts of constant depths, it may be said that, *in principle*, VELTKAMP solved the problem of calculating wind effects caused by an arbitrary stationary wind field over an arbitrarily shaped sea with an arbitrary depth configuration.

LAUWERIER [46, 47, 32, 48, 49], has treated the theory of non-stationary wind effects for some simply-shaped seas, e.g. an infinitely large shallow sea, a half-plane sea, an infinitely long channel, a rectangular sea and a funnel-shaped sea. His results, found by means of Laplace transformations (the backward transformations leading to extremely difficult mathematical problems), are however of so complicated form that it is doubtfull whether they can be applied or generalized to more practical problems.

VELTKAMP [50] has shown that the elementary solutions of the equations of motion and the equation of continuity for the case of no wind over a rectangular sea of constant depth, viz. the Kelvin- and Poincaré-waves, form a *complete* set of functions. This implies that any wind-induced current system and the corresponding surface topography may be thought of as being composed of a series of Kelvin- and Poincaré waves.

CHAPTER 1

GENERAL EQUILIBRIUM THEORY

1.1 THE EQUATIONS OF WIND EFFECT AND VOLUME TRANSPORT

1.1.1 INTRODUCTORY REMARKS

The effect of the wind field over a sea on the water level at a particular place is a function of time, $h\ (t)$, which may be thought to be composed of an "equilibrium effect" $h_0\ (t)$ and a "perturbation term" $c_1\ (t)$, so that:

$$h\ (t) = h_0\ (t) + c_1\ (t).$$

$h_0\ (t)$ is the effect that would be present if the wind field prevailing at the time t were stationary all the time. Obviously, $h_0\ (t)$ depends only on the wind field prevailing at the time t over the sea area involved. An empirical study by SCHALKWIJK [1] and a routine practice of forecasting sea level heights along the Netherlands coast have shown that for a sea area like the southern North Sea a good first approximation of the perturbation term c_1 is given by $c_1\ (t) = h_0\ (t - t^*) - h_0\ (t)$, so that the wind effect may be written as:

$$h\ (t) = h_0\ (t - t^*) + c_2\ (t),$$

where t^* is a time lag and $c_2\ (t)$ is a correction term, involving inertia effects, such as the effects of "overshooting" or "undershooting" of a maximum or a minimum of the equilibrium effect, and oscillatory after-effects during the decaying state of a storm surge. SCHALKWIJK has established rules for the "overshooting" effect" as well as for the oscillatory after-effects.

Another method of approximating $h\ (t)$ if $h_0\ (t)$ is given, is to treat $h\ (t)$ as the response of a one-dimensional damped harmonic oscillator to an external force proportional to $h_0\ (t)$:

$$\frac{d^2h}{dt^2} + a'\frac{dh}{dt} + b'\frac{dh}{dt} = b'h_0\,(t)$$

where a' and b' are parameters to be found empirically for any particular place; a' increases with the wind speed (WEENINK [17]). This equation can easily be solved by numerical integration or by means of an electric model.

For the Netherlands coast, which is considered in particular in the present study, the corrections to be applied to the "equilibrium effect" are in general relatively small and both practical methods described above give sufficiently accurate results for a place like Hook of Holland. This publication mainly concerns the problem of finding the *equilibrium* wind effect from the wind field. For a particular case (the twin storm surges of 21–24 December 1954 in the North Sea) the values 0.12 $(\text{hour})^{-1}$ and 0.041 $(\text{hour})^{-2}$ were found for a' and b', respectively, at Hook of Holland.

Physically one cannot, of course, expect $h(t)$ to be determined by the equilibrium effect at the place under consideration alone (although this may, for forecasting purposes, be practically right in most cases). GROEN [51] has developed a method of computing the wind effect $h(x, y; t)$ by means of successive approximations, where the development in time of the whole field $h_0(x, y; t)$ is used for finding the next approximation.

1.1.2 DERIVATION OF THE EQUATIONS OF WIND EFFECT AND VOLUME TRANSPORT

The wind-induced disturbance of the sea level can be described mathematically by means of the equations of motion and of continuity for a vertical column of water extending from the bottom of the sea to the sea surface, and the boundary conditions. The equations of motion are here deduced from the equations of Navier-Stokes for a turbulent incompressible fluid, on a rotationg earth assuming the velocity distribution to be such that only those frictional terms need be considered that result from friction between horizontal layers of fluid. This assumption gives very satisfying results in problems of ocean currents in general. Moreover, the field accelerations and the vertical velocities are neglected as is mostly allowed in problems like this. We then obtain (see e.g. SVERDRUP [52], chapter 13).

$$\frac{\partial v_x}{\partial t} = -\frac{1}{\varrho}\frac{\partial p}{\partial x} + c\, v_y + \frac{\partial}{\partial z}\left(\nu \frac{\partial v_x}{\partial z}\right), \tag{1}$$

$$\frac{\partial v_y}{\partial t} = -\frac{1}{\varrho}\frac{\partial p}{\partial y} - c\, v_x + \frac{\partial}{\partial z}\left(\nu \frac{\partial v_y}{\partial z}\right), \tag{2}$$

$$0 = -\frac{1}{\varrho}\frac{\partial p}{\partial z} - g. \tag{3}$$

For the meaning of the symbols used see list on page 108.

The vertical components of the accelerations are neglected since they are only important in short period surface waves which shall not be dealt with here.

The equation of continuity can be written as follows:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = -\frac{\partial v_z}{\partial z}. \tag{4}$$

In this equation it is not permissible to neglect the vertical component of the velocity, since it is this component that links the currents to the disturbance, h, of the sea-level relative to the sea-level in the case of no motion.

The plane occupied by the sea surface in the case of complete rest is chosen as zero surface for z. This surface $z = 0$ is level, i.e. perpendicular to the direction of gravity, as is evident from the equations (1), (2) and (3) where substituting $v_x = 0$ and $v_y = 0$ makes the horizontal components of the pressure gradient vanish. Thus the isobaric surfaces are horizontal. If we assume that the sea surface is an isobaric surface, then this surface must also be horizontal, as we wished to show.

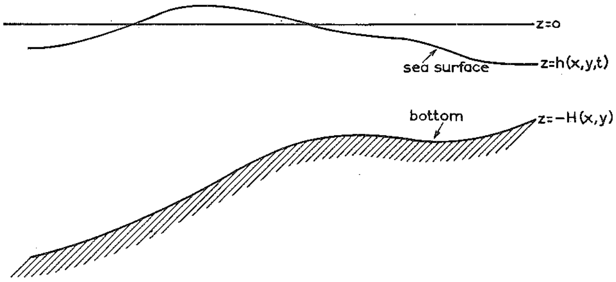

FIG. 1.1 a. Vertical section of a sea.

Let furthermore the topography of the bottom of the sea be given by $z = -H(x, y)$, see figure 1.1 a.

Next consider the pressure distribution beneath the sea surface. For that purpose equation (3) is solved by integrating vertically from z' to z, taking a constant density ϱ:

$$p(z) = p(z') - \int_{z'}^{z} \varrho g d\zeta = p(z') - \varrho g (z - z'), \qquad (5)$$

where z' may be an arbitrary function of x and y. If the pressure distribution in the surface $z = z'$ is known, the pressure everywhere can be computed by means of equation (5).

Let $z = z'$ be the sea surface:

$$z' = h(x,y). \qquad (6)$$

The pressure distribution in this surface is the atmospheric pressure distribution at sea level, which is assumed to be known. From (5) we deduce the components of the horizontal pressure gradient:

$$\frac{\partial p(z)}{\partial x} = \frac{\partial p(h)}{\partial x} + \varrho g \frac{\partial h}{\partial x}, \qquad (7)$$

$$\frac{\partial p(z)}{\partial y} = \frac{\partial p(h)}{\partial y} + \varrho g \frac{\partial h}{\partial y}. \qquad (8)$$

It is evident from (7) and (8) that the horizontal component of the pressure gradient is independent of depth, at least under the assumption of a constant density.

Integrating (1) and (2) vertically from the bottom to the sea surface, making use of (7) and (8), we obtain:

$$\frac{\partial S_x}{\partial t} = -\frac{(H+h)}{\varrho}\frac{\partial p(h)}{\partial x} - (H+h)\,g\frac{\partial h}{\partial x} + c\,S_y + \left(\nu\frac{\partial v_x}{\partial z}\right)_h - \left(\nu\frac{\partial v_x}{\partial z}\right)_{-H}, \tag{9}$$

and

$$\frac{\partial S_y}{\partial t} = -\frac{(H+h)}{\varrho}\frac{\partial p(h)}{\partial y} - (H+h)\,g\frac{\partial h}{\partial y} - c\,S_x + \left(\nu\frac{\partial v_y}{\partial z}\right)_h - \left(\nu\frac{\partial v_y}{\partial z}\right)_{-H}, \tag{10}$$

where S_x and S_y represent the x- and y-components, respectively, of the vector $\vec{S}$, the volume transport, defined by

$$\vec{S} = \int_{-H}^{h} \vec{v}\, d\zeta. \tag{11}$$

The terms $(\nu\, \partial v_x/\partial z)_h$ and $(\nu\, \partial v_y/\partial z)_h$ represent, after multiplication by ϱ, the x- and y-components, respectively, of the stress vector in the water at the sea surface, $\varrho\, (\nu\, \partial\vec{v}/\partial z)_h$. Since the stress is vertically continuous at the sea surface, they also represent the x- and y-components τ_{sx} and τ_{sy}, respectively of the wind stress vector $\vec{\tau}_s$ in the lowest layers of the atmosphere:

$$\vec{\tau}_s = \varrho\left(\nu\frac{\partial\vec{v}}{\partial z}\right)_h. \tag{12}$$

$\vec{\tau}_s$ may be assumed to be known from the wind field, as will be explained in section 3.2.

Similarly, the terms $-(\nu\, \partial v_x/\partial z)_{-H}$ and $-(\nu\, \partial v_y/\partial z)_{-H}$, after multiplication by ϱ, represent the x- and y-components, respectively, of the bottom stress vector $\vec{\tau}_r$:

$$\vec{\tau}_r = -\varrho\left(\nu\frac{\partial\vec{v}}{\partial z}\right)_{-H}. \tag{13}$$

Concerning the bottom stress it should be noticed that it must not be considered as a given function of x and y, but that it depends on the unknown vertical velocity profile, which in its turn is closely connected with the bottom friction. The problem of the determination of the bottom friction is in fact a three-dimensional problem, involving the solution of the three-dimensional velocity field from the equations of motion and continuity, taking as a boundary condition at the bottom: $\vec{v} = 0$. We shall, however, reduce this very complicated problem to the problem of computing the two-dimensional field of the volume transport. An assumption must then be made concerning the bottom stress. As such we take the following linear expression:

$$\vec{\tau}_r = +\varrho r\,\overline{\vec{v}} + n\,\vec{\tau}_s = +\varrho r\frac{\vec{S}}{H+h} + n\,\vec{\tau}_s, \tag{14}$$

where r and n are parameters, which, in general, are functions of place and time and $\vec{v}$ is the depth-mean velocity. In par. 3.3 this assumption will be explained. By this linear relationship between $\vec{\tau}_r$ and $\vec{S}$ the equations (9) and (10) become linear in S_x and S_y. With respect to h, however, the equations are still non-linear. To linearize them with respect to h too $(H+h)$ is replaced by H, assuming that the disturbance h is small as compared with depth. This is in general true in marginal seas, like the North Sea, except in a very shallow coastal strip. Strictly speaking, therefore, the linearized equations may only be applied in a sea with vertical boundaries and of sufficient depth. For the present it will be assumed that the linear theory may be used. This implies that the disturbance h may be split up into a tidal effect, a wind effect and a pressure effect, produced respectively by the tide, the wind field and the atmospheric pressure field. These parts, then, are independent of each other and may be computed separately. The joint effect of wind and pressure is called *atmospheric effect*. SCHALKWIJK [1], among others, has shown that in the North Sea the pressure effect is, in general, small compared with the wind effect.

In the present investigation we shall confine ourselves to *wind*-induced disturbances of the sea level, *wind effects*, henceforth denoted by h. Then the linearized equations for the volume transport become:

$$\frac{\partial S_x}{\partial t} = -gH\frac{\partial h}{\partial x} + c\,S_y - \frac{r}{H}S_x + \frac{1}{\varrho}\tau_x, \tag{15}$$

$$\frac{\partial S_y}{\partial t} = -gH\frac{\partial h}{\partial y} - c\,S_x - \frac{r}{H}S_y + \frac{1}{\varrho}\tau_y, \tag{16}$$

where now τ_x and τ_y represent the x- and y-components, respectively, of the "apparent wind stress" vector $\vec{\tau}$, which is defined as the sum of $\vec{\tau}_s$, the pure wind stress vector, and $n\,\vec{\tau}_s$, denoting that part of the bottom stress (14) that is proportional to $\vec{\tau}_s$ and independent of $\vec{S}$. The equations (15) and (16) express the fact that the acceleration of a vertical column of water (as a whole) is due to the pressure gradient force, the deflecting force of the earth's rotation (the Coriolis force), the bottom friction and the wind stress. Although $r\,\vec{S}/H$ is, strictly speaking, according to (14), not the whole bottom stress (a minor part of it having been absorbed by the "apparent wind stress"), I shall for the sake of brevity call it *the* bottom stress.

The equation of continuity for a vertical column of water is found by integrating both sides of (4) from the bottom to the surface:

$$\frac{\partial S_x}{\partial x} + \frac{\partial S_y}{\partial y} = -v_z(h) + v_z(-\mathrm{H}) = -\frac{\partial h}{\partial t} - v_x\frac{\partial h}{\partial x} - v_y\frac{\partial h}{\partial y} + v_z(-H). \tag{17}$$

For the *equilibrium* state (*quasi-stationary* state) we have:

$$gH\frac{\partial h}{\partial x}+\frac{r}{H}S_x-c\,S_y=\frac{1}{\varrho}\tau_x, \tag{18}$$

$$gH\frac{\partial h}{\partial y}+\frac{r}{H}S_y+c\,S_x=\frac{1}{\varrho}\tau_y, \tag{19}$$

obtained from (15) and (16) by putting the accelerations equal to zero.

Neglecting the vertical component of the velocity at the bottom as well as at the sea surface, which is permissible since the bottom slope and the slope of the sea surface are small, we find from the equation of continuity (17), for the stationary state:

$$\frac{\partial S_x}{\partial x}+\frac{\partial S_y}{\partial y}=0. \tag{20}$$

Equation (20) implies the existence of a stream function Φ, defined by:

$$\left.\begin{aligned} S_x&=-\frac{\partial\Phi}{\partial y}\\ S_y&=+\frac{\partial\Phi}{\partial x}\end{aligned}\right\}. \tag{21}$$

Substituting these expressions for S_x and S_y in (18) and (19), we obtain, after dividing both members of the equations by gH:

$$\frac{\partial h}{\partial x}-\frac{r}{gH^2}\frac{\partial\Phi}{\partial y}-\frac{c}{gH}\frac{\partial\Phi}{\partial x}=\frac{\tau_x}{\varrho g H}, \tag{22}$$

$$\frac{\partial h}{\partial y}+\frac{r}{gH^2}\frac{\partial\Phi}{\partial x}-\frac{c}{gH}\frac{\partial\Phi}{\partial y}=\frac{\tau_y}{\varrho g H}. \tag{23}$$

It is evident from these equations that the wind effect difference between any two places can be computed if the wind field (the wind stress $\vec{\tau}$) and the field of the volume transport (the stream function Φ) are known. The stream function, however, connot be considered as being an independent quantity, since it is determined by the wind field and the boundary conditions. The differential equation that describes the dependence of Φ on wind stress can easily be deduced from (22) and (23) by differentiating these equations with respect to y and x, respectively, and subtracting the equations, thus obtained, thus eliminating the wind effect h. This yields:

$$\frac{\partial}{\partial x}\left(\frac{r}{H^2}\frac{\partial\Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{r}{H^2}\frac{\partial\Phi}{\partial y}\right)+\frac{\partial}{\partial y}\left(\frac{c}{H}\frac{\partial\Phi}{\partial x}\right)-\frac{\partial}{\partial x}\left(\frac{c}{H}\frac{\partial\Phi}{\partial y}\right)=\frac{\partial}{\partial x}\left(\frac{\tau_y}{\varrho H}\right)-\frac{\partial}{\partial y}\left(\frac{\tau_x}{\varrho H}\right). \tag{24}$$

1.2 Boundary conditions

In order to be able to solve Φ from equation 1.1 (24) the boundary conditions must be specified. I shall not try to solve this equation for a sea of a completely arbitrary shape, but confine myself here to a marginal sea (like the North Sea, which in fact I have in mind) called B and having a wide connection with a very large and deep sea basin A (the ocean) and a coastline interrupted by the entrance to a relatively narrow strait through which water can flow in or out (see figure 1.2 a).

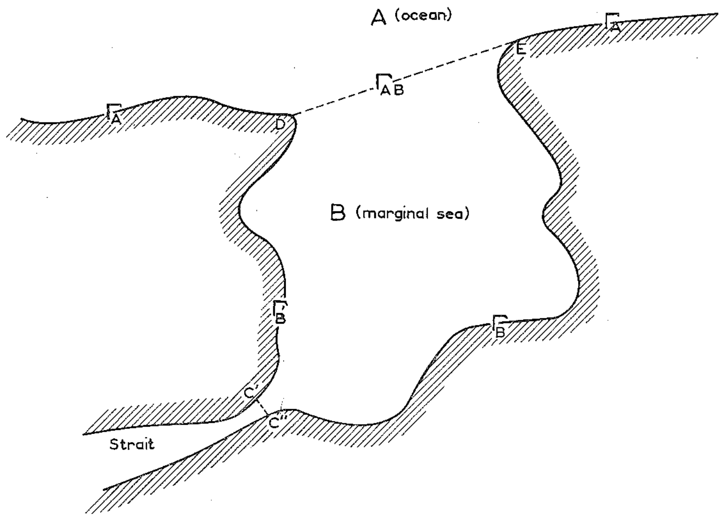

Fig. 1.2 a. Example of a marginal sea of an ocean.

For solving Φ the ocean, the marginal sea and the strait must be considered as a single region since a change in the state of one of the parts has a bearing on the sea regions connected with it. Along an uninterrupted coastline Γ the value of the stream function is a constant, since the component of the volume transport vector normal to the coast vanishes:

$$0 = [\overleftrightarrow{ds}_\Gamma \,.\, \vec{S}] = S_y dx_\Gamma - S_x dy_\Gamma = \left(\frac{\partial \Phi}{\partial x}\right)_\Gamma dx_\Gamma + \left(\frac{\partial \Phi}{\partial y}\right)_\Gamma dy_\Gamma = (d\Phi)_\Gamma, \qquad (1)$$

where [.] is the symbol for the vectorial product and $\overleftrightarrow{ds}_\Gamma$ is a line element along the coastline Γ, with components dx_Γ and dy_Γ. From (1) we obtain:

$$\Phi_\Gamma = \text{constant}, \qquad (2)$$

as was stated.

In a sea having its bottom sloping towards the coast (beach) the stream function has moreover to satisfy the condition that the derivative of Φ normal to the coast vanishes, for, since the depth becomes zero when approaching the coastline, the volume transport will vanish too if we exclude infinite velocities. For a closed sea this implies that the current system cannot be described by a differential equation of the second order like 1.1 (24) since the solution of such an equation is completely determined by one single boundary condition. In general it will not be possible to satisfy the second boundary condition unless the order of the differential equation is more than two.

An equation of a higher order is indeed obtained if we drop the restriction that h should be small as compared with H, thus returning to non-linear equations. This difficult problem will not be dealt with: it will be assumed that the depth does not vanish at the coastline. In 1.3.4. an estimate of the effect of simplification will be given. The boundary of the ocean (A) be Γ_A as far as it consists of a coastline and Γ_{AB} where it borders on the region B. Γ_{AB} may be chosen arbitrarily: the splitting up of the whole sea into two parts, one being called the ocean and the other the marginal sea, is not unique, in general, while the depth varies gradually and the coastline is of an irregular shape.

Suppose that Φ is known at Γ_A, viz.:

$$\Phi\,(\Gamma_A) = f_A, \tag{3}$$

where f_A represents a known function of place along Γ_A. In the following a subscript A or B will denote the region to which a certain quantity refers.

Further, it is assumed that the stream function is known and constant on both coastlines bordering this marginal sea. The values of the constant will, in general, be different for the two coastlines, indicating that water flows into or out of the sea, through the strait separating both coastlines. The quantities of fresh water from rivers or rain flowing into the sea, which are very small as compared with the quantities involved in sea currents are neglected. The difference between the values of the stream function on the two coastlines is a measure of the total transport of water through the strait, as is evident from the definition of the stream function. The variation of the stream function across the opening to the strait may be chosen arbitrarily, provided the stream function has a finite gradient in each point of the borderline between sea and strait, and has, at both ends of this borderline, the prescribed values. This is permissible as long as we are interested in the current pattern only at relatively large distances from the strait (compared with the width of the strait), since in this case the strait may be considered as a point-source or -sink. The only important thing then is the total amount of water transport through the strait.

Thus, the stream function at Γ_B is supposed to be given:

$$\Phi\,(\Gamma_B) = f_B, \tag{4}$$

where Γ_B consists of both coastlines and the borderline with the strait.

In principle, the stream function can now be solved for the whole sea from the differential equation 1.1 (24) and the boundary conditions (3) and (4), if the wind stress field is known for the whole sea area considered. VELTKAMP [45] has shown, that if the ocean is very deep as compared with the marginal sea, and the depth in each of them is constant, a good approximation of Φ_B for the case of equilibrium can be obtained by solving Φ from 1.1 (24) with the boundary condition (4) at Γ_B and the condition:

$$h = 0 \text{ at } \Gamma_{AB}. \tag{5}$$

This condition is not yet an explicit boundary condition for 1.1 (24), but it can easily be converted into one by using the component along Γ_{AB} of the equation of motion, which in analogy to 1.1 (22) becomes:

$$\frac{\partial h}{\partial s} - \frac{r}{gH^2_{AB}} \frac{\partial \Phi_B}{\partial n} - \frac{c}{gH_{AB}} \frac{\partial \Phi_B}{\partial s} = \frac{\tau_{AB}}{\varrho g H_{AB}} \text{ at } \Gamma_{AB}, \tag{6}$$

where τ_{AB} denotes the component of the wind stress parallel to Γ_{AB}, H_{AB} the depth in region B in the immediate vicinity of Γ_{AB} and $\partial/\partial s$ and $\partial/\partial n$ denote differentiation in the direction parallel or normal to Γ_{AB}, respectively; $\vec{dn}$ is orientated in such a way with respect to $\overleftrightarrow{ds}$, that the vector product $[\vec{ds} \,.\, \vec{dn}]$ is directed vertically upwards. Condition (5) implies that the derivative of h along Γ_{AB} is zero. Together with (6), this yields the following boundary condition for Φ at Γ_{AB}:

$$\frac{r}{H_B} \frac{\partial \Phi_B}{\partial n} + c \frac{\partial \Phi_B}{\partial s} + \frac{\tau_{AB}}{\varrho} = 0 \text{ at } \Gamma_{AB}. \tag{7}$$

Now assume that the approximate condition (5) may also be used if the depths of the ocean and of the sea are not constant, provided the depths of the ocean are great as compared with those of the marginal sea. The fact that the wind effect along Γ_{AB} is practically zero is very important for this purpose since the computation of wind effects in this marginal sea may now be confined to the wind field over that sea. Mathematically it can be shown that there is only one solution of equation 1.1 (24) which satisfies the boundary conditions (4) and (7).

The solution of Φ determines the gradient of the wind effect, as follows from 1.1 (22) and 1.1 (23). Consequently, the wind effect itself is also fixed if the wind effect is known at one point. Since the windeffect at Γ_{AB} is known the equilibrium wind effect in a marginal sea from the wind stress field over that sea can, in principle, be solved.

1.3 SPLITTING-UP OF THE WIND EFFECT INTO PARTS

1.3.1 INTRODUCTION AND PRINCIPLE OF THE SPLITTING-UP

In the paragraphs 1.1 and 1.2 it has been shown that the central problem in the computation of wind effects consists of the determination of the stream function Φ from 1.1 (24). For an arbitrary coastline and an arbitrary bottom topography this is a complicated problem, which can only be solved by numerical methods. It is therefore useful to split up the problem into elementary parts. For the following it is convenient to combine the equations of motion 1.1 (22) and 1.1 (23) into a vector equation:

$$\nabla h = \frac{\vec{\tau}}{\varrho g H} + \frac{c}{gH} \nabla \Phi - \frac{r}{gH^2} [\vec{k} \cdot \nabla \Phi], \qquad (1)$$

where ∇ (nabla) is the symbol for the gradient operator and $\vec{k}$ denotes the unit vector pointing vertically upwards. From (1) it is obvious that the wind effect can be split up into a direct effect of the wind stress, the static effect h_τ, determined by the term $\vec{\tau}/\varrho g H$ of (1), and a current effect h_Φ, determined by the terms containing Φ. Concerning this splitting-up it must be remarked, however, that each term on the right of equation (1) is not curl-free, in general, although the sum of the three terms is. The sum of the terms containing Φ is only curl-free if the vector $\vec{\tau}/\varrho g H$ is so. If it is wished to compute the difference in effect between two places in this particular (curl-free) case, the term $\vec{\tau}/\varrho g H$ may be integrated along an arbitrarily chosen path connecting these places. If, however, $\vec{\tau}/\varrho g H$ is not curl-free, the difference in static effect will depend on the path of integration that has been followed. In order to define a difference in static effect unambiguously, an appointment must be made about the path of integration. The path of integration is chosen here as follows: after having defined a cartesian system of coordinates (x, y), first follow a line $x =$ constant, until the y of the endpoint is reached and next integrate in the x-direction up to the second point. For an arbitrary sea, however, it will not always be possible to fix a path in this way without crossing land. This can be avoided by defining the path of integration in such cases as follows (islands are left out of consideration): follow the line $x =$ constant through the starting point until a coast is reached; then follow the coastline up to the y of the endpoint; next integrate in the x-direction up to the endpoint, if no coastline is met; if again a coast is met, one should once more proceed as just described, etc.

1.3.2 Static wind effect

The difference of static wind effect between any two points P and Q, having the coordinates x_P, y_P and x_Q, y_Q respectively, can be written as follows, if we assume $y_P < y_Q$ and if no coast crosses the paths of integration:

$$h_\tau(\mathrm{Q}) - h_\tau(\mathrm{P}) = \int_{y_P}^{y_Q} \left(\frac{\tau_y}{\varrho g H}\right)_{x_P} dy + \int_{x_P}^{x_Q} \left(\frac{\tau_x}{\varrho g H}\right)_{y_Q} dx, \qquad (2)$$

where the indices x_P and y_Q of the integrands mean: $x = x_P$ and $y = y_Q$, respectively.

In case of a homogeneous wind field (2) can be simplified to:

$$h_\tau(Q) - h_\tau(P) = \frac{\tau_y (y_Q - y_P)}{\varrho g \bar{H}_y} + \frac{\tau_x (x_Q - x_P)}{\varrho g \bar{H}_x}, \tag{3}$$

where $\bar{H}_x$ and $\bar{H}_y$ represent the harmonic averages of the depths along the straight lines (x_P, x_P) — (x_P, y_Q) and (x_P, y_Q) — (x_Q, y_Q), respectively. With a view to an application, further on (section 2.2) this difference of static wind effect is split up into two parts; if we denote by $\bar{H}$ the harmonic mean depth of the sea or of a part of it, (3) can be transformed to:

$$h_\tau(Q) - h_\tau(P) = \frac{\tau_y (y_Q - y_P)}{\varrho g \bar{H}} + \frac{\tau_x (x_Q - x_P)}{\varrho g \bar{H}} + \frac{\tau_y (y_Q - y_P)}{\varrho g \bar{H}} \times \frac{\bar{H} - \bar{H}_y}{\bar{H}_y} +$$

$$+ \frac{\tau_x (x_Q - x_P)}{\varrho g \bar{H}} \times \frac{\bar{H} - \bar{H}_x}{\bar{H}_x}. \tag{4}$$

The first and the second term in the righthand side of (4) represent the difference of static effect between P and Q if the depth of the sea is constant and equal to $\bar{H}$, the third and fourth give the correction to be applied to this difference, due to the non-constancy of the depth.

1.3.3 Current effect

As to this part of the wind effect the same comments can be made concerning the path of integration, as have been made in connection with the static wind effect. The path of integration must be the same as for the static wind effect because otherwise the sum of both will not be equal to the difference in wind effect between P and Q ,in general. The current effect difference between P and Q can be written as follows:

$$h_\Phi(Q) - h_\Phi(P) = \int_{y_P}^{y_Q} \left(\frac{c}{gH} \frac{\partial \Phi}{\partial y} - \frac{r}{gH^2} \frac{\partial \Phi}{\partial x} \right)_{x_P} dy +$$

$$+ \int_{x_P}^{x_Q} \left(\frac{c}{gH} \frac{\partial \Phi}{\partial x} + \frac{r}{gH^2} \frac{\partial \Phi}{\partial y} \right)_{y_Q} dx. \tag{5}$$

If P and Q lie on the same coast line, (5) has to be replaced by:

$$h_{\Phi}(Q) - h_{\Phi}(P) = -\int_{s_P}^{s_Q} \frac{r}{gH^2} \frac{\partial \Phi}{\partial n} ds, \tag{6}$$

where $\partial\Phi/\partial n$ denotes the derivative of Φ normal to the coast and landward, and s a length coordinate along the coast orientated in such a way that $\vec{ds} = [\vec{dn} \,.\, \vec{k}]$. In (6) the terms with c are lacking because the coast is a stream line (Φ = constant), as is known (section 1.2).

