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L. OTTO

RESULTS OF CURRENT OBSERVATIONS AT THE NETHERLANDS LIGHTVESSELS OVER THE PERIOD 1910–1939

PART I

TIDAL ANALYSIS AND THE MEAN RESIDUAL CURRENTS

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CONTENTS

LIST OF SYMBOLS

A amplitude of a constituent tide, indicated by a subscript.
a half the long axis of the tidal current ellipse, or maximum value of the tidal current.
b half the short axis of the tidal current ellipse, or value of the tidal current during the turnings of the tide.
E argument at 1 January at 00. G.M.T. for the constituent tide, indicated by a subscript.
f node factor.
g modified phase or epoch of a constituent tide, indicated by a subscript.
I obliquity of the lunar orbit with respect to the earth's equator.
m difference between the equilibrium arguments of the M_2 and S_2 tides, in groups of 36°, called: double lunar phase.
n equilibrium argument of the M_2 tide, in groups of 18° or 36°, called: tide phase.
p number of the month (January=0).
R residual current.
t time.
U time of the day, expressed in groups of 4 hours, taking together the equal hours ante and post meridiem.
u part of the argument of a constituent tide (indicated by a subscript), depending upon variations in the obliquity of the lunar orbit.
V sea current; with a subscript: that part of the current that is caused by the indicated constituent tide.
W number of the watch (period of 4 hours).
χ direction of the maximum tidal current.
χ_R direction of the residual current.
κ kappa number, phase lag or epoch of a constituent tide, indicated by a subscript.
φ equilibrium argument of the M_2 tide.
ψ equilibrium argument of the MS_f tide.
ω angular velocity of a constituent tide, indicated by a subscript.

1 INTRODUCTION

1.1 General remarks on tides and currents

The most important contribution to the currents at the Netherlands light-vessels in the North Sea is by the tidal currents. These currents arise as a result of the tidal motion and, like the vertical tide, can be regarded as consisting of a number of water movements which vary as cosine functions with different periods and which are superimposed one upon the other. Apart from these periodical currents, there is also present in the total current a non-periodical component, the residual current, which is caused, directly or indirectly, by the wind field over the North Sea and the Atlantic Ocean and by density differences in the sea water.
The various periodical tidal motions, the so-called constituent or component tides, which together constitute the total tide, can be of three types:
1. Astronomical tides. These are the constituent tides that arise under the influence of the tidal forces of the sun and the moon. The periods of these constituent tides can be determined on the basis of astronomical calculations.
2. Shallow water tides. These are constituent tides that arise by the deformation of the tide in shallow water.
3. Meteorological tides. These can arise where there is a periodical variation of meteorological factors, particularly wind, atmospheric pressure and sea water temperature. The periods of these tides are linked with the length of the day or of the year.
The problem of deriving the amplitudes of the various constituent tides purely theoretically has not yet been solved. The amplitude of the tide depends not only on the variations of the tidal forces, but also on the dimensions of the oceans and of the sea area under consideration.
The various constituent tides are indicated by a symbol. The tides dealt with in this study, apart from those of par. 4.8, are the following:

Symbol	Angular speed	Period	Character
MS_f	1.0159° hr^{-1}	$14\frac{3}{4}$ day	shallow-water tide
μ_2 (or $2MS_2$)	27.9682°	12h.52 min.	semi-diurnal lunar correction tide
M_2	28.9841°	12h.12 min.	semi-diurnal lunar tide
S_2	30.0000°	12h.	semi-diurnal solar tide
M_4	57.9682°	6h.12 min.	shallow-water tides
MS_4	58.9841°	6h.06 min.	shallow-water tides
S_4	60.0000°	6h.	shallow-water tides
M_6	86.9523°	4h.08 min.	shallow-water tides
$2MS_6$	87.9682°	4h.06 min.	shallow-water tides
S_6	90.0000°	4h.	shallow-water tides

There are various methods of calculating from the observations for a constituent tide the amplitude and the phase lag in relation to a certain initial moment.
In the case of the vertical tide and of purely alternating currents we can analyse the observational data directly. The current, however, usually changes direction during a tidal period. The current then has to be resolved into two components (for instance the N–S and E–W directions), after which the analysis can be carried out for each of the two components. If subsequently the contributions of a constituent tide to both directions are combined, we find that the current-vector describes an ellipse. In this way for each constituent tide a current ellipse can be found.
The variation with time as a result of a constituent tide, for instance the M_2 tide, of the sea level or the current component in a certain direction is generally written in the form:

$$V_{M_2} \quad \text{or} \quad H_{M_2} = A_{M_2} \cos\left(\omega_{M_2} t - \kappa_{M_2}\right) \; ;$$

in which ω is the angular frequency of the tide (in this case $\omega_{M_2} = 28.9841°$ per hour) and κ is the so-called phase lag or epoch of the tide with respect to the moment at which the tide potential reaches its maximum value at the point of observation.
These moments can be calculated for the different constituent tides with the aid of astronomical and nautical tables.
Often, instead of κ the so-called "modified epoch" g is used; this is the phase with respect to the moment at which the tide potential reaches its maximum value for another meridian. The time is then also reckoned from that moment. In the present study we shall use for the time G.M.T. (Greenwich Mean Time) and for g the value with respect to the meridian of Greenwich.
The amplitude A_{M_2} for a particular year can be found from the mean amplitude, a value given in the tide tables, by multiplication by the so-called node factor f (see par. 4.7). This node factor is different for different years and is very close to 1.
Tides and tidal currents in general will not be dealt with any further here. The reader is referred to the relevant chapters of the various handbooks on oceanography, to the works of Marmer (1926), Thorade (1941), and for a more profound study to that of Doodson and Warburg (1941). For Dutch readers the book *Overzicht der Getijleer* (1949), published by the Ministry of the Navy, is recommended.

1.2 Previous investigations in the North Sea

The first measurements of tidal currents in the North Sea date from the end of the last century. These first studies of Stessels (1867), Bernelot Moens and

Tutein Nolthenius (1883), van Heerdt (1890), Petit (1892, 1894) and Phaff (1899) give the results of the measurements as means per tide phase, sometimes subdivided in spring-tides, neap-tides and the intermediate periods. Van der Stok (1905) was the first to apply a harmonic analysis to such observations. Other results of harmonic analyses of tidal current observations have been published in the issues of the *Bulletin Hydrographique* for the years 1910–1911, 1912–1913 and 1913–1914, and also by Smith (1910), Wind c.s. (1912), Doodson (1930) and Mandelbaum (1934).

The residual current, i.e. the mean current that remains after elimination of the various constituent tides, has usually also been calculated as a result of such analyses. In addition to the above-mentioned studies, other investigations have also provided a considerable amount of information about the residual current. Although the residual current over nearly the whole of the North Sea is slight as compared with the tidal current, the knowledge of it is very important for the study of the circulation of the North Sea.

1.3 Objects of the present investigation

Although on the one hand the currents of the M_2 tide in the North Sea are fairly well known, as evidenced, for example, by the often very detailed current atlases that have been published of this area by different institutes, and on the other hand a great deal of data is available on the residual current, there is still a need for more complete analyses like the present one.

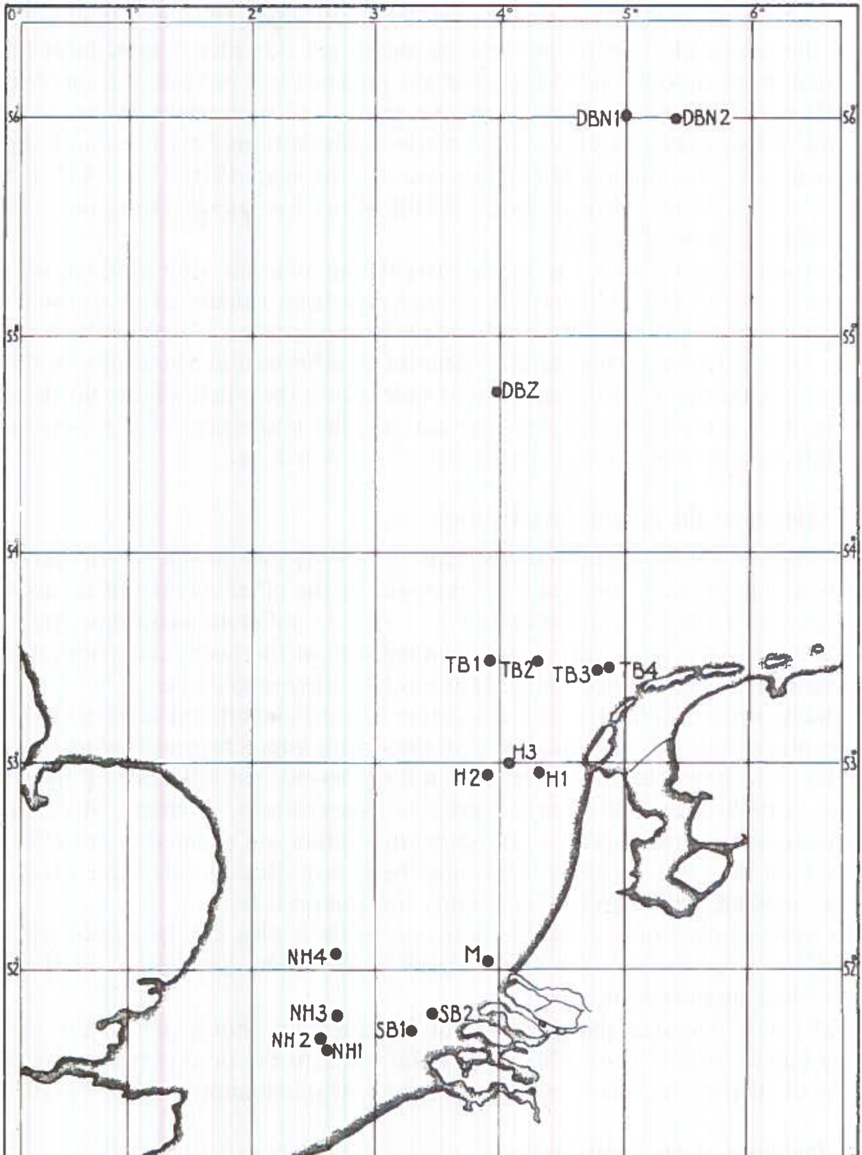
In the first place, an analysis to determine various constituent tides, including tides other than the M_2 tide, can give important supplementary information. As the data have been collected over a long period, the influence of disturbances, which often bias shorter series of observations, is small. Moreover, measurements were elaborated of points that either are situated in areas from which till now not so many data have been published, or that are situated close together, which gives a possibility for comparison.

The same applies to the residual current, while it also can be important to ascertain whether there are any changes in the mean circulation, compared with older observations.

Finally it is intended that the present investigation should serve as a basis for a further study of the influences of the wind on the tidal currents and the residual current, such as those carried out by Mandelbaum (1934, 1955, 1957).

1.4 Positions of the lightvessels

During the period 1910–1939 the lightvessels did not always occupy the same positions. It seemed best as a rule to distinguish between these different positions, and only by way of exception to combine observations made at points more than 1 nautical mile apart.


0°
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DBZ
TB1
TB2
TB3
TB4
H3
H2
H1
M
NH4
NH3
NH2
NH1
SB1
SB2

This makes it possible to discover local differences in the currents. On the other hand, if such a distinction is not made, it might happen that apparent variations of the current in the course of time would be found, that are in reality caused by local differences.

In the following two cases observations at positions more than 1 nautical mile apart have been combined.