1.3.4 Effect of the shallow coastal strip on the water level at the coast

To compute differences of static wind effect or of current effect by means of the formulas of 1.3.2 and 1.3.3 the restriction that h must be small compared with H must be borne in mind. This implies that these formulas may not be used for the computation of wind effects in the vicinity of beachy coasts, since there the depth becomes even zero when approaching the coast. Application of the formulas of section 1.3.2 would then yield an infinite static wind effect at the coast unless the wind stress should vanish there too. The latter, of course, is not to be expected in nature. But even if this were so the static effect and the current effect computed by those formulas would be much too large since the condition $h \ll H$ would still be violated then.

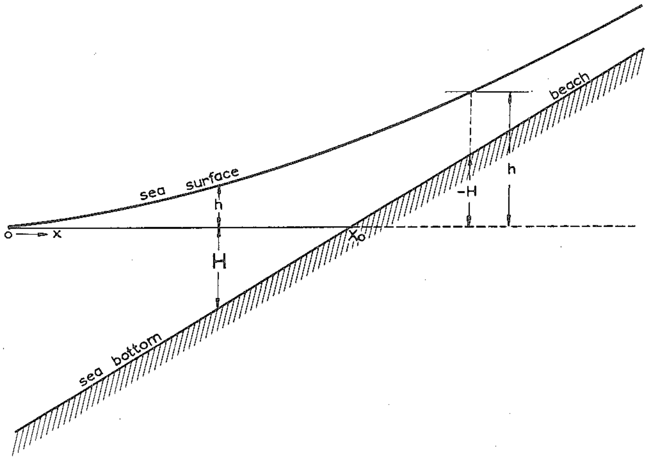

FIG. 1.3 a. Wind effect on a sloping beach caused by a landward wind.

It will be shown now that the non-linear equation for the wind effect which is obtained if the condition $h \ll H$ is dropped can easily be solved exactly in the one-dimensional case if the volume transport is zero. This case corresponds to the problem of computing differences of wind effect for an infinitely long straight beachy coast having a uniform bottom slope perpendicular to the coast, under the assumption that the Coriolis force may be neglected. It should be borne in mind that in the non-linear case the wind effect cannot be split up into a static effect and a current effect which are independent of each other.

Let the x-axis be perpendicular to the coast and let the undisturbed depth be given by:

$$H(x) = \beta (x_0 - x), \tag{7}$$

where β denotes the slope of the bottom; x is zero at a distance x_0 from the undisturbed coastline (the borderline of the water surface) and the depth there is: $H(0) = \beta x_0$ (see figure 1.3 a).

In the stationary state the horizontal component of the pressure gradient force exerted on the vertical column of water of height $H + h$ will exactly balance the apparent wind stress, provided there is no volume transport ($\vec{S} = 0$). So, if the wind stress $\vec{\tau}$ is at right angles to the coast we have:

$$\tau = \varrho g (H + h) \frac{dh}{dx}. \tag{8}$$

Equation (8) can easily be solved by interchanging the dependent and independent variables h and x.

We thus obtain:

$$h(x) - h(0) = \frac{\tau}{\varrho g \beta} \ln \frac{H(0) + h(0) - \dfrac{\tau}{\varrho g \beta}}{H(x) + h(x) - \dfrac{\tau}{\varrho g \beta}}. \tag{9}$$

From (9) it is obvious that:

$$\lim_{x \to \infty} (H + h) = \frac{\tau}{\varrho g \beta} \neq 0. \tag{10}$$

It appears that, if there is a wind blowing towards a coast having a sloping beach, a coastline can not exist under the simple conditions of the present theory. For large positive values of x the sea surface will be practically parallel to the bottom, the depth being such that the corresponding pressure gradient force exactly balances the apparent wind stress.

This limiting value of the depth of the water layer is exactly equal to the difference of static effect that would occur between $x = x_0$ and any other x, say x_1, in a sea of constant depth $H(x_1)$ under equal circumstances:

$$\frac{\tau}{\varrho g \beta} = \frac{\tau (x_0 - x_1)}{\varrho g H (x_1)}. \tag{11}$$

In nature, of course, a coastline exists at finite values of x. This shows that other mechanisms become important in the vicinity of the coast, such as the formation of waves and breakers, field accelerations, porosity of the bottom, instability of the solution in the direction parallel to the coast (rip currents). In the meanwhile, the limiting "depth" calculated above would be very thin, as may be seen from the following examples. For a bottom slope of 1/100 and wind velocities of 1, 10 and 20 m/sec the limiting layer thickness would be 0.003; 0.3 and 1.2 cm, respectively.

As a suitable measure of the wind effect on a beachy coast choose the wind effect at $x = x_0$, for which by substituting x_0 for x in (9) is obtained:

$$h(x_0) - h(0) = \frac{\tau x_0}{\varrho g H(0)} \ln \frac{H(0) + h(0) - \frac{\tau x_0}{\varrho g H(0)}}{h(x_0) - \frac{\tau x_0}{\varrho g H(0)}}. \tag{12}$$

For reasonable values of τ, x_0 and $H(o)$ e.g. 16 dn/cm^2 (which corresponds to a wind velocity of about 20 m/sec), 2 km and 30 m respectively, this yields:

$$h(x_0) - h(0) = 6.3 \frac{\tau x_0}{\varrho g H(0)} = 6.7 \text{ cm}. \tag{13}$$

This means that for a bottom having a slope of 1.5/100 the difference of static windeffect will be 6.3 times as much as it would be if in the coastal strip of 2 km width the depth remained constant, i.e. equal to the depth prevailing at 2 km distance from the coast. Thus we have an additional wind effect amounting to 5.3 $\tau x_0/\varrho g H(0)$.

For a sea like the North Sea, this additional wind effect will usually be a small fraction of the total wind effect that would appear at the coast if the coastal strip had a constant depth, since the fetch of the wind over the North Sea is of the order of some hundreds of kilometers, which is very large as compared with the narrow coastal strip. If the additional wind effect is expressed in the unit used further on, viz. $\tau a/\varrho g H_S$, where $a = 575$ km and $H_S = 43$ m, we obtain, for the case under consideration (bottom slope = 1.5/100, $x_0 =$ 2 km):

$$h(x_0) - h(0) = 0.026 \frac{\tau a}{\varrho g H_S}. \tag{14}$$

The effect will actually be smaller than this, because of the other forces involved, as explained above.

The general two-dimensional, non-linear problem of the wind effect in the shallow coastal strip is much more complicated than the simple one-dimensional theory given here. I shall not tackle this problem (where also the coupling between wind effect and tide should be considered), but assume that also in the general case the additional wind effect will be small as compared with the total wind effect, so that it may be neglected.

1.4 Splitting-up the current effect

On account of the linearity of the equations of motion 1.1(22) and 1.1(23), or 1.3(1), the stream function Φ may be thought to be composed of stream functions Φ_i ($i = 1,2,3$) corresponding to current fields induced by different "causes". Consequently, the problem of solving Φ from 1.1(24) can be split up into simpler problems and at the same time a better insight is obtained into the relative importance of the different factors involved.

First split up the righthand member of the equation 1.1(24) as follows:

$$\frac{\partial}{\partial x}\left(\frac{\tau_y}{\varrho H}\right)-\frac{\partial}{\partial y}\left(\frac{\tau_x}{\varrho H}\right)=\frac{1}{\varrho H}\left(\frac{\partial \tau_y}{\partial x}-\frac{\partial \tau_x}{\partial y}\right)+\left(\frac{\tau_y}{\varrho}\frac{\partial H^{-1}}{\partial x}-\frac{\tau_x}{\varrho}\frac{\partial H^{-1}}{\partial y}\right), \qquad (1)$$

where the first term to the right is proportional to curl $\vec{\tau}$ and the second term on the right depends on the slope of the bottom. Next we write Φ as the sum of three stream functions Φ_i ($i = 1,2,3$):

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3, \qquad (2)$$

where Φ_1, Φ_2 and Φ_3 are defined as follows.
Φ_1 is a solution of the differential equation

$$\frac{\partial}{\partial x}\left(\frac{r}{H^2}\cdot\frac{\partial \Phi_1}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{r}{H^2}\frac{\partial \Phi_1}{\partial y}\right)+\frac{\partial}{\partial y}\left(\frac{c}{H}\frac{\partial \Phi_1}{\partial x}\right)-\frac{\partial}{\partial x}\left(\frac{c}{H}\frac{\partial \Phi_1}{\partial y}\right)=\frac{1}{\varrho H}\left(\frac{\partial \tau_y}{\partial x}-\frac{\partial \tau_x}{\partial y}\right), \qquad (3)$$

obtained by substituting for the righthand member of 1.1(24) the first term of the righthand member of equation (1). The stream function Φ_1 describes a current field induced by the vorticity of the wind stress (curl $\vec{\tau}$); for this reason call Φ_1 the *wind-vorticity stream function* and the corresponding current (field) the *wind-vorticity current (field)*, or shortly the vorticity stream function and the vorticity current (field) respectively. The boundary conditions which Φ_1 has to satisfy will be specified below. The stream function Φ_2 is a solution of the differential equation

$$\frac{\partial}{\partial x}\left(\frac{r}{H^2}\frac{\partial \Phi_2}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{r}{H^2}\frac{\partial \Phi_2}{\partial y}\right)+\frac{\partial}{\partial y}\left(\frac{c}{H}\frac{\partial \Phi_2}{\partial x}\right)-\frac{\partial}{\partial x}\left(\frac{c}{H}\frac{\partial \Phi_2}{\partial y}\right)=\frac{1}{\varrho}\left(\tau_y\frac{\partial H^{-1}}{\partial x}-\tau_x\frac{\partial H^{-1}}{\partial y}\right), \qquad (4)$$

obtained by substituting for the righthand member of 1.1(24) the second term of the righthand member of (1).

Φ_2 describes that part of the total current that is caused by the wind stress component parallel to the local depth line, since the righthand member of (4) represents the vectorial product of $\vec{\tau}$ and $\nabla\, 1/H$, which means that only the component of $\vec{\tau}$ perpendicular to $\nabla\, 1/H$, or parallel to the local depth line contributes to Φ_2. For this reason we call Φ_2 the *bottom-slope stream function,* and the corresponding current the *bottom-slope current*, or shortly the slope stream function and the slope current, respectively. The boundary conditions for Φ_2 will be specified below.

Finally, Φ_3 is a solution of the differential equation:

$$\frac{\partial}{\partial x}\left(\frac{r}{H^2}\frac{\partial \Phi_3}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{r}{H^2}\frac{\partial \Phi_3}{\partial y}\right)+\frac{\partial}{\partial y}\left(\frac{c}{H}\frac{\partial \Phi_3}{\partial x}\right)-\frac{\partial}{\partial x}\left(\frac{c}{H}\frac{\partial \Phi_3}{\partial y}\right)=0. \tag{5}$$

Φ_3 will be called *leak-stream function* and the corresponding current the *leak current*, as will become clear when the boundary conditions will have been specified.

For Φ the boundary conditions are given by 1.2 (4) and 1.2(7) at Γ_B and Γ_{AB} respectively. Bij (2) they can be written as follows:

$$\Phi_1 + \Phi_2 + \Phi_3 = f_B \text{ at } \Gamma_B, \tag{6}$$

and

$$\left(\frac{r}{H_B}\frac{\partial \Phi_1}{\partial n}+c\frac{\partial \Phi_1}{\partial s}\right)+\left(\frac{r}{H_B}\frac{\partial \Phi_2}{\partial n}+c\frac{\partial \Phi_2}{\partial s}\right)+\left(\frac{r}{H_B}\frac{\partial \Phi_3}{\partial n}+c\frac{\partial \Phi_3}{\partial s}\right)+\frac{1}{\varrho}\tau_{AB}=0 \text{ at } \Gamma_{AB}. \tag{7}$$

This way of writing the boundary conditions suggests the following boundary conditions for Φ_1, Φ_2 and Φ_3:

For Φ_1:

$$\Phi_1 = 0 \text{ at } \Gamma_B, \tag{8}$$

$$\frac{r}{H_B}\frac{\partial \Phi_1}{\partial n}+c\frac{\partial \Phi_1}{\partial s}=0 \text{ at } \Gamma_{AB}. \tag{9}$$

For Φ_2:

$$\Phi_2 = 0 \text{ at } \Gamma_B, \tag{10}$$

$$\frac{r}{H_B}\frac{\partial \Phi_2}{\partial n}+c\frac{\partial \Phi_2}{\partial s}+\frac{1}{\varrho}\tau_{AB}=0 \text{ at } \Gamma_{AB}. \tag{11}$$

For Φ_3 the boundary conditions are now fixed too, since for the sum Φ the boundary conditions are given by (6) and (7).

Thus, we obtain for Φ_3:

$$\Phi_3 = f_B \text{ at } \Gamma_B, \tag{12}$$

$$\frac{r}{H_B}\frac{\partial \Phi_3}{\partial n}+c\frac{\partial \Phi_3}{\partial s}=0 \text{ at } \Gamma_{AB}. \tag{13}$$

It will be evident, now, why Φ_3 was called the leak stream function: it is completely determined by the leak current through the narrow sea strait, as follows from (12) and the definition of f_B in section 1.2.

1.5 SOLUTION OF THE EQUATION FOR THE VOLUME TRANSPORT

1.5.1 FORMAL SOLUTION

In this section a differential equation of the type of 1.1 (24) will be formally solved. The righthand member of the equation is assumed to be an arbitrary function of x and y, so that at the same time a formal solution of the equations 1.4 (3), 1.4 (4) and 1.4 (5) is found. Supposing that nowhere in the region considered r/H^2 becomes zero, we may transform the lefthand member of these differential equations by multiplying by H^2/r into the form

$$\mathcal{L}[\Phi] \equiv \triangle\, \Phi + (\vec{A} \,.\, \nabla\, \Phi), \tag{1}$$

where $\triangle$ is the Laplace operator $\partial^2/\partial x^2 + \partial^2/\partial y^2$, (.) is the symbol for scalar vector-multiplication, and $\vec{A}$ stands for

$$\vec{A} = -2 \nabla \ln \frac{H}{H_S} + \frac{c H_S}{r} \times \frac{H}{H_S} \left[\vec{k} \,.\, \nabla \ln \frac{H}{H_S} \right]; \tag{2}$$

H_S represents an arbitrary constant depth, for the present, so that H/H_S is a dimensionless quantity. The differential equation to be solved has now the following form:

$$\mathcal{L}[\Phi] = \Omega\,(x, y), \tag{3}$$

where $\Omega\,(x, y)$ is a function of x and y, which for the three current components defined in section 1.4 takes the following forms:

$$\Omega = \Omega_1 \equiv \frac{H}{\varrho r} \left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} \right), \tag{4}$$

$$\Omega = \Omega_2 \equiv \frac{H}{\varrho r} \left(\tau_x \frac{\partial \ln H}{\partial y} - \tau_y \frac{\partial \ln H}{\partial x} \right) \tag{5}$$

$$\text{and } \Omega = \Omega_3 \equiv 0, \tag{6}$$

respectively; these expressions are obtained by multiplying the righthand members of 1.4(3), 1.4(4) and 1.4(5) bij H^2/r.

In this section, however, the function $\Omega\,(x, y)$ is left completely arbitrary.

From a mathematical point of view it would be very convenient if the differential expression (1) were "self-adjoint", because the corresponding eigenfunctions defined by the differential equation

$$\mathcal{L}[\Phi] + \lambda\, \Phi = 0, \tag{7}$$

would then form a complete set of orthonormal functions and the solution of (3) could then be written as the sum of a series of eigenfunctions.

$\mathscr{L}[\Phi]$ is called self-adjoint if it is identical to the adjoint differential form $\mathscr{M}[\Phi]$, which is defined by:

$$\mathscr{M}[\Phi] \equiv \triangle \Phi - (\nabla \cdot \Phi \vec{A}). \tag{8}$$

For an arbitrary $\vec{A}$, the differential form $\mathscr{L}[\Phi]$ will not be self-adjoint, however. Indeed, from (1) and (8) it is evident that $\mathscr{L}[\Phi]$ will be self-adjoint only if any function Φ satisfies the equation:

$$(\vec{A} \cdot \nabla \Phi) + (\nabla \cdot \Phi \vec{A}) = 0, \tag{9}$$

which yields $\vec{A} \equiv 0$. This is only true if the depth is constant, as is evident from (2). Thus, for an arbitrary depth pattern $\mathscr{L}[\Phi]$ will not be self-adjoint. It is possible, however, to transform $\mathscr{L}[\Phi]$ into a self-adjoint expression by means of a transition to new variables ξ and η instead of x and y:

$$\xi = \xi(x, y) \text{ and } \eta = \eta(x, y). \tag{10}$$

Substituting the new variables in (1) a new differential form is obtained:

$$\mathscr{L}'[\Phi] \equiv (\nabla \xi)^2 \frac{\partial^2 \Phi}{\partial \xi^2} + 2(\nabla \xi \cdot \nabla \eta) \frac{\partial^2 \Phi}{\partial \xi \partial \eta} + (\nabla \eta)^2 \frac{\partial^2 \Phi}{\partial \eta^2} + \\ + (\vec{A} \cdot \nabla \xi) \frac{\partial \Phi}{\partial \xi} + (\vec{A} \cdot \nabla \eta) \frac{\partial \Phi}{\partial \eta}. \tag{11}$$

The adjoint form is now:

$$\mathscr{M}'[\Phi] \equiv \frac{\partial^2}{\partial \xi^2}\{(\nabla \xi)^2 \Phi\} + 2 \frac{\partial^2}{\partial \xi \partial \eta}\{(\nabla \xi \cdot \nabla \eta) \Phi\} + \\ + \frac{\partial^2}{\partial \eta^2}\{(\nabla \eta)^2 \Phi\} - \frac{\partial}{\partial \xi}\{(\vec{A} \cdot \nabla \xi) \Phi\} - \frac{\partial}{\partial \eta}\{(\vec{A} \cdot \nabla \eta) \Phi\}. \tag{12}$$

$\mathscr{L}'[\Phi]$ is self-adjoint if it is identical to $\mathscr{M}'[\Phi]$; this will be so if ξ and η satisfy the following differential equations, which are easily deduced from (11) and (12):

$$\left.\begin{aligned} (\vec{A} \cdot \nabla \xi) &= \frac{\partial}{\partial \xi}(\nabla \xi \cdot \nabla \xi) + \frac{\partial}{\partial \eta}(\nabla \xi \cdot \nabla \eta), \\ \text{and} \quad (\vec{A} \cdot \nabla \eta) &= \frac{\partial}{\partial \xi}(\nabla \xi \cdot \nabla \eta) + \frac{\partial}{\partial \eta}(\nabla \eta \cdot \nabla \eta). \end{aligned}\right\} \tag{13}$$

In principle ξ and η can be solved from (13), but, because of the non-linearity of these equations, this will be extremely difficult, unless numerical methods are used. For obtaining a better insight into the problems, however, numerical methods are not suitable because the values of the parameters will then have to be chosen at the beginning of the computation. For this reason and for overcoming the difficulties of solving the equations (13), we shall not try to make the differential equation (3) self-adjoint for a completely arbitrary depth configuration, but we shall look for those depth configurations, that is to say for that form of $\vec{A}$, that makes (3) self-adjoint without too complicated computations. If the trivial case of $\vec{A}$ being identically equal to zero is excluded, which corresponds to a constant depth (as seen above), the only way is to look for a suitable integrating factor $\mu(x, y)$, by which $\mathcal{L}[\Phi]$ has to be multiplied in order to obtain a self-adjoint form. The adjoint form of

$$\mu \mathcal{L}[\Phi] \equiv \mu \triangle \Phi + \mu (\vec{A} \cdot \nabla \Phi) \tag{14}$$

is

$$\triangle (\mu \Phi) - (\nabla \cdot \mu \Phi \vec{A}). \tag{15}$$

Equating (14) and (15) the differential equation is obtained which μ has to satisfy in order to make (14) self-adjoint:

$$\nabla \ln \mu = \vec{A}. \tag{16}$$

From (16) it follows that $\vec{A}$ must be curl-free. Assuming that c and r are constants, this implies that, according to (2), the depth must satisfy the equation:

$$\triangle H = 0, \tag{17}$$

in order to make (14) self-adjoint. In section 3.3 it will be shown that r is not a true constant since it depends on the wind, the vertical eddy viscosity, the roughness of the bottom, the depth and the velocity of the tidal currents. For given circumstances, however, and for mathematical convenience, we may take it as constant. As to c, we may take it to be constant for the North Sea, since it varies in fact between the values $2\,\omega \sin 50°$ and $2\,\omega \sin 60°$, ω being the angular velocity of the earth's rotation; that means that it may be taken to be $2\,\omega \sin 55° \approx 1.2 \times 10^{-4}\ \text{sec}^{-1}$ for the North Sea with a maximum inaccuracy of 6 percent. If we now assume that (17) is satisfied, (3) can by making use of (16), be transformed into

$$(\nabla \cdot \mu \nabla \Phi) = \mu \Omega. \tag{18}$$

The factor μ is easily found from (16):

$$\mu(x, y) = \mu(x_0, y_0) \exp \left\{ \int_{(x_0, y_0)}^{(x, y)} (\vec{A} \cdot \vec{ds}) \right\}, \tag{19}$$

supposing that μ is known in an arbitrary point (x_0, y_0). The path of integration in (19) may be chosen arbitrarily, since $\vec{A}$ is curl-free.

The next step will be to determine the eigenfunctions $\Phi_{(n)}$ of the equation:

$$(\nabla \cdot \mu \nabla \Phi) + \lambda \mu \Phi = 0. \tag{20}$$

This equation has only non-zero solutions for special values of λ, the eigenvalues λ_n. Any solution of (18) can then be written as the sum of a series of eigenfunctions with coefficients q_n:

$$\Phi = \sum_{n=1}^{\infty} q_n \Phi_{(n)}. \tag{21}$$

Let the expansion of Ω into a series of eigenfunctions be:

$$\Omega = \sum_{n=1}^{\infty} c_n \Phi_{(n)}, \tag{22}$$

with known coefficients c_n. Substituting the expressions (21) and (22) into (18) and making use of (20) we obtain:

$$-\sum_{n=1}^{\infty} q_n \lambda_n \Phi_{(n)} = \sum_{n=1}^{\infty} c_n \Phi_{(n)}, \tag{23}$$

which, because of the orthogonality of the eigenfunctions, yields:

$$q_n = -\frac{c_n}{\lambda_n}. \tag{24}$$

Thus a solution of the differential equation (18) has been found, at least formally:

$$\Phi = -\sum_{n=1}^{\infty} \frac{c_n}{\lambda_n} \Phi_{(n)}. \tag{25}$$

1.5.2 Two applications of 1.5.1

Let us first apply the formal theory of 1.5.1 to the problem of computing the current field in a closed rectangular sea with straight depth lines parallel to one of the sides. A cartesian system of coordinates with axes parallel to the sides of the rectangle and with the origin at the point halfway one of the sides (see figure 1.5 a) is chosen. From the figure it is evident that, in first approximation, the North Sea may be considered as having a rectangular shape, but with an open connection to the ocean instead of being closed.

Let a and b be the width and the length of the rectangle, and let the depth pattern be given by

$$\frac{H}{H_S} = d_1 + d_2 \frac{y}{b}, \tag{26}$$

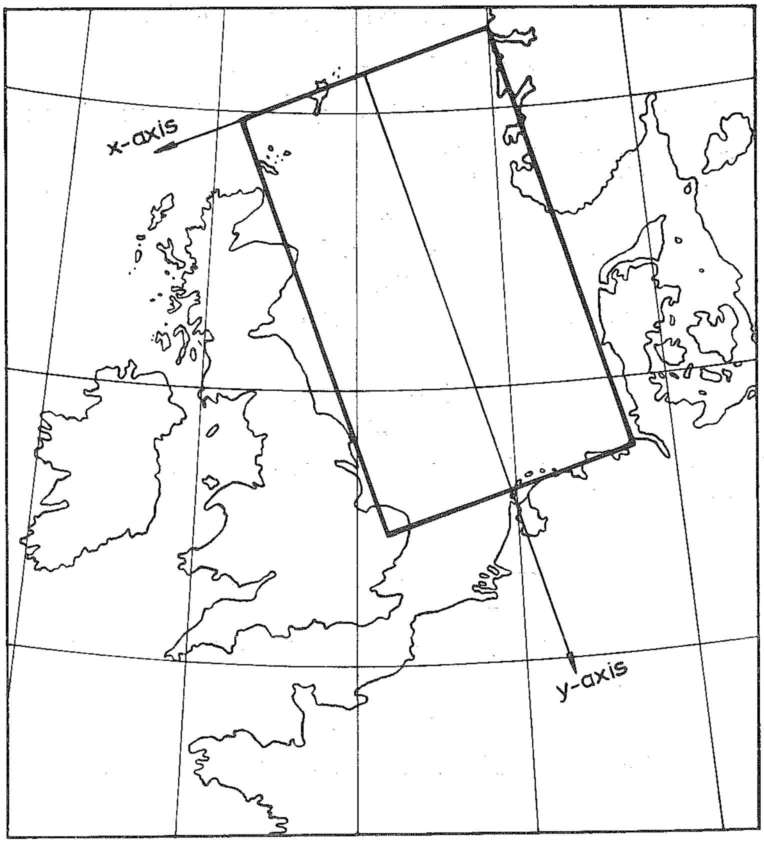

FIG. 1.5 a. Model of the North Sea.

where d_1 and d_2 are dimensionless numbers. This depth profile resembles roughly that of the North Sea, if suitable values of d_1 and d_2 are chosen, and gives depth lines parallel to the x-axis. Moreover, (26) is that solution of (17) that only depends on y. In figure 1.5 b the actual mean depth profile of the North Sea in the longitudinal direction is drawn as a full line; for each y the average depth of four equidistant points, dividing the width of the sea into five equal parts, has been plotted. The best fit of formula (26) to the actual depth profile is obtained by taking for d_1, d_2 and H_S the values 4.60, —4.23 and 35 m respectively.

Then (26) becomes:

$$\frac{H(y)}{H_S} = 4.60 - 4.23\frac{y}{b}. \tag{27}$$

The depth profile according to (27) has been drawn in figure 1.5 b as a

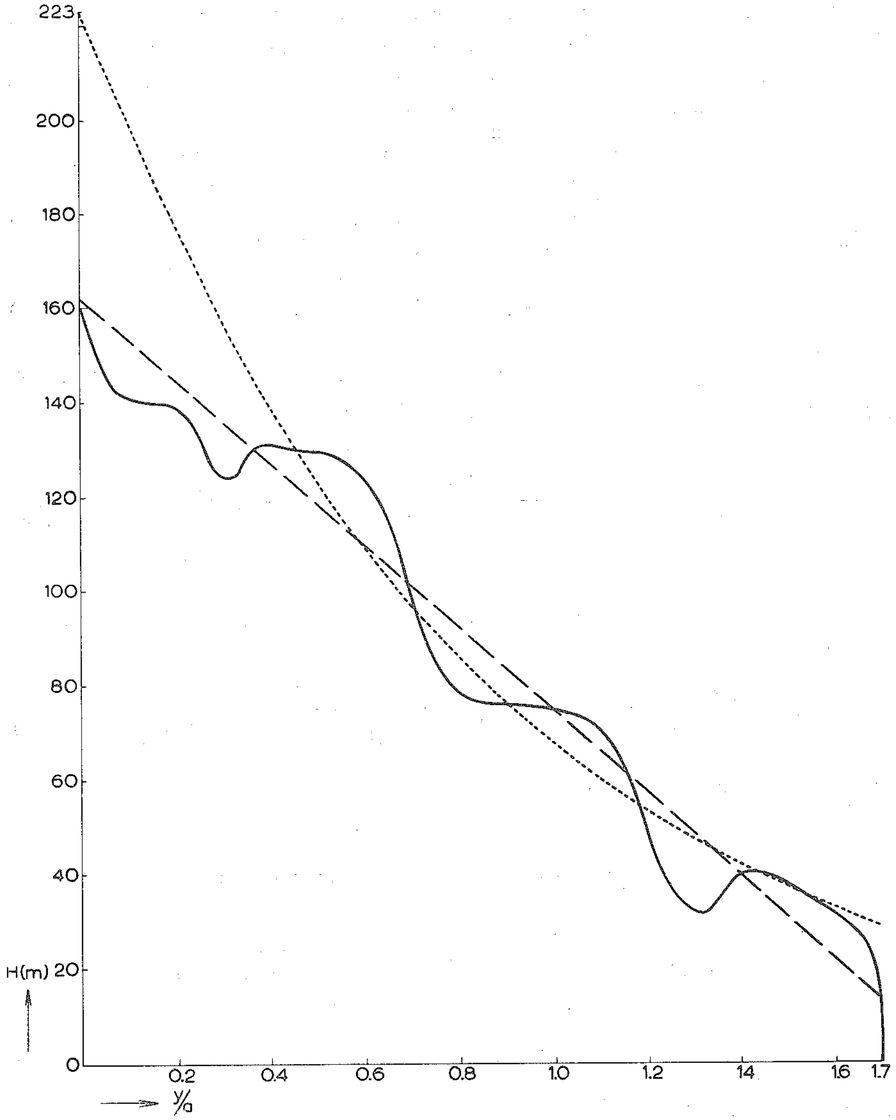

FIG. 1.5 b. Depth profile of the North Sea.
——— Actual mean depth profile.
— — — — Depth profile according to formula 1.5 (27).
.... Depth profile according to formula 2.1 (1).

broken line. It may be remarked that the actual profile can also be approximated with reasonable accuracy by an exponential curve (see figure 1.5 b, dashed line). An exponential model profile would give an advantage if a simpler law of bottom friction were valid: it would then make the equation (3) self-adjoint. The modified law of bottom friction meant is such that r/H is a constant parameter, instead of r, which means that the bottom friction must be proportional to $\vec{S}$ in stead of to $\vec{v}$, as is evident from 1.1 (14). In the theoretical part of this study, the approximation given by (26), which is consistent with the more realistic bottom friction law 1.1 (14) with constant r will be employed.