Terschellinger Bank: 3–8–1917 till 3–1–1918, position 53°29′N, 4°02′E
3–1–1918 till 27–11–1918, position 53°29′N, 3°52′E

Noord Hinder: 1–1–1931 till 1–1–1937, position 51°38′N, 2°34′E
1–1–1937 till 3–9–1939, position 51°39′N, 2°34′E

In the other cases the distance between the positions of which the observations have been combined is less than 1 nautical mile, while moreover in none of these cases the local situation was such that it was expected that the currents at the combined positions would differ considerably.

The different groups of observations that are analysed are presented in Table I.

TABLE I

Lightvessel	Group	Period	(mean) position	Greatest distance apart in nautical miles
Noord Hinder	1	1–1–1910 till 28–10–1914 19–1–1920 till 1–1–1931	51°35.5′N 2°36.5′E	0.1
	2	1–1–1931 till 3–9–1939	51°38.7′N 2°34′E	1.0
	3	28–10–1914 till 17–6–1916	51°47′N 2°41′E	0
	4	17–6–1916 till 13–12–1917	52°05.2′N 2°39.9′E	0
Schouwen Bank	1	1–1–1915 till 16–2–1917	51°41.8′N 3°17.4′E	0
	2	1–1–1910 till 1–1–1915 1–9–1921 till 25–6–1934	51°47.1′N 3°27.4′E	0.3
Maas	1	1–1–1911 till 30–5–1917 28–1–1919 till 3–9–1939	52°01.8′N 3°53.7′E	0.6
Haaks	1	1–1–1910 till 6–8–1914 28–10–1919 till 3–9–1939	52°57.5′N 4°18.8′E	0.9
	2	2–1–1918 till 28–10–1919	52°57.2′N 3°55′E	0.5
	3	6–8–1917 till 2–1–1918	53°00′N 4°05′E	0

TABLE I (continued)

Lightvessel	Group	Period		(mean) position	Greatest distance apart in nautical miles
Terschellinger Bank	1	3–8–1917	till 27–11–1918	53° 29′ N 3° 55′ E	6.0
	2	9–2–1917	till 3–8–1917	53° 29′ N 4° 17′ E	0
	3	7–8–1929	till 3–9–1939	53° 27′ N 4° 46.9′ E	0.3
	4	1–1–1910 1–1–1916 18–8–1921	till 6–8–1914 till 9–2–1917 till 7–8–1929	53° 27′ N 4° 51.9′ E	0.4
Doggersbank Noord	1	2–8–1917	till 22–7–1921	56° 00′ N 5° 00′ E	0
	2	12–3–1917	till 2–8–1917	55° 59.5′ N 5° 24.1′ E	0
Doggersbank Zuid	1	14–6–1918	till 26–7–1921	54° 45′ N 3° 58.5′ E	0

The "mean position" is the time-average of the various positions of which the observations have been combined. The last column of the table gives the greatest distance in nautical miles between the combined positions. In fig. 1 the various positions listed in Table I are shown.

During these periods it sometimes happened that a lightvessel was absent from its station for a short time, for instance owing to repairs.

2 ACCURACY

2.1 Method of observation

The current was usually measured by means of a hand-log. This consists of a small piece of wood known as a chip, that floats on the surface, and is so weighted that it stands vertically in the water. This chip is paid out on a line provided with markers at fixed distances. Before measuring the current one waits till the log, after having been paid out, has drifted away some distance from the ship, in order to avoid the influence of the ship on the current. The measuring consists in determining the distance which the log drifts during a fixed time interval, by counting the markers of the line that pass by in that interval. In the present case the time interval was determined with the aid of a log glass. The direction of the current was determined by taking the bearing with the ship's compass.
Both the log line as well as the log glass were regularly checked. It was also possible to determine the current with sufficient accuracy at night, as the sea around a lightvessel is then fairly well illuminated.
In some cases it was not possible to measure the current in this way. This could happen when the current was slight (during the turning of the tide) and especially when, under influence of the wind, the ship was not yet heading the current and consequently the current was running underneath the ship from abeam or astern. In such cases the current was estimated, either by measuring the time that a wooden block took to drift a certain distance (between two markers on board the ship) or by determining the angle at which a suspended weighted line stood in the water.
Although this last method in particular is rather inaccurate, it is unlikely that this has had much influence on the results. This method was used only at low current velocities (less than $\frac{1}{2}$ knot or 25 cm/sec). The interval in which the currents were noted down was $\frac{1}{8}$ knot (or about 6 cm/sec). It seems reasonable to assume that this accuracy could be attained at these low velocities by experienced observers.

2.2 Inacurracy introduced by rounding off

Apart from the above mentioned possible source of errors, another cause of inaccuracy lies in the way in which the currents are read off and noted down.
The current was measured to an accuracy of $\frac{1}{8}$ knot and the direction was determined in double points. Under these circumstances a maximum error of $\pm\frac{1}{16}$ knot (about ± 3 cm/sec) in the velocity and of $\pm 11\frac{1}{4}^{\circ}$ in the direction can be expected.

It is evident, however, from the distribution of the observations over the different velocities, that the observers had a preference for rounding off to an even number of times $\frac{1}{8}$ knot. Thus the rounding-off inaccuracy for the current velocities per observation can be greater than $\frac{1}{16}$ knot. The distribution of the observations over the velocity-intervals shows that in about 60% of the cases in which the velocity should have been rounded off to an odd number of times $\frac{1}{8}$ knot the current has been incorrectly rounded off to an even number of times $\frac{1}{8}$ knot. Therefore the rounding off inaccuracy per individual observation is estimated to be at the most $\pm 1.6 \times \frac{1}{16}$ knot $= \pm \frac{1}{10}$ knot or ± 5 cm/sec. No such preference in the rounding-off of the measurement for the current directions is apparent from the distribution of the observations over the directions.

In the calculation of mean values from a large number of observations the accuracy of the result will increase with the number of observations, if the error is not systematic. The mean error in a calculation over n observations is assumed to be $1/\sqrt{n}$ times the error of the individual observations.

The observations were as a rule made every four hours, so that there were six observations a day. Disregarding interruptions in the observations (because of bad weather for instance) we have therefore about 180 observations per month.

For one month of observations the mean rounding-off error in the final result will therefore be about:

± 0.4 cm/sec for the current strength,

$\pm 50'$ for the current direction.

For a number of q months the errors will be $1/\sqrt{q}$ times these values.

2.3 Rounding-off error in the calculation of north and east components

In the calculations no use was made of the data as recorded directly, only the north and east components of the currents entered into the computation. The values of these components, as used for the calculation with punched cards, have, however, now been rounded off again to whole numbers of $\frac{1}{8}$ knot. This means therefore that each individual current observation has an extra rounding-off error in the north and east components. When the observations in a group to be averaged show a small spreading over the different current strengths and directions the mean of the north and east components of that group can have an error which in certain cases may approach $0.5 \times \frac{1}{8}$ knot.

It had therefore to be investigated whether the spreading of the individual observations in a group (with a certain tide phase and in a certain month) over the different current strengths and directions was such as to make the error in the group average small enough.

This was investigated in the case of the lightvessels Noord Hinder (almost

alternating current) and Terschellinger Bank (clearly rotary tidal current) for the tide phases in which the tidal current attains a maximum and for those in which the tide is turning. These investigations were carried out for the months of March and May (small steadiness of the wind with high and low mean wind force respectively) and July and November (great steadiness of the wind with low and high mean wind force respectively).
This investigation showed that the mean error that can be expected from this resolution into north and east components is at most $\pm 0.11 \times \frac{1}{8}$ knot, i.e. at most ± 0.7 cm/sec per group. In a calculation over 20 groups (tide phases) of the constituent tides and the residual current, the final error may be assumed to be $1/\sqrt{20} \times 0.7$ cm/sec, or $\pm$ 0.15 cm/sec because the rounding-off errors per group are not correlated. In a division into 10 groups (tide phases) the spreading per group will be greater, and thus the error per group will be smaller than for a division into 20 groups. This effect will compensate for the smaller number of groups.

2.4 Total error introduced by both rounding-off procedures

The accuracy of the results of the calculations can be estimated from the data given in pars. 2.2 and 2.3. Since the smallest number of months over which calculations have been made is three, the error discussed in par. 2.2 can be put at ± 0.2 cm/sec in the current strength. The error discussed in par. 2.3 is taken to be ± 0.15 cm/sec, so that the final error can be assumed to be less or equal to ± 0.35 cm/sec.
This total error does not include the errors of the measuring method as discussed in par. 2.1. In so far as they are systematic it is not possible to determine them afterwards. One can at most obtain a general idea of their possible magnitude from reports on the measuring procedure followed.
The non-systematic errors are considered to be small compared with the rounding-off interval.
The accuracy of the calculated average current direction and of the times of maximum current strength is dependent on the relative error of the calculated components and thus varies from case to case. In the results the current strengths will be given to an accuracy of 0.1 cm/sec, and the directions and phase angles in degrees. Comparison of the results with each other and with the results of other investigations shows that in general it is useful to present the results with this degree of accuracy.

3 DISTRIBUTION OF THE OBSERVATIONS OVER THE DIFFERENT WIND AND CURRENT CONDITIONS

3.1 Distribution over different wind conditions

A point that deserves further study, before elaboration of the data can be started, is the question whether the observations are evenly distributed over the various possible conditions of wind and tide-generating forces. If this is not the case, this must be borne in mind in the discussion of the results.
First of all we shall consider the distribution of the observations over the different wind conditions. The first factor that can be of influence is a forced discontinuation of the measurements in heavy seas. In the observational material there are relatively few cases where observations have been made at a Beaufort number of more than 5. The results of the elaboration will thus be only representative for the lower wind forces. It is expected that the results of the tide-analysis will not differ much from what would be applicable for an average over all wind conditions (although a certain influence of the wind is probable, in view of the studies by Mandelbaum (1934, 1957)), but on the results of the calculation of the mean residual current this will certainly have an influence.
We can further ascertain the mean wind force above which the observations were discontinued. We know how many observations could in principle have been made, and how many have been made in reality. From this follows the percentage of watches during which no measurements could be carried out, and from this and the wind frequencies as given by Verploegh (1958) the average wind force can be found that corresponds as limit with this percentage.
The results are given in Table II.

TABLE II

	Number of watches	Number of current observations	Percentage of watches with current observations	Mean limit for the wind force
Noord Hinder	55171	44137	80.0	4.6 Bft.
Schouwen Bank	38162	33852	88.7	5.0
Maas	59102	50221	85.0	4.4
Haaks	52961	43102	81.4	4.4
Terschellinger Bank	51530	41302	80.2	4.2

However, it is not only important that at high wind forces the observations were discontinued. but it is also evident that this was more likely to happen during the turnings of the tide than during the times of maximum current. It is often more difficult to make a reliable measurement during the turning of the tide than during full current, as already stated in par. 2.1. In the event of high wind forces it is also much more difficult to estimate the current in the manner set out in that paragraph. It is evident that, at a wind force of Bft 5 and higher during the turning of the tide 5% to 27% fewer observations were made than at maximum current, depending on the lightship. The result is that the means of the current measurements made during the turnings of the tide will differ more than those for the other tide phases from the true mean for all wind conditions.

If we take into account not only the wind force but also the wind direction, then it is to be expected that the fetch of the wind has an influence (as it is mainly the state of the sea that hampers the observations), so that observations can, in general, be made with higher wind speeds in the case of offshore winds than when the wind blows all the way over the sea. This has indeed been found to be so, particularly in the case of the lightvessels Terschellinger Bank and Haaks. Here relatively few observations at windforce 5 Bft and higher have been made for wind directions between west and northwest and a relatively large number for wind directions between southeast and east, compared with the frequency distributions of the wind as given by Verploegh (1958). At the other lightvessels this effect is less pronounced, possibly as a result of the more complicated local conditions due, for instance, to banks and shoals.