Substituting H/H_S according to (26) in (2), we obtain:

$$\vec{A} = -\frac{d_2}{b}\left(\frac{c\,H_S}{r}\vec{i} + 2\frac{H_S}{H}\vec{j}\right), \tag{28}$$

where $\vec{i}$ and $\vec{j}$ represent the unit vectors in the x- and y-directions, respectively. According to (19), this expression for $\vec{A}$ yields:

$$\mu(x, y) = \frac{H(o)^2}{H(y)^2}\cdot e^{-\frac{d_2}{b}\frac{cH_S}{r}x} = \left\{\frac{\frac{d_1 b}{d_2}}{\frac{d_1 b}{d_2} + y}\right\}^2 e^{-\frac{d_2}{b}\frac{cH_S}{r}x}, \tag{29}$$

if we take for (x_0, y_0) the point (o,o) and assume μ (o,o) to be equal to 1. Then (18) becomes:

$$\left(\nabla\cdot\frac{H(o)^2}{H(y)^2}\cdot e^{-\frac{d_2}{b}\frac{cH_S}{r}x}\,\nabla\Phi\right) = \frac{H(o)^2}{H(y)^2}\cdot e^{-\frac{d_2}{b}\frac{cH_S}{r}x}\cdot\Omega(x, y). \tag{30}$$

Let the boundary condition be:

$$\Phi = 0. \tag{31}$$

Now assume that $\Phi(x, y)$ can be expanded into a series of eigenfunctions of (20):

$$\Phi(x, y) = \sum_{k=1}^{\infty}\sum_{l=1}^{\infty} p_{kl}\,X_k(x)\,Y_l(y) = \sum_{k=1}^{\infty}\sum_{l=1}^{\infty} p_{kl}\,\Phi_{kl}, \tag{32}$$

where

$$\Phi_{kl} \equiv X_k(x)\,Y_l(y) \tag{33}$$

is an eigenfunction of (20) and, consequently, satisfies the equation:

$$\left(\nabla\cdot\frac{H(o)^2}{H(y)^2}e^{-\frac{d_2}{b}\frac{cH_S}{r}x}\,\nabla\Phi_{kl}\right) + \lambda_{kl}\frac{H(o)^2}{H(y)^2}e^{-\frac{d_2}{b}\frac{cH_S}{r}x}\cdot\Phi_{kl} = 0. \tag{34}$$

The corresponding boundary condition is:

$$\Phi_{kl} = 0. \tag{35}$$

Equation (34) can be split up into two ordinary differential equations of the second order in x and y respectively:

$$X''_k - \frac{d_2}{b}\frac{cH_S}{r}X'_k + \lambda_{xk}X_k = 0, \tag{36}$$

and

$$Y''_l - 2\frac{d_2}{b}\frac{H_S}{H(y)}Y'_l + \lambda_{yl}Y_l = 0; \tag{37}$$

λ_{xk} and λ_{yl} are eigenvalues of (36) and (37), respectively. These eigenvalues are coupled to each other by the condition:

$$\lambda_{xk} + \lambda_{yl} = \lambda_{kl}, \tag{38}$$

The boundary condition (35) can be separated into two boundary conditions, corresponding to (36) and (37) respectively:

$$X_k\left(\frac{a}{2}\right) = X_k\left(-\frac{a}{2}\right) = 0, \tag{39}$$

and

$$Y_l(o) = Y_l(b) = 0. \tag{40}$$

Solving (36) and (37) with the conditions (39) and (40), we obtain the following expressions for the eigenfunctions:

$$X_k(x) = A_{xk}\cos(2k+1)\frac{\pi}{a}x \,.\, e^{\frac{d_2}{2b}\frac{cH_S}{r}x}, \tag{41}$$

and

$$Y_l(y) = A_{yl}\begin{vmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{vmatrix}, \tag{42}$$

where A_{xk} and A_{yl} represent normalizing factors, and K_{11}, K_{12}, K_{21} and K_{22} are defined by:

$$K_{11} = \cos b\frac{d_2}{d_1}\sqrt{\lambda_{yl}} + b\frac{d_2}{d_1}\sqrt{\lambda_{yl}}\sin b\frac{d_2}{d_1}\sqrt{\lambda_{yl}}$$

$$K_{12} = \sin b\frac{d_2}{d_1}\sqrt{\lambda_{yl}} - b\frac{d_2}{d_1}\sqrt{\lambda_{yl}}\cos b\frac{d_2}{d_1}\sqrt{\lambda_{yl}}$$

$$K_{21} = \left\{\cos\left(y + b\frac{d_2}{d_1}\right)\sqrt{\lambda_{yl}} + \left(y + b\frac{d_2}{d_1}\right)\sqrt{\lambda_{yl}}\sin\left(y + b\frac{d_2}{d_1}\right)\sqrt{\lambda_{yl}}\right\}$$

$$K_{22} = \left\{\sin\left(y + b\frac{d_2}{d_1}\right)\sqrt{\lambda_{yl}} - \left(y + b\frac{d_2}{d_1}\right)\sqrt{\lambda_{yl}}\cos\left(y + b\frac{d_2}{d_1}\right)\sqrt{\lambda_{yl}}\right\}.$$

The functions Φ_{kl} constitute an orthonormal set of functions with a weight function $\mu(x, y)$; this means that for all combinations of the subscripts the following relation holds:

$$\int_{-a/2}^{a/2} \int_{0}^{b} \mu(x, y)\, \Phi_{kl}(x, y)\, \Phi_{mn}(x, y)\, dxdy = \delta_{km} \cdot \delta_{ln}, \tag{43}$$

where δ is Kronecker's symbol. The eigenvalue belonging to Φ_{kl} is:

$$\lambda_{kl} = (2k+1)^2 \frac{\pi^2}{a^2} + \frac{d_2^{\,2}\, c^2\, H_S^{\,2}}{4 r^2 b^2} + \lambda_{yl}, \tag{44}$$

where λ_{yl} ($l = 1, 2, \ldots$) are the roots of the transcendental equation obtained by substituting $Y_l(y)$, according to (42), and its first and second derivatives in (37). These roots can only be determined approximately by numerical or graphical methods.

Since the eigenfunctions and eigenvalues are known, the stream function can now be written as the sum of a series of eigenfunctions, like (25). Thus for any Ω having an H satisfying (17) the corresponding stream function can be computed.

From (32) and (41) we deduce that the stream function Φ, the solution of (3), may be written as:

$$\Phi(x, y) = \left(\frac{cb}{d} + y\right) e^{\frac{d_2 c H_S}{2br} x} \sum_k \sum_l p_{kl} A_{xk} Y_l(y) \cos (2k+1) \frac{\pi x}{a}. \tag{45}$$

The above series represents an even function of x. If we next assume that Φ has only one extremum for each y, it will be evident from the exponential function in (45) that this extremum is to be found on that side of the line $x = 0$ where the product $d_2 x$ is positive. This means that the stream function has an extremum on the righthand side of the sea looking in the longitudinal direction from shallower to deeper parts, provided that the stream function has *one* extremum for each y and that the sea is situated on the northern hemisphere.

As a second example of the theory given in section 1.5.1 we shall compute the stream function for an infinitely long straight channel, assuming that no wind is blowing and that the depth is a linear function of y, the coordinate in the longitudinal direction of the channel:

$$H = H(y) = H(0) + dy \qquad (d > 0). \tag{46}$$

As x-axis we take the left hand edge of the channel, looking from shallow to deep; let the width of the channel be a.

A solution of the differential equation for the stream function, which only depends on x is sought.

The differential equation to be solved is obtained by putting $\partial\Phi_3/\partial y$ equal to zero in 1.4 (5):

$$\frac{d^2\Phi_3}{dx^2} - \frac{cd}{r}\frac{d\Phi_3}{dx} = 0. \tag{47}$$

The boundary conditions are:

$$\left.\begin{array}{l}\Phi_3\,(0) = 0 \\ \Phi_3\,(a) = \varphi_a\end{array}\right\}, \tag{48}$$

where φ_a represents an arbitrary given value which indicates the total flow of water through a cross section of the channel. Solving (47) with the conditions (48), we obtain:

$$\Phi_3\,(x) = \varphi_a \frac{e^{\frac{cd}{r}x} - 1}{e^{\frac{cd}{r}a} - 1}. \tag{49}$$

From this solution we see that, irrespective of the direction of the stream (that is: of the sign of φ_a), the current velocity $d\Phi_3/dx$ increases with x. In other words: there is a crowding of the stream lines towards the righthand side of the channel, looking from shallow to deep.

Now, if we let the slope of the bottom tend to zero, (49) tends to a linear function of x. Thus, the effect of the bottom slope is to shift the stream lines towards the righthand side of the channel, as is evident from the presence of the exponential function in (49).

Similarly we can deduce from (45), that a vanishing slope of the bottom ($d_2 \rightarrow 0$) causes the stream function to have an extremum at $x = 0$, that is, in the middle of the sea, whereas in a sea with a sloping bottom the extremum lies on the righthand side of the middle. Thus, in this case too, the effect of a bottom slope is a shifting of the stream lines towards the right.

Thus far, the stream functions have been computed by analytical methods. In practice however, numerical methods will be used e.g. the relaxation method, when solving the differential equation for the stream function, because, firstly, the analytical methods are mostly rather lengthy because of the slow convergency of the series involved, and secondly, these methods can only be used with succes for simply shaped regions, since only then can the eigenfunctions be written down explicitly. For a sea having a more complicated shape the eigenfunctions must be determined numerically; but then it is of course much better to attack the whole problem by direct numerical methods. The North Sea can only in first approximation be considered as a rectangle, especially if we are interested in the wind effects along the southern coast. Moreover, the North Sea has an open communication with the ocean, which involves a complicated boundary condition (1.2 (7)), leading to very complicated problems which have only been solved for a few special cases (e.g. by VELTKAMP [44, 45] and LAUWERIER [43]).

In this section, however, the differential equation for the stream function has been solved by analytical methods because this enables us to derive general properties of the flow patterns, e.g. the asymmetric crowding of stream lines and the position of the extremum of the stream function. It has been shown that in a sea with a sloping bottom the stream pattern can be obtained from the solution for a sea with a constant depth by a shifting of the stream lines towards the right. In the following paragraph this property will be described and analyzed more thoroughly.

1.5.3 THE SHIFTING OF THE STREAM LINES

Before dealing with this subject I shall first give an elementary explanation of the flow itself. For briefness' sake it will be confined to a homogeneous wind field over a closed sea of rectangular shape; furthermore suppose the depth lines to run parallel to the shorter side of the rectangle. Looking towards the deeper part of the sea, the righthand side of the rectangle will be called the east side, the deeper part the north side, and so on. If no currents would appear, a homogeneous west wind would, on the east coast cause a smaller wind effect in the north than it would in the south, as in obvious from 1.3 (2), since without currents, there would be symmetry with respect to the longitudinal axis.

Along the east coast the surface would then have a slope towards the north, and along the west coast a slope towards the south. In the case of no current there would be no forces to balance the pressure gradient forces due to these slopes. This is impossible in the stationary state, so that there must be a flow. This flow tends to diminish the slopes along the east and west coasts. It must be such that the integral of the bottom stress divided by gH, all along the coast, equals the corresponding integral of the wind stress, also divided by gH. This can easily be verified from 1.3 (1) by taking the contour integral, all around the sea, of both members of the equation, bearing in mind that $\oint (\nabla h \, . \, \vec{ds})$ and $\oint (c \, \nabla \, \Phi/gH) \, . \, \vec{ds})$ vanish identically, the first because a gradient vector is always curl-free and the second because the coast is a stream line.

Now, we shall proceed to the discussion of the phenomenon of the shifting of the stream lines towards the east. This asymmetrical behaviour of the stream line pattern will be shown to be a general feature, related to the asymmetry in the wind-induced ocean current pattern [53]. It will be shown qualitatively that all kinds of currents, also those which are not caused by the slope of the bottom, are influenced by the mere presence of the bottom slope in such a way that the stream line pattern is shifted towards the right, looking from shallow to deep. For the sake of brevity only a special case of the theory described in section 1.5.1., viz. the bottom-slope current caused by a homogeneous west wind will be dealt with here. In passing it should be noticed that the shifting of the stream lines towards the east would also occur if an east wind were blowing over the sea; only the direction of the flow would be reversed. Returning to the former case, consider two east-west cross-sections, close to each other, forming a narrow strip. As explained in the beginning of this section, a circulation will be established with a southerly current in the western part and a northerly current in the eastern part of the sea.

Since the sea is supposed to be closed the total flow through a cross-section must be zero: the total flow towards the south in the western part must be equal to the northward flow in the eastern part. Hence, there must, on each cross-section, be a point where the north-south component of the volume transport vanishes. (The flow is considered two-dimensionally, i.e. at every place only the vertically integrated current, the volume transport, is considered). These points form a line which intersects both cross-sections of the strip. The part of this strip to the left of this line will henceforth be denoted by G_l, the other part by G_r.

By taking the curl of both members of 1.3 (1) it follows that the curls of the bottom-stress force, the wind-stress force and the Coriolis force, converted into forces per unit of mass by dividing them by ϱH, balance each other; in other words: the couples exerted by these forces per unit of mass balance each other. Now first consider the couples of the various forces, separately. Commence by considering the wind-stress couple. Along the north side of a column of water, the wind-stress force per unit of mass, $\vec{\tau}/H$, is smaller than along the south side, because of the greater depth in the north. Hence the wind-stress couple will try to make the column of water rotate anti-clockwise. This wind-stress couple is constant along a cross-section of the sea because the depth lines run east-west and the wind-stress field has been assumed to be homogeneous.

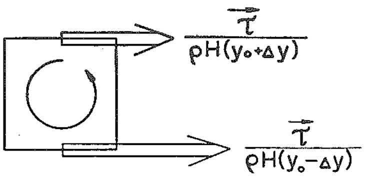

FIG. 1.5 c. Wind-stress couple.

In figure 1.5 c the foregoing has been illustrated for a cubic element of water, having its north-side at $y_0 + \triangle y$ and its south side at $y_0 - \triangle y$. The corresponding depths are $H(y_0 + \triangle y)$ and $H(y_0 - \triangle y)$.

The Coriolis force per unit of mass will be greater in the south than it will in the north, for particles at the same distance from the east and west coast, because of the smaller depth in the south (provided the north and south coasts are at sufficiently large distances from the cross-section considered, so that these coasts have no influence of any importance on the currents near the cross-section). This means that a southward flow gives rise to a Coriolis couple that will tend to make a water column rotate clockwise, and that a northward flow gives rise to a Coriolis couple tending to cause an anti-clockwise rotation. In figure 1.5 d this has been illustrated; a single-shafted arrow indicates a velocity, a double-shafted arrow a force, in this case: the Coriolis force.

The bottom-stress couple does not depend on the current itself, but on the shear of the current. This bottom-stress couple tends to cause a clockwise or anti-clockwise rotation of a column of water, dependent on whether the current shear is positive or negative (fig. 1.5 e).

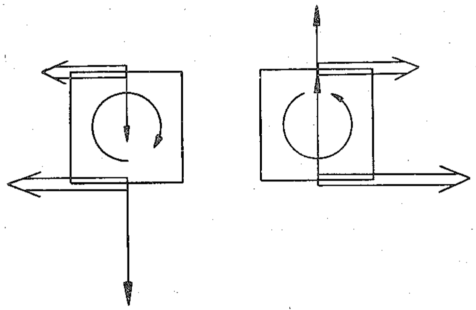

FIG. 1.5 d. Coriolis couple.

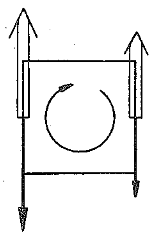

FIG. 1.5 e. Bottom-stress couple.

Now, the integrals of the sum of the three forces involved, viz. $\tau/\varrho H$, $c\nabla\Phi/\varrho H$ and $r[\vec{k}\nabla\Phi]/\varrho H$ over the contours of the strips G_l and G_r are zero in the equilibrium state, as follows from 1.3 (1). If we compare G_l and G_r, we see that the integrals of the Coriolis force over the contours of G_l and G_r are of equal magnitude but have opposite signs, viz. positive and negative for G_l and G_r respectively. The integrals of $\vec{\tau}/\varrho H$ and the $\vec{\tau}$-couples on the contrary have the same sign for both G_l and G_r, viz. negative. The bottom-stress couples and the corresponding contour integrals are positive for both G_l and G_r as is evident bearing in mind that to these integrals the only contributions come from the integrals of the bottom-stress force along the west and the east coast, respectively, since along the remaining parts of the contours no component of bottom stress along the contour is present, either because the bottom-stress is normal to the contour or because the current vanishes, as is the case at the border between G_l and G_r.

Since for G_l as well as for G_r the sum of the three integrals is zero, the contour integral of the bottom-stress force will for G_r have to be equal to the sum of the two negative contour integrals of the wind-stress force and the Coriolis force, and for G_l equal to the sum of a positive and a negative integral. Hence, the value of the contour integral of the bottom-stress force must for G_r be larger than for G_l. Since both are positive, it may be concluded that along the east coast the northward current is stronger than the southward current along the west coast. This has been illustrated in figure 1.5 f.

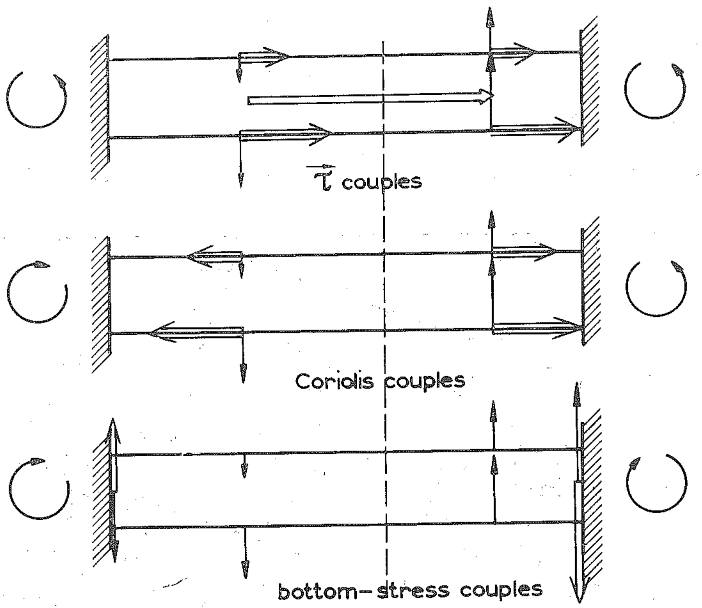

FIG. 1.5 f. Illustration of the crowding of stream lines in the "eastern" part of a sea.

If the wind direction is reversed, i.e. if an east wind is blowing, the conclusion that the strongest currents appear along the east coast remains valid, since of all the couples the sign is reversed, whereas the absolute values remain the same.

The shifting of the stream lines towards the east occurs not only in the case of bottom-slope currents but for all kinds of currents, e.g. a "leak" current. To explain this, let us assume for instance a southerly leak current. Then there will, in analogy to the foregoing in a belt across the sea, be a Coriolis couple acting anti-clockwise, supposing that the depth of the sea increases from south to north and that the sea is situated on the northern hemisphere.

This Coriolis couple must balance the bottom-stress couple, in such a way that the latter must have an opposite orientation; this can only be so if the velocity increases from west to east. A northerly current would do the same.

CHAPTER 2

APPLICATION OF THE THEORY TO THE NORTH SEA

2.1 Introduction

The methods developed in chapter 1 make it possible, at least in principle, to compute the equilibrium wind effects that would be caused by an arbitrary wind field over a sea, having an arbitrary shape and depth configuration. The current field can be computed from the wind field by means of the method of section 1.5 and then the wind effect determined by means of formula 1.3 (1). In practice, however, one has to model the wind field, since the exact calculations would be much too lengthy to be useful for prognostic purposes.

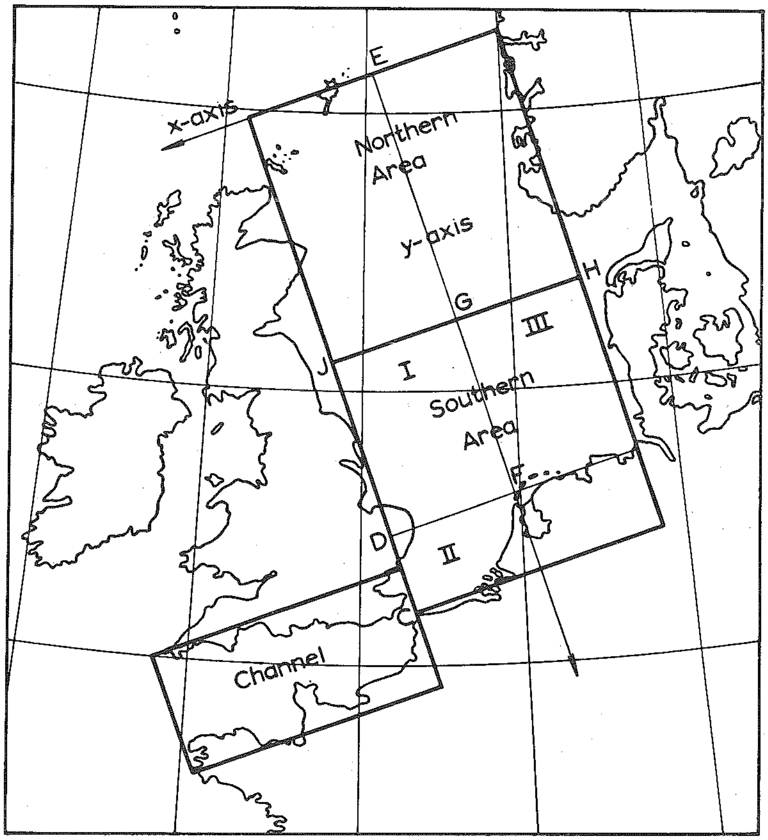

Fig. 2.1 a. Model of the North Sea.

SCHALKWIJK has modelled the wind field very drastically: he divided the North Sea, to which we shall confine ourselves henceforth, into two parts, viz. a northern part N, and a southern part S. He has composed empirical relations between the wind effect at Hook of Holland and the mean wind vectors, of the wind fields over the three areas N, S and the Channel C.

Afterwards it has appeared that the partition of the North Sea into two parts is sometimes too rough and therefore here S will be subdivided into three sub-areas (see figure 2.1 a), I, II and III respectively.

The theoretical formulas for the wind effects at some places along the south coast of the North Sea, viz.: Calais, Flushing, Hook of Holland, Den Helder, Eierlandse Gat, Borkum and Cuxhaven will be composed in this chapter. For that purpose a system of cartesian coordinates, having their origin at a point of the northern boundary of the North Sea, halfway between Scotland and Norway (point E in figure 2.1 a) will be employed; this boundary line has been chosen as x-axis, while the y-axis is formed by the line connecting the origin with the point F of figure 2.1 a, F being situated near the Eierlandse Gat; x has been chosen positive west of the y-axis, and y is positive south of the x-axis. The coast lines of the North Sea have been schematized by straight lines.

The actual depth configuration has been simplified to the configuration described by the following formula, giving the depth in meters:

$$H = H(y) = H_{\mathrm{F}}\, e^{-2.04\left(\frac{y}{b}-1\right)} = 29 e^{-2.04\left(\frac{y}{b}-1\right)} =$$

$$= 223 e^{-2.04\frac{y}{b}} = H_{\mathrm{E}}\, e^{-2.04\frac{y}{b}}, 0 \leqslant y \leqslant b, \tag{1}$$

where $H_{\mathrm{F}} = 29$ m represents the depth at the line $y = y_{\mathrm{F}}$ and $H_{\mathrm{E}} = 223$ m represents the depth at the x-axis. The depth has been assumed to depend on y only, so that the depth lines run parallel to the x-axis. It has been shown in figure 1.5 b that this formula, represented in figure 1.5 b by the dotted curve, gives a fairly good approximation of the actual depth configuration which is represented in figure 1.5 b by a full-drawn line. Especially in the southern part of the North Sea this exponential depth profile gives a better description of the actual depth distribution than the linear depth profile given by formula 1.5 (27), as is evident from figure 1.5 b. The fact that the depth profile according to (1) does not make the differential equation 1.1 (24) self-adjoint is no drawback, since the equation will be solved numerically. For $y > y_{\mathrm{F}} = b$ assume:

$$H = H_{\mathrm{F}} = 29 \text{ meter}. \tag{2}$$

For $y < 0$ assume that the depth is infinite (the ocean). The coasts are supposed to have vertical walls; this simplification does not introduce serious errors, as has been shown in section 1.3.4. In table 2.1 α the coordinates of some places are given; here, a represents the width of the modelled sea, EF lies halfway between the east- and west coasts.

In the following sections the stream functions and their contributions to the wind effects and the static effects will be computed for each of the wind fields consisting of a homogeneous wind field over one of the sub-areas into which the North Sea is divided and no wind over the remaining ones. In the last section of this chapter these effects will be added, thus obtaining the formulas for the total wind effect for the places listed in table 2.1 α. Suppose that over each of the parts into which the North Sea has been divided, a homogeneous wind is blowing. Then the total wind effect at an arbitrary place may be thought to be composed of the contributions which each of the sub-areas would yield separately in the case of no wind elsewhere, thanks to the supposed linearity of the equations governing the wind effects.

TABLE 2.1 α

Place	coordinates	
	x/a	y/a
E	0.00	0.00
G	0.00	1.00
C = Calais	0.50	2.00
V = Flushing	0.26	2.00
H = Hook of Holland	0.14	1.93
He = Den Helder	0.02	1.77
F = Eierlandse Gat	0.00	1.71
B = Borkum	—0.23	1.71
Cu = Cuxhaven	—0.47	1.71

These partial wind effects resulting from the wind over one of the sub-areas can again be supposed to be composed of the static effect and the three current effects. Concerning the current effects it should be noticed that the Channel region makes itself felt only by means of the "leak" current. In the case of wind over area II only, there will be no bottom-slope current, since the depth has been assumed to be constant there. Winds over the other areas will induce all three types of currents. Concerning the wind-vorticity currents it should be noticed that the vorticity field of the wind stress is singular along the borders of the areas considered since at these borders the wind stress is assumed to be discontinuous in these modelled wind fields.

2.2 Static effect

According to section 1.2 the difference in static effect between any two places in the sea is equal to the integral of $\vec{\tau}/\varrho g H$ from the first point to the second one. Since, however, $\vec{\tau}/\varrho g H$ is not curl-free, in general, an appointment about the path of integration must be made. Henceforth the appointment made in section 1.3.1 will be employed. As starting point E is chosen, the origin of the coordinate system. The difference in static effect, therefore, will always be that between any point P in the North Sea and E; in the following this difference will be called the static effect in P, since at E the wind effect is assumed to be zero. For an arbitrary point P on the south coast the static effect can be written as:

$$h_\tau (\mathrm{P}) = \int\limits_{y_\mathrm{E}}^{y_\mathrm{F}} \frac{\tau_y}{\varrho g H} dy + \int\limits_{\sigma_\mathrm{F}}^{\sigma_\mathrm{P}} \frac{\tau_\sigma}{\varrho g H_\mathrm{F}} d\sigma, \tag{1}$$

where τ_σ represents the component of the wind stress parallel to the coast and $d\sigma$ represents a length element of the coast line.