No important influence on the results of the tide analysis is to be expected from higher wind forces of the winds blowing from south and east. But it may be anticipated that the average residual current found from our measurements will as a result have a somewhat too small component in the east direction and a somewhat too large component in the north direction.

An estimate of the magnitude of the effects dealt with in this section is possible only if the influence of the wind on the current has been investigated (part II of this investigation). We shall confine ourselves here to merely indicating these effects.

3.2 Distribution of the observations over the various constituent tides

The distribution of the observations over the various constituent tides should also be investigated. The important question is whether, in the selection of the observations according to a certain phase of the constituent tide to be investigated (the M_2 tide), the phases of the other tides are so distributed over each of those selected groups that in the average over such a group all these latter tides disappear and consequently no disturbing contributions thereof remain.

The current observations were made at 4-hourly intervals. This means that only some discrete tide-phases of the solar tide S_2 and the related overtides occur in the entire material. Also, for a certain phase of the M_2 tide, the compound tides resulting from the interaction of the M_2 and S_2 tides occur only in certain phases. Calculated over a long period, all phases of the other tides occur with equal probability, and it can be shown that in a long series of observations at any rate, these tides will have no influence on the mean current per tide phase of the M_2 tide.
Therefore it has only to be ascertained whether the S_2 tide and the related overtides and compound tides exercise a disturbing influence, when attempting to calculate the M_2 tide.
The angular velocities of the different constituents mentioned in the table in par. 1.1 can be written as linear functions with integer coefficients of the angular velocities of the M_2 and MS_f tide. We can write the current components in the north- or east-direction (positive or negative) as follows:

$$\begin{aligned} V = R &+ A_{MS_f}\cos(\psi-\kappa_{MS_f}) + A_{M_2}\cos(\varphi-\kappa_{M_2}) + A_{S_2}\cos(\varphi+\psi-\kappa_{S_2}) + \\ &+ A_{\mu_2}\cos(\varphi-\psi-\kappa_{\mu_2}) + A_{M_4}\cos(2\varphi-\kappa_{M_4}) + A_{MS_4}\cos(2\varphi+\psi-\kappa_{MS_4}) + \\ &+ A_{S_4}\cos(2\varphi+2\psi-\kappa_{S_4}) + A_{M_6}\cos(3\varphi-\kappa_{M_6}) + \\ &+ A_{2MS_6}\cos(3\varphi+\psi-\kappa_{2MS_6}) + A_{S_6}\cos(3\varphi+3\psi-\kappa_{S_6}) + G\,. \end{aligned} \quad (1)$$

In this equation we use the following symbols

V = component of the total current in N or E direction (positive or negative)
R = component of the residual current.
A = amplitude of the current component of a certain constituent tide indicated by the subscript.
κ = kappa-number, i.e. phase-lag of the current component for the constituent tide indicated by the subscript.
φ = equilibrium argument of the M_2 constituent, increasing by 28.9841° per hour.
ψ = equilibrium argument of the MS_f constituent, increasing by 1.0159° per hour.
G = sum of the current components of all other constituent tides.

In this investigation the symbol R has been used for the residual current in order to indicate that the annual and semi-annual variations of the current have been included therein. The S_6 tide cannot be separated from R (see further on in this paragraph). Usually the symbol A_0 or S_0 is used.
Because of the practice of observing the current at fixed moments (every 4 hours, at the end of a watch period), there is a certain relation between the values of φ and ψ that can occur together in an observation.

The observations were made at the moments 0, 4, 8, 12, 16 and 20 hours, in G.M.T. (Greenwich Mean Time) or in Amsterdam time (= G.M.T. +20 minutes), on the end of a watch numbered 0 to 5.
Since the term $\varphi+\psi$ increases by 30° per hour and the observations were made 4 hours apart, in the observational material only combinations $\varphi+\psi$ differing by 120° will occur.
Thus:

$$\varphi+\psi=\delta+W.120^\circ \tag{2}$$

Here W is the number of the watch, consequently an integer from 0 to 5, and $\delta/30^\circ$ gives the difference in hours between the local time (for the meridian of the point of observation) and the time at which the observation has been made.
With formula (2) formula (1) now becomes:

$$\begin{aligned} V= R &+ A_{MS_f}\cos(-\varphi+\delta+W.120^\circ-\kappa_{MS_f})+A_{M_2}\cos(\varphi-\kappa_{M_2})+\\ &+A_{S_2}\cos(\alpha+W.120^\circ-\kappa_{S_2})+A_{\mu_2}\cos(2\varphi-\delta-W.120^\circ-\kappa_{\mu_2})+\\ &+A_{M_4}\cos(2\varphi-\kappa_{M_4})+A_{MS_4}\cos(\varphi+\delta+W.120^\circ-\kappa_{MS_4})+\\ &+A_{S_4}\cos(2\delta+2W.120^\circ-\kappa_{S_4})+A_{M_6}\cos(3\varphi-\kappa_{M_6})+\\ &+A_{2MS_6}\cos(2\varphi+\delta+W.120^\circ-\kappa_{2MS_6})+\\ &+A_{S_6}\cos(3\delta+3W.120^\circ-\kappa_{S_6})+G \end{aligned} \tag{3}$$

If we calculate the mean of all values of the current with the phase of the M_2 tide in a small interval of a width $2\Delta\varphi$, thus between $\varphi-\Delta\varphi$ and $\varphi+\Delta\varphi$, it is found that, with equal numbers of observations on the different watches, thus with equal numbers of observations for the different values of W, the contributions of the MS_f, S_2 μ_2, MS_4, S_4 and $2MS_6$ tides disappear, since:

$$\sum_{k=0}^{5}\cos(\xi+k.120^\circ)=2\sum_{n=0}^{2}\cos(\xi+n.120^\circ)=0$$
$$\sum_{k=0}^{5}\cos(\xi+k.240^\circ)=2\sum_{n=0}^{2}\cos(\xi-n.120^\circ)=0\,.$$

Apart from the tides M_2, M_4, and the residual current R, there will be only a contribution of the S_6 tide, the mean of which is $A_{S_6}\cos(3\delta-\kappa_{S_6})$, and which cannot by further analysis be separated from the residual current R.
In general, however, at a certain tide phase of the M_2 tide the numbers of observations per watch will not always be equal. Since of the above constituent tides the S_2 tide has the greatest amplitude, the effect of this tide on the mean values over the different tide phases resulting from the unequal distribution of observations over the watches, will be investigated further.

Let us assume at the tide phase φ a total of P observations have been made, of which $\frac{1}{6}P+\Delta_W$ have been made at the watch W.
Then according to this assumption:

$$\sum_{W=0}^{5} \Delta_W = 0 .$$

After calculation of the mean for all observations with tide phase φ there will remain in this mean a residue of the S_2 tide equal to:

$$Q = \frac{1}{P} \sum_{W=0}^{5} \Delta_W A_{S_2} \cos(\delta + W.120^\circ - \kappa_{S_2}) .$$

This can be converted as follows:

$$Q = \frac{A_{S_2}}{P} \{\tfrac{3}{2}(\Delta_0+\Delta_3)\cos(\delta-\kappa_{S_2}) - \tfrac{1}{2}\sqrt{3}(\Delta_1-\Delta_2+\Delta_4-\Delta_5)\sin(\delta-\kappa_{S_2})\} .$$

For a given watch number W, the quotient Δ_W/P will vary with the tide phase φ and thus Q will also vary with φ. The total residue of the S_2 tide Q can be split into three terms, one that is constant for all values of φ, one that varies with φ like a cosine function, and one that comprises all other (periodical or irregular) fluctuations of Q with φ.

$$Q = Q_0 + Q_1 \cos(\varphi - \varepsilon) - Q_2(\varphi) .$$

The first term Q_0 produces an error in the calculation of the residual current, the second term produces an error in the calculation of the M_2 tide.
If we divide in this way

$$\frac{\Delta_0+\Delta_3}{P} = X_0 + X_1 \cos(\varphi-\beta) + X_2(\varphi)$$

and

$$\frac{\Delta_1-\Delta_2+\Delta_4-\Delta_5}{P} = Y_0 + Y_1 \cos(\varphi-\gamma) + Y_2(\varphi)$$

then

$$Q_0 = A_{S_2}\{\tfrac{3}{2}X_0 \cos(\delta-\kappa_{S_2}) - \tfrac{1}{2}\sqrt{3}\, Y_0 \sin(\delta-\kappa_{S_2})\}$$
$$Q_1 \leqq A_{S_2}\{\tfrac{3}{2}X_1 \cos(\delta-\kappa_{S_2}) - \tfrac{1}{2}\sqrt{3}\, Y_1 \sin(\delta-\kappa_{S_2})\} .$$

The observations investigated show that X_0 and Y_0 are generally very small (about 0.002), so that in view of the moderate amplitude of the S_2 tide the influence of Q_0 on the calculation of the residual current is negligible.
Since mainly during the turnings of the tide the deviations of the numbers of observations per watch number are fairly large, X_1 and Y_1 are more important. For the longer observation series X_1 is found to be about 0.008 and Y_1 about

0.022. From this it follows, with aid of the values of A_{S_2} calculated later on, that

$$Q_1 \leqq 0.02 A_{S_2} \leqq 0.4 \text{ cm/sec} .$$

Thus for the longer observation series Q_1 is at most of the same order as the total inaccuracy (par. 2.4), so that no further account is taken of this. In shorter series of observations, however, there is a good chance that Q_1 becomes more important, as small deviations have a relatively large influence. It is for this reason that a different procedure has been followed for the calculation of the M_2 tide for these observations (par. 4.2).

4 CALCULATIONS

4.1 Working up and sorting of the observations

The individual current observations (resolved into N and E components) were transferred to punched cards together with other data on tide phase and the wind.
With the aid of these punched cards various sortings and calculations were possible, which served as the basis for further tide analyses.
The observations can be sorted according to:
1) the equilibrium argument of the M_2 tide at the moment of observation, calculated with respect to the Greenwich meridian, making 20 or 10 groups of 18° or 36° respectively, to be called tide phases (n);
2) the difference between the equilibrium arguments of the S_2 and the M_2 tides at the moment of observation, making 10 groups of 36°, to be called double lunar phases (m);
3) the time of day at which the observations have been made, expressed in 6 watches (W) or, when the equal hours ante and post meridiem are taken together, making 3 groups, (U);
4) the different months, making 12 groups (p).
Since the equilibrium argument with respect to the Greenwich meridian is used, we shall further use, instead of the phase lag with respect to the meridian of the place of observation, κ, the phase lag with respect to the Greenwich meridian g. Thus the zero-point of the tidal cycle is the moment at which the argument of the equilibrium tide is zero for the Greenwich meridian.