We shall apply (1) to different kinds of wind fields.

a. Static effect due to a homogeneous wind field over area N

Formula (1) can in this case be simplified to the following expression, valid for any point P on the south coast of the North Sea:

$$h^N{}_\tau (\mathrm{P}) = \int\limits_{y_\mathrm{E}}^{y_\mathrm{G}} \frac{\tau^N{}_y}{\varrho g H} dy = \frac{\tau^N{}_y}{\varrho g} \int\limits_{y_\mathrm{E}}^{y_\mathrm{G}} \frac{dy}{H} = \frac{\tau^N{}_y a}{\varrho g H_N} = \frac{\tau^N a}{\varrho g H_N} \cos \alpha_N = A_N \cos \alpha_N, \tag{2}$$

where a denotes the distance EG, which is the width of the sea in our model, and H_N represents the harmonic mean depth along EG. According to (2) a homogeneous wind over the northern area yields the same static effect for all points along the south coast. H_N can be computed from formula 2.1 (1):

$$H_N = 117 \text{ m}. \tag{3}$$

b. Static effect on the south coast due to a homogeneous wind field over area S

In this case the static effect can be expressed by:

$$h^S{}_\tau (\mathrm{P}) = \int\limits_{y_\mathrm{G}}^{y_\mathrm{F}} \frac{\tau^S{}_y}{\varrho g H} dy + \int\limits_{\sigma_\mathrm{F}}^{\sigma_\mathrm{P}} \frac{\tau^S{}_\sigma}{\varrho g H} d\sigma. \tag{4}$$

For places on the south coast east of F this can be written:

$$h^S_\tau(\mathrm{P}) = \int_{y_G}^{y_F} \frac{\tau^S_y}{\varrho g H} dy - \int_{x_F}^{x_P} \frac{\tau^S_x}{\varrho g H} dx. \tag{5}$$

For places on the south coast west of F we obtain:

$$h^S_\tau(\mathrm{P}) = \int_{y_G}^{y_F} \frac{\tau^S_y}{\varrho g H} dy + \int_{y_F}^{y_P} \frac{\tau^S_y}{\varrho g H_F} dy + \int_{x_F}^{x_P} \frac{\tau^S_x}{\varrho g H_F} dx, \tag{6}$$

making use of the fact that $\vec{\tau}/\varrho g H_F$ in area II is curl-free, on account of the constant depth, so that the path of the integration from F to P may be chosen arbitrarily. By means of (5) and (6) the static effect on the south coast in the case of a homogeneous wind over the area S can be computed. For that purpose we transform (5) and (6) into one formula:

$$h^S_\tau(x_P, y_P) = \left[\left\{0.71 + (y_P - y_F)\frac{H_S}{H_F}\right\}\cos\alpha_S + x_P \frac{H_S}{H_F}\sin\alpha_S\right]\frac{\tau^S a}{\varrho g H_S}, \tag{7}$$

where H_S represents the harmonic depth of the area S and α the angle between the direction of the wind and the y-axis, so that τ_x and τ_y are equal to $\tau \sin\alpha$ and $\tau \cos\alpha$, respectively (see figure 2.2a).

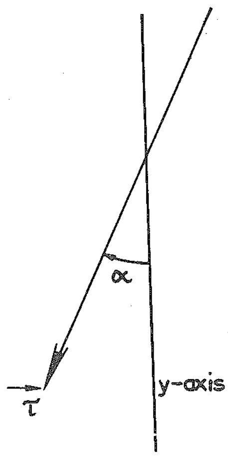

FIG. 2.2 a.
Orientation of angle between wind direction and y-axis.

For H_S we obtain, by integration of the inverse of the depth, according to 2.1 (1):

$$H_S = 43 \text{ m}. \tag{8}$$

With $H_F = 29$ meter, (7) can now be written as follows:

$$h^S{}_\tau(P) = [\{0.71 + 1.48\,(y_P - y_F)\}\cos\alpha_S + 1.48\,x_P \sin\alpha_S]\frac{\tau^S a}{\varrho g H_S}. \quad (9)$$

Substituting the coordinates of the places of table 2.1.α into (9), we finally arrive at the following formulas for the static effect:

$$h^S{}_\tau(C) = (1.13\cos\alpha_S + 0.74\sin\alpha_S)B_S \quad (10\,a)$$
$$h^S{}_\tau(V) = (1.13\cos\alpha_S + 0.38\sin\alpha_S)B_S \quad (10\,b)$$
$$h^S{}_\tau(H) = (1.03\cos\alpha_S + 0.21\sin\alpha_S)B_S \quad (10\,c)$$
$$h^S{}_\tau(He) = (0.79\cos\alpha_S + 0.03\sin\alpha_S)B_S \quad (10\,d)$$
$$h^S{}_\tau(F) = (0.70\cos\alpha_S + 0.00\sin\alpha_S)B_S \quad (10\,e)$$
$$h^S{}_\tau(B) = (0.70\cos\alpha_S - 0.34\sin\alpha_S)B_S \quad (10\,f)$$
$$h^S{}_\tau(Cu) = (0.70\cos\alpha_S - 0.70\sin\alpha_S)B_S \quad (10\,g)$$

where B_S stands for $\tau^S a/\varrho g H_S$.

c. Static effect due to a homogeneous wind field over area I

In this case too the formulas (5) and (6) are employed which can now be simplified to:

$$h^I{}_\tau(P) = \int_{y_G}^{y_F} \frac{\tau_y}{\varrho g H}\,dy, \quad (11)$$

where an appointment must be made as to the meaning of τ_y on GF, since GF lies between an area with $\tau_y \neq 0$ and an area where $\tau_y = 0$. There is a choice of three possibilities:

1° take τ_y on GF equal to τ_y of area I;

2° take τ_y on GF equal to zero;

3° take τ_y on GF equal to half the value of τ_y of area I.

These alternatives are equivalent to GF belonging to area I or to area III, of half to I and half to III. Here the third possibility is chosen. This implies following the same practice for the computation of current effects.

Finally formula (11) takes the form:

$$h^I{}_\tau(P) = \int_{y_G}^{y_F} \frac{\frac{1}{2}\tau^I{}_y}{\varrho g H}\,dy = (0.35\cos\alpha_I + 0.00\sin\alpha_I)\,B_I, \quad (12)$$

where B_I stands for $\tau^I a/\varrho g H_S$.

Formula (12) is valid for any point P on the south coast of the North Sea.

d. Static effect due to a homogeneous wind field over area II

In this case the formulas (5) and (6) become:

$$h^{II}{}_{\tau}(P) = 0, \text{ if } x_P < 0, \tag{13}$$

and

$$h^{II}{}_{\tau}(P) = \int_{y_F}^{y_P} \frac{\tau^{II}{}_y}{\varrho g H_F} dy + \int_{x_F}^{x_P} \frac{\tau^{II}{}_x}{\varrho g H_F} dx, \text{ if } x_P > 0. \tag{14}$$

These expressions yield the following formulas for the static effect, where B_{II} stands for $\tau^{II} a/\varrho g H_S$.

$$h^{II}{}_{\tau}(C) = (0.43 \cos \alpha_{II} + 0.74 \sin \alpha_{II})\, B_{II} \tag{15 a}$$
$$h^{II}{}_{\tau}(V) = (0.43 \cos \alpha_{II} + 0.38 \sin \alpha_{II})\, B_{II} \tag{15 b}$$
$$h^{II}{}_{\tau}(H) = (0.33 \cos \alpha_{II} + 0.21 \sin \alpha_{II})\, B_{II} \tag{15 c}$$
$$h^{II}{}_{\tau}(He) = (0.09 \cos \alpha_{II} + 0.03 \sin \alpha_{II})\, B_{II} \tag{15 d}$$
$$h^{II}{}_{\tau}(F) = (0.00 \cos \alpha_{II} + 0.00 \sin \alpha_{II})\, B_{II} \tag{15 e}$$
$$h^{II}{}_{\tau}(B) = (0.00 \cos \alpha_{II} + 0.00 \sin \alpha_{II})\, B_{II} \tag{15 f}$$
$$h^{II}{}_{\tau}(Cu) = (0.00 \cos \alpha_{II} + 0.00 \sin \alpha_{II})\, B_{II} \tag{15 g}$$

e. Static effect due to a homogeneous wind field over area III

In view of the aforementioned appointment concerning the value of the wind stress on GF, write (5) and (6) in this case as follows:

$$h^{III}{}_{\tau}(P) = \int_{y_G}^{y_F} \frac{\frac{1}{2}\tau_y{}^{III}}{\varrho g H} dy, \text{ if } x_P > 0, \tag{16}$$

and

$$h^{III}{}_{\tau}(P) = \int_{y_G}^{y_F} \frac{\frac{1}{2}\tau^{III}{}_y}{\varrho g H} dy - \int_{x_F}^{x_P} \frac{\tau^{III}{}_x}{\varrho g H_F} dx, \text{ if } x_P < 0. \tag{17}$$

These formulas yield the following expressions for the static effect, where $B_{III} = \tau^{III} a/\varrho g H_S$.

$$h^{III}{}_{\tau}(C) = (0.35 \cos \alpha_{III} + 0.00 \sin \alpha_{III})\, B_{III} \tag{18 a}$$
$$h^{III}{}_{\tau}(V) = (0.35 \cos \alpha_{III} + 0.00 \sin \alpha_{III})\, B_{III} \tag{18 b}$$
$$h^{III}{}_{\tau}(H) = (0.35 \cos \alpha_{III} + 0.00 \sin \alpha_{III})\, B_{III} \tag{18 c}$$
$$h^{III}{}_{\tau}(He) = (0.35 \cos \alpha_{III} + 0.00 \sin \alpha_{III})\, B_{III} \tag{18 d}$$
$$h^{III}{}_{\tau}(F) = (0.35 \cos \alpha_{III} + 0.00 \sin \alpha_{III})\, B_{III} \tag{18 e}$$
$$h^{III}{}_{\tau}(B) = (0.35 \cos \alpha_{III} - 0.34 \sin \alpha_{III})\, B_{III} \tag{18 f}$$
$$h^{III}{}_{\tau}(Cu) = (0.35 \cos \alpha_{III} - 0.70 \sin \alpha_{III})\, B_{III} \tag{18 g}$$

f. Static effect due to a homogeneous wind field over area C

Since in this case the wind stress along the path of integration from E to the south coast of the North Sea is equal to zero, the static effect is zero for every point P on the south coast.

2.3 Wind-vorticity current effect

The currents caused by the vorticity of the wind stress can, in principle, be computed for any wind field by means of the method developed in section 1.5.1. We shall, however, not use this method here because it is too lengthy for practical purposes. The current field is computed by means of the numerical method which has been developed by Southwell [54] and which is called the relaxation method. This method is especially suitable for solving Laplace- and Poisson-equations. The equation to be solved, however, is of the more general form 1.5 (3). If we wish to make use of the relaxation method we may write the equation for the stream function in the form of a Poisson equation:

$$\triangle \Phi_1 = \frac{H}{\varrho r}(\vec{k} \,.\, \mathrm{rot}\, \vec{\tau}) + s, \tag{1}$$

where s represents a perturbation term which is a function of Φ and according to 1.5 (1) and 1.5 (2) can be written:

$$s = 2\left(\nabla \ln \frac{H}{H_S} \,.\, \nabla \Phi_1\right) - \frac{cH_S}{r} \,.\, \frac{H}{H_S}([\vec{k} \,.\, \nabla \ln H] \,.\, \nabla \Phi_1). \tag{2}$$

Since Φ is unknown, s is also unknown, so that we cannot obtain the solution by means of the relaxation method in one step, but have to proceed by an iteration method. First we solve the zero-order approximation of Φ_1, which we call Φ_1 (0) and which is defined as a solution of:

$$\triangle \Phi_1^{(0)} = \frac{H}{\varrho r}(\vec{k} \,.\, \mathrm{rot}\, \vec{\tau}). \tag{3}$$

This $\Phi_1^{(0)}$ (0) substituted into (2) yields the zero order approximation of the perturbation, $s^{(0)}$. Substituting next $s^{(0)}$ for s in (1), a Poisson equation for the first order approximation is obtained of Φ_1, called $\Phi_1^{(1)}$:

$$\triangle \Phi_1^{(1)} = \frac{H}{\varrho r}(\vec{k} \,.\, \mathrm{rot}\, \vec{\tau}) + s^{(0)}. \tag{4}$$

Proceeding in this way, we may solve the n^{th} order approximation of Φ_1 from the Poisson-equation:

$$\triangle \Phi_1^{(n)} = \frac{H}{\varrho r}(\vec{k} \,.\, \mathrm{rot}\, \vec{\tau}) + s^{(n-1)}. \tag{5}$$

This procedure has to be continued until $\Phi_1^{(n+1)}$ differs from $\Phi_1^{(n)}$ by less than the required accuracy. In practice this method converges rapidly to the solution; in most of the cases dealt with here, it suffices to go to $n = 2$ or 3. For the convergence of this procedure it is necessary that s is sufficiently small as compared with Φ_1.

As explained before (section 2.1) only wind-vorticity currents due to the singularity of the wind-stress shear along the boundaries of each area need be considered. Below the current fields caused by these singularities will be treated successively.

Before this a few words about the boundary conditions. Along the coasts and across the Straits of Dover we assume $\Phi_1 = 0$. Strictly speaking the stream function can only be solved if the North Sea and the Atlantic Ocean are considered as one single area. Assuming, however, that the ocean is very deep as compared with the depth of the North Sea, the currents in the ocean may be left out of consideration, if the equation 1.4 (9) is taken as boundary condition along the borderline between the North Sea and the Ocean. This boundary condition, however, leads to very complicated problems, and therefore we shall take as boundary condition along this line: $\Phi_1 = 0$, which is equivalent to the assumption that this line is a stream line. It will yet be shown that this simplification does not affect results seriously, as far as concerns the currents caused by the wind-stress shears along the boundaries of the southern area (S, I, II or III) only, since it appears that for these cases the currents are very weak in the vicinity of the northern boundary of the North Sea.

As to a wind shear along the northern boundary of area N notice that this shear may be considered as not causing a wind-vorticity current; all of the effect due to the jump of the depth along that boundary may be ascribed to the corresponding slope current (caused by the infinitely steep slope of the jump of the depth).

Indeed, from the differential equation governing the stream function it is obvious that the distribution of the wind stress over the infinitely deep ocean is irrelevant for the stream function since in the righthand side member of that equation $\vec{\tau}$ only appears in the ratio $\vec{\tau}/H$. But then it may be assumed that the wind stress over the ocean is equal to that over area N, which means that no wind-vorticity current need be considered that is caused by the wind shear at the northern boundary of area N.

The total wind-vorticity current due to a homogeneous wind over one of the areas can be split up into partial wind-vorticity currents caused by the wind shears along each of the straight boundaries of the area considered. The figures 2.3 a to 2.3 e inclusive show the current fields due to the wind shears along the lines JH, JG, GH, GF and DF, respectively, by means of stream lines. The numbers in the stream lines indicate the values of the stream function expressed in the unit $\tau a H_S / 2\pi\varrho r$.

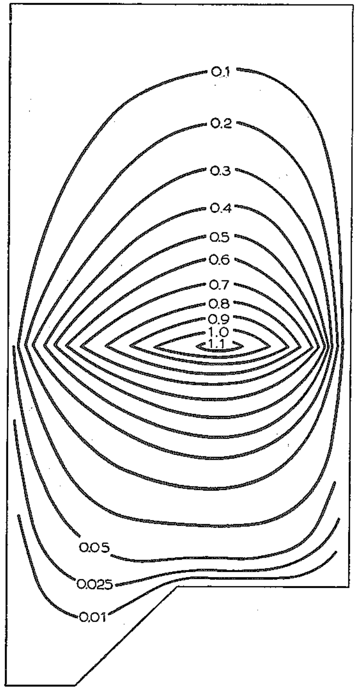

FIG. 2.3 a. Wind-vorticity stream line pattern due to wind shear along J H.

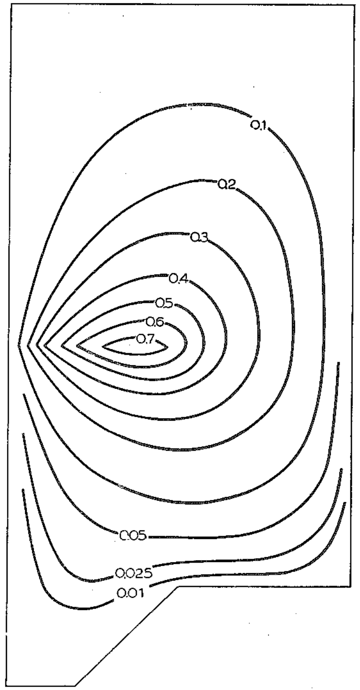

FIG. 2.3 b. Wind-vorticity stream line pattern due to wind shear along J G.

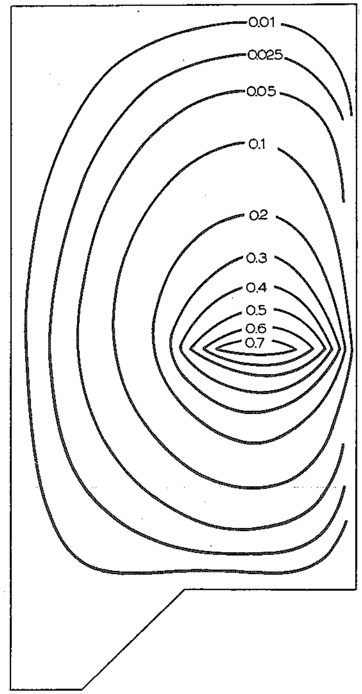

FIG. 2.3 c. Wind-vorticity stream line pattern due to wind shear along G H.

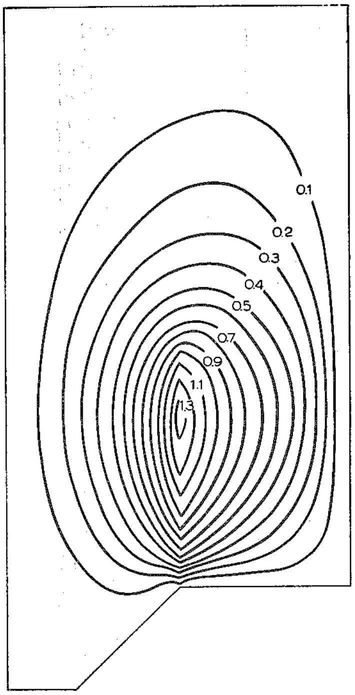

FIG. 2.3 d. Wind-vorticity stream line pattern due to wind shear along G F.

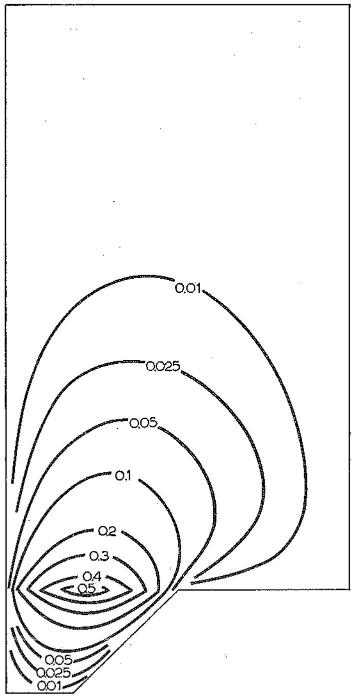

FIG. 2.3 e. Wind-vorticity stream line pattern due to wind shear along D F.

The current effect due to these partial-wind vorticity currents can be computed by means of formula 1.3 (5), which, for places on the south coast of the North Sea, can be transformed into:

$$h^i_1\,(\mathrm{P}) = \int\limits_{y_\mathrm{E}}^{y_\mathrm{F}} \left(\frac{c}{gH}\frac{\partial \Phi_1^{(i)}}{\partial y}\right)_{x=0} dy - \int\limits_{y_\mathrm{E}}^{y_\mathrm{F}} \left(\frac{r}{gH^2}\frac{\partial \Phi_1^{(i)}}{\partial x}\right)_{x=0} dy - \int\limits_{s_\mathrm{F}}^{s_\mathrm{P}} \frac{r}{gH^2_\mathrm{F}}\frac{\partial \Phi_1^{(i)}}{\partial n}\, ds, \qquad (6)$$

where i denotes which partial current field is meant. In table 2.3 α these partial vorticity-current effects are given expressed in the unit $\tau a/\varrho g H_S$, where τ denotes the absolute value of the jump of the wind stress at the borderline in question. The numbers given denote the effects due to a positive wind shear.

TABLE 2.3 α. *Wind-vorticity current effects*

Place	Singular-shear lines				
	JH	JG	GH	GF	DF
Calais	+ 0.13	+ 0.14	— 0.01	— 0.29	+ 0.34
Flushing	+ 0.13	+ 0.14	— 0.01	— 0.29	+ 0.31
Hook of Holland	+ 0.12	+ 0.14	— 0.02	— 0.28	+ 0.24
Den Helder	+ 0.11	+ 0.13	— 0.02	— 0.23	+ 0.10
Eierlandse Gat	+ 0.11	+ 0.13	— 0.02	— 0.20	+ 0.08
Borkum	+ 0.07	+ 0.11	— 0.04	— 0.02	+ 0.05
Cuxhaven	+ 0.05	+ 0.10	— 0.05	+ 0.05	+ 0.05

From this table we deduce, by adding the partial wind-vorticity current effects corresponding to the straight boundary lines of each of the areas, the total wind-vorticity current effects due to a homogeneous wind over each of the areas. Thus we obtain:

a. Wind-vorticity current effect due to a homogeneous wind field over area N

$$h^N{}_1\,(\mathrm{C}) = (0.00 \cos \alpha_N + 0.13 \sin \alpha_N)\, B_N \tag{7 a}$$
$$h^N{}_1\,(\mathrm{V}) = (0.00 \cos \alpha_N + 0.13 \sin \alpha_N)\, B_N \tag{7 b}$$
$$h^N{}_1\,(\mathrm{H}) = (0.00 \cos \alpha_N + 0.12 \sin \alpha_N)\, B_N \tag{7 c}$$
$$h^N{}_1\,(\mathrm{He}) = (0.00 \cos \alpha_N + 0.11 \sin \alpha_N)\, B_N \tag{7 d}$$
$$h^N{}_1\,(\mathrm{F}) = (0.00 \cos \alpha_N + 0.11 \sin \alpha_N)\, B_N \tag{7 e}$$
$$h^N{}_1\,(\mathrm{B}) = (0.00 \cos \alpha_N + 0.07 \sin \alpha_N)\, B_N \tag{7 f}$$
$$h^N{}_1\,(\mathrm{Cu}) = (0.00 \cos \alpha_N + 0.05 \sin \alpha_N)\, B_N \tag{7 g}$$

where B_N denotes $\tau^N a/\varrho g H_S$.

b. Wind-vorticity current effect due to a homogeneous wind field over area S

Since these effects are caused by the shear of the wind stress along JH, as are the effects of the section *a*, we obtain for these effects, by reversing the sign of the formulas (7 a) to (7 g) inclusive:

$$h^S{}_1\,(\mathrm{C}) = (0.00 \cos \alpha_S - 0.13 \sin \alpha_S)\, B_S \tag{8 a}$$
$$h^S{}_1\,(\mathrm{V}) = (0.00 \cos \alpha_S - 0.13 \sin \alpha_S)\, B_S \tag{8 b}$$
$$h^S{}_1\,(\mathrm{H}) = (0.00 \cos \alpha_S - 0.12 \sin \alpha_S)\, B_S \tag{8 c}$$
$$h^S{}_1\,(\mathrm{He}) = (0.00 \cos \alpha_S - 0.11 \sin \alpha_S)\, B_S \tag{8 d}$$
$$h^S{}_1\,(\mathrm{F}) = (0.00 \cos \alpha_S - 0.11 \sin \alpha_S)\, B_S \tag{8 e}$$
$$h^S{}_1\,(\mathrm{B}) = (0.00 \cos \alpha_S - 0.07 \sin \alpha_S)\, B_S \tag{8 f}$$
$$h^S{}_1\,(\mathrm{Cu}) = (0.00 \cos \alpha_S - 0.05 \sin \alpha_S)\, B_S \tag{8 g}$$

c. Wind-vorticity current effect due to a homogeneous wind field over area I

$$h^{I}_{1}\,(C) = (0.29 \cos \alpha_I + 0.20 \sin \alpha_I)\, B_I \qquad (9\text{ a})$$
$$h^{I}_{1}\,(V) = (0.29 \cos \alpha_I + 0.17 \sin \alpha_I)\, B_I \qquad (9\text{ b})$$
$$h^{I}_{1}\,(H) = (0.28 \cos \alpha_I + 0.10 \sin \alpha_I)\, B_I \qquad (9\text{ c})$$
$$h^{I}_{1}\,(He) = (0.23 \cos \alpha_I - 0.03 \sin \alpha_I)\, B_I \qquad (9\text{ d})$$
$$h^{I}_{1}\,(F) = (0.20 \cos \alpha_I - 0.05 \sin \alpha_I)\, B_I \qquad (9\text{ e})$$
$$h^{I}_{1}\,(B) = (0.02 \cos \alpha_I - 0.06 \sin \alpha_I)\, B_I \qquad (9\text{ f})$$
$$h^{I}_{1}\,(Cu) = (-\,0.05 \cos \alpha_I - 0.05 \sin \alpha_I)\, B_I \qquad (9\text{ g})$$

d. Wind-vorticity current effect due to a homogeneous wind field over area II

$$h^{II}_{1}\,(C) = (0.00 \cos \alpha_{II} - 0.34 \sin \alpha_{II})\, B_{II} \qquad (10\text{ a})$$
$$h^{II}_{1}\,(V) = (0.00 \cos \alpha_{II} - 0.31 \sin \alpha_{II})\, B_{II} \qquad (10\text{ b})$$
$$h^{II}_{1}\,(H) = (0.00 \cos \alpha_{II} - 0.24 \sin \alpha_{II})\, B_{II} \qquad (10\text{ c})$$
$$h^{II}_{1}\,(He) = (0.00 \cos \alpha_{II} - 0.10 \sin \alpha_{II})\, B_{II} \qquad (10\text{ d})$$
$$h^{II}_{1}\,(F) = (0.00 \cos \alpha_{II} - 0.08 \sin \alpha_{II})\, B_{II} \qquad (10\text{ e})$$
$$h^{II}_{1}\,(B) = (0.00 \cos \alpha_{II} - 0.05 \sin \alpha_{II})\, B_{II} \qquad (10\text{ f})$$
$$h^{II}_{1}\,(Cu) = (0.00 \cos \alpha_{II} - 0.05 \sin \alpha_{II})\, B_{II} \qquad (10\text{ g})$$

e. Wind-vorticity current effect due to a homogeneous wind field over area III

$$h^{III}_{1}\,(C) = (-0.29 \cos \alpha_{III} + 0.01 \sin \alpha_{III})\, B_{III} \qquad (11\text{ a})$$
$$h^{III}_{1}\,(V) = (-0.29 \cos \alpha_{III} + 0.01 \sin \alpha_{III})\, B_{III} \qquad (11\text{ b})$$
$$h^{III}_{1}\,(H) = (-0.28 \cos \alpha_{III} + 0.02 \sin \alpha_{III})\, B_{III} \qquad (11\text{ c})$$
$$h^{III}_{1}\,(He) = (-0.23 \cos \alpha_{III} + 0.02 \sin \alpha_{III})\, B_{III} \qquad (11\text{ d})$$
$$h^{III}_{1}\,(F) = (-0.20 \cos \alpha_{III} + 0.02 \sin \alpha_{III})\, B_{III} \qquad (11\text{ e})$$
$$h^{III}_{1}\,(B) = (-0.02 \cos \alpha_{III} + 0.04 \sin \alpha_{III})\, B_{III} \qquad (11\text{ f})$$
$$h^{III}_{1}\,(Cu) = (+\,0.05 \cos \alpha_{III} + 0.05 \sin \alpha_{III})\, B_{III} \qquad (11\text{ g})$$

2.4 Slope-current effect

The bottom-slope currents caused by the wind- stress component parallel to the depth lines, viz. the partial slope-current fields caused by the uniform winds over each of the areas of the North Sea separately, are also computed by means of the relaxation method. In the figures 2.4 a to 2.4 d inclusive the slope-current fields caused by a homogeneous wind field over areas N, S, I and III, respectively are given by means of stream lines. The numbers on the stream lines denote the value of the stream function expressed in the unit $\tau a H_S / 2\pi\varrho r$.

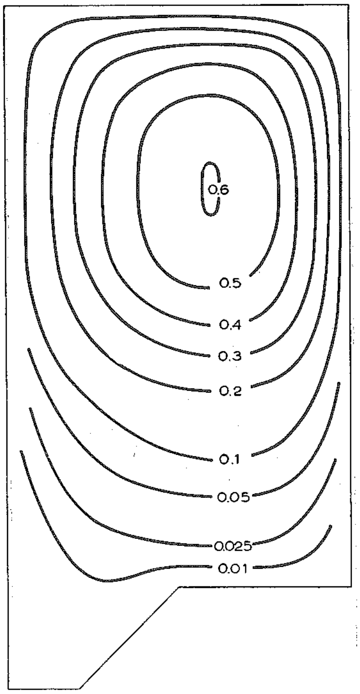

FIG. 2.4 a. Slope-current stream line pattern due to a uniform wind over area *N*.

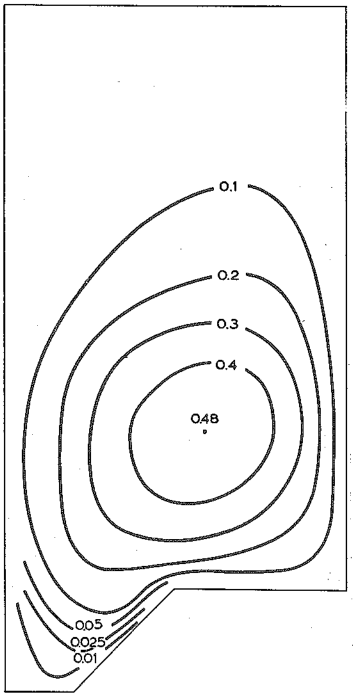

FIG. 2.4 b. Slope-current stream line pattern due to a uniform wind over area *S*.

These current fields have been computed for an enclosed sea. This does not introduce a serious error in the computation of the current effects for those cases which correspond to a homogeneous wind field over one of the southern areas (I, III or *S*) because of the weak currents prevailing on the northern boundary. It can be expected, however, that the current field given in figure 2.4 a, which denotes the slope-current field caused by a homogeneous wind over the northern part of the North Sea, if its northern boundary is a streamline (enclosed sea), will be quite different from the current field which will occur in the actual case of the sea having a wide connection with the ocean to the North, because of the strong currents prevailing on the northern boundary in this case. Nevertheless the current effects belonging to the current field of fig. 2.4 a are computed here, but a correction which has to be determined empirically is applied in order to find the influence of the open connection

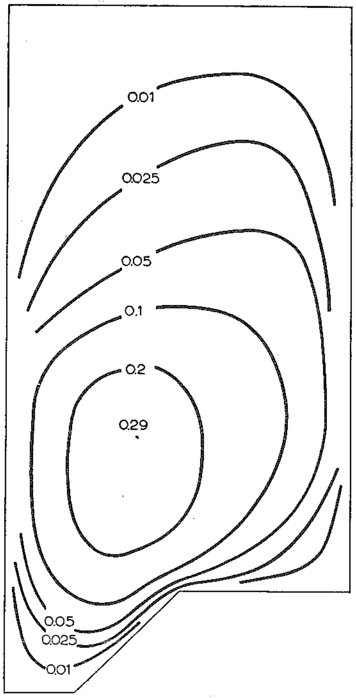

FIG. 2.4 c. Slope-current stream line pattern due to a uniform wind over area I.

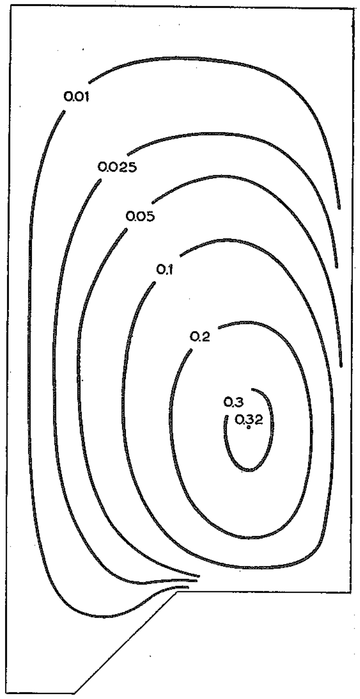

FIG. 2.4 d. Slope-current stream line pattern due to a uniform wind over area III.

with the ocean on the slope-current effects. It is assumed that this correction is proportional to $A_N \sin \alpha_N$, say equal to $q(H_N/H_S)\, A_N \sin \alpha_N$. The value of the parameter q will be determined in section 2.6.2 by comparing the theoretical wind effect formula for Hook of Holland, containing q, with the empirical one found by SCHALKWIJK. Furthermore it will be assumed that q is a constant for all places on the southern coast of the North Sea. This seems permissible in first approximation, since VELTKAMP [2] has shown that for a sea of constant depth and of similar dimensions as those of the North Sea it is true to a high degree of approximation.

A uniform wind over area II does not involve a slope current because of the supposed constancy of the depth in that area.

In the following formulas the slope-current effects due to a homogeneous wind field over each of the areas N (enclosed sea), S, I and III, are given separately. These effects have been computed by means of the following formula, which can be derived from formula 1.3 (5), and which is completely similar to formula 2.3 (6):

$$h^i_2\,(\mathrm{P}) = \int\limits_{y_\mathrm{E}}^{y_\mathrm{F}} \left(\frac{c}{gH}\,\frac{\partial \Phi_2^{(i)}}{\partial y}\right)_{x=0} dy - \int\limits_{y_\mathrm{E}}^{y_\mathrm{F}} \left(\frac{r}{gH^2}\,\frac{\partial \Phi_2^{(i)}}{\partial x}\right)_{x=0} dy - \int\limits_{s_\mathrm{F}}^{s_\mathrm{P}} \frac{r}{gH_\mathrm{F}^2} \cdot \frac{\partial \Phi_2^{(i)}}{\partial n}\, ds, \qquad (1)$$

where i again denotes which partial field is meant.

a. Slope-current effect due to a homogeneous wind field over area N (enclosed sea)

$$h^N_2\,(\mathrm{C}) = (-0.08 \sin \alpha_N)\, B_N = -0.08\, \frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (2\,\mathrm{a})$$

$$h^N_2\,(\mathrm{V}) = (-0.08 \sin \alpha_N)\, B_N = -0.08\, \frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (2\,\mathrm{b})$$

$$h^N_2\,(\mathrm{H}) = (-0.08 \sin \alpha_N)\, B_N = -0.08\, \frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (2\,\mathrm{c})$$

$$h^N_2\,(\mathrm{He}) = (-0.07 \sin \alpha_N)\, B_N = -0.07\, \frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (2\,\mathrm{d})$$

$$h^N_2\,(\mathrm{F}) = (-0.07 \sin \alpha_N)\, B_N = -0.07\, \frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (2\,\mathrm{e})$$

$$h^N_2\,(\mathrm{B}) = (-0.06 \sin \alpha_N)\, B_N = -0.06\, \frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (2\,\mathrm{f})$$

$$h^N_2\,(\mathrm{Cu}) = (-0.05 \sin \alpha_N)\, B_N = -0.05\, \frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (2\,\mathrm{g})$$

b. Slope-current effect due to a homogeneous wind field over area S

$$h^S_2\,(\mathrm{C}) = (-0.21 \sin \alpha_S)\, B_S \qquad (3\,\mathrm{a})$$

$$h^S_2\,(\mathrm{V}) = (-0.20 \sin \alpha_S)\, B_S \qquad (3\,\mathrm{b})$$

$$h^S_2\,(\mathrm{H}) = (-0.19 \sin \alpha_S)\, B_S \qquad (3\,\mathrm{c})$$

$$h^S_2\,(\mathrm{He}) = (-0.14 \sin \alpha_S)\, B_S \qquad (3\,\mathrm{d})$$

$$h^S_2\,(\mathrm{F}) = (-0.11 \sin \alpha_S)\, B_S \qquad (3\,\mathrm{e})$$

$$h^S_2\,(\mathrm{B}) = (\ \ 0.00 \sin \alpha_S)\, B_S \qquad (3\,\mathrm{f})$$

$$h^S_2\,(\mathrm{Cu}) = (+0.09 \sin \alpha_S)\, B_S \qquad (3\,\mathrm{g})$$

c. Slope-current effect due to a homogeneous wind field over area I

$$h^{\mathrm{I}}{}_2\,(\mathrm{C}) = (-0.21 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (4\,\mathrm{a})$$
$$h^{\mathrm{I}}{}_2\,(\mathrm{V}) = (-0.20 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (4\,\mathrm{b})$$
$$h^{\mathrm{I}}{}_2\,(\mathrm{H}) = (-0.19 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (4\,\mathrm{c})$$
$$h^{\mathrm{I}}{}_2\,(\mathrm{He}) = (-0.14 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (4\,\mathrm{d})$$
$$h^{\mathrm{I}}{}_2\,(\mathrm{F}) = (-0.12 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (4\,\mathrm{e})$$
$$h^{\mathrm{I}}{}_2\,(\mathrm{B}) = (-0.07 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (4\,\mathrm{f})$$
$$h^{\mathrm{I}}{}_2\,(\mathrm{Cu}) = (-0.05 \sin \alpha_{\mathrm{I}})\, B_{\mathrm{I}} \qquad (4\,\mathrm{g})$$

d. Slope-current effect due to a homogeneous wind field over area III

$$h^{\mathrm{III}}{}_2\,(\mathrm{C}) = (0.00 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (5\,\mathrm{a})$$
$$h^{\mathrm{III}}{}_2\,(\mathrm{V}) = (0.00 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (5\,\mathrm{b})$$
$$h^{\mathrm{III}}{}_2\,(\mathrm{H}) = (0.00 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (5\,\mathrm{c})$$
$$h^{\mathrm{III}}{}_2\,(\mathrm{He}) = (0.00 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (5\,\mathrm{d})$$
$$h^{\mathrm{III}}{}_2\,(\mathrm{F}) = (0.01 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (5\,\mathrm{e})$$
$$h^{\mathrm{III}}{}_2\,(\mathrm{B}) = (0.07 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (5\,\mathrm{f})$$
$$h^{\mathrm{III}}{}_2\,(\mathrm{Cu}) = (0.14 \sin \alpha_{\mathrm{III}})\, B_{\mathrm{III}} \qquad (5\,\mathrm{g})$$

e. Slope-current effect due to a homogeneous wind field over area N (open sea)

By adding the correction term $q\,(H_N/H_S)\,A_N \sin \alpha_N$ to the formulas of *a.* we obtain the slope-current effects referring to an open sea:

$$h^{N}{}_2\,(\mathrm{C}) = (q-0.08)\,\frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (6\,\mathrm{a})$$
$$h^{N}{}_2\,(\mathrm{V}) = (q-0.08)\,\frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (6\,\mathrm{b})$$
$$h^{N}{}_2\,(\mathrm{H}) = (q-0.08)\,\frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (6\,\mathrm{c})$$
$$h^{N}{}_2\,(\mathrm{He}) = (q-0.07)\,\frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (6\,\mathrm{d})$$
$$h^{N}{}_2\,(\mathrm{F}) = (q-0.07)\,\frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (6\,\mathrm{e})$$
$$h^{N}{}_2\,(\mathrm{B}) = (q-0.06)\,\frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (6\,\mathrm{f})$$
$$h^{N}{}_2\,(\mathrm{Cu}) = (q-0.05)\,\frac{H_N}{H_S} \sin \alpha_N\, A_N \qquad (6\,\mathrm{g})$$

2.5 LEAK-CURRENT EFFECT

2.5.1 THE LEAK CURRENT

Corresponding to the general definition of the leak-current field given in section 1.4, as the solution of a homogeneous partial differential equation of the second order of the type 1.4 (5) satisfying the boundary conditions 1.4 (12) and 1.4 (13), the leak-current field in our North Sea model is defined as the solution of that same differential equation 1.4 (5) satisfying the boundary conditions:

$$\Phi_3 = 0, \text{ at the North Sea coast of Britain,} \tag{1}$$

$$\Phi_3 = \Phi_c, \text{ at the continental North Sea coast,} \tag{2}$$

$$\text{and } \frac{r}{H_B} \cdot \frac{\partial \Phi_3}{\partial y} + c \frac{\partial \Phi_3}{\partial x} = 0, \text{ at } y = 0; \tag{3}$$

while the Straits of Dover are considered as a point sink having a strength of Φ_c. By means of the relaxation method we can compute the leak-current field if Φ_c is known, that is if the strength of the leak current through the Straits of Dover is known. This is not the case beforehand. Φ_c is determined by comparison with the empirical formulas of SCHALKWIJK by means of the method described in the next section. The shape of the leak-current pattern is independent of Φ_c. In figure 2.5 a the leak-current field is pictured by means of stream lines. The numbers on the stream lines denote the values of the stream function, the unit being Φ_c.

The boundary condition (3) implies that the stream lines intersect the line $y = 0$ at an angle having its tangens equal to $r/cH(0)$. The points $(a/2, 0)$ and $(-a/2, 0)$ are singular points. For the behaviour of the stream function in the vicinity of these points see VELTKAMP [55]. The depth configuration of 2.1 (1) has been used. If the depth field is extrapolated according to this formula for negative values of y, and the coast lines are prolonged rectilinearly towards that side, the leak-current field becomes as illustrated in figure 2.5 b. From both figures the crowding of the stream lines towards the east is evident. Moreover, we see from a comparison of the two stream line patterns that the influence of the northern boundary of the North Sea extends only to about half way in the North Sea; the current field in the southern part is unaffected by the condition at the northern boundary.

The actual leak-current field will show a pattern which lies between these two extreme cases, since the actual depth profile does not show an infinite jump such as has been assumed in figure 2.5 a, nor such a gradual slope as has been assumed in figure 2.5 b.

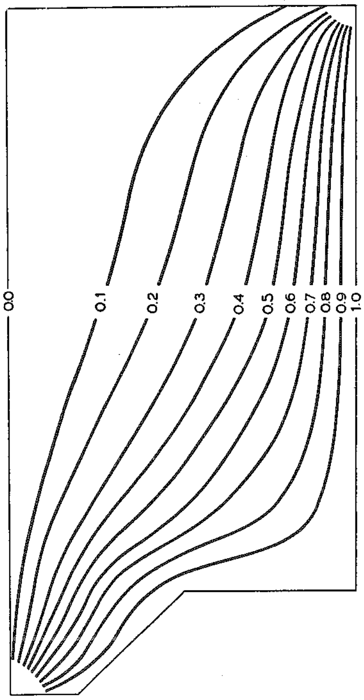

FIG. 2.5 a. Leak-current stream line pattern corresponding to boundary condition 2.5 (3).

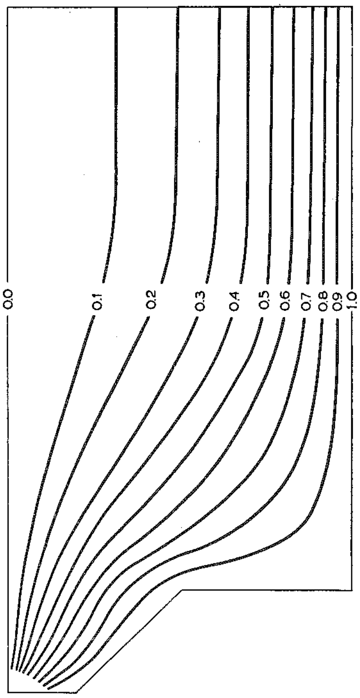

FIG. 2.5 b. Leak-current stream line pattern corresponding to $\partial \Phi_3 / \partial y = 0$ at the northern boundary.

2.5.2 THE LEAK FACTOR

In 2.5.1 the leak-current field has been computed under the supposition that the strength of the leak current through the Straits of Dover is known. This strength, however, is determined by the wind field over the North Sea and the Channel. In this paragraph we shall derive the relation between the wind field and the strength of the leak current. For that purpose it is assumed that the leak current is proportional to the difference between the sea levels which, influenced by the winds over the North Sea and the Channel, would appear on both sides of an imaginary dam across the Straits of Dover:

$$\Phi_c = K\,(h^* - h^{**}), \tag{4}$$

where h^* and h^{**} are these imaginary levels, on the North Sea side and the Channel side respectively, and where K denotes a factor of proportionality, or the strength of the leak current through the Straits of Dover which would be established by a unit difference of sea level in the "closed case".

Now the *leak effect* is the change in sea level due to the removal of the imaginary dam across the Straits of Dover; this change is caused by the leak current and can, for a point P on the south coast of the North Sea, be written, by means of 1.3 (5), as follows:

$$h_3\,(\mathrm{P}) = \int\limits_{y_\mathrm{E}}^{y_\mathrm{F}} \left(\frac{c}{gH}\frac{\partial \Phi_3}{\partial y}\right)_{x=0} dy - \int\limits_{y_\mathrm{E}}^{y_\mathrm{F}} \left(\frac{r}{gH^2}\frac{\partial \Phi_3}{\partial x}\right)_{x=0} dy - \int\limits_{s_\mathrm{F}}^{s_\mathrm{P}} \frac{r}{gH^2{}_\mathrm{F}}\frac{\partial \Phi_3}{\partial n}\, ds. \tag{5}$$

Since the differential equation which governs the leak-current field is linear, the stream function Φ_3 can be written

$$\Phi_3 = f\,(x, y)\; \Phi_c = f\,(x, y)\; K\,(h^* - h^{**}), \tag{6}$$

where $f\,(x, y)$ is the leak-current field corresponding to a leak current of unit strength; $f\,(x, y)$ is shown in the figures 2.5 a and 2.5 b. Substituting (6) in (5) gives:

$$h_3\,(\mathrm{P}) = g\,(\mathrm{P})\,.\,K\,(h^* - h^{**}), \tag{7}$$

if, for short, g (P) is written for the known expression:

$$g\,(\mathrm{P}) = \int\limits_{y_\mathrm{E}}^{y_\mathrm{F}} \frac{c}{gH}\left(\frac{\partial f}{\partial y}\right)_{x=0} dy - \int\limits_{y_\mathrm{E}}^{y_\mathrm{F}} \frac{r}{gH^2}\left(\frac{\partial f}{\partial x}\right)_{x=0} dy - \int\limits_{s_\mathrm{F}}^{s_\mathrm{P}} \frac{r}{gH^2{}_\mathrm{F}}\frac{\partial f}{\partial n}\, ds. \tag{8}$$

Let now the *leak factor* for a place P be defined by:

$$L\,(\mathrm{P}) = -g\,(\mathrm{P})\,.\,K. \tag{9}$$

Then the leak factors for the various places along the south coast of the North Sea can be computed except for the unknown factor K. Thus, if the leak factor for one place is known, the leak factor for any other place can be computed. In paragraph 4.2.1.2 the leak factor for Hook of Holland will be computed by comparing the theoretical wind effect formula with the empirical relation given by SCHALKWIJK; the result is: L (H) $= 0.46$. From (9) we obtain:

$$L(\mathrm{P}) = \frac{g\,(\mathrm{P})}{g\,(\mathrm{H})}\,.\,L\,(\mathrm{H}). \tag{10}$$

By means of (10) and the known value of the leak factor for Hook of Holland, the leak factors for other places can be computed; some results are given in table 2.5 α.

TABLE 2.5 α

Place	Leak factor
Straits of Dover	L (S) = 0.50
Calais	L (C) = 0.59
Flushing	L (V) = 0.53
Hook of Holland	L (H) = 0.46
Den Helder	L (He) = 0.36
Eierlandse Gat	L (F) = 0.34
Borkum	L (B) = 0.25
Cuxhaven	L (Cu) = 0.23

By the leak factor for the Straits of Dover is meant the average of the leak factors of all points of a line across the Straits of Dover; let the stream lines in the Straits be equidistant then this average leak factor is equal to the leak factor of the point S halfway on that line. The difference between the leak factors at Dover and at Calais is entirely due to the Coriolis force, which makes the sea surface in the Straits of Dover slope across the stream.

By means of formula (8) only g (P) and thus L (P) for a place on the south coast of the North Sea can be computed. For a place Q in the sea itself the following term must be added which takes into account the effect of the slope of the sea surface which is due to the presence of the Coriolis force and which is at right angles to the current direction:

$$h_3\,(\mathrm{Q}) - h_3\,(\mathrm{P}) = \int_{n_\mathrm{P}}^{n_\mathrm{Q}} \frac{c}{gH} \frac{\partial f}{\partial n}\, dn, \qquad (11)$$

where now P denotes the point where the orthogonal trajectory of the leak stream lines through Q intersects the south coast.

Using this formula for the computations of h_3 in the middle of the Straits of Dover and assuming a linear variation of Φ_3 across the Straits, gives:

$$h_3\,(\mathrm{S}) - h_3\,(\mathrm{C}) = \frac{c}{2gH_F}\, \Phi_c = \frac{c}{2gH_F} \cdot K(h^* - h^{**}). \qquad (12)$$

From this formula follows the amount to be added to the leak factor of Calais in order to obtain the leak factor in the middle of the Straits of Dover.

The leak factors have been found without knowledge of the value of K. These values of the leak factor can be verified by computing the leak-current field in the Channel too and then integrating from ocean to ocean through the North Sea and the Channel via the Straits of Dover; in this way K may be determined (integrating along one of the coasts) from:

$$h^* - h^{**} = -\int_{ocean}^{ocean} \frac{r}{gH^2} \frac{\partial \Phi_3}{\partial n} ds = -K(h^* - h^{**}) \int_{ocean}^{ocean} \frac{r}{gH^2} \frac{\partial f}{\partial n} ds, \qquad (13)$$

the left hand equation being derived by integrating both sides of the equation of motion along one of the coasts, taking into account that the integrals of the Coriolis force and of the pressure gradient force vanish, the first because the Coriolis force is perpendicular to the stream line and the second because it is assumed that the levels of both oceans are equal; $h^* - h^{**}$ then represents the integral of the component of $\vec{\tau}/\varrho g H$ along the stream line, from ocean to ocean.

From (13) K can easily be deduced:

$$\frac{1}{K} = -\int_{ocean}^{ocean} \frac{r}{gH^2} \frac{\partial f}{\partial n} ds. \qquad (14)$$

An estimate of K from (14) derived from a roughly drawn leak-current field in the Channel and inserted in (9), yields for the leak factor in the middle of the Straits of Dover the value 0.47, which is close to the value given in table 2.5 α.

2.5.3 The leak-current effect

The leak factor defined in section 2.5.2 enables us to compute the leak-current effect, or the "leak effect", i.e. the changes in sea level due to the removal of the imaginary dam across the Straits of Dover. From (7) and (9) it is evident that this effect can be written as:

$$h_3\,(\mathrm{P}) = -L\,(\mathrm{P})\,(h^* - h^{**}). \qquad (15)$$

Assuming that at the boundaries where the North Sea and the Channel meet the ocean the wind effects are zero, h^* and h^{**}, i.e. the levels on both sides of the imaginary dam across the Straits of Dover must be entirely due to the winds over the North Sea and the Channel respectively.

The effect of the wind over the Channel on the sea level at a place P on the south coast of the North Sea can then be described by:

$$h^C{}_3\,(\mathrm{P}) = L\,(\mathrm{P})\,h^{**}. \qquad (16)$$

In section 2.6.3 it will be seen that $h^C{}_3$ (P) can be computed without knowledge of h^{**}, which is an advantage, since otherwise the static effect and the current field in the Channel area would have to be computed.

In order to be able to compute the leak effect due to wind over the North Sea the relation between the wind field over the North Sea and h^* must be known. The latter quantity may be thought to be composed of the contributions of the static effect, the wind-vorticity current effect and the slope-current effect. Thus the contribution of a homogeneous wind field over area i ($i = N$, S or I, II or III) to the leak effect in P can be written:

$$h^i_3\,(\mathrm{P}) = -L\,(\mathrm{P})\, h^{*i} = -L\,(\mathrm{P})\;(h^{*i}_\tau + h^{*i}_1 + h^{*i}_2), \tag{17}$$

where h^{*i} denotes the wind effect in the middle of the closed Straits of Dover (North Sea side) caused by a homogeneous wind field over area i; h^{*i}_τ, h^{*i}_1 and h^{*i}_2 represent the static effect, the wind-vorticity current effect and the slope current effect into which h^{*i} can be split up.

In these considerations of the leak effects the "Bernoulli acceleration" has been neglected, although it could be imagined that especially in the narrow Straits of Dover this acceleration might be of some importance. BOWDEN [33] has shown empirically that this effect may indeed be neglected.

Finally, the leak effects for the places aforementioned will be computed for the cases of a homogeneous wind field over each of the areas N, S, I, II and III respectively.

Area N

From (17) it is evident that h^{*N} must be known in order to be able to compute h^N_3 (P) and that h^{*N} consists of the contributions h^{*N}_τ, h^{*N}_1 and h^{*N}_2.

Now, the first two contributions can be computed from 2.2 (2) and 2.3 (7a), since, as to the static wind effect, the wind-vorticity- and slope-current effects the place C (Calais) may be identified with S, the place in the middle of the Straits of Dover, because of the narrowness of the Straits. Only when considering the leak current a distinction between C and S must be made. The slope-current effect may be taken from formula 2.4 (6a).

Thus we have:

$$h^{*N}_\tau = (1.00 \cos \alpha_N + 0.00 \sin \alpha_N)\, A_N \tag{2.2(2)}$$

$$h^{*N}_1 = \left(0.00 \cos \alpha_N + 0.13\, \frac{H_N}{H_S} \sin \alpha_N\right) A_N \tag{2.3(7a)}$$

$$h^{*N}_2 = \left\{0.00 \cos \alpha_N + (q - 0.08)\, \frac{H_N}{H_S} \sin \alpha_N\right\} A_N. \tag{2.4(6a)}$$

Adding these partial effects we obtain:

$$h^{*N} = \left\{1.00 \cos \alpha_N + (0.05 + q)\, \frac{H_N}{H_S} \sin \alpha_N\right\} A_N. \tag{18}$$

Multiplying h^{*N} according to (18) by the leak factors of table 2.5α the following leak effects due to a homogeneous wind field over area N are obtained:

$$h^N{}_3\,(S) \;= \left\{-0.47 \cos \alpha_N - 0.47\,(0.05 + q)\,\frac{H_N}{H_S} \sin \alpha_N\right\} A_N \qquad (19)$$

$$h^N{}_3\,(C) \;= \left\{-0.56 \cos \alpha_N - 0.56\,(0.05 + q)\,\frac{H_N}{H_S} \sin \alpha_N\right\} A_N \qquad (19\text{ a})$$

$$h^N{}_3\,(V) \;= \left\{-0.50 \cos \alpha_N - 0.50\,(0.05 + q)\,\frac{H_N}{H_S} \sin \alpha_N\right\} A_N \qquad (19\text{ b})$$

$$h^N{}_3\,(H) \;= \left\{-0.44 \cos \alpha_N - 0.44\,(0.05 + q)\,\frac{H_N}{H_S} \sin \alpha_N\right\} A_N \qquad (19\text{ c})$$

$$h^N{}_3\,(He) = \left\{-0.34 \cos \alpha_N - 0.34\,(0.05 + q)\,\frac{H_N}{H_S} \sin \alpha_N\right\} A_N \qquad (19\text{ d})$$

$$h^N{}_3\,(F) \;= \left\{-0.32 \cos \alpha_N - 0.32\,(0.05 + q)\,\frac{H_N}{H_S} \sin \alpha_N\right\} A_N \qquad (19\text{ e})$$

$$h^N{}_3\,(B) \;= \left\{-0.24 \cos \alpha_N - 0.24\,(0.05 + q)\,\frac{H_N}{H_S} \sin \alpha_N\right\} A_N \qquad (19\text{ f})$$

$$h^N{}_3\,(Cu) = \left\{-0.22 \cos \alpha_N - 0.22\,(0.05 + q)\,\frac{H_N}{H_S} \sin \alpha_N\right\} A_N \qquad (19\text{ g})$$

Area S

In the same way as in the foregoing section, by adding the formulas 2.2 (10a), 2.3 (8a) and 2.4 (3a) for h^{*S} is obtained:

$$h^{*S} = (1.13 \cos \alpha_S + 0.40 \sin \alpha_S)\, B_S, \qquad (20)$$

and, by multiplying this by the leak factors, we find:

$$h^S{}_3\,(S) \;= (-0.53 \cos \alpha_S - 0.19 \sin \alpha_S)\, B_S \qquad (21)$$
$$h^S{}_3\,(C) \;= (-0.63 \cos \alpha_S - 0.23 \sin \alpha_S)\, B_S \qquad (21\text{ a})$$
$$h^S{}_3\,(V) \;= (-0.57 \cos \alpha_S - 0.20 \sin \alpha_S)\, B_S \qquad (21\text{ b})$$
$$h^S{}_3\,(H) \;= (-0.50 \cos \alpha_S - 0.18 \sin \alpha_S)\, B_S \qquad (21\text{ c})$$
$$h^S{}_3\,(He) = (-0.39 \cos \alpha_S - 0.14 \sin \alpha_S)\, B_S \qquad (21\text{ d})$$
$$h^S{}_3\,(F) \;= (-0.37 \cos \alpha_S - 0.13 \sin \alpha_S)\, B_S \qquad (21\text{ e})$$
$$h^S{}_3\,(B) \;= (-0.28 \cos \alpha_S - 0.10 \sin \alpha_S)\, B_S \qquad (21\text{ f})$$
$$h^S{}_3\,(Cu) = (-0.25 \cos \alpha_S - 0.09 \sin \alpha_S)\, B_S \qquad (21\text{ g})$$

Area I

By adding 2.2 (12), 2.3 (9a) and 2.4 (4a) we find:

$$h^{*I}\,(S) \;= (0.50 \cos \alpha_I - 0.01 \sin \alpha_I)\, B_I, \qquad (22)$$

and for the leak effects:

$$h^{I}_{3}\,(S) = (-0.24 \cos \alpha_{I} + 0.01 \sin \alpha_{I})\, B_{I} \qquad (23)$$
$$h^{I}_{3}\,(C) = (-0.28 \cos \alpha_{I} + 0.01 \sin \alpha_{I})\, B_{I} \qquad (23\ a)$$
$$h^{I}_{3}\,(V) = (-0.25 \cos \alpha_{I} + 0.01 \sin \alpha_{I})\, B_{I} \qquad (23\ b)$$
$$h^{I}_{3}\,(H) = (-0.22 \cos \alpha_{I} + 0.00 \sin \alpha_{I})\, B_{I} \qquad (23\ c)$$
$$h^{I}_{3}\,(He) = (-0.17 \cos \alpha_{I} + 0.00 \sin \alpha_{I})\, B_{I} \qquad (23\ d)$$
$$h^{I}_{3}\,(F) = (-0.16 \cos \alpha_{I} + 0.00 \sin \alpha_{I})\, B_{I} \qquad (23\ e)$$
$$h^{I}_{3}\,(B) = (-0.12 \cos \alpha_{I} + 0.00 \sin \alpha_{I})\, B_{I} \qquad (23\ f)$$
$$h^{I}_{3}\,(Cu) = (-0.11 \cos \alpha_{I} + 0.00 \sin \alpha_{I})\, B_{I} \qquad (23\ g)$$