4.2 Calculation of the M_2 tide

The M_2 tide was calculated for all positions given in Table I. For the positions of the lightvessels Noord Hinder no. 1 and 2, Schouwen Bank no. 2, Maas no. 1, Haaks no. 1 and Terschellinger Bank no. 3 and 4 the M_2 tide was also calculated for pairs of two consecutive months. For these latter positions the method used was as follows.
Since all contributions of tides other than the M_2 tide and the connected higher harmonics M_4 and M_6 disappear in the mean current per tide phase n (par. 3.2), the mean current per tide phase in the north or east direction is:

$$\overline{V}_n = R + A_{M_2} \cos(n.18° - g_{M_2}) + A_{M_4} \cos(n.36° - g_{M_4}) + \\ + A_{M_6} \cos(n.54° - g_{M_6}) + A_{S_6} \cos(g_{S_6}) \quad (4)$$

From this it follows that

$$A^2_{M_2} = \left(\tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_n \cos n.18^\circ\right)^2 + \left(\tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_n \sin n.18^\circ\right)^2 \qquad (5A)$$

$$\sin g_{M_2} = \frac{1}{10A_{M_2}} \sum_{n=0}^{19} \overline{V}_n \sin n.18^\circ$$

$$\cos g_{M_2} = \frac{1}{10A_{M_2}} \sum_{n=0}^{19} \overline{V}_n \cos n.18^\circ \qquad (5B)$$

For the other lightvessel positions the M_2 tide was calculated only for the the whole year. Since there is a smaller number of observations for these positions on account of the shorter time that these positions were occupied, another procedure was followed in the calculation. In the first place a division into 10 tide phases n was used. Furthermore there was the chance that the measuring period would be too short to eliminate entirely the effect of the S_2 tide as described in par. 3.2.

Therefore the mean current per tide phase n and per double lunar phase m was taken as basis. This mean current $\overline{V}_{nm}$ is:

$$\begin{aligned}\overline{V}_{nm} = R + A_{MS_f} \cos(m.36^\circ - g_{MS_f}) + A_{M_2} \cos(n.36^\circ - g_{M_2}) + \\ + A_{S_2} \cos(n.36^\circ + m.36^\circ - g_{S_2}) + A_{\mu_2} \cos(n.36^\circ - m.36^\circ - g_{\mu_2}) + \\ + A_{M_4} \cos(n.72^\circ - g_{M_4}) + A_{MS_4} \cos(n.72^\circ + m.36^\circ - g_{MS_4}) + \\ + A_{S_4} \cos(n.72^\circ + m.72^\circ - g_{S_4}) + A_{M_6} \cos(n.108^\circ - g_{M_6}) + \\ + A_{2MS_6} \cos(n.108^\circ + m.36^\circ - g_{2MS_6}) + \\ + A_{S_6} \cos(n.108^\circ + m.108^\circ - g_{S_6}) \qquad (6)\end{aligned}$$

From this the mean current for the various values of n and m can be calculated:

$$\overline{V}_n = \frac{1}{N} \sum_{m=0}^{9} \overline{V}_{nm} ,$$

N being the number of values of m that occur with a certain value of n.

If all combinations of n and m occur, it is clear that in this mean only R, M_2 and its overtides remain.

However, because of the method of observing at 4-hourly intervals, not all combinations of m and n occur, while moreover for the group of observations with a certain value of m not every value of the double lunar phase occurs within the interval between the limits $(m-\frac{1}{2}).36^\circ$ and $(m+\frac{1}{2}).36^\circ$, so that the mean for this interval is not equal to m.

If we define all phases with respect to the meridian of Greenwich and if the times of the watches are in G.M.T., formula (2) becomes

$$\varphi + \psi = W.120^\circ .$$

For a certain value of n(n is an integer), we have:

$$n-\tfrac{1}{2}\leqq \psi/36^\circ < n+\tfrac{1}{2}\,.$$

Or:

$$n-\tfrac{1}{2}\leqq W.3\tfrac{1}{3}-\psi/36^\circ < n+\tfrac{1}{2}\,.$$

All values of $\psi/36^\circ$ lying between $m-\frac{1}{2}$ and $m+\frac{1}{2}$ are taken together in the group with double lunar phase m.
The following scheme indicates for the different values of U, which is, accordingly to par. 4.1 related to W by $U=W$ or $W-3$, the values of m that go with a certain tide phase n and the mean values m^* of the double lunar phase and n^* of the tide phase.

U	lower limit of $\psi/36^\circ$	upper limit of $\psi/36^\circ$	m	lower limit of $\psi/36^\circ$ at given m	upper limit of $\psi/36^\circ$ at given m	m^*	lower limit of $\varphi/36^\circ$ at given n	upper limit of $\varphi/36^\circ$ at given n	n^*
0	$-n-\frac{1}{2}$	$-n+\frac{1}{2}$	$-n$	$-n-\frac{1}{2}$	$-n+\frac{1}{2}$	$-n$	$n-\frac{1}{2}$	$n+\frac{1}{2}$	n
1	$-n+2\frac{5}{6}$	$-n+3\frac{5}{6}$	$-n+3$	$-n+2\frac{5}{6}$	$-n+3\frac{1}{2}$	$-n+3\frac{1}{6}$	$n-\frac{1}{6}$	$n+\frac{1}{2}$	$n+\frac{1}{6}$
			$-n+4$	$-n+3\frac{1}{2}$	$-n+3\frac{5}{6}$	$-n+3\frac{4}{6}$	$n-\frac{1}{2}$	$n-\frac{1}{6}$	$n-\frac{2}{6}$
2	$-n+6\frac{1}{6}$	$-n+7\frac{1}{6}$	$-n+6$	$-n+6\frac{1}{6}$	$-n+6\frac{1}{2}$	$-n+6\frac{2}{6}$	$n+\frac{1}{6}$	$n+\frac{1}{2}$	$n+\frac{2}{6}$
			$-n+7$	$-n+6\frac{1}{2}$	$-n+7\frac{1}{6}$	$-n+6\frac{5}{6}$	$n-\frac{1}{2}$	$n+\frac{1}{6}$	$n-\frac{1}{6}$

If m or m^* are 10 or more, it is clear that 10 should be subtracted, as m is always smaller than 10.
If we substitute in formula (6) m^* and n^* for m and n and then calculate the average to obtain $\overline{V}_n$, we have:

$$\begin{aligned}\overline{V}_n = R &- 0.23A_{MS_f}\cos(-n.36^\circ - g_{MS_f}) + 0.99A_{M_2}\cos(n.36^\circ - g_{M_2}) - \\ &- 0.20A_{S_2}\cos g_{S_2} - 0.25A_{\mu_2}\cos(n.72^\circ - g_{\mu_2}) + \\ &+ 0.96A_{M_4}\cos(n.72^\circ - g_{M_4}) - 0.19A_{MS_4}\cos(n.36^\circ - g_{MS_4}) - \\ &- 0.20A_{S_4}\cos g_{S_4} + 0.90A_{M_6}\cos(n.108^\circ - g_{M_6}) - \\ &- 0.11A_{2MS_6}\cos(n.72^\circ - g_{2MS_6}) + A_{S_6}\cos g_{S_6} \end{aligned} \qquad (7)$$

Thus the contributions of the MS_f and MS_4 tides remain in the result of the calculation of the M_2 tide. Since the amplitudes of these constituent tides are small in comparison to that of the M_2 tide, these contributions can be disregarded.

Thus we have

$$A^2_{M_2} = \left(\tfrac{1}{5}\sum_{n=0}^{9}\frac{1}{0.99}\,\overline{V}_n\cos n.36^\circ\right)^2 + \left(\tfrac{1}{5}\sum_{n=0}^{9}\frac{1}{0.99}\,\overline{V}_n\sin n.36^\circ\right)^2 \qquad (8A)$$

$$\left.\begin{aligned} \sin g_{M_2} &= \tfrac{1}{5}\cdot\frac{1}{A_{M_2}}\sum_{n=0}^{9}\frac{1}{0.99}\,\overline{V}_n\sin n.36^\circ \\ \cos g_{M_2} &= \tfrac{1}{5}\cdot\frac{1}{A_{M_2}}\sum_{n=0}^{9}\frac{1}{0.99}\,\overline{V}_n\cos n.36^\circ \end{aligned}\right\} \qquad (8B)$$

4.3 Calculation of the M_4 and M_6 tides

The M_4 tide was calculated for the lightvessel positions Noord Hinder 1 and 2, Schouwen Bank 2, Maas 1, Haaks 1 and Terschellinger Bank 3 and 4. From formula (4) it follows that

$$A^2_{M_4} = \left(\tfrac{1}{10}\sum_{n=0}^{19}\overline{V}_n\cos n.36^\circ\right)^2 + \left(\tfrac{1}{10}\sum_{n=0}^{19}\overline{V}_n\sin n.36^\circ\right)^2 \qquad (9A)$$

$$\left.\begin{aligned} \sin g_{M_4} &= \tfrac{1}{10}\cdot\frac{1}{A_{M_4}}\sum_{n=0}^{19}\overline{V}_n\sin n.36^\circ \\ \cos g_{M_4} &= \tfrac{1}{10}\cdot\frac{1}{A_{M_4}}\sum_{n=0}^{19}\overline{V}_n\cos n.36^\circ \end{aligned}\right\} \qquad (9B)$$

For the same positions it was possible to calculate also the M_6 tide with the formulas

$$A^2_{M_6} = \left(\tfrac{1}{10}\sum_{n=0}^{19}\overline{V}_n\cos n.54^\circ\right)^2 + \left(\tfrac{1}{10}\sum_{n=0}^{19}\overline{V}_n\sin n.54^\circ\right)^2 \qquad (10A)$$

$$\left.\begin{aligned} \sin g_{M_6} &= \tfrac{1}{10}\cdot\frac{1}{A_{M_6}}\sum_{n=0}^{19}\overline{V}_n\sin n.54^\circ \\ \cos g_{M_6} &= \tfrac{1}{10}\cdot\frac{1}{A_{M_6}}\sum_{n=0}^{19}\overline{V}_n\cos n.54^\circ \end{aligned}\right\} \qquad (10B)$$

In the cases where A_{M_6} appeared to be smaller than the inaccuracy the M_6 tide is not included in the tables representing the results of the analysis.

4.4 Calculation of the S_2 tide

The S_2 tide was calculated for the lightvessel positions Noord Hinder 1 and 2, Schouwen Bank 2, Maas 1, Haaks 1, Terschellinger Bank 3 and 4, Doggersbank Z 1 and Doggersbank N 1.

Here the mean current per tide phase n per watch U (a.m. and p.m. combined) was taken as a basis. Now the mean of all components with certain values of n and U is:

$$\overline{V}_{nU}=R+A_{M_2}\cos(n.18^\circ-g_{M_2})+A_{M_4}\cos(n.36^\circ-g_{M_4})+$$
$$+A_{M_6}\cos(n.54^\circ-g_{M_6})+A_{S_2}\cos(U.120^\circ-g_{S_2})+$$
$$+A_{S_4}\cos(U.240^\circ-g_{S_4})+A_{S_6}\cos(U.360^\circ-g_{S_6})+$$
$$+A_{MS_f}\cos(-n.18^\circ+U.120^\circ-g_{MS_f})+$$
$$+A_{\mu_2}\cos(n.36^\circ-U.120^\circ-g_{\mu_2})+A_{MS_4}\cos(n.18^\circ+U.120^\circ-g_{MS_4})+$$
$$+A_{2MS_6}\cos(n.36^\circ+U.120^\circ-g_{2MS_6}) \tag{11}$$

From this it follows, that

$$\overline{V}_U=\tfrac{1}{20}\sum_{n=0}^{19}\overline{V}_{nU}=R+A_{S_2}\cos(U.120^\circ-g_{S_2})+A_{S_4}\cos(U.240^\circ-g_{S_4})+$$
$$+A_{S_6}\cos(U.360^\circ-g_{S_6}) \tag{12}$$

Now from formula (12) it follows that:

$$\left.\begin{aligned}\tfrac{2}{3}\sum_{U=0}^{2}\overline{V}_U\sin U.120^\circ&=A_{S_2}\sin g_{S_2}-A_{S_4}\sin g_{S_4}\\ \tfrac{2}{3}\sum_{U=0}^{2}\overline{V}_U\cos U.120^\circ&=A_{S_2}\cos g_{S_2}+A_{S_4}\cos g_{S_4}\end{aligned}\right\} \tag{13}$$