Area II

By adding 2.2 (15a) and 2.3 (10a) we obtain:

$$h^{*II}\,(S) = (0.43 \cos \alpha_{II} + 0.40 \sin \alpha_{II})\, B_{II}, \qquad (24)$$

and for the leak effects:

$$h^{II}_{3}\,(S) = (-0.20 \cos \alpha_{II} - 0.19 \sin \alpha_{II})\, B_{II} \qquad (25)$$
$$h^{II}_{3}\,(C) = (-0.24 \cos \alpha_{II} - 0.23 \sin \alpha_{II})\, B_{II} \qquad (25\ a)$$
$$h^{II}_{3}\,(V) = (-0.22 \cos \alpha_{II} - 0.20 \sin \alpha_{II})\, B_{II} \qquad (25\ b)$$
$$h^{II}_{3}\,(H) = (-0.19 \cos \alpha_{II} - 0.18 \sin \alpha_{II})\, B_{II} \qquad (25\ c)$$
$$h^{II}_{3}\,(He) = (-0.15 \cos \alpha_{II} - 0.14 \sin \alpha_{II})\, B_{II} \qquad (25\ d)$$
$$h^{II}_{3}\,(F) = (-0.14 \cos \alpha_{II} - 0.13 \sin \alpha_{II})\, B_{II} \qquad (25\ e)$$
$$h^{II}_{3}\,(B) = (-0.11 \cos \alpha_{II} - 0.10 \sin \alpha_{II})\, B_{II} \qquad (25\ f)$$
$$h^{II}_{3}\,(Cu) = (-0.10 \cos \alpha_{II} - 0.09 \sin \alpha_{II})\, B_{II} \qquad (25\ g)$$

Area III

By adding 2.2 (18a), 2.3 (11a) and 2.4 (5a) we obtain:

$$h^{*III}\,(S) = (0.20 \cos \alpha_{III} + 0.01 \sin \alpha_{III})\, B_{III}, \qquad (26)$$

and for the leak effects:

$$h^{III}_{3}\,(S) = (-0.09 \cos \alpha_{III} - 0.01 \sin \alpha_{III})\, B_{III} \qquad (27)$$
$$h^{III}_{3}\,(C) = (-0.11 \cos \alpha_{III} - 0.01 \sin \alpha_{III})\, B_{III} \qquad (27\ a)$$
$$h^{III}_{3}\,(V) = (-0.10 \cos \alpha_{III} - 0.01 \sin \alpha_{III})\, B_{III} \qquad (27\ b)$$
$$h^{III}_{3}\,(H) = (-0.09 \cos \alpha_{III} - 0.00 \sin \alpha_{III})\, B_{III} \qquad (27\ c)$$
$$h^{III}_{3}\,(He) = (-0.07 \cos \alpha_{III} - 0.00 \sin \alpha_{III})\, B_{III} \qquad (27\ d)$$
$$h^{III}_{3}\,(F) = (-0.07 \cos \alpha_{III} - 0.00 \sin \alpha_{III})\, B_{III} \qquad (27\ e)$$
$$h^{III}_{3}\,(B) = (-0.05 \cos \alpha_{III} - 0.00 \sin \alpha_{III})\, B_{III} \qquad (27\ f)$$
$$h^{III}_{3}\,(Cu) = (-0.04 \cos \alpha_{III} - 0.00 \sin \alpha_{III})\, B_{III} \qquad (27\ g)$$

2.6 FORMULAS FOR THE TOTAL WIND EFFECT

2.6.1 INTRODUCTION

In sections 1.3 and 1.4 it has been shown that the total wind effect in an arbitrary place P can be thought to be composed of the static wind effect h_τ (P) and the three current effects, viz. the wind-vorticity current effect h_1 (P), the slope-current effect h_2 (P), and the leak-current effect h_3 (P):

$$h\,(\mathrm{P}) = h_\tau\,(\mathrm{P}) + h_1\,(\mathrm{P}) + h_2\,(\mathrm{P}) + h_3\,(\mathrm{P}). \tag{1}$$

The wind effect may, however, also be considered to consist of the contributions due to the wind over each area separately:

$$h\,(\mathrm{P}) = h^N\,(\mathrm{P}) + h^C\,(\mathrm{P}) + \underbrace{h^{\mathrm{I}}\,(\mathrm{P}) + h^{\mathrm{II}}\,(\mathrm{P}) + h^{\mathrm{III}}\,(\mathrm{P})}_{h^S\,(\mathrm{P})}, \tag{2}$$

where the upper index indicates from which area the contribution comes. Thus the wind effect can be split up into 16 parts:

$$\begin{aligned} h\,(\mathrm{P}) = {} & h^N{}_\tau\,(\mathrm{P}) + & & + h^{\mathrm{I}}{}_\tau\,(\mathrm{P}) + h^{\mathrm{II}}{}_\tau\,(\mathrm{P}) + h^{\mathrm{III}}{}_\tau\,(\mathrm{P}) \\ & + h^N{}_1\,(\mathrm{P}) + & & + h^{\mathrm{I}}{}_1\,(\mathrm{P}) + h^{\mathrm{II}}{}_1\,(\mathrm{P}) + h^{\mathrm{III}}{}_1\,(\mathrm{P}) \\ & + h^N{}_2\,(\mathrm{P}) + & & + h^{\mathrm{I}}{}_2\,(\mathrm{P}) + h^{\mathrm{II}}{}_2\,(\mathrm{P}) + h^{\mathrm{III}}{}_2\,(\mathrm{P}) \\ & + h^N{}_3\,(\mathrm{P}) + & h^C{}_3\,(\mathrm{P}) & + h^{\mathrm{I}}{}_3\,(\mathrm{P}) + h^{\mathrm{II}}{}_3\,(\mathrm{P}) + h^{\mathrm{III}}{}_3\,(\mathrm{P}) = \\ & \;\; h^N\,(\mathrm{P}) + & h^C\,(\mathrm{P}) & + h^{\mathrm{I}}\,(\mathrm{P}) + h^{\mathrm{II}}\,(\mathrm{P}) + h^{\mathrm{III}}\,(\mathrm{P}). \end{aligned} \tag{3}$$

Since it is intended to compose diagrams for each area which will give the relation between the wind field over that area and the corresponding contribution to the wind effect at a few places P, instead of (3) the following formula will be used which represents the wind effect contribution of area i to the total wind effect as the sum of the static wind effect, the vorticity-current effect, the slope-current effect and the leak-current effect, due to a homogeneous wind field over area i and no wind elsewhere:

$$h^{(i)}\,(\mathrm{P}) = h^{(i)}{}_\tau\,(\mathrm{P}) + h^{(i)}{}_1\,(\mathrm{P}) + h^{(i)}{}_2\,(\mathrm{P}) + h^{(i)}{}_3\,(\mathrm{P}), \; (i = N, C, S, \mathrm{I}, \mathrm{II}, \mathrm{III}). \tag{4}$$

In the sections 2.2, 2.3, 2.4 and 2.5 each of these terms has been computed separately for the places considered. In the following sections these contributions will be added so as to give the total wind effect formulas.

2.6.2 AREA N

Adding the contributions 2.2 (2), 2.3 (7c), 2.4 (2c) and 2.5 (19c) for Hook of Holland is obtained:

$$h^N(H) = \{0.54 \cos \alpha_N + (0.046 + 1.45\,q) \sin \alpha_N\} A_N. \tag{5}$$

SCHALKWIJK has shown empirically that h^N is zero for $\alpha_N = 97°$. Substituting this value of α_N in (5) and putting the result equal to zero an equation for q is obtained which yields:

$$q = 0.014. \tag{6}$$

Substituting this value of q in the expressions of the leak effects 2.5 (19) and adding to the expressions thus obtained the corresponding static wind effect, and current effects according to 2.2 (2), 2.3 (7) and 2.4 (2), finally the following expressions for the wind effects due to a homogeneous wind field over the northern area of the North Sea are obtained:

$$h^N\,(\mathrm{S}) = (0.53 \cos \alpha_N + 0.03 \sin \alpha_N)\, A_N \tag{7}$$
$$h^N\,(\mathrm{C}) = (0.44 \cos \alpha_N + 0.03 \sin \alpha_N)\, A_N \tag{7 a}$$
$$h^N\,(\mathrm{V}) = (0.50 \cos \alpha_N + 0.03 \sin \alpha_N)\, A_N \tag{7 b}$$
$$h^N\,(\mathrm{H}) = (0.56 \cos \alpha_N + 0.02 \sin \alpha_N)\, A_N \tag{7 c}$$
$$h^N\,(\mathrm{He}) = (0.66 \cos \alpha_N + 0.03 \sin \alpha_N)\, A_N \tag{7 d}$$
$$h^N\,(\mathrm{F}) = (0.68 \cos \alpha_N + 0.03 \sin \alpha_N)\, A_N \tag{7 e}$$
$$h^N\,(\mathrm{B}) = (0.76 \cos \alpha_N + 0.01 \sin \alpha_N)\, A_N \tag{7 f}$$
$$h^N\,(\mathrm{Cu}) = (0.78 \cos \alpha_N + 0.00 \sin \alpha_N)\, A_N \tag{7 g}$$

2.6.3 AREA *C*

As has been said already in section 2.1 the wind over the Channel can only cause wind effects along the North Sea coasts by means of the leak current through the Straits of Dover. These wind effects can be computed by 2.5 (16), if h^{**} is known. In principle h^{**} could be computed like h^*. Instead of doing so h^{**} will be computed by comparing the theoretical formula for h^C at Hook of Holland with the empirical relation between the wind over the Channel and its effect at Hook of Holland. Applying 2.5 (16) to Hook of Holland, h^{**} can be computed since $h^C{}_3$ (H) is known from SCHALKWIJK's empirical relation (see formula (10c)):

$$h^{**} = \frac{h^C\,(\mathrm{H})}{L\,(\mathrm{H})}. \tag{8}$$

Substituting this expression for h^{**} in 2.5 (16), we obtain:

$$h^C\,(\mathrm{P}) = \frac{L\,(\mathrm{P})}{L\,(\mathrm{H})}\, h^C\,(\mathrm{H}). \tag{9}$$

Applying (9) to the various places and making use of the values of the leak factors according to table 2.5 α, the following expressions for the wind effects due to a homogeneous wind field over the Channel relative to the corresponding wind effect at Hook of Holland are obtained:

$$h^C\,(\mathrm{S}) = 1.07\,h^C\,(\mathrm{H}) = -(3.22\cos\alpha_C + 8.85\sin\alpha_C)\,10^{-6}\,V^2_{sC} \quad (10)$$
$$h^C\,(\mathrm{C}) = 1.27\,h^C\,(\mathrm{H}) = -(3.82\cos\alpha_C + 10.50\sin\alpha_C)\,10^{-6}\,V^2_{sC} \quad (10\,a)$$
$$h^C\,(\mathrm{V}) = 1.14\,h^C\,(\mathrm{H}) = -(3.43\cos\alpha_C + 9.45\sin\alpha_C)\,10^{-6}\,V^2_{sC} \quad (10\,b)$$
$$h^C\,(\mathrm{H}) = 1.00\,h^C\,(\mathrm{H}) = -(3.01\cos\alpha_C + 8.27\sin\alpha_C)\,10^{-6}\,V^2_{sC} \quad (10\,c)$$
$$h^C\,(\mathrm{He}) = 0.77\,h^C\,(\mathrm{H}) = -(2.31\cos\alpha_C + 6.37\sin\alpha_C)\,10^{-6}\,V^2_{sC} \quad (10\,d)$$
$$h^C\,(\mathrm{F}) = 0.73\,h^C\,(\mathrm{H}) = -(2.20\cos\alpha_C + 6.04\sin\alpha_C)\,10^{-6}\,V^2_{sC} \quad (10\,e)$$
$$h^C\,(\mathrm{B}) = 0.55\,h^C\,(\mathrm{H}) = -(1.66\cos\alpha_C + 4.55\sin\alpha_C)\,10^{-6}\,V^2_{sC} \quad (10\,f)$$
$$h^C\,(\mathrm{Cu}) = 0.50\,h^C\,(\mathrm{H}) = -(1.50\cos\alpha_C + 4.14\sin\alpha_C)\,10^{-6}\,V^2_{sC} \quad (10\,g)$$

where h^C is expressed in cm and V_{sC} in cm/sec; V_{sC} represents 0.75 × the gradient wind speed over the Channel.

2.6.4 Areas S, and I, II and III

Adding the corresponding formulas of 2.2 (10), 2.3 (8), 2.4 (3) and 2.5 (21) the following expressions for the wind effects due to a homogeneous wind field over the area S are obtained:

$$h^S\,(\mathrm{S}) = (0.57\cos\alpha_S + 0.20\sin\alpha_S)\,B_S \quad (11)$$
$$h^S\,(\mathrm{C}) = (0.46\cos\alpha_S + 0.16\sin\alpha_S)\,B_S \quad (11\,a)$$
$$h^S\,(\mathrm{V}) = (0.53\cos\alpha_S - 0.16\sin\alpha_S)\,B_S \quad (11\,b)$$
$$h^S\,(\mathrm{H}) = (0.50\cos\alpha_S - 0.29\sin\alpha_S)\,B_S \quad (11\,c)$$
$$h^S\,(\mathrm{He}) = (0.38\cos\alpha_S - 0.36\sin\alpha_S)\,B_S \quad (11\,d)$$
$$h^S\,(\mathrm{F}) = (0.32\cos\alpha_S - 0.36\sin\alpha_S)\,B_S \quad (11\,e)$$
$$h^S\,(\mathrm{B}) = (0.42\cos\alpha_S - 0.51\sin\alpha_S)\,B_S \quad (11\,f)$$
$$h^S\,(\mathrm{Cu}) = (0.44\cos\alpha_S - 0.75\sin\alpha_S)\,B_S \quad (11\,g)$$

From 2.2 (12), 2.3 (9), 2.4 (4) and 2.5 (23) is found in the same way:

$$h^I\,(\mathrm{S}) = (0.32\cos\alpha_I - 0.00\sin\alpha_I)\,B_I \quad (12)$$
$$h^I\,(\mathrm{C}) = (0.26\cos\alpha_I - 0.00\sin\alpha_I)\,B_I \quad (12\,a)$$
$$h^I\,(\mathrm{V}) = (0.30\cos\alpha_I - 0.02\sin\alpha_I)\,B_I \quad (12\,b)$$
$$h^I\,(\mathrm{H}) = (0.33\cos\alpha_I - 0.09\sin\alpha_I)\,B_I \quad (12\,c)$$
$$h^I\,(\mathrm{He}) = (0.35\cos\alpha_I - 0.17\sin\alpha_I)\,B_I \quad (12\,d)$$
$$h^I\,(\mathrm{F}) = (0.33\cos\alpha_I - 0.17\sin\alpha_I)\,B_I \quad (12\,e)$$
$$h^I\,(\mathrm{B}) = (0.21\cos\alpha_I - 0.13\sin\alpha_I)\,B_I \quad (12\,f)$$
$$h^I\,(\mathrm{Cu}) = (0.15\cos\alpha_I - 0.10\sin\alpha_I)\,B_I \quad (12\,g)$$

From 2.2 (15), 2.3 (10) and 2.5 (25) we find:

$$h^{II}\,(S) = (0.22 \cos \alpha_{II} + 0.20 \sin \alpha_{II})\, B_{II} \tag{13}$$
$$h^{II}\,(C) = (0.18 \cos \alpha_{II} + 0.16 \sin \alpha_{II})\, B_{II} \tag{13 a}$$
$$h^{II}\,(V) = (0.20 \cos \alpha_{II} - 0.14 \sin \alpha_{II})\, B_{II} \tag{13 b}$$
$$h^{II}\,(H) = (0.13 \cos \alpha_{II} - 0.22 \sin \alpha_{II})\, B_{II} \tag{13 c}$$
$$h^{II}\,(He) = (-0.07 \cos \alpha_{II} - 0.21 \sin \alpha_{II})\, B_{II} \tag{13 d}$$
$$h^{II}\,(F) = (-0.14 \cos \alpha_{II} - 0.22 \sin \alpha_{II})\, B_{II} \tag{13 e}$$
$$h^{II}\,(B) = (-0.11 \cos \alpha_{II} - 0.15 \sin \alpha_{II})\, B_{II} \tag{13 f}$$
$$h^{II}\,(Cu) = (-0.10 \cos \alpha_{II} - 0.14 \sin \alpha_{II})\, B_{II} \tag{13 g}$$

The formulas 2.2 (18), 2.3 (11), 2.4 (5) and 2.5 (27) yield:

$$h^{III}\,(S) = (0.03 \cos \alpha_{III} + 0.00 \sin \alpha_{III})\, B_{III} \tag{14}$$
$$h^{III}\,(C) = (0.02 \cos \alpha_{III} + 0.00 \sin \alpha_{III})\, B_{III} \tag{14 a}$$
$$h^{III}\,(V) = (0.03 \cos \alpha_{III} + 0.00 \sin \alpha_{III})\, B_{III} \tag{14 b}$$
$$h^{III}\,(H) = (0.04 \cos \alpha_{III} + 0.02 \sin \alpha_{III})\, B_{III} \tag{14 c}$$
$$h^{III}\,(He) = (0.10 \cos \alpha_{III} + 0.02 \sin \alpha_{III})\, B_{III} \tag{14 d}$$
$$h^{III}\,(F) = (0.13 \cos \alpha_{III} + 0.03 \sin \alpha_{III})\, B_{III} \tag{14 e}$$
$$h^{III}\,(B) = (0.32 \cos \alpha_{III} - 0.23 \sin \alpha_{III})\, B_{III} \tag{14 f}$$
$$h^{III}\,(Cu) = (0.39 \cos \alpha_{III} - 0.51 \sin \alpha_{III})\, B_{III} \tag{14 g}$$

In the above formulas the theoretical dependency of the wind effects on the wind stress vector has been expressed. However, this vector cannot be read from the weather maps directly. What is needed is a direct relationship between the gradient wind and the corresponding wind effects. This will be dealt with in the next chapter. Besides we shall derive in some detail the assumptions made concerning the bottom friction, this being the second external factor governing the water movements in the sea, be it of less importance than the wind stress.

Finally the leak capacity, which is the third external parameter involved, will be computed independently from Bowden's investigations of the currents through the Straits of Dover in order to verify the value of the leak capacity that will be found from comparison between the theoretical and empirical wind effect formulas for Hook of Holland.

CHAPTER 3

WIND STRESS AND BOTTOM FRICTION

3.1 INTRODUCTION

In the preceding theoretical formulas for wind effect computation three parameters are involved, viz. a wind stress parameter, a bottom stress parameter and a parameter for the leak capacity. In this chapter these three external parameters will be considered in detail.

Starting with the wind stress it should be remembered that the formulas derived in the preceding chapter express the relationship between the wind stress vector corresponding to a homogeneous wind field over one of the areas involved and the corresponding wind effect at a specific place. For practical use of these formulas, however, it is necessary to know the relationship between the gradient wind vector over the area considered and the corresponding wind effects, since it is this wind vector which can easily be read from the weather maps. SCHALKWIJK (l.c.) has correlated the wind effects at Hook of Holland with 0.75 × the gradient wind over each of the three areas into which he subdivided the whole area of the North Sea plus the Channel. This has the advantage that for northwesterly storms in wintertime on an average the wind effects have in fact been correlated with the actual winds at about 10 meters height above sea level, since in an atmosphere with an unstability which is normal for northwesterly storms over the North Sea during wintertime the wind speed at 10 meters above sea level is roughly 0.75 × the gradient wind speed. In this investigation the same choice will be kept. Thus henceforth the wind effects will be correlated with a wind speed V_s defined by:

$$V_s = 0.75\ V_{\text{grad}}. \qquad (1)$$

Now the relation between the wind-stress vector $\vec{\tau}$ and $\vec{V_s}$ must be known. This relation depends on the thermal structure of the atmosphere, or, what is closely related thereto, on the difference between the temperature of the air at say 10 meters above sea level, T_a, and the sea surface temperature T_s.

This difference $T_a - T_s$ will in general be different for different storms, and therefore it will be essentially better to take into account this thermal dependency of the relation between $\vec{\tau}$ and $\vec{V_s}$ for each storm separately. Especially for storms from easterly directions this method is advisable, because of the great scatter in the values of $T_a - T_s$ for different storms. For storms from westerly directions during the storm season, however, the scatter is rather small so that for these storms it will suffice to establish the relationship between $\vec{\tau}$ and $\vec{V_s}$ for the mean difference $T_a - T_s$ corresponding to any specific wind direction.

In these cases it would be an excessive refinement to take into account the actual value of $T_a - T_s$ for the storm in question, since there is already a relatively large inaccuracy in the wind effect computations because of the lack of accurate knowledge of the wind field over the North Sea and of the value of $T_a - T_s$ to be taken. Therefore the mean value of $T_a - T_s$ for each direction of the wind will now be employed here, so that the stability of the atmosphere is supposed to be dependent on the direction of the wind only. This implies that in some extreme cases with a stability (or instability) widely differing from the mean (in-)stability for that direction, a correction to the wind effects computed from the formulas which will be developed must be applied. Thus it will be assumed that the relation between $\vec{\tau}$ and $\vec{V_s}$ also depends on the direction of the wind only. This relation will be dealt with in detail in the following paragraph.

It should be remembered that the vector $\vec{\tau}$ of the foregoing chapters does not represent the pure wind stress, but that it also comprises part of the bottom stress, namely that part which is also present if the mean current velocity (averaged over the vertical) vanishes (see section 1.1). This part amounts, however, to only a small fraction of the wind stress, at least at sea. SCHALKWIJK (l.c.) assumed it to be about 0.10. For a purely laminar flow it would be 0.50; for turbulent flows values of 0.01 to 0.30 have been given in literature.

In section 3.3 a detailed treatment of this subject will be given. Moreover the linear law for the bottom stress (1.1 (14)) will be justified.

Finally, in section 3.4 the leak capacity of the Straits of Dover will be considered to some extent.

3.2 WIND STRESS

The relation between wind stress and wind velocity is very complicated. For practical purposes such as the problem of the computation of wind effects to which the present investigation is confined an empirical quadratic law may be used with a reasonable accuracy. But then a factor of proportionality must be used which is not a constant, but which depends on the height to which the wind speed refers, on the thermal stratification of the atmosphere and on the nature of the surface to which the stress is applied. Consider first the wind stress on a fixed horizontal surface and afterwards proceed to the more complex problem of the stress on a mobile surface such as that of the sea.

The general expression for the stress across a horizontal surface in a horizontal turbulent flow of a fluid having a density ϱ, is, if molecular viscosity is neglected relative to REYNOLDS' stresses,

$$\tau = \varrho \, \overline{v'w'}, \tag{1}$$

where v' en w' represent the velocity fluctuation parallel to the direction of the main flow and vertically upwards, respectively [56]. By introducing a "mixing length" l PRANDTL has transformed (1) for an adiabatic atmosphere or fluid, i.e. in case of zero stability, assuming isotropic turbulence, into:

$$\tau = \varrho\, l^2 \left(\frac{\partial v}{\partial z}\right)^2, \tag{2}$$

where $v(z)$ denotes the mean velocity of the flow at a height z above the boundary surface in question.

It has been shown that near a rough surface we have in first approximation:

$$l = \varkappa\,(z + z_0), \tag{3}$$

where $\varkappa$ represents the constant of VON KÁRMÁN, which has the value of 0.40 approximately, and z_0 is a measure of the roughness of the surface, the socalled *roughness parameter*.

Next a "friction velocity" v_* is defined by:

$$\tau \equiv \varrho\, v_*^2,$$

and τ is assumed to be nearly constant, i.e. independent of height, in the lowest 20 or 30 meters of the atmosphere. Eliminating τ and l from (2), (3) and (4), a differential equation for $v\,(z)$ is obtained wich has the following solution:

$$v\,(z) = \frac{v_*}{\varkappa} \ln \frac{z + z_0}{z_0}, \tag{5}$$

if $v\,(0) = 0$ is taken as boundary condition. If now v_* is eliminated from (4) and (5) the following expression for τ is obtained:

$$\tau = \varrho \frac{\varkappa^2}{\left(\ln \frac{z + z_0}{z_0}\right)^2} \cdot v\,(z)^2 \equiv \varrho\,\gamma^2{}_0\,(z)\; v\,(z)^2, \tag{6}$$

where $\gamma^2{}_0\,(z)$, the socalled "*drag coefficient*" for an adiabatic atmosphere, has been defined by:

$$\gamma^2{}_0\,(z) \equiv \frac{\varkappa^2}{\left(\ln \frac{z + z_0}{z_0}\right)^2}. \tag{7}$$

According to (6) the wind stress in an adiabatic atmosphere or fluid (i.e. zero stability) depends quadratically on the mean wind speed and the resistance parameter depends only on z and the roughness length z_0. In case of air flowing over a rigid surface z_0 is a constant, the value of which can be determined by (5) from measurements of the mean velocity at two heights.

The value of z_0 thus found appears to depend on the nature of the boundary surface only. For flow over land the quadratic relation (6) has been verified empirically. For flow over a water surface there is, until now, no unanimity as to the relation between wind stress, wind speed and roughness of the surface. Theoretically as well as empirically this is still an unsolved problem. CHARNOCK [57] assumes that the roughness of the sea surface increases monotonously with the friction velocity according to the following formula (derived by dimensional reasoning):

$$v_*^{\,2} = g z_0 + \text{const.} \tag{8}$$

If it is assumed that z_0 is very small relative to z, this yields for the velocity profile:

$$V(z) = \frac{v_*}{\varkappa} \ln \frac{gz}{v_*^{\,2}} + \text{const.} \tag{9}$$

This relation between v_* and V is non-linear, which implies that the relation between τ and V is no longer simply quadratic. MUNK [58] too has made plausible, by theoretical reasoning, that the relation is not quadratic. He argues that τ consists of two parts: 1st. the "pure" wind stress, which is quadratic in V, and 2nd. the "form drag" (which represents the horizontal component of the mean normal pressure force and which depends therefore on the spectrum of the slopes of the sea surface), which appears to be proportional to V^3. Thus MUNK's hypothesis leads to the relation:

$$\tau = k_1 V^2 + k_2 V^3, \tag{10}$$

where k_1 and k_2 are constants.

Several authors have also shown empirically that over sea a simple quadratic law according to (6) with a γ^2 depending on z and z_0 only, is not observed in nature. This means that, if we write:

$$\tau = \varrho\, \gamma(z)^2\, V(z)^2, \tag{11}$$

then γ^2 would even in the adiabatic case still depend on $V(z)$. From measurements of wind stresses over a water surface in wind tunnels FRANCIS [59] derives a $\gamma^2(z)$ proportional to V. ROLL [60] and DURST [61] arrive at a similar result by totally different ways. KEULEGAN [62] and NEUMANN [63] on the contrary found that $\gamma^2(z)$ decreases with increasing $V(z)$. VAN DORN [64] could describe his observations of wind effects in a pond best by the following formulas:

$$\left.\begin{aligned} \tau &= l_1 V^2, \text{ for } V < V_c \\ \text{and} \quad \tau &= l_1 V^2 + l_2 (V - V_c)^2, \text{ for } V \geqslant V_c \end{aligned}\right\}, \tag{12}$$

where the first term on the righthand side of both formulas represents the "pure" stress and the second term of the second formula represents the "form drag"; l_1 and l_2 are constants and V_c is a critical velocity below which the "form drag" vanishes.

HUNT [27], in an elaborate study, came to the following relation between V and z:

$$V - 65 = \frac{v_*}{\varkappa} \ln \frac{z}{1.5} \text{ (cgs units)}, \tag{13}$$

which yields, if we still assume an adiabatic atmosphere,

$$\tau = \varrho \frac{\varkappa^2}{\left(\ln \frac{z}{1.5}\right)^2} (V - 65)^2. \tag{14}$$

From these contradictory results of the various authors it is clear that there is still a great lack of knowledge of the relation sought. MONTGOMERY [65] rightly declared in 1952 that "at no wind speed the resistance coefficient γ^2 is known confidently within half its value".

At present most oceanographers tend to agree with an increase of γ^2 with V in an adiabatic atmosphere.

There are various methods by which τ may be determined; e.g. from

1° measurement of the vertical wind profile;

2° measurement of the slope of the sea surface;

3° measurement of the simultaneous fluctuations v' and w' of the flow (see (1));

4° measurement of the angle of deviation between the wind at a height of e.g. 10 meters, and the direction of the isobars.

5° direct measurement of the stress.

Of these methods, the first seems to me to give the most reliable results. The second method, which has also often been used (e.g. by COLDING [66], LORENTZ [26], and HELA [67]) is difficult because of the influence of current effects which have to be eliminated.

If even for an adiabatic atmosphere our knowledge of γ^2 is still insufficient, this is the more so for a non-adiabatic atmosphere.