It is not possible to calculate the S_2 and S_4 tides separately from formulas (13). However, it is to be expected that the amplitude of the S_2 tide will be much greater than that of the S_4 tide. (The ratio of the amplitudes of the M_4 and M_2 tides is about 0.06). Therefore, subject to an estimated maximum error of 6%, we can say that

$$A_{S_2}^2=\left\{\tfrac{2}{3}\sum_{U=0}^{2}\overline{V}_U\cos U.120^\circ\right\}^2+\left\{\tfrac{2}{3}\sum_{U=0}^{2}\overline{V}_U\sin U.120^\circ\right\}^2 \tag{14A}$$

$$\begin{aligned}\sin g_{S_2}&=\tfrac{2}{3}\frac{1}{A_{S_2}}\sum_{U=0}^{2}\overline{V}_U\sin U.120^\circ\\ \cos g_{S_2}&=\tfrac{2}{3}\frac{1}{A_{S_2}}\sum_{U=0}^{2}\overline{V}_U\cos U.120^\circ\end{aligned} \tag{14B}$$

4.5 Calculation of the MS_4 and MS_f tides

From formula (11) it follows, that

$$\tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_{nU} \cos n.18^\circ = A_{M_2} \cos g_{M_2} + A_{MS_f} \cos (U.120^\circ - g_{MS_f}) + $$
$$+ A_{MS_4} \cos (-U.120^\circ + g_{MS_4})$$

and:

$$\tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_{nU} \sin n.18^\circ = A_{M_2} \sin g_{M_2} + A_{MS_f} \sin (U.120^\circ - g_{MS_f}) + $$
$$+ A_{MS_4} \sin (-U.120^\circ + g_{MS_4})$$

From this it follows that

$$C_1 = \tfrac{2}{3}\sum_{U=0}^{2} \left\{ \tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_{nU} \cos (n.18^\circ) \right\} \cos U.120^\circ =$$
$$= A_{MS_f} \cos g_{MS_f} + A_{MS_4} \cos g_{MS_4} \tag{15A}$$

$$C_2 = \tfrac{2}{3}\sum_{U=0}^{2} \left\{ \tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_{nU} \cos (n.18^\circ) \right\} \sin U.120^\circ =$$
$$= A_{MS_f} \sin g_{MS_f} + A_{MS_4} \sin g_{MS_4} \tag{15B}$$

$$C_3 = \tfrac{2}{3}\sum_{U=0}^{2} \left\{ \tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_{nU} \sin (n.18^\circ) \right\} \cos U.120^\circ =$$
$$= -A_{MS_f} \sin g_{MS_f} + A_{MS_4} \sin g_{MS_4} \tag{15C}$$

$$C_4 = \tfrac{2}{3}\sum_{U=0}^{2} \left\{ \tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_{nU} \sin (n.18^\circ) \right\} \sin U.120^\circ =$$
$$= A_{MS_f} \cos g_{MS_f} - A_{MS_4} \cos g_{MS_4} \tag{15D}$$

From these formulas the constants of the MS_f tide are found as follows:

$$\left.\begin{aligned} A_{MS_f} \cos g_{MS_f} &= \frac{C_1 + C_4}{2} \\ A_{MS_f} \sin g_{MS_f} &= \frac{C_2 - C_3}{2} \end{aligned}\right\} \tag{16}$$

and those of the MS_4 tide as follows:

$$\left.\begin{aligned} A_{MS_4} \cos g_{MS_4} &= \frac{C_1 - C_4}{2} \\ A_{MS_4} \sin g_{MS_4} &= \frac{C_2 + C_3}{2} \end{aligned}\right\} \tag{17}$$

The amplitudes of the MS_f and MS_4 tides at certain lightvessel positions were found to be smaller than the inaccuracy. The tables give the values for these constituent tides only, where the amplitude exceeds the inaccuracy.

4.6 Calculation of the μ_2 and $2MS_6$ tides

From formula (11) it follows that

$$\tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_{nU} \cos n.36^\circ = A_{M_4} \cos g_{M_4} + A_{\mu_2} \cos(U.120^\circ + g_{\mu_2}) + \\ + A_{2MS_6} \cos(-U.120^\circ + g_{2MS_6})$$

and

$$\tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_{nU} \sin n.36^\circ = A_{M_4} \sin g_{M_4} + A_{\mu_2} \sin(U.120^\circ + g_{\mu_2}) \\ + A_{2MS_6} \sin(-U.120^\circ + g_{2MS_6}) .$$

From these formulas it can be derived that:

$$D_1 = \tfrac{2}{3}\sum_{U=0}^{2}\left\{\tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_{nU} \cos n.36^\circ\right\} \cos U.120^\circ = \\ = A_{\mu_2} \cos g_{\mu_2} + A_{2MS_6} \cos g_{2MS_6} \tag{18A}$$

$$D_2 = \tfrac{2}{3}\sum_{U=0}^{2}\left\{\tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_{nU} \cos n.36^\circ\right\} \sin U.120^\circ = \\ = -A_{\mu_2} \sin g_{\mu_2} + A_{2MS_6} \sin g_{2MS_6} \tag{18B}$$

$$D_3 = \tfrac{2}{3}\sum_{U=0}^{2}\left\{\tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_{nU} \sin n.36^\circ\right\} \cos U.120^\circ = \\ = A_{\mu_2} \sin g_{\mu_2} + A_{2MS_6} \sin g_{2MS_6} \tag{18C}$$

$$D_4 = \tfrac{2}{3}\sum_{U=0}^{2}\left\{\tfrac{1}{10}\sum_{n=0}^{19} \overline{V}_{nU} \sin n.36^\circ\right\} \sin U.120^\circ = \\ = A_{\mu_2} \cos g_{\mu_2} - A_{2MS_6} \cos g_{2MS_6} \tag{18D}$$

The constants of the μ_2 tide follow from:

$$\left.\begin{aligned} A_{\mu_2} \cos g_{\mu_2} &= \frac{D_1 + D_4}{2} \\ A_{\mu_2} \sin g_{\mu_2} &= \frac{D_3 - D_2}{2} \end{aligned}\right. \tag{19}$$

and those of the $2MS_6$ tide from:

$$A_{2MS_6} \cos g_{2MS_6} = \frac{D_1 - D_4}{2}$$

$$A_{2MS_6} \sin g_{2MS_6} = \frac{D_2 + D_3}{2} \qquad (20)$$

Only in cases where the $2MS_6$ tide has an amplitude greater than the inaccuracy have the constants of this tide been included in the tables of the results.

4.7 Influence of the node factor

When analysing tidal data of a series of observations that consists of several years, attention should be paid to the effect of the variation of the node factor f on the constituent tides of lunar or mixed origine. For information on the significance of this node factor the reader is referred to the works by Schureman (1941), Doodson and Warburg (1941) and to *Overzicht der Getijleer* (1949).
The node factor is used with a mean amplitude of a constituent tide in order to obtain the amplitude of this tide for a given year. Conversely, the node factor gives the mean amplitude of a tide from the amplitude calculated from data of one particular year.
The node factor f varies as a function of I, the obliquity of the lunar orbit with respect to the earth's equator. I varies very slowly, with a period of $18\frac{2}{3}$ year.
For the different constituent tides f is a different function of I.
An analysis over N years gives a mean amplitude $\bar{A}$ of a constituent tide that is related to the true mean amplitude A by:

$$\bar{A} \cos \varphi = \frac{1}{N} \sum_{n=K}^{K+N} A f_n \cos \varphi = A \cos \varphi \cdot \frac{1}{N} \sum_{n=K}^{K+N} f_n$$

With the aid of tables for f as published by Schureman (1941) the value of the correction factor

$$\frac{1}{N} \sum_{n=K}^{K+N} f_n$$

has been calculated for the various periods of the lightvessel positions mentioned in Table I. This correction has been applied to the amplitudes calculated for the various constituent tides.

4.8 Analysis of S_1, K_1, P_1, K_2, T_2 and R_2 tides. Method used for analysis

From the data under consideration the tidal constants of a group of other constituent tides can be obtained. These tides are the following

Symbol	Angular speed	Period	Character
S_1	15.0000° hr^{-1}	1 day	solar diurnal tide (mainly meteorological tide)
K_1	15.0411°	23h.56 min.	solar/lunar diurnal tide (declination tide)
P_1	14.9589°	24h.04 min.	solar diurnal tide (declination tide)
K_2	30.0821°	11h.58 min.	solar/lunar semidiurnal tide (declination tide)
T_2	29.9589°	12h.01 min.	solar semidiurnal tide (declination tide)
R_2	30.0411°	11h.59 min.	

Five of these six tides have approximately a period of 1 day (K_1 and P_1) or of $\frac{1}{2}$ day (K_2, T_2, R_2). In the course of one year K_1 and R_2 will gain 1 period. K_2 will gain 2 periods and P_1 and R_2 will lag 1 period behind.
Using a division of the data into groups according to the watch number and the number of the month, we can find these constituent tides if the observations extend over a period of sufficiently long duration. This procedure has been used by van der Stok to calculate these constituent tides for the Noord Hinder lightvessel over an observation period of 5 years. For a watch number W and a month number p (January$=0$) the mean value for the tide over a long series of years, in which the effect of other tides can be eliminated, is approximately:

$$
\begin{aligned}
G_{pW} = {} & A_{S_1}\cos(W.60^\circ - E_{S_1}) + A_{S_2}\cos(W.120^\circ - E_{S_2}) + \\
& + A_{K_1}\frac{12\sin 15^\circ}{\pi}\cos(W.60^\circ - E_{K_1} + 15^\circ + p.30^\circ) + \\
& + A_{P_1}\frac{12\sin 15^\circ}{\pi}\cos(W.60^\circ - E_{P_1} - 15^\circ - p.30^\circ) + \\
& + A_{K_2}\frac{6\sin 30^\circ}{\pi}\cos(W.120^\circ - E_{K_2} + 30^\circ + p.60^\circ) + \\
& + A_{T_2}\frac{12\sin 15^\circ}{\pi}\cos(W.120^\circ - E_{T_2} - 15^\circ - p.30^\circ) + \\
& + A_{R_2}\frac{12\sin 15^\circ}{\pi}\cos(W.120^\circ - E_{R_2} + 15^\circ + p.30^\circ) \qquad (21
\end{aligned}
$$

Iere E is the argument of the tide at 1 January at 00 G.M.T. for the meridian ıf Greenwich. This argument E may contain, apart from the phase lag g for he relevant tide, also a constant angle and an angle that depends on the vari- .tion in the obliquity of the lunar orbit.

From this equation A and E can be calculated for the various constituent ides. The procedure is such that first for all different values of p a harmonic nalysis is made for the variation of G_{pW} with W and then the values found .re analysed for their variation with p. The procedure has been described by 'an der Stok (1905).

As S_2 has already been analysed according to the procedure described in ›ar. 4.4, we shall pay no further attention to the second term of formula (23). The magnitude of the S_1 tide as deduced from astronomical considerations is light. It may, however, be important as a meteorological tide. It has therefore ›een analysed, but only for the lightvessel Maas the amplitude of the tidal :urrent is more than 1 cm/sec. As already mentioned in par. 5.1, for smaller ımplitudes the relative inaccuracy will have too much influence, so the value ›f S_1 for the other lightvessels cannot be given here with sufficient accuracy.

I.9 Elimination of the effects on K_1 and K_2 of the variations of amplitude and phase-angle with the period of $18\frac{2}{3}$ years

The calculation of the K_1 and K_2 tides from equation (21) gives some difficulty f the series of observations that is analyzed comprises several years. This is ›ecause for these tides E consists of an angle V_0, the phase lag g and an angle hat depends on the variation of the obliquity of the lunar orbit, commonly ndicated by the letter u. Since u varies slowly in the course of the years with ı period of $18\frac{2}{3}$ year, it follows that the constants of K_1 and K_2 cannot be :alculated directly from this equation if the series of observations is of more han one or two years duration.

For the year with number K the average value of the contribution of the K_1 ide to the tidal motion over all observations with watch number W and nonth number p is

$$A_K \frac{12 \sin 15^\circ}{\pi} f_K(K_1) \cos \{W.60^\circ + V_0(K_1) + 15^\circ + \\ + p.30^\circ + U_K(K_1) - g_{K_1}\}$$

ınd a similar expression can be given for the contribution of the K_2 tide. This expression can be written as:

$$B f_K \cos (W.60^\circ + p.30^\circ - \vartheta + u_K)$$

ts mean value over the years K' till K'' will be

$$\frac{1}{K''-K'} B \sum_{K=K'}^{K''} f_K \cos(W.60^\circ + p.30^\circ - \vartheta + u_K) =$$

$$= \frac{1}{K''-K'} \left\{ B \cos(W.60^\circ + p.30^\circ - \vartheta) \sum_{K=K'}^{K'} f_K \cos u_K - \right.$$

$$\left. - B \sin(W.60^\circ + p.30^\circ - \vartheta) \sum_{K=K'}^{K''} f_K \sin u_K \right\}$$

We can write

$$\frac{1}{K''-K'} \sum_{K=K'}^{K''} f_K \cos u_K = F(K', K'') \cos \eta(K', K'')$$

and

$$\frac{1}{K''-K'} \sum_{K=K'}^{K''} f_K \sin u_K = F(K', K'') \sin \eta(K', K'') .$$

Then the expression for the contribution of the K_1 tide becomes

$$BF(K', K'') \cos \{W.60^\circ + p.30^\circ - \vartheta + \eta(K', K'')\} .$$

This expression can be analysed according to the procedure mentioned in the foregoing paragraph. The results of the analysis have to be corrected to give the amplitude and phase lag of the K_1 tide. The same applies to the analysis of the K_2 tide.
The correction to be applied to the amplitude consists of a division by F where

$$F = \frac{1}{K''-K'} \sqrt{\left(\sum_{K=K'}^{K''} f_K \cos u_K \right)^2 + \left(\sum_{K=K'}^{K''} f_K \sin u_K \right)^2} . \qquad (22$$

The correction to be applied to the phase lag consists of an addition of η where

$$\eta = \operatorname{arctg} \frac{\sum_{K=K'}^{K''} f_K \sin u_K}{\sum_{K=K'}^{K''} f_K \cos u_K} . \qquad (23$$

F and η can be calculated for each series of observations from tables of the yearly values of f and u for the constituents K_1 and K_2 such as those given by Schureman (1941).

.10 Calculation of the constants determining the shape of the tidal ellipse

'he tidal constants for the north and east components of the different con-tituent tides give the variations of the tidal current in these directions. As lready said in par. 1.1, the end point of the current vector resulting from hese two components describes an ellipse. This ellipse can be calculated from he constants found in the preceding paragraphs in a manner that is in prin-iple the same as the method used by van der Stok (1905).
et us assume that the N and E components of the current of a certain onstituent tide are represented by:

$$V_N = A_N \cos(\omega t - g_N)$$
$$V_E = A_E \cos(\omega t - g_E)\,.$$

The ellipse described by the end-point of the current vector V can be found y rotating the axes of the co-ordinates in such a way that the new axes oincide with the long and short axes of the ellipse. The ellipse can then be vritten as follows:

$$\frac{V_x^2}{a^2} + \frac{V_y^2}{b^2} = 1\,,$$

vhere V_x and V_y are the vector components according to the new co-ordinate ixes and $2a$ and $2b$ are the long and short axes of the ellipse, respectively. Furthermore:

$$V_x = a \cos(\omega t - g_a)\,,$$
$$V_y = -b \sin(\omega t - g_a)\,.$$

For $\omega t = g_a$ the current is at its maximum. If a and b have the same sign, the vector revolves in a positive direction (with the sun), if they have opposite signs, the vector revolves in a negative direction (against the sun).
Irrespective of the signs we find a and b as well as the angle α between the direction of a and the meridian (reckoned positive from north over east to south and west) from the equations:

$$\left.\begin{aligned} A_N^2 + A_E^2 &= a^2 + b^2 \\ A_N^2 - A_E^2 &= (a^2 - b^2)\cos 2\alpha \\ 2A_N A_E \cos(g_N - g_E) &= (a^2 - b^2)\sin 2\alpha \end{aligned}\right\} \qquad (24)$$

Furthermore g_a can be found from:

$$\left.\begin{aligned} \sin(2g_a - g_N - g_E) &= -\cos 2\alpha \sin(g_N - g_E) \\ \cos(2g_a - g_N - g_E) &= \frac{a^2 + b^2}{a^2 - b^2}\cos(g_N - g_E) \end{aligned}\right\} \qquad (25)$$

4.11 Calculation of the residual current

The residual current follows from formula (4) if we neglect the contribution of the S_6 tide:

$$R = \frac{1}{20} \sum_{n=0}^{19} \overline{V}_n \quad \text{or} \quad R = \frac{1}{10} \sum_{n=0}^{9} \overline{V}_n, \tag{26}$$

depending on a division of the tidal period into 20 or into 10 groups.
We can estimate the error introduced by neglecting the contribution of the S_6 tide. The maximum contribution occurs when $g_{S_6} = 0°$ or $180°$; it is equal to the amplitude of the S_6 tide. If we assume the ratio of the amplitudes of the S_6 and S_2 tides to be the same as the ratio between those of the M_6 and M_2 tides, we find for the amplitude $A_{S_6} \approx 0.5$ cm/sec. Therefore the maximum influence of the S_6 tide is estimated to be ± 0.5 cm/sec.
For those positions where the period during which observations have been made was of sufficient length, the average residual current was calculated for three-month periods, or per month. It would have been possible to calculate from the annual variations of R the annual and semi-annual constituents S_a and S_{sa}. However, because practical use of this analysis was thought to be small, this has been omitted here.

5 RESULTS

5.1 General remarks

The results of the analyses are presented in the Tables III to VI.
In studying the results it should be borne in mind that differences of results for a particular lightvessel at different positions need not entirely be due to difference in place, since there can also have been variations with time. The periods of the observations as given in Table I should therefore not be lost sight of.
Table III gives the values for the M_2 tide and the residual current, as calculated from all the observations at the different positions.
Tables IVa to i give for the various positions with a longer measuring period the analysed constituent tides in so far as their amplitudes are greater than 1 cm/sec. At smaller amplitudes the inaccuracy was considered to be relatively too great.
For the lightvessels Schouwen Bank, Maas and Haaks the constituent tides S_2, MS_f, μ_2, MS_4 and $2MS_6$ have been calculated only for the period after 1930. This is because of a practical difficulty. Before this year the moments of observation were rekoned in "Amsterdam Time" (=G.M.T.+20 minutes), after this year in G.M.T. An analysis over all the observations would have meant a calculation for both periods separately. As no important variations in the tides are expected, it was decided to analyse the second period only.
Table V gives the residual current per period of 3 months for all the positions with a measuring period of more than 1 year.
Table VI gives the residual current per month only for the positions with longer measuring periods.

5.2 Symbols used

The following symbols are used in the tables.

ω = angular speed in degrees per hour of the constituent tide concerned.
A_N = amplitude of the north component of the tidal current in cm/sec.
A_E = amplitude of the east component of the tidal current in cm/sec.
g_N = phase lag of the north component of the tidal current with respect to the Greenwich meridian and for G.M.T.
g_E = phase lag of the east component of the tidal current.
g_a = phase lag of the component of the current along the long axis of the tidal current ellipse.
a = half the long axis of the tidal current ellipse, or maximum value of the tidal current, in cm/sec.

b = half the short axis of the tidal current ellipse, or value of the tidal current during the turnings of the tide, in cm/sec.

α = direction of the long axis of the tidal current ellipse, with respect to north, or direction of the maximum tidal current (reckoned positive from north via east to south and west).

R_N = north component of the residual current in cm/sec.

R_E = east component of the residual current in cm/sec.

R = absolute value of the residual current in cm/sec.

α_R = direction of the residual current with respect to north.

rotation = sense of rotation (positive if from north over east to south and west).

alt. = alternating.

TABLE III

M$_2$ *constituent tide and residual current*

Lightvessel		Position	A_N	A_E	g_N	g_E	a	b	α	g_a	rotation	R_N	R_E	R	α_R
Noord Hinder	1	51°35.5′N 2°36.5′E	41.1	41.1	47°	38°	57.9	5.0	45°	42°	–	1.2	0.9	1.5	36°
	2	51°38.7′N 2°34′E	43.0	43.0	48°	46°	60.8	0.5	45°	47°	–	3.3	3.2	4.6	44°
	3	51°47′N 2°41′E	42.8	42.8	59°	43°	60.0	8.1	45°	51°	–	2.2	1.7	2.8	37°
	4	52°05.2′N 2°39.9′E	44.1	43.9	65°	53°	62.1	6.5	45°	59°	–	1.5	1.4	2.0	43°
Schouwen Bank	1	51°41.8′N 3°17.4′E	32.9	51.6	66°	43°	60.4	10.9	58°	49°	–	7.5	–6.0	9.4	321°
	2	51°47.1′N 3°27.4′E	45.2	47.4	66°	58°	65.4	2.8	46°	62°	–	3.1	2.3	3.9	37°
Maas	1	52°01.8′N 3°53.7′E	39.6	59.3	66°	73°	71.2	3.6	56°	71°	+	8.7	5.0	10.0	30°
Haaks	1	52°57.5′N 4°18.8′E	37.9	33.7	109°	124°	50.2	6.4	41°	116°	+	3.9	2.7	4.7	34°
	2	52°57.2′N 3°55′E	34.2	30.2	87°	111°	44.7	9.4	41°	98°	+	1.8	–0.1	1.8	356°
	3	53°00′N 4°05′E	27.6	31.0	101°	119°	41.0	6.6	48°	111°	+	1.5	0.1	1.5	3°
Terschellinger-Bank	1	53°29′N 3°55′E	20.5	28.7	99°	162°	30.9	17.0	64°	147°	+	2.3	6.0	6.4	69°
	2	53°29′N 4°17′E	25.6	41.5	135°	158°	47.9	8.6	59°	152°	+	3.2	4.6	5.6	55°
	3	53°27′N 4°46.9′E	32.5	44.4	148°	172°	54.0	10.7	55°	164°	+	4.7	4.4	6.4	43°
	4	53°27′N 4°51.9′E	31.4	54.8	161°	174°	62.8	6.3	61°	171°	+	4.5	4.8	6.6	47°
Doggersbank Z	1	54°45′N 3°58.5′E	2.0	11.5	5°	215°	11.6	1.0	98°	198°	–	0.8	0.6	1.0	39°
Doggersbank N	1	56°00′N 5°00′E	4.8	12.7	113°	216°	12.8	4.7	96°	219°	+	1.2	1.9	2.2	56°
	2	55°59.5′N 5°24.1′E	5.5	11.0	94°	219°	11.5	4.3	109°	226°	+	2.1	1.1	2.3	28°