Theoretically this problem has been tackled by e.g. ROSSBY *and* MONTGOMERY [68], DEACON [69], LETTAU [70] and BUSINGER [71]. As long as there is no adequate theory of turbulence in the atmosphere, the results remain more or less hypothetical, while measurements are far too unreliable, too scarce and too incomplete to admit an unambiguous interpretation. The present views appear to indicate that the wind stress increases with increasing stability if the wind speed at say 10 meters above sea level is given. At first sight this result seems strange, for the turbulence and hence also the mixing length l tend to decrease with increasing stability. This would involve a decreasing transport of momentum too, i.e. a decreasing wind stress, if at the same time the velocity shear should not increase at an even greater rate. The latter according to (2) causes τ to increase with increasing stability.

As to the numerical values of γ^2 the data of Hunt appear to be the most reliable at present. From his results the following figures are borrowed. If the air-sea temperature difference amounts to $-3°$ Celsius which corresponds to the mean instability of the air in northwesterly storms over the North Sea in wintertime, the value of γ^2 for $z = 8$ m and a mean wind velocity of 17 m/sec, which is the mean value of all wind speeds used by SCHALKWIJK for the construction of his wind effect diagrams, appears to be:

$$\gamma^2_{-3}\,(8) = 3.5 \times 10^{-3}. \tag{15}$$

This value, together with an air density of 1.25×10^{-3} gr/cm^3, yields the following formula for τ_s, the wind stress:

$$\tau_s = 4.35 \times 10^{-6}\, V(8)^2, \text{ if } T_a - T_s = -3°\text{ C}, \tag{16}$$

where $V(8)$ denotes the mean wind speed at a height of 8 meters above sea level, to which height HUNT has reduced all his results.

By means of the logarithmic velocity profile this formula is reduced to the height of 10 meters above sea level, which is the level of reference in the present investigation and at which, under the conditions mentioned, the velocity is assumed to be $0.75 \times$ the gradient wind speed. This yields:

$$\tau_s = 4.1 \times 10^{-6}\, V(10)^2 \text{ if } T_a - T_s = -3°\text{ C}. \tag{17}$$

The apparent wind stress, however, comprises a small contribution due to the bottom friction, as has been explained already in sections 1.1 and 3.1 and as will be explained in more detail in the next paragraph. There it will be explained too that a reasonable estimate of that contribution amounts to 7 per cent of the true wind stress under conditions prevailing in the North Sea. Hence, substituting V_s for $V(10)$, as is permissible in the circumstances under consideration for the apparent wind stress may be written:

$$\tau = 4.4 \times 10^{-6}\, V_s^2 \text{ if } T_a - T_s = -3°\text{ C}. \tag{18}$$

In order to obtain an idea of the opposite effects of a decrease of the mixing length and a simultaneous increase of the shear, as given by the ratio $V(10)/V_s$, in table 3.2α for different values of $T_a - T_s$ the corresponding values of the coefficient c_s defined by:

$$\tau_s = \varrho' c_s\, V_s^2 \tag{19}$$

have been given.

In the same table also the two factors which form together c_s:

$$c_s = \gamma\,(10)^2 \left(\frac{V(10)}{V_s}\right)^2, \tag{20}$$

and which have opposite tendencies with varying value of $T_a - T_s$, are found. The air density ϱ' has been assumed to be constant and equal to 1.25×10^{-3} gr/cm^3.

TABLE 3.2α

$T_a - T_s$	$10^6.\varrho'\gamma^2\,(10)$	$\left(\frac{V(10)}{V_s}\right)^2$	$10^6\,\varrho'\,c_s$
— 6.0	3.8	1.17	4.45
— 3.0	4.1	1.00	4.1
0.0	4.4	0.83^5	3.65
+ 3.0	4.75	0.66	3.15

The values of $\gamma^2\,(10)$ in this table have been derived from HUNT, those of $V(10)/V_s$ from JOHNSON [72].

From table 3.2α we see indeed that the factors of c_s have opposite tendencies, the shear factor being the dominating one. The relation between $\varrho' c_s$ and T_a—T_s of table 3.2α can be expressed in the following formula:

$$\tau_s = 3.62\{1 - 0.044\,(T_a - T_s)\}\,.\,10^{-6}\,V_s^2. \tag{21}$$

For the effective wind stress this yields:

$$\tau = 1.07\,\tau_s = 3.88\{1 - 0.044\,(T_a - T_s)\}\,10^{-6}\,V_s^2. \tag{22}$$

This formula enables us to compute the wind effects also for a storm having a thermal stratification which deviates considerably from the average, other conditions remaining the same.

Finally the average relation between τ and V_s in dependence on the direction of the wind will be derived. For that purpose the mean value of $T_a - T_s$ for each direction of the wind during storm situations in wintertime must be known. SCHALKWIJK (l.c. fig. 26) has described this dependence already for the storms he considered, blowing from directions between southwest and north. The present author has supplemented these data by those of 67 storms of the period from 1945 to 1951 inclusive. In this way the empirical relation illustrated in figure 3.2a was obtained. This figure shows the relation between the mean value of $T_a - T_s$ denoted by ϑ, and the direction of the wind α, the latter being given relative to the direction of the longitudinal axis of the North Sea. From this relation by means of (22), the dependence of τ on V_s^2 for any α can easily be derived:

$$\tau = f(\alpha)\,V_s^2. \tag{23}$$

The function $f(\alpha)$, which is called *wind stress factor*, has also been plotted in fig. 3.2a.

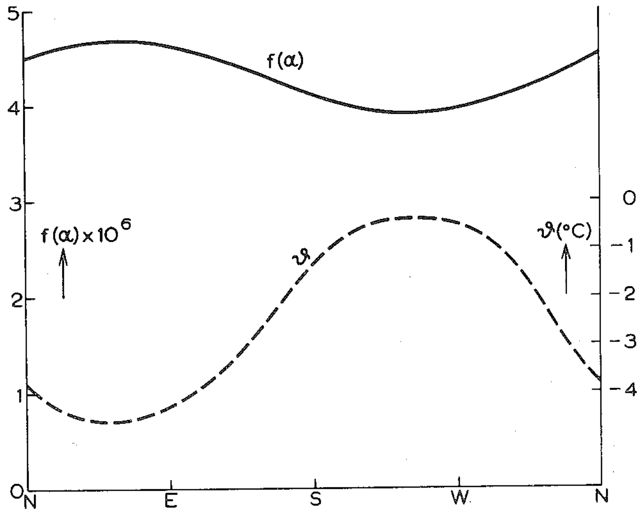

FIG. 3.2 a. Graph of the wind stress factor f (α) for area *S*, as a function of wind direction (α) for strong winds during the storm surge season (November through February), and corresponding values of the mean difference of air temperature *minus* sea surface temperature (ϑ).

3.3 BOTTOM FRICTION

It has been shown empirically as well as theoretically that in turbulent flow along a rough wall the frictional force per unit area, the stress, depends quadratically on the mean current velocity at some distance from the wall. This is also true for the stress τ_b at the bottom of the sea, the bottom stress:

$$\vec{\tau}_b = -k\,(z_b)\;v\,(z_b)\;\overrightarrow{v\,(z_b)}, \tag{1}$$

where k denotes a factor of proportionality which depends on the roughness of the bottom and on z_b, the distance from the bottom to which the mean velocity refers. For $z_b = 75$ cm a reasonable value of k in the North Sea appears to be

$$k\,(75\text{ cm}) = 2.4 \times 10^{-3} \tag{2}$$

according to BOWDEN *and* FAIRBURN [73].

Even if, however, the roughness of the bottom were known exactly everywhere in the North Sea, we could not use (1) for the computation of the bottom stresses in the present problem, since in this problem $v\,(z_b)$ is not known, but depends on the bottom stress itself as well as on the wind stress, the depth, the vertically averaged current, the Coriolis force, the field accelerations, the presence of tidal currents which are superposed on the mean flow etc.

If we neglect the small effects of the Coriolis force and of field accelerations on the bottom stress under the circumstances prevailing in the North Sea, there remains to be established the relationship between v (75), the mean velocity $\bar{v}$ = averaged vertically from surface to bottom, the wind stress τ_s and the velocity v_t of the tidal current near the bottom, say at 75 cm above it. It is evident then that $\vec{\tau}_b$ will be a function of place and time, which will be different for different storms. It is not the purpose, however, to make a detailed study of these relations, but only to derive a simple formula which gives a sufficiently good approximation of reality. BOWDEN [33, 75], REID [74] and SCHÖNFELD [42] have shown independently that the bottom stress in first approximation consists of a part proportional to the wind stress τ_s and a part proportional to the mean velocity $\bar{v}$ defined above. In the following BOWDEN's notation will be used.

He has shown [75] that in a sea where the wind-induced currents may be considered as a perturbation on dominating tidal streams, as is in first approximation true for the North Sea and the Channel, the quadratic law for the bottom stress (1) may be linearized in the following way. Let

$$v_b = v_0 + v_t \cos \sigma_t t, \;\; v_t \gg v_0, \tag{3}$$

represent the total velocity near the bottom, say at 75 cm above it, v_0 and v_t being the drift-current velocity and the amplitude of the tidal current at the same height above the bottom; σ_t denotes the angular frequency of the tide and t the time. BOWDEN expands the expression for τ_b obtained by substituting (3) in (1) into a Fourier series in order of multiples of $\sigma_t\, t$.

The non-periodic component τ^*_b in this expansion then becomes:

$$\tau^*_b = f k \varrho v_t v_0, \tag{4}$$

where f represents a factor which depends on the ratio v_t/v_0, and which for values of this ratio larger than 1.5 may be taken to be constant and equal to 1.27. From the drastically simplified equations of motion BOWDEN next derives that v_0 may be thought to be composed of two terms, the most important of which is proportional to $\bar{v}$, whereas the second one, which can be considered as a correction to the first one, is proportional to τ_s. On account of the proportionality between τ^*_b and v_0, a similar relation holds for τ^*_b:

$$\tau^*_b = \varrho r \bar{v} + n \tau_s, \tag{5}$$

where n and r appear to depend on the velocity of the tidal current, on the depth and on the eddy viscosity. The part of τ_b that depends on τ_s has already been used in the foregoing section as a "correction" to be applied to τ_s in order to obtain the apparent wind stress. For n BOWDEN gives $n = 0.12$ as a reasonable value. This value, however, has been derived from measurements in very shallow water during periods with low wind speeds. The relation for *n*, according to BOWDEN, is:

$$n = \frac{1}{2 + \dfrac{6\nu}{fkH\nu_t}}, \tag{6}$$

where ν represents the kinematic eddy viscosity coefficient and H the depth. If we compute ν by means of the empirical formula (SVERDRUP p. 494):

$$\nu = 4.3\ V^2\ (V > 6\ \text{m/sec}), \tag{7}$$

and if we take the wind speed at 10 meters above the sea surface $V = 20$ m/sec we find $\nu = 1700$ cm²/sec. The value of ν_t has been taken from current measurements made on board the Netherlands lightships in the North Sea, at a depth of about 6 meters. The mean amplitude of the tidal currents appears to be of the order of magnitude of 1 m/sec. Using the vertical current distribution as deduced by BOWDEN from current measurements, we derive from the above velocity amplitude an amplitude of the tidal currents at 75 cm above the sea bottom of 65 cm/sec. If finally we take for f and k, the values mentioned before we obtain from (6):

$$n \approx 0.07 \tag{8}$$

as a value which is thought to be more or less representative for the southern North Sea.

The coefficient r in (5) has also been expressed in ν by BOWDEN:

$$r = \frac{3\nu}{1 + \dfrac{3\nu}{fkH}}. \tag{9}$$

The same values of ν, f, k and H as used before, yield:

$$r = 0.17\ \text{cm/sec}. \tag{10}$$

A totally different method for the determination of the value of r is found in an analysis by BOWDEN of the flow through the Straits of Dover. He has correlated the flow through the Straits of Dover with the wind and the slope of the sea surface. As a measure for this slope he has taken the difference $h' - h''$ between the mean sea surface heights at the cross-sections Aldeburgh-Flushing (h') and Shoreham-Dieppe (h''). This difference is due to the bottom friction and therefore we have:

$$h' - h'' = r \int_{A-F}^{S-D} \frac{v(s)}{gH}\, ds \tag{11}$$

where the path of integration is the central stream line and $v(s)$ represents the depth-mean velocity at the point having a value s for the length coordinate along this stream line. BOWDEN found empirically that the drop of sea level $h' - h''$ in absence of wind is connected to the current velocity v in the Straits of Dover by the formula:

$$v(\mathrm{S}) = 0.70\,(h' - h''), \text{ (c.g.s. units)} \tag{12}$$

where S refers to the point in the middle of the Straits of Dover. From (11) and (12) we obtain:

$$r = \frac{v(\mathrm{S})}{0.70 \int\limits_{A-F}^{S-D} \frac{v(s)}{gH}\, ds}. \tag{13}$$

In section 2.5 (2.5 (6)) we have shown that the leak current at any place in the area under consideration is proportional to the total flow through the Straits of Dover, thus also proportional to v(S). But then the integral in the denominator of (13) is also proportional to v(S). Thus the righthand side of (13) is independent of v(S). We can compute the leak-current field between the cross-sections A — F and S — D for, say, unit total transport of the leak current ($\Phi_c = 1$) and from this field the denominator of (13) can be computed. Doing so we find:

$$r = 0.31 \text{ cm/sec.} \tag{14}$$

This value of r is of the same order of magnitude as the value given by (10), although they differ by about a factor 2. Because of the many simplifications and assumptions made in both methods the agreement is, after all, not too bad. We shall take for r the average of (10) and (14):

$$r = 0.24 \text{ cm/sec.} \tag{15}$$

All current effects have been computed with this value of r, and so have all theoretical wind effect formulas of the preceding chapter. In the next chapter it will be shown that the wind effect formulas so computed agree quite well with the empirical data. Therefore it may be concluded that the value of r that has been used is sufficiently accurate.

3.4 LEAK CAPACITY

In section 2.5 it has been shown that the leak factor of any place in the North Sea can be computed without knowledge of the value of the "leak capacity" K (defined by 2.5 (4)), supposing that the leak factor of one specific place is known. As such a place Hook of Holland has been chosen. The value of the leak factor of Hook of Holland will be derived in section 4.2 by comparing the theoretical wind effect formula with SCHALKWIJK's empirical formula for area S.

From the value of L (H) found there, the leak capacity can be computed by means of formula 2.5 (9); thus we obtain:

$$K = 0.70 \times 10^{10} \text{ cm}^2/\text{sec}. \qquad (1)$$

It would be a support to the theory developed here, and of the value of L (H), if the value of K could also be derived from measurements of the leak current through the Straits of Dover. For this purpose an investigation by BOWDEN [33], in which he empirically derived the following formula:

$$v = 0.37\, V^2 \cos \varphi + 0.70\, (h' - h''), \qquad (2)$$

is very suitable; in this formula v represents the mean current velocity (averaged over the cross-section of the Straits of Dover), in cm/sec, V the wind speed in m/sec, φ the angle between the wind direction and the axis of the Straits of Dover, and h' and h'' the mean sea level heights in cm over the cross-sections Aldeburgh-Flushing and Shoreham-Dieppe ($h' - h''$ is a measure of the slope of the sea surface in the Straits of Dover). Assuming that this formula is also valid if $V = 0$, we obtain for Φ_c, by multiplying both members of (2) by the area of the cross-section of the Straits (1.2×10^{10} m^2) and putting $V = 0$:

$$\Phi_c = 0.84 \times 10^{10}\, (h' - h'') \text{ (c.g.s. units)}. \qquad (3)$$

BOWDEN has shown that probably the coefficient 0.84×10^{10} must be multiplied by a factor 1.25 in order to reduce the empirical formula (2) to an equilibrium state. Since the wind effect formulas derived in chapter 2 refer to an equilibrium (3) must be replaced by:

$$\Phi_c = 1.05 \times 10^{10}\, (h' - h'') \text{ (c.g.s. units)} \qquad (4)$$

in order to secure the comparability between the "leak capacity" to be derived from BOWDEN's material and the "leak capacity" derived from SCHALKWIJK's material.

From (4) a value of the "leak capacity" can be computed if the relation between $h' - h''$ and $h^* - h^{**}$ is known. This relation has been derived as follows.

Assume that no wind is blowing over the North Sea and the Channel. Let first the Straits of Dover be closed and let the sea level heights in the whole Channel be h^* and h^{**} respectively. If now the closing dam is supposed to be removed a leak current is established which changes h everywhere by a certain amount: the leak effect (see section 2.5). Because of the capacity of the oceans, the leak effects at the northern boundary of the North Sea and the western boundary of the Channel may be supposed to be zero.

The mean leak-current velocity of a cross-section over the North Sea may be computed from continuity considerations if the total stream transport through a specific cross-section is known (the mean current velocity being inversely proportional to the area of the cross-section). Thus, starting from an arbitrary total water transport through the Straits of Dover, for each cross-section the mean current velocity can be computed. But then also the slope of the sectionally mean sea level from ocean to ocean can be computed, assuming equilibrium between the mean pressure gradient force due to the surface slope and the mean bottom friction force over a cross-section. In this way the topography of the mean sea level along a longitudinal section from ocean to ocean has been computed; see fig. 3.4 a. From this graph we read:

$$h' - h'' = 0.54\,(h^* - h^{**}). \tag{5}$$

From (4) and (5) we obtain:

$$\Phi_c = 0.57 \times 10^{10}\,(h^* - h^{**})\ \text{(c.g.s. units)} \tag{6}$$

which gives the following value of the leak capacity:

$$K = 0.6 \times 10^{10}\ \text{cm}^2/\text{sec}. \tag{7}$$

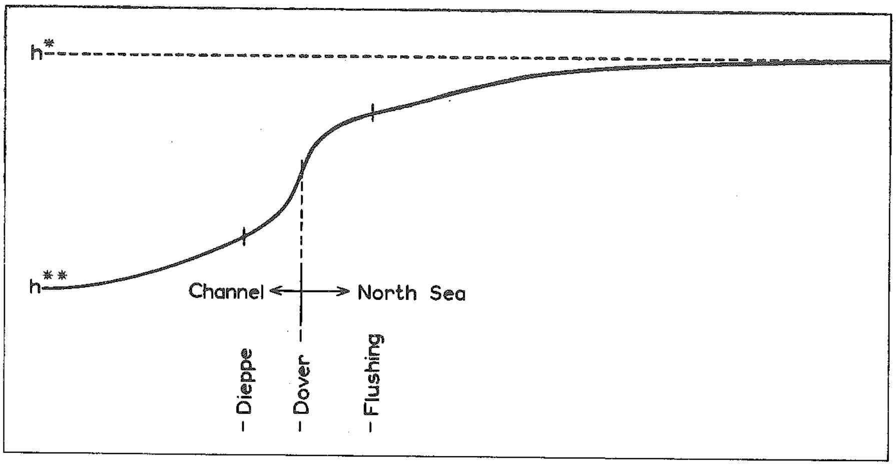

FIG. 3.4 a. Mean sea level height along longitudinal section through North Sea and Channel.

This value is in fair agreement with the value which has been derived from L (H) and given in (1), the more so since (7) has been derived from an estimated leak-current field.

CHAPTER 4

COMPARISON BETWEEN THEORY AND OBSERVATIONS

4.1 INTRODUCTION

In this chapter some of the theoretical formulas for the computation of wind effects, as deduced in chapter 2 will be compared with the corresponding empirical data. Since we are especially interested in the wind effects along the Netherlands coast, we shall only try to verify the formulas for Hook of Holland, as being representative of the southwestern part of this coast and of Borkum as representative of its northern part. If these formulas are confirmed by observations, it may safely be assumed that the formulas for the other places along the Netherlands coast will also be sufficiently reliable.

For Hook of Holland there are the empirical formulas that have been established by SCHALKWIJK for the computation of wind effects due to homogeneous wind fields over the southern part and the northern part of the North Sea and over the Channel.

We shall only compare the empirical formulas for the North Sea areas with theory and derive the unknown parameter L (H), the leak factor for Hook of Holland, and the parameter relating the wind stress to the "wind speed" V_s for a certain wind direction e.g. $\alpha_S = 0$, from the comparison with the theoretical formula for the southern area. The empirical formula for the Channel area will only be used for finding similar formulas for other places than Hook of Holland.

As to Borkum, we have at our disposal the empirical diagrams for the determination of a quantity O (Dutch "opzet") which have been constructed by TOMCZAK [15]. This quantity is defined as the height difference between the observed high water and the corresponding predicted astronomical high water, which are not reached simultaneously, in general, whereas the wind effect is defined as the difference in height of the observed sea level and the astronomical level predicted for the same moment. Any comparison of TOMCZAK's diagrams with our theoretical results must be performed, however, with great caution, since O is, in general, not equal to the wind effect. A second drawback of TOMCZAK's diagrams for a comparison with the theoretical wind effect formulas is that they give O in relation to the mean wind as observed on the German lightships in the German Bight, instead of to the mean wind over the southern area of the North Sea, which is used by us.

Besides TOMCZAK's data we shall make use of an analysis of 67 storm situations made by the author.

In this analysis too, however, values of O were used instead of wind effects, but these values were correlated with the mean wind over each of SCHALKWIJK's North Sea areas.

Summarizing we may say that the empirical data we have for Borkum are not very suitable for comparison with theory.

The comparisons to be made will afford a test of the theoretical considerations and hypotheses used. It is true that from the comparison with the empirical data the value of the leak factor for Hook of Holland can be derived but this does not by itself guarantee a complete agreement between theory and observations, since the wind effect curve at a fixed wind speed varies roughly sinusoidally with the direction of the wind and such a curve needs at least two parameters, e.g. amplitude and phase. Thus we could choose the leak factor so as to give complete agreement of, say, the amplitudes and verify whether then the phases too would agree. In the following paragraph a slightly different method will be used. If the leak factor for Hook of Holland has been determined, the leak factors for all other places on the coast under consideration are fixed too, according to what has been said in 2.5.2.

A comparison of the theoretical and empirical formulas for Borkum would thus give a still sharper test of the theory, if only the empirical data of Borkum were somewhat more suitable for the purpose.

4.2 Hook of Holland

4.2.1 Area S

4.2.1.1 Empirical knowledge

From sea level observations at Hook of Holland made during thirteen storm periods Schalkwijk derived the atmospheric effects by eliminating the astronomical tide. The astronomical tide was for each half hour of the periods considered computed by means of the method of harmonic analysis. The height difference between observed and predicted astronomical sea levels, considered as a function of time (the graph of this function will be called the residual curve), still contained a tidal oscillation.

By means of the method of overlapping means this tidal constituent was largely eliminated for rather straight parts of the curve. For parts of the curve with strong curvature a different method was used, because the first method would imply a reduction of the extrema. The second method used was based on a straightening of strong curvatures by first eliminating the tidal oscillation roughly by a "free hand" smoothing, and then applying the method of overlapping means to the difference between the residual curve and the smoothed curve. The result of this was added afterwards to the smoothed curve.

In this way Schalkwijk obtained for each storm the atmospheric-effect curve. From these he then eliminated the inertial oscillations in a similar way as the tidal oscillations were removed.

For the mean period of the damped inertial oscillations of the storm surges studied, SCHALKWIJK found 41 hours, which is of the same order of magnitude as the theoretical period of the longest longitudinal oscillation of the North Sea. TOMCZAK found empirically a period of 37 hours. If we omit the three far outlying values of periods greater than 50 hours in SCHALKWIJK's table 8 ([1] l.c. pag. 53) the mean period becomes 38 hours, in good agreement with TOMCZAK's value. The remaining curves represent the equilibrium atmospheric-effect curves. From these the atmospheric pressure effect was eliminated, which amounts maximally to two or three decimeters. This effect was determined by correlating the deviations from the astronomical tide for calm periods with the air pressure at Den Helder.

In this way SCHALKWIJK finally arrived at equilibrium wind effect curves. Selecting from these curves those parts where the wind effect remained stationary for some time, and comparing these wind effects with the corresponding gradient winds SCHALKWIJK found that the equilibrium wind effect at Hook of Holland (H) depends quadratically on the gradient wind speed over the sea area involved, at least for velocities up to 20 m/sec. As to the relation between the wind effect and the direction of the wind over area S he did not find a pure sine function. He could describe his data by the following expression:

$$h\,(\mathrm{H}) = (a_N \cos \alpha_N + b_N \sin \alpha_N)\, V^2_{sN} + (a_C \cos \alpha_C + b_C \sin \alpha_C)\, V^2_{sC} + p(\alpha_S)\, V^2_{sS}, \quad (1)$$

where the subscripts N, C and S refer to the three areas considered; V_{sN}, V_{sC} and V_{sS} denote 0.75 × the gradient wind over the areas N, C and S respectively; the a's and b's are constants and p (α_S) represents a function of the direction of the wind over area S. The function p (α_S) is represented graphically in figure 4.1 a.

Applying formula (1) to those cases, in which the wind effect increases or decreases, SCHALKWIJK found a time lag between the gradient wind and the corresponding wind effect, which amounted to 2.2 hours for the rising part of the wind effect curve and 2.8 hours for the descending part of the curve.

From figure 4.1 a we see that p (α_S) does not represent a pure sine function. As to the contributions of the areas N and C SCHALKWIJK assumed a sine function; this is permissible since these contributions are mostly small as compared with that of area S.

Figure 4.1 a also shows (by means of dots) the empirical values on which SCHALKWIJK has based part of this curve for area S. Each dot represents the average of a number of observations for neighbouring wind directions. SCHALKWIJK remarks that the curve is only reliable for directions between $\alpha_S = 225°$ and $\alpha_S = 15°$, through west, since only the values in this sector have been based on reliable wind data. It appears to the present author, however, that also the value of p (α_S) for $\alpha_S = 239°$ is an outlyer. First, it has been computed from only a small number of data. Moreover, these data are relatively inaccurate compared with the points corresponding to the more "dangerous directions", because residuals of inertial effects, astronomical tides, external effects, effects of the inhomogeneity of the wind field etc. ("noise") are relatively large as compared with the effect to be found. In the second place would the maintaining of that value lead to an improbably high value of the leak factor for Hook of Holland, as follows from formula (3), if for α_S that value is substituted for which $p(\alpha_S) = 0$, according to SCHALKWIJK, and L (H) is solved from the equation thus found. For these reasons it appears to be necessary to reject this value.

As a cause of the asymmetry of the curve SCHALKWIJK mentions the effect of the stability of the atmosphere, or $T_a - T_s$, on the wind stress. Moreover, the fetch might be of importance since it governs the wave height and, consequently, the roughness of the sea surface. According to present views, however, the fetch has no appreciable influence on the wind stress, since there are arguments which tend to indicate that the roughness of the sea surface is mainly due to the very small capillary waves having heights of about 1 cm. (HUNT [27], ROLL [60], MUNK [58], a.o.); such small waves need a fetch which is negligible relative to the dimensions of the sea.

4.2.1.2 FITTING THE THEORETICAL FORMULA TO THE EMPIRICAL DATA

In Chapter 1 it has been shown that the wind effect formulas are composed of formulas for the static wind effect, the wind-vorticity-current effect, the slope-current effect and the leak-current effect. Adding the three first-mentioned effects we obtain the wind effects as would be observed if the Straits of Dover were closed. These effects can easily be computed by adding the corresponding formulas of 2.2, 2.3 and 2.4. Also h^* (S) can be found in this way.

The leak effects are then to be found by multiplying h^* (S) by the leak factors for the places concerned, as has been explained in section 2.5. Subtracting these leak effects from the wind effects for the closed case we obtain the total wind effect formulas. For Hook of Holland this procedure yields, in the case of a homogeneous wind field over the area S and no wind elsewhere, the following formula, which still contains the leak factor of Hook of Holland as an unknown quantity:

$$h_S(\mathrm{H}) = \{(1.03 - 1.13\,L(\mathrm{H}))\cos\alpha_S - (0.10 + 0.40\,L(\mathrm{H}))\sin\alpha_S\}\frac{\tau^S a}{\varrho g H_S}. \qquad (2)$$

For τ^S we may, according to 3.2 (23), write $f(\alpha_S) \,.\, V_{SS}{}^2$; the function $f(\alpha_S)$, according to section 3.2, has been given in figure 3.2 a. It would be valuable if from SCHALKWIJK's material an independent determination of the wind stress parameter could be derived. This will be done by putting

$$f(\alpha) = f(0) \,.\, \chi(\alpha) \qquad (3)$$

and leaving f (0) to be determined together with L (H). $\chi(\alpha)$ can easily be derived from figure 3.2 a by dividing $f(\alpha)$ by the value of $f(0)$ used there. Using for a and H_S the values 5.75×10^7 cm and 4.3×10^3 cm respectively, we obtain for $a/\varrho g H_S$ the value:

$$\frac{a}{\varrho g H_S} = 13.8 \text{ c.g.s. units.}$$

Substituting this into the righthand member of (2) and comparing the formula thus obtained with the righthand member of (1), with $V_{sN} = 0$ and $V_{sC} = 0$ we obtain the following equation:

$$\{(1.03 - 1.13\, L\,(\mathrm{H}))\cos\alpha_S - (0.10 + 0.40\, L\,(\mathrm{H}))\sin\alpha_S\}\, f(0) = \frac{p\,(\alpha_S)}{13.8\, \chi\,(\alpha_S)}. \quad (4)$$

Now the best values of $L\,(\mathrm{H})$ and $f(0)$ can easily be determined by means of the method of least squares by not using SCHALKWIJK's curve for $p\,(\alpha_S)$ but the ten empirical values which we have accepted as reliable (figure 4.1 a). The best fit of the time function of the lefthand member is then obtained by taking:

$$L\,(\mathrm{H}) = 0.46, \quad (5)$$

and

$$f(0) = 0.0040. \quad (6)$$

This value of $f(0)$ is in good agreement with the value derived in section 3.2, viz. 0.0041.