TABLE IVa *Noord Hinder 1* 51°35.5′ N 2°36.5′ E

	ω	A_N	A_E	g_N	g_E	a	b	α	g_a	rotation
M_2 (year)	28.984°	41.1	41.1	47°	38°	57.9	5.0	45°	42°	–
(J+F)		41.8	41.3	48°	38°	58.6	4.7	45°	43°	–
(M+A)		43.0	42.4	47°	38°	60.2	4.7	45°	43°	–
(M+J)		42.7	42.7	48°	37°	60.2	5.3	45°	42°	–
(J+A)		41.2	41.7	49°	38°	58.4	5.3	45°	43°	–
(S+O)		39.0	38.9	50°	39°	54.9	4.9	45°	45°	–
(N+D)		39.4	39.6	43°	34°	55.8	4.5	45°	39°	–
MS_f	1.016°									
P_1	14.959°									
S_1	15.000°									
K_1	15.041°	0.7	0.8	172°	220°	1.0	0.4	50°	200°	+
μ_2	27.968°	3.3	3.5	136°	124°	4.8	0.5	47°	130°	–
T_2	29.959°	1.3	0.3	133°	109°	1.3	0.1	12°	132°	–
S_2	30.000°	10.9	11.1	107°	98°	15.5	1.2	45°	102°	–
R_2	30.041°	1.3	0.4	330°	10°	1.3	0.2	14°	332°	+
K_2	30.082°	3.5	3.9	121°	104°	5.2	0.8	48°	112°	–
M_4	57.968°	2.3	2.4	342°	341°	3.3	0.0	46°	342°	–
MS_4	58.984°	2.1	2.0	8°	26°	2.9	0.4	44°	17°	+
M_6	86.952°									
$2MS_6$	87.968°									

TABLE IVb *Noord Hinder 2* 51°38.7′ N 2°34′ E

	ω	A_N	A_E	g_N	g_E	a	b	α	g_a	rotation
M_2 (year)	28.984°	43.0	43.0	48°	46°	60.8	0.5	45°	47°	–
(J+F)		44.2	44.2	46°	44°	62.4	0.5	45°	45°	–
(M+A)		42.2	42.1	48°	46°	59.5	0.5	45°	47°	–
(M+J)		42.6	42.7	48°	46°	60.2	0.0	45°	47°	–
(J+A)		42.7	42.9	49°	47°	60.4	1.0	45°	48°	–
(S+O)		42.7	42.7	48°	47°	60.3	0.5	45°	48°	–
(N+D)		44.0	43.8	46°	45°	60.3	1.0	45°	46°	–
MS_f	1.016°	1.0	1.0	19°	19°	1.4	0.0	45°	19°	alt.
P_1	14.959°									
S_1	15.000°									
K_1	15.041°	1.2	1.1	161°	185°	1.6	0.3	42°	172°	+
μ_2	27.968°	2.5	2.7	122°	122°	3.7	0.0	47°	122°	alt.
T_2	29.959°									
S_2	30.000°	11.3	11.2	109°	108°	15.9	0.1	45°	109°	–
R_2	30.041°									
K_2	30.082°	3.5	3.0	111°	108°	4.6	0.1	40°	110°	–
M_4	57.968°	2.1	2.0	346°	347°	2.9	0.0	43°	346°	–
MS_4	58.984°	1.9	1.9	49°	50°	2.7	0.0	45°	50°	+
M_6	86.952°	1.2	1.3	341°	319°	1.7	0.3	44°	330°	–
$2MS_6$	87.968°									

ABLE IVc *Schouwen Bank 2* 51°47.1′N 3°27.4′E

	ω	A_N	A_E	g_N	g_E	a	b	α	g_a	rotation
1$_2$ (year)	28.984°	45.2	47.4	66°	58°	65.4	2.8	46°	62°	−
(J+F)		46.5	48.8	64°	56°	67.3	3.7	46°	60°	−
(M+A)		45.8	47.9	65°	59°	66.2	2.9	46°	62°	−
(M+J)		45.7	48.5	65°	58°	66.5	3.8	47°	61°	−
(J+A)		44.8	47.5	68°	60°	65.2	4.1	47°	64°	−
(S+O)		43.6	45.7	68°	60°	63.1	4.7	46°	64°	−
(N+D)		44.7	46.5	65°	57°	64.3	4.7	46°	61°	−
1S$_f$	1.016°	1.0	1.2	171°	175°	1.6	0.1	40°	173°	+
ʼ$_1$	14.959°									
$_1$	15.000°									
$_1$	15.041°									
$_2$	27.968°	3.2	4.0	163°	153°	5.1	0.4	52°	157°	−
$_2$	29.959°	0.8	0.9	115°	128°	1.2	0.1	48°	122°	+
$_2$	30.000°	12.0	12.2	123°	122°	17.1	0.1	46°	122°	−
$_2$	30.041°	0.7	0.8	306°	309°	1.1	0.0	49°	308°	+
$_2$	30.082°	3.7	2.6	122°	120°	4.5	0.0	35°	122°	−
1$_4$	57.968°	3.9	4.3	40°	36°	5.7	0.1	48°	24°	−
1S$_4$	58.984°	2.3	2.3	72°	74°	3.3	0.1	45°	73°	+
1$_6$	86.952°									
MS$_6$	87.968°	1.2	0.6	135°	161°	1.3	0.2	25°	140°	+

ABLE IVd *Maas 1* 52°01.8′N 3°53.7′E

	ω	A_N	A_E	g_N	g_E	a	b	α	g_a	rotation
1$_2$ (year)	28.984°	39.6	59.3	66°	73°	71.2	3.6	56°	71°	+
(J+F)		39.2	59.9	63°	70°	71.4	4.0	57°	68°	+
(M+A)		40.6	60.3	66°	72°	72.5	4.1	56°	70°	+
(M+J)		43.2	63.4	68°	75°	76.6	4.4	56°	72°	+
(J+A)		39.1	57.7	68°	74°	69.7	3.6	56°	73°	+
(S+O)		37.1	55.8	67°	73°	66.9	3.3	56°	71°	+
(N+D)		38.0	58.3	64°	70°	65.6	3.2	57°	69°	+
1S$_f$	1.016°	0.8	1.5	172°	335°	1.7	0.2	118°	294°	+
ʼ$_1$	14.959°									
$_1$	15.000°	0.7	0.9	218°	232°	1.1	0.1	52°	230°	+
$_1$	15.041°	0.7	0.7	207°	188°	1.0	0.2	45°	198°	−
$_2$	27.968°	6.6	4.9	184°	194°	8.2	0.7	36°	188°	+
$_2$	29.959°	0.6	0.9	11°	60°	1.0	0.4	61°	48°	+
$_2$	30.000°	10.9	17.0	127°	129°	20.2	0.4	57°	128°	+
$_2$	30.041°	1.1	1.0	185°	265°	1.1	0.9	30°	210°	+
$_2$	30.082°	2.6	4.6	128°	123°	5.3	0.2	60°	124°	−
1$_4$	57.968°	3.4	4.9	82°	49°	5.7	1.6	56°	59°	−
1S$_4$	58.984°	1.9	2.3	149°	87°	2.6	1.5	56°	108°	−
1$_6$	86.952°	1.4	1.4	107°	118°	2.0	0.3	45°	112°	+
MS	87.968°	3.5	1.5	170°	189°	3.8	0.5	22°	173°	+

TABLE IVe *Haaks* 1 52° 57.5′ N 4° 18.8′ E

	ω	A_N	A_E	g_N	g_E	a	b	α	g_a	rotation
M_2 (year)	28.984°	37.9	33.7	109°	124°	50.3	6.4	41°	116°	+
(J+F)		37.9	33.8	107°	122°	50.4	6.2	42°	114°	+
(M+A)		38.2	34.4	110°	124°	51.0	6.4	42°	116°	+
(M+J)		40.2	35.8	111°	126°	53.4	7.0	42°	117°	+
(J+A)		39.2	34.2	109°	124°	51.7	6.5	41°	115°	+
(S+O)		35.3	31.9	108°	124°	47.1	6.3	42°	115°	+
(N+D)		35.5	31.3	108°	123°	46.9	6.1	41°	114°	+
MS_f	1.016°	1.3	0.9	322°	317°	1.6	0.1	35°	320°	−
P_1	14.959°									
S_1	15.000°									
K_1	15.041°									
μ_2	27.968°	4.2	4.3	223°	247°	5.9	1.2	46°	235°	+
T_2	29.959°	0.6	0.9	190°	197°	1.1	0.0	56°	195°	+
S_2	30.000°	8.9	7.8	162°	183°	11.6	2.1	41°	171°	+
R_2	30.041°									
K_2	30.082°	2.7	1.8	176°	196°	3.2	0.5	33°	182°	+
M_4	57.968°	3.2	1.9	201°	209°	3.7	0.2	31°	203°	+
MS_4	58.984°	2.6	2.3	264°	264°	3.5	0.0	42°	264°	alt.
M_6	86.952°	1.1	1.2	224°	172°	1.5	0.7	52°	193°	−
$2MS_6$	87.968°	0.8	1.0	285°	252°	1.2	0.4	52°	272°	−

TABLE IVf *Terschellinger Bank* 3 53° 27′ N 4° 46.9′ E

	ω	A_N	A_E	g_N	g_E	a	b	α	g_a	rotation
M_2 (year)	28.984°	32.5	44.4	148°	172°	54.0	10.7	55°	164°	+
(J+F)		31.0	41.0	146°	169°	50.4	10.0	54°	161°	+
(M+A)		32.1	44.0	147°	171°	53.4	10.8	55°	163°	+
(M+J)		34.9	48.7	151°	175°	58.8	11.6	55°	167°	+
(J+A)		34.7	46.8	152°	174°	57.3	10.4	54°	167°	+
(S+O)		29.9	41.6	150°	170°	50.6	8.7	55°	164°	+
(N+D)		30.7	42.8	147°	170°	51.8	9.9	55°	162°	+
MS_f	1.016°	0.8	1.0	356°	274°	1.0	0.8	74°	286°	+
P_1	14.959°									
S_1	15.000°									
K_1	15.041°									
μ_2	27.968°	3.8	5.4	247°	272°	6.5	1.4	55°	264°	+
T_2	29.959°	0.5	0.9	287°	237°	1.0	0.4	67°	246°	−
S_2	30.000°	7.4	10.2	220°	241°	12.4	2.2	55°	234°	+
R_2	30.041°	0.9	0.8	321°	253°	1.0	0.7	36°	295°	−
K_2	30.082°	1.5	3.6	218°	212°	3.9	0.4	67°	213°	−
M_4	57.968°	2.6	2.7	233°	249°	3.7	0.5	46°	241°	+
MS_4	58.984°	2.4	2.4	280°	269°	3.4	0.3	45°	274°	−
M_6	86.952°	1.2	1.8	324°	312°	2.1	0.2	58°	316°	−
$2MS_6$	87.968°	1.2	1.7	55°	11°	1.9	0.7	58°	24°	−

TABLE IVg *Terschellinger Bank* 4 53°27′N 4°51.9′E

	ω	A_N	A_E	g_N	g_E	a	b	α	g_a	rotation
M_2 (year)	28.984°	31.4	54.8	161°	174°	62.8	6.3	61°	171°	+
(J+F)		31.3	56.5	161°	174°	64.3	6.0	61°	171°	+
(M+A)		31.3	55.5	164°	176°	63.4	5.8	61°	173°	+
(M+J)		33.0	57.6	160°	175°	66.0	7.1	61°	171°	+
(J+A)		32.6	54.4	161°	174°	63.1	6.4	59°	170°	+
(S+O)		29.5	50.8	161°	175°	58.4	6.1	60°	171°	+
(N+D)		30.3	53.8	160°	172°	61.5	5.3	61°	169°	+
MS_f	1.016°									
P_1	14.959°									
S_1	15.000°									
K_1	15.041°									
μ_2	27.968°	4.1	7.1	259°	275°	8.1	1.0	60°	271°	+
T_2	29.959°	1.3	2.0	211°	216°	2.4	0.1	57°	214°	+
S_2	30.000°	7.3	12.7	220°	232°	14.6	1.2	60°	229°	+
R_2	30.041°	1.0	1.0	319°	64°	1.1	0.9	135°	102°	+
K_2	30.082°	2.2	4.6	243°	245°	5.1	0.0	64°	244°	+
M_4	57.968°	1.6	1.9	270°	267°	2.5	0.5	49°	268°	–
MS_4	58.984°									
M_6	86.952°	1.2	2.6	328°	336°	2.9	0.2	66°	335°	+
$2MS_6$	87.968°	0.8	2.6	113°	57°	2.6	0.7	80°	60°	–