Substituting (5) in (2) we obtain:

$$h_S\,(\mathrm{H}) = (0.50 \cos\alpha_S - 0.29 \sin\alpha_S)\, f^*\,(\alpha_S)\; V^2_{sS} \text{ (c.g.s. units)}, \quad (7)$$

where $f^*\,(\alpha_S)$ has been written for:

$$f^*\,(\alpha_S) = 13.8\, f\,(\alpha_S). \quad (8)$$

Figure 4.1 a shows also the wind effect curve according to (7) for a wind speed of 10 m/sec. The agreement between this curve and the ten empirical points on which it has been based is very good, about as good as the curve drawn by SCHALKWIJK for wind directions between $\alpha_S = 250°$ and $\alpha_S = 30°$ (via 360°), if we discard the point at 239°. On the whole we may say that the positive parts of both curves agree quite well. The negative parts, however, differ considerably. The negative part of SCHALKWIJK's curve is rather unreliable in the present author's opinion because:

1° it has been based on three storm surges only;

2° the corresponding wind effects were small (deviations of less than 5 dm from the undisturbed sea level), and therefore relatively inaccurate;

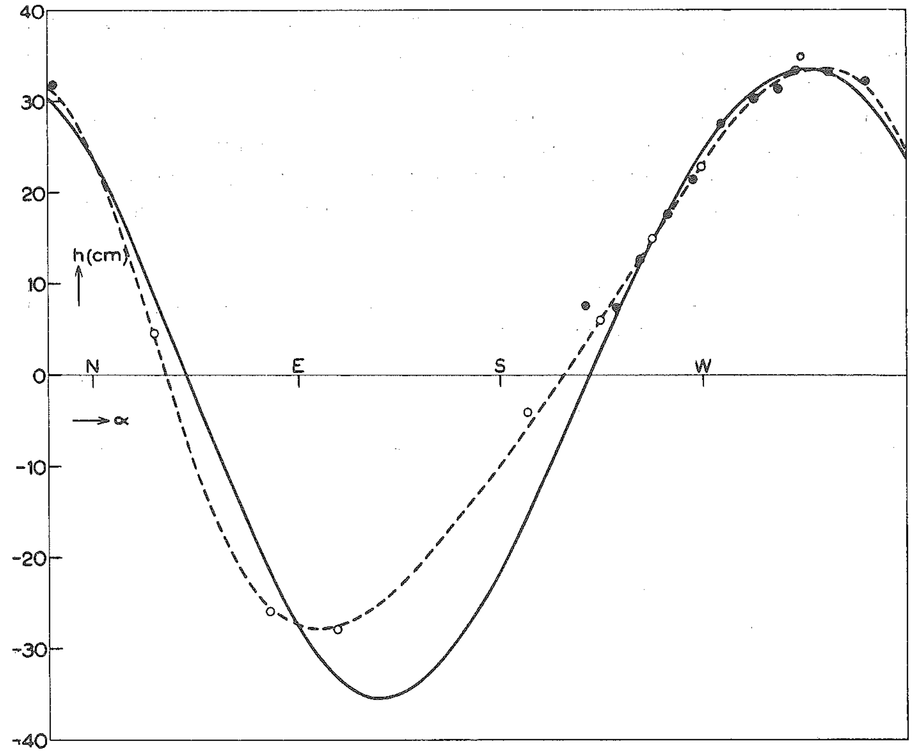

FIG. 4.1 a. Wind effect at Hook of Holland caused by a uniform wind $V_S = 10$ m/sec over area S, according to Schalkwijk (— — — —) and according to the present investigation (————). The full dots (●) and open circles (o) represent average wind effect values computed from actual gradient wind speeds as derived from weather maps, and computed from estimated winds respectively. The full dot closest to South is rejected in the present investigation.

3° the points used have not been computed from the actual gradient wind speeds as derived from weather maps, but from estimated winds;

4° the few storms may have had $T_a - T_s$ values that differed considerably from the climatological $T_a - T_s$ mean values for storms from the directions under consideration, for which the $T_a - T_s$ values used in this publication are a better estimate because they have been based on larger material; the sampling effect for small samples is relatively large. SCHALKWIJK did not publish the $T_a - T_s$ values for these storms, so that it cannot easily be checked whether this effect may account for the discrepancy.

Moreover, it may be brought forward against such a strong asymmetry as appears in the negative part of SCHALKWIJK's curve, that, if it were real, a similar asymmetry would probably also be observed in the Borkum curve, which is contrary to the facts, as will be shown in the next section. Our conclusion is, therefore, that (7) is in better agreement with the facts and with average conditions of stability, i.e. average values of $T_a - T_s$, than SCHALKWIJK's curve. For storms having a strongly deviating stability a correction has to be applied to the wind effects as computed by our formulas. Such a correction can be computed from the following formula, which has been derived from 3.2 (22), taking into account the proportionality between τ and h:

$$\frac{\delta h}{h} = -\frac{0.044}{1 - 0.044\,\overline{(T_a - T_s)_\alpha}}\{(T_a - T_s) - \overline{(T_a - T_s)_\alpha}\}, \tag{9}$$

where δh represents the correction to be applied to the wind effect if the stability corresponds to $T_a - T_s$ instead of $\overline{(T_a - T_s)_\alpha}$, the temperatures being expressed in degrees Celsius. This means roughly that the relative change of the wind effect caused by a change in $T_a - T_s$ of 1 degree Celsius amounts to about 5 per cent.

4.2.2 AREA N

In section 2.6.2 we have derived theoretically the wind effect formulas for a homogeneous wind field over the northern area, giving its contribution to the wind effect. There we have made use of one empirical datum only, viz. the direction of the wind that gives no effect at Hook of Holland. If we now apply the formula derived for Hook of Holland to the situation of a homogeneous wind over the northern area blowing in the longitudinal direction of the sea ($\alpha_N = 0$), we obtain theoretically:

$$h^N\,(\mathrm{H}) = 0.56\,\frac{\tau^N a}{\varrho g H_N}. \tag{10}$$

Using the values 5.75×10^7 cm, and 1.17×10^4 cm for a and H_N (see 2.2 (3)) and formula 3.2 (17) for τ^N with a wind speed of 10 m/sec, we obtain from (10) a wind effect of 11 cm, whereas SCHALKWIJK's formula under similar conditions yields 5 cm, which is less than half the theoretical value. There are several possible causes for this discrepancy:

1° the actual effective depth of the northern area is greater than 117 m, because of the deep trough off the Norwegian coast, which has not been taken into account in our model.

2° SCHALKWIJK applied a time lag of three hours for area N as well as for area S, whereas it is probable that for the northern area a much greater time lag should be used. Correlating a wind effect with the wind of a wrong point of time will give a mean wind effect which is too small as compared with the equilibrium wind effect.

3° SCHALKWIJK's formula for the wind effect due to a wind over the northern area is necessarily not very accurate because it has been determined as a small differential effect.

The first argument implies that the ratio a/H_N in the righthand member of (10) would actually be smaller than has been assumed here. From the foregoing it appears that if an effective value of a/H_N is used, that is 45 per cent of the value used before, formula (10) will agree with SCHALKWIJK's.

If we wish to take into account the second argument we should have to work up the whole empirical material introducing the hypothesis of a greater time lag from the very beginning.

CORKAN [3] has shown that the time lag will amount to about 9 hours for winds over the northern area with respect to the corresponding wind effects along the south coast of the North Sea. Moreover, because of the great time lag, it becomes more probable that the equilibrium state will not be reached in many cases. We shall not, however, study these problems of time lags and the equilibrium state, because SCHALKWIJK's method yields sufficiently reliable results as has been proved by many years of routine forecasting. Until it has been shown which of the first two possible causes of the discrepancy is the most important one, it seems better to use SCHALKWIJK's formula in stead of formula (10), even if applied with a time lag of say 9 hours. As to the third argument we remark that the accuracy could only be increased by working up a much larger observational material.

4.2.3 AREA C

In 2.5 (16) we have expressed the wind effect of area C at a place in the North Sea by means of the leak factor for that place and h^{**}, the wind effect on the Channel side of our imaginary dam across the Straits of Dover.

In principle h^{**} could be computed in a similar way as h^{*} has been computed, viz. by computing the static wind effect, the slope-current effect and the wind-vorticity-current effect caused by a wind over the Channel area. This procedure has not been followed however, firstly because these computations are rather lengthy, secondly because it is not to be expected that the results of these computations would agree with SCHALKWIJK's empirical formula, for reasons similar to those which lead to a discrepancy between the theoretical equilibrium formula and SCHALKWIJK's formula for area N, viz. the time lag used by SCHALKWIJK being too small and the equilibrium state not being reached. As long as no further empirical investigation has been made with respect to the time lag to be used for the Channel area it seems best to use SCHALKWIJK's formula for the computation of wind effects due to the wind over the Channel.

The theory developed in section 2.6.3 is then still valid as a basis for extrapolation of SCHALKWIJK's formula to other places along the Netherlands coast.

4.3 BORKUM

4.3.1 EMPIRICAL KNOWLEDGE

For Borkum we have two kinds of empirical data on the effects of the wind on the sea level. Unfortunately none of these is very suitable with regard to the verification of our theoretical formulas, which have been derived in section 2.6. Yet some of the empirical relations may be used for that purpose, as will be shown.

The first kind of empirical relation consists of a table compiled by TOMCZAK (not published), giving the relation between the mean wind over the German Bight as derived from lightship observations and the corresponding difference between the heights of the observed high water and the corresponding predicted astronomical high water. This height difference is denoted by O (Dutch: "opzet"). In general, it differs from the wind effect (which is the height difference between the observed and the astronomically predicted sea level) because the times of the observed high water and the astronomical high water will, in general, not coincide during storm surges. TOMCZAK has not correlated O at Borkum and the wind in the German Bight directly but via O at Cuxhaven and the wind over the German Bight and then correlated the difference in O between Borkum and Cuxhaven with the same wind. The latter relation has been published. Figure 4.3 a shows the empirical curve representing O at Borkum as dependent on the direction of the wind involved for a wind force of 5 Beaufort, according to TOMCZAK. By "wind involved" is meant the mean wind that blew over the German Bight 3 or 4 hours before the time of the observed high water at Borkum.

The second kind of empirical data consists of O values of 35 storms of the period 1947 to 1951 inclusive. For each of these storms the mean wind vectors of the areas N, S and C were taken from three-hourly weather maps. For each O value that weather map was taken as the corresponding one that was nearest in time to three hours before the time of the observed high water. The difference in time between the time of observed high water and the time of the weather map thus could vary from $1\frac{1}{2}$ to $4\frac{1}{2}$ hours. Those storms were selected which developed gradually, so that for these cases the wind effects may be supposed not to be seriously affected by inertial effects. Then it is permissible to compare the observed O data with the values computed by the equilibrium theory. Moreover, the effect of the scattering of the time lags used will then be small. From the weather maps the mean gradient wind vectors over the three areas S, N and C were derived. From the observed O values the effects of

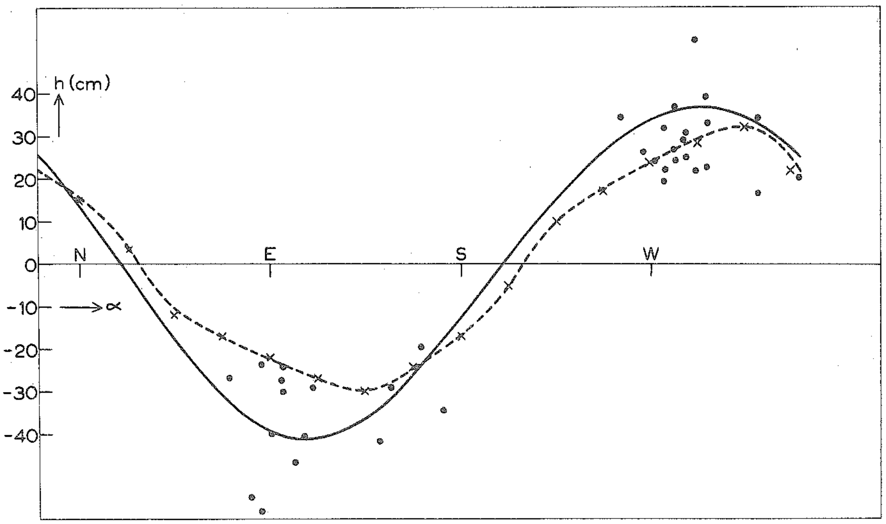

FIG. 4.3 a. Wind effect at Borkum caused by a uniform wind $V_S = 10$ m/sec over area S according to the present investigation (————) and "opzet" O at Borkum caused by a wind of 5 Beaufort over the German Bight according to Tomczak (— — — —). The dots (◉) represent O values of 35 storms of the period 1947 to 1951 inclusive, the crosses (×) mean O values according to Tomczak.

the winds over the northern area and the Channel area, computed by the formulas 2.6 (7f) and 2.6 (10f), respectively, as well as the effect of air pressure, computed by SCHALKWIJK's formula (pag. 73), were subtracted, in order to obtain that part of O that can be ascribed to the wind over the southern area. The value thus obtained was finally reduced to a wind speed $V_s = 10$ m/sec. The reduced values have, in figure 4.3 a, been plotted against the direction of the wind.

The points thus obtained show a rather large scatter, which may have been caused by:

1° the possibility that O still contains a contribution of the astronomical tide;

2° the time lag used not being constant;

3° the fact that the wind fields over the North Sea were insufficiently known;

4° occasional non-stationary effects not having been eliminated.

Nothwithstanding the large scatter it can be seen from figure 4.3 a that on an average our points agree with TOMCZAK's curve, at least for positive wind effects.

For negative wind effects, however, our points tend to indicate larger values than does TOMCZAK's graph. A possible explanation for this discrepancy could be that these points represent O-values reduced to a wind speed $V_s = 10$ m/sec, whereas TOMCZAK's graph refers to a *wind force* of 5 Beaufort, corresponding to a wind speed of 10 m/sec (at 10—15 m height). It is however a well-known fact that the surface wind and, hence, the BEAUFORT estimate increases with decreasing stability if the gradient wind remains the same. Hence, in a very unstable atmosphere the same Beaufort estimate corresponds to a smaller gradient wind, and hence a smaller value of V_s, than in a less unstable atmosphere.

Since we do not know what has been the mean value of T_a—T_s in the situations with negative O used by TOMCZAK, we cannot correct for this effect, the less so since TOMCZAK has used the winds over the German Bight, where in general T_a—T_s will deviate from the mean of T_a—T_s over the whole southern area. Finally the mean winds over the whole southern area of the North Sea may differ from the mean winds over the German Bight, to which TOMCZAK's data refer.

4.3.2 COMPARISON BETWEEN THEORY AND OBSERVATIONS

In section 2.6.4 we derived the following formula for the relation between the wind over the southern part of the North Sea and the corresponding wind effect at Borkum (2.6 (11 f)):

$$h^S\,(\mathrm{B}) = (0.42 \cos \alpha_S - 0.51 \sin \alpha_S)\, f^*\,(\alpha_S)\, V^2_{sS} =$$

$$= 0.66 \cos (\alpha_S + 51°) .\, f^*\,(\alpha_S)\, V^2_{sS}. \qquad (1)$$

Figure 4.3 a shows the curve representing the wind effect according to this formula for a wind speed of 10 m/sec. The agreement between this curve and the empirical points is rather good, especially for negative wind effects; for positive wind effects the observations appear to indicate somewhat lower values i.e. smaller deviations from the undisturbed sea level.

This may, however, be explained by the following facts:

1° O values are compared with wind effects, these two being essentially different quantities. Since the wind fields used are related to the times of *observed* high water instead of astronomical high water, it is, in general, to be expected that for positive wind effects, the wind effects are larger than the corresponding O values, as becomes evident from figure 4.3 b, whereas for negative wind effects the O values will be larger, in general (see fig. 4.3 c).

2° the empirical data have been derived from observations of the sea level on the southern beach of Borkum. Hence it may be expected that for northerly winds the sea level on the south coast will be lower than on the north coast, because of a local wind influence. Similarly southerly winds will cause a higher wind effect on the south coast than on the north coast.

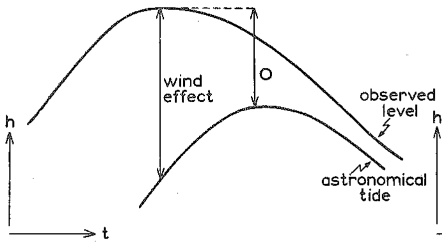

FIG. 4.3 b. Example of graphs of observed sea level and astronomical tide level near high water in the case of positive wind effects, showing that then the wind effect at the time of observed high water is *larger* than the "opzet" O.

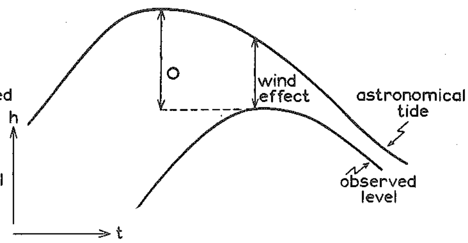

FIG. 4.3 c. Example of graphs of observed sea level and astronomical tide level near high water in the case of negative wind effects, showing that then the wind effect at the time of observed high water is *smaller* than the "opzet" O.

1° and 2° give deviations in the same sense for positive wind effects and in the opposite sense for negative wind effects; this may be an explanation of the facts that the positive part of the theoretical curve yields somewhat higher values of wind effects than the corresponding O values, and that for the negative part O and wind effect values agree quite well with each other.

The directions of zero wind effect and zero O agree quite well.

4.4 CONCLUSION

On the whole it may be said that the theoretical wind effect formulas for Borkum as well as for Hook of Holland, have been confirmed empirically at least for winds over the southern area. This result is the more valuable since there were only two unknown parameters available, viz. L (H) and f (0). f (0) appeared to agree with the value derived from HUNT's [27] and JOHNSON's [72] investigations.

Hence, the confirmation of the theory implies also a confirmation of the value used for the third parameter, viz. the bottom friction coefficient.

This is very valuable since the value of this parameter which has been used in the computations of the current effects was the weakest link in the chain of our arguments, based as it was on two values, which differed by a relatively large amount (section 3.3), obtained by different methods. This is the more so, because it can be shown that the current effects depend rather strongly on the value of the bottom stress parameter: in first approximation the current effect changes proportional to a change in r.

4.5 APPLICATION OF THE THEORY TO THE STORM SURGE OF 1 FEBRUARY 1953

Up to the present we have not verified the wind effect formulas for the areas I, II and III. For such a verification strongly inhomogeneous wind fields are required. Because of the very high correlation between the mean wind vectors of these areas many data have to be used in order to secure a significant conclusion as to the validity of the theoretical formulas. As an illustration the formulas for these sub-areas will be applied to the storm surge of February 1st. 1953 for Hook of Holland and Borkum. This storm surge is especially suitable for the purpose because of the inhomogeneity of the wind field, as is evident from tables 4.4 α and 4.4 β where the wind speeds (in m/sec) as well as the wind directions are given for January 31st. 21 G.M.T., February 1st. 0 G.M.T. and 3 G.M.T. Also the corresponding theoretical equilibrium wind effects are given (in decimeters).

These wind effects have been computed from formulas 2.6 (12 c), 2.6 (12 f), 2.6 (13 c), 2.6 (13 f), 2.6 (14 c) and 2.6 (14 f). A correction has been applied in order to allow for an inertial effect, viz. the socalled overshooting effect, for which SCHALKWIJK (l.c. pag. 65) has found the following empirical rule: the overshooting effect is equal to 2.2 times the maximum preceding increase per hour of the wind effect.

Comparing the computed effects with the observed effects we see that they agree quite well, especially if we take into account the inaccuracy of the wind data over the North Sea, and bear in mind that an error of 1 m/sec in a wind speed of 30 m/sec causes an error in the computed wind effect of about 7 per cent.

TABLE 4.4 α. (Hook of Holland)

Date	Time (G.M.T.)	V_{sI}	α_I	V_{sII}	α_{II}	V_{sIII}	α_{III}	h_I	h_{II}	h_{III}	h_N	h_C	h_{eq}	δh	h_{comp}	h_{obs}
31–1–'53	2100	27	345	23.5	325	0	—	15	7	0	3	1	26	—	—	—
1–2–'53	0000	30	350	25.5	335	0	—	18	8	0	2	0	28	5	33	31
1–2–'53	0300	26	345	24	320	19	345	14	8	1	2	0	25	—	—	—

TABLE 4.4 β (Borkum)

Date	Time (G.M.T.)	V_{sI}	α_I	V_{sII}	α_{II}	V_{sIII}	α_{III}	h_I	h_{II}	h_{III}	h_N	h_C	h_{eq}	δh	h_{comp}	h_{obs}
31–1–'53	2100	27	345	23.5	325	0	—	10	0	0	4	0	14	—	—	—
1–2–'53	0000	30	350	25.5	335	0	—	12	– 1	0	3	0	14	—	—	—
1–2–'53	0300	26	345	24	320	19	345	9	0	8	3	0	20	5	25	24

V_{sI}, V_{sII} and V_{sIII} in meters per second.

α_I, α_{II} and α_{III} in degrees ($\alpha = 0$ represents the longitudinal direction of the North Sea).

All heights are expressed in decimeters.

h_{eq} is the equilibrium wind effect, h_{comp} the computed wind effect and h_{obs} the observed wind effect; δh means "overshooting effect".

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LIST OF SYMBOLS

Symbol		Description	dimension
x, y, z	=	orthogonal cartesian coordinates, z positive upwards	cm
$\vec{v}$	=	vector of velocity	cm/sec
v_x, v_y, v_z	=	x, y and z components of $\vec{v}$	cm/sec
$\vec{S}$	=	volume transport vector	cm^2/sec
S_x, S_y	=	x and y components of $\vec{S}$	cm^2/sec
V_{grad}	=	gradient wind velocity	cm/sec
V_s	=	3/4 of gradient wind velocity	cm/sec
t	=	time coordinate	sec
ϱ	=	density of sea water, taken constant = 1.03	gr/cm^3
p	=	pressure	$gr/cm\ sec^2$
c	=	Coriolis parameter = $2\ \omega \sin \varphi$	1/sec
ω	=	angular velocity of the earth (in radians per second)	1/sec
φ	=	geographical latitude	dimensionless
ν	=	kinematic coefficient of eddy viscosity	cm^2/sec
g	=	acceleration of gravity	cm/sec^2
H	=	depth with respect to mean sea level	cm
$\bar{H}$	=	harmonic mean of H	cm
H_S, H_N, H_C	=	harmonic mean depth of the southern and the northern area of the North Sea model and of the Channel, respectively	cm
h	=	height of the sea surface above mean sea level, disturbance height, or wind effect	cm
h_τ	=	static wind effect	cm
h_1, h_2, h_3	=	wind-vorticity-current effect, bottom-slope-current effect and leak-current effect, respectively	cm
h^*, h^{**}	=	wind effects that would prevail on the North Sea side and the Channel side of an imaginary closing dam across the Straits of Dover	cm
O	=	"Opzet", height difference between the observed high water and the corresponding predicted astronomical high water	cm

		dimension
$\vec{\tau}_s$	= true wind stress	gr/cm sec^2
$\vec{\tau}_r$	= true bottom stress	gr/cm sec^2
$\vec{\tau} = (1+n)\,\vec{\tau}_s$	= apparent wind stress	gr/cm sec^2
τ_x, τ_y	= x and y components of $\vec{\tau}$, respectively	gr/cm sec^2
r	= bottom friction parameter	cm/sec
n	= factor by which $\vec{\tau}_s$ must be multiplied in order to obtain that part of $\vec{\tau}_r$ which depends directly on $\vec{\tau}_s$	dimensionless
Φ	= stream function of volume transport	cm^3/sec
Φ_1, Φ_2, Φ_3	= stream function of wind-vorticity current, bottom-slope current and leak current	cm^3/sec
Φ_c	= value of the stream function at the continental North Sea coast	cm^3/sec
α	= angle between longitudinal axis of the North Sea and wind direction	dimensionless
a	= width of the modelled North Sea	cm
K	= leak capacity	cm^2/sec
L (P)	= leak factor for a place P	dimensionless
T_a	= air temperature at 10 meters above sea level	degree
T_s	= sea surface temperature	degree
∇	= gradient operator = $\vec{i}\,\partial/\partial x + \vec{j}\,\partial/\partial y$	1/cm
$\triangle$	= Laplace operator	1/cm^2

SUMMARY

This publication deals with the theoretical development of a method of computing the effect of the wind on the sea levels in a shallow partly-enclosed sea. This method may be called an equilibrium method, since its basic idea is that the main part of the wind effect is what may be called the equilibrium effect (i.e. the effect that would be present at a certain moment if the wind prevailing at that moment had been stationary all the time) and that the actual wind effect may be derived from the equilibrium effect by applying certain corrections taking into account the effects of accelerations.

The basis of this method has been laid by W. F. SCHALKWIJK, who constructed graphs for finding the equilibrium effect at Hook of Holland from the pressure field over the North Sea and the Channel. In the present study this method is extended in two respects. Firstly, it is extended to other places of the Netherlands coast. Secondly, it is developed in such a way that inhomogeneities of the wind field over the southern half of the North Sea are now taken into account, this field being divided into three subfields, whereas in the original SCHALKWIJK method it was treated as a whole.

The dependence of the equilibrium effect, or equilibrium height, on the wind field is found as follows. In a shallow sea with sufficiently strong tidal currents the equations of motion may be integrated vertically and be linearized with a fair degree of approximation. In the equilibrium state the slope of the sea surface is then determined by the wind stress on the water and by the volume transport of the current, which gives a Coriolis force and a bottom friction stress (the latter being supposed to depend linearly on the volume transport of the current). Using as a boundary condition that at the northern boundary of the North Sea the equilibrium effect is kept zero by the ocean, one can then compute the height at an arbitrary place as the sum of (1) a part depending on the wind stress field only, and independent of the current field, and (2) a part depending on the current field. These two parts may be called the static effect and the current effect respectively.

The current transport can be computed from the wind stress field by means of the equation obtained from the two-dimensional vector equation of motion (mentioned above) by taking the curl of both its members, combined with the proper boundary conditions. For the latter it makes a great difference whether the sea is closed on one side or has an opening acting as a "leak" in case of a storm surge. It may easily be seen that the total current transport may be regarded as composed of three components: a wind-vorticity current (which vanishes identically with the vorticity of the wind stress), a bottom-slope current (which vanishes identically with the bottom slope) and a "leak" current. Correspondingly, the current effect may also be understood as composed of a "wind-vorticity effect", a "bottom-slope effect" and a "leak effect". In

applying this theory to the North Sea, the wind field acting upon the water may, for practical computational purposes, be schematized into a pattern of sub-fields, few in number, each of which has a uniform wind. Since the equations involved are linear, the wind effect at a particular place may be considered as being built up from the contributions of the sub-fields, taken separately.

This mathematical model has three parameters that have to be determined empirically, viz. (a) the proportionality factor linking the bottom friction stress to the current transport, (b) a "leak parameter" linking the "leak effect" to the sum of the other effects, and (c) the "drag coefficient" linking the wind stress to the square of the gradient wind velocity, this parameter being supposed to be a climatological function of wind-direction.

If, then, a quantitative relation between the wind over the sea under consideration and the wind effect at one particular place has been determined empirically, the above parameters may be computed by means of the method of least squares and formulas for the equilibrium wind effect at any other place of the coast may be computed. This has been done for various places of the continental coast of the southern North Sea in a slightly different way (in order to avoid computational difficulties), the empirical basis being given mainly by SCHALKWIJK's data for Hook of Holland.

Van de reeks MEDEDELINGEN EN VERHANDELINGEN *zijn bij het Staatsdrukkerij- en Uitgeverijbedrijf nog verkrijgbaar de volgende nummers:*

23, 25, 26, 27, 29*b*, 30, 31, 33, 34*b*, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48.

alsmede:

49. A. Labrijn. Het klimaat van Nederland gedurende de laatste twee en een halve eeuw. — The climate of the Netherlands during the last two and a half centuries. 1945. (114 blz. met 6 fig. en 1 kaart) . . . . f 1,15

50. J. P. M. Woudenberg. Het verband tussen het weer en de opbrengst van wintertarwe in Nederland. — The correlation between weather and yield of wheat in the Netherlands. 1946. (43 blz. met 6 fig.) 0,70

51. S. W. Visser. Weersverwachtingen op langen termijn in Nederland. — Long range weather forecasts in the Netherlands. 1946. (143 blz. met 25 fig.) 2,05

52. R. J. v. d. Linde en J. P. M. Woudenberg. Een methode ter bepaling van de breedte van een schaduw in verband met den tijd van een jaar en de oriëntatie van het beschaduwde object. — A method for determining the daily variation in width of a shadow in connection with the time of the year and the orientation of the overshadowing object. 1946. (6 blz. met 2 fig, en 2 kaarten) 0,40

53. A. Labrijn. Het klimaat van Nederland. Temperatuur, neerslag en wind. — The climate of the Netherlands. Temperature, precipitations and wind. 1946. (71 blz. met 1 kaart) 2,50

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