TABLE IVh *Doggersbank Zuid* 1 54°45′N 3°58.5′E

	ω	A_N	A_E	g_N	g_E	a	b	α	g_a	rotation
M_2	28.984°	2.0	11.5	5°	215°	11.6	1.0	98°	198°	–
P_1	14.959°									
S_1	15.000°									
K_1	15.041°									
μ_2	27.968°									
T_2	29.959°									
S_2	30.000°	0.8	3.7	20°	278°	3.8	0.3	92°	277°	–
R_2	30.041°	0.0	2.2	—	310°	2.2	0.0	90°	310°	alt.
K_2	30.082°	0.2	2.8	359°	242°	2.8	0.2	92°	242°	–

TABLE IVi *Doggersbank Noord 1 56°00′N 5°00′E*

	ω	A_N	A_E	g_N	g_E	a	b	α	g_a	rotation
M_2	28.984°	4.7	12.7	113°	216°	12.8	4.6	96°	219°	+
P_1	14.959°	1.1	0.9	61°	41°	1.4	0.2	39°	53°	−
S_1	15.000°									
K_1	15.041°	1.7	0.7	80°	60°	1.8	0.2	22°	77°	−
μ_2	27.968°									
T_2	29.959°									
S_2	30.000°	1.9	4.7	186°	286°	4.7	1.9	85°	274°	+
R_2	30.041°									
K_2	30.082°	0.2	1.6	262°	192°	1.6	0.2	88°	192°	−

TABLE V *Residual current per 3 months*

		Noord Hinder 1	Noord Hinder 2	Noord Hinder 3	Noord Hinder 4	Schouwen Bank 1	Schouwen Bank 2	Maas 1	Haaks 1	Haaks 2	Tersch. Bank 1	Tersch. Bank 3	Tersch. Bank 4	Doggersbank Z. 1	Doggersbank N. 1
Dec. Jan. Feb.	R_N	1.9	3.2	3.4	1.3	6.4	3.6	9.7	3.8	1.7	2.2	4.5	5.9	0.7	0.3
	R_E	1.4	3.2	3.2	1.4	−3.8	2.8	6.3	2.6	0.3	4.9	4.2	5.7	0.3	1.5
	R	2.3	4.5	4.7	2.0	7.4	4.6	11.6	4.6	1.7	5.3	6.2	8.2	0.7	1.6
	α_R	36°	45°	44°	47°	329°	38°	33°	34°	10°	66°	43°	44°	22°	80°
March April May	R_N	0.9	3.7	2.2	1.5	8.7	3.7	9.2	4.1	1.8	2.3	4.3	4.4	0.9	0.8
	R_E	0.7	3.7	1.0	0.9	−7.9	2.6	5.4	2.7	0.0	3.4	4.7	4.4	0.5	0.6
	R	1.1	5.3	2.4	1.7	11.7	4.5	10.7	4.9	1.8	4.1	6.4	6.2	1.0	1.0
	α_R	37°	45°	26°	33°	318°	35°	30°	34°	0°	56°	47°	45°	31°	36°
June July Aug.	R_N	1.0	3.4	2.6	1.9	7.3	2.9	7.8	4.2	2.0	1.7	5.0	4.2	0.4	1.8
	R_E	0.7	3.2	2.0	1.9	−6.9	2.3	4.3	3.3	−1.1	6.5	4.9	5.2	0.9	2.6
	R	1.2	4.6	3.3	2.7	10.0	3.7	9.0	5.3	2.2	6.7	7.0	6.7	1.0	3.2
	α_R	37°	43°	37°	45°	317°	38°	29°	39°	331°	75°	44°	51°	66°	56°
Sept. Oct. Nov.	R_N	1.3	3.1	1.0	1.3	7.6	2.4	7.8	3.2	1.7	3.1	4.0	3.6	1.2	2.1
	R_E	0.9	3.0	0.4	1.3	−5.4	1.7	3.8	1.8	0.3	9.1	3.7	3.7	0.8	2.8
	R	1.6	4.3	1.1	1.8	9.3	3.0	8.6	3.7	1.7	9.6	5.4	5.2	1.4	3.5
	α_R	36°	45°	21°	45°	324°	35°	26°	29°	9°	71°	43°	46°	35°	53°

TABLE VI *Residual current per month*

		J	F	M	A	M	J	J	A	S	O	N	D
N. Hinder 1	R_N	1.7	2.0	0.9	1.1	0.6	1.1	0.9	1.0	1.2	1.1	1.5	1.9
	R_E	1.2	1.6	0.9	1.0	0.1	1.0	0.3	0.8	1.0	0.8	1.0	1.3
	R	2.1	2.6	1.3	1.5	0.6	1.5	0.9	1.3	1.5	1.3	1.8	2.3
	α_R	36°	38°	45°	41°	8°	44°	21°	40°	39°	36°	32°	35°
N. Hinder 2	R_N	3.8	2.3	4.0	3.4	3.8	3.5	3.4	3.2	2.6	2.9	3.7	3.5
	R_E	3.7	2.3	3.9	3.4	3.8	3.2	3.1	3.2	2.5	2.8	3.7	3.4
	R	5.3	3.2	5.6	4.9	5.4	4.7	4.6	4.6	3.6	4.0	5.2	4.9
	α_R	44°	45°	44°	45°	45°	43°	42°	45°	44°	45°	45°	44°
Schouwen Bank 2	R_N	3.8	3.7	3.5	4.0	3.6	3.0	2.8	2.8	2.3	2.6	2.3	3.4
	R_E	2.8	2.9	2.5	2.9	2.4	2.8	1.8	2.3	1.3	2.2	1.7	2.7
	R	4.7	4.7	4.3	4.9	4.3	4.1	3.3	3.6	2.6	3.4	2.8	4.4
	α_R	37°	38°	36°	36°	34°	42°	32°	40°	29°	39°	36°	38°
Maas 1	R_N	10.4	10.4	10.2	8.7	8.8	8.1	7.8	7.6	7.4	7.5	8.4	8.3
	R_E	7.0	7.0	5.9	5.4	4.9	4.7	4.3	3.9	3.1	3.8	4.5	5.0
	R	12.5	12.5	11.8	10.2	10.0	9.4	8.9	8.6	8.0	8.4	9.5	9.7
	α_R	34°	34°	30°	32°	29°	30°	29°	27°	23°	27°	28°	31°
Haaks 1	R_N	4.0	4.0	4.3	3.8	4.3	4.2	4.9	3.4	3.3	3.5	2.8	3.4
	R_E	2.9	2.2	2.7	2.7	2.8	3.5	3.6	2.9	1.9	2.0	1.6	2.6
	R	5.0	4.6	5.1	4.6	5.1	5.5	6.0	4.5	3.8	4.0	3.2	4.3
	α_R	37°	29°	32°	36°	33°	40°	36°	40°	30°	29°	29°	37°
Tersch. Bank 3	R_N	4.6	4.4	5.0	4.0	3.9	4.0	5.8	5.2	3.7	3.7	4.4	4.5
	R_E	5.0	3.6	5.7	4.4	3.9	4.3	6.1	4.2	3.7	3.3	3.9	4.2
	R	6.8	5.6	7.6	6.0	5.5	5.9	8.4	6.7	5.3	5.0	5.9	6.2
	α_R	47°	39°	49°	47°	45°	47°	47°	39°	45°	42°	41°	43°
Tersch. Bank 4	R_N	6.7	5.9	4.7	3.8	4.7	3.8	3.8	5.0	3.8	2.5	4.5	5.0
	R_E	6.5	6.0	4.0	3.7	5.5	5.0	4.8	5.7	3.8	3.2	4.0	4.5
	R	9.3	8.4	6.2	5.4	7.2	6.3	6.2	7.6	5.5	4.0	6.0	6.8
	α_R	44°	45°	40°	44°	49°	52°	51°	49°	45°	52°	42°	42°

6 DISCUSSION OF THE RESULTS

6.1 Comparison of different constituent tides with each other

According to Hansen (1938) the ratio between the amplitudes and the difference between the phase angles of two constituent tides having about the same period, such as the M_2, S_2 and μ_2 or the M_4 and MS_4 tides, are nearly constant for a certain sea area. In the following table this is checked for the calculated values of a and g_a, while the table also gives some values for the vertical tide of some coastal stations, calculated with tidal constants derived from the *Getijtafels voor Nederland* (Tidal tables for the Netherlands) published by the "Rijkswaterstaat".

TABLE VII

	a_{M_2}/a_{S_2}	$g_{S_2}-g_{M_2}$	a_{M_2}/a_{μ_2}	$g_{\mu_2}-g_{M_2}$	a_{M_2}/a_{K_2}
Noord Hinder 1	3.7	60°	12.0	90°	11.1
Noord Hinder 2	3.8	62°	16.2	75°	13.2
Vlissingen	3.6	58°	12.8	103°	12.6
Schouwen Bank 2	3.8	60°	12.8	95°	13.4
Hoek van Holland	4.0	61°	9.2	118°	13.9
Maas 1	3.5	57°	8.6	117°	13.4
IJmuiden	3.9	69°	7.3	98°	13.2
Haaks 1	4.3	55°	8.5	119°	15.7
Den Helder	3.6	68°	7.5	88°	11.8
Terschellinger Bank 3	4.3	70°	8.3	100°	13.8
Terschellinger Bank 4	4.3	58°	7.7	100°	12.3
Doggersbank Z 1	3.2	79°			4.2
Doggersbank N 1	2.8	55°			8.0

The mutual relationships of the constants for the different constituent tides are indeed in good agreement with this rule. Only for the tides with small amplitudes is the agreement not so good. Here the inaccuracy has probably too much influence, relatively.

6.2 Long-period variations of tidal currents

We shall compare the calculated tidal constants of the M_2 tide with the results of other analyses for the same lightvessels.
Tidal calculations have been made by van der Stok (1905) for the lightvessel Noord Hinder for the years 1890 and 1894 and for the other lightvessels for the years 1898 and 1899. The issues of the "*Bulletin Hydrographique*" for the years 1912–1913 and 1913–1914 give results of tidal current analyses of ob-

servations over periods of 10 and 14 days, respectively, at the lightvessel Terschellinger Bank and the lightvessels Haaks and Noord Hinder. These are observations of the current at 10 meters depth.

These data and the results published here show differences in certain respects. In order to study the possibility of variation with time of certain tidal constants the tidal constants of the M_2 tide at the lightvessels Schouwen Bank, Maas and Haaks were calculated separately for the period before and after 1930 (making use of a division of the material that was necessary because of the change from Amsterdam Time to G.M.T.)

Table VIII gives the results for the M_2 constituent tide. It can be seen that the various values of g_a are in good mutual agreement, except for the result for 1912. As the period of observation in the latter case was short, it is clear that the possibility of a disturbance is great.

$g_{M_2}-g_{M_2}$	a_{M_2}/a_{R_2}	$g_{R_2}-g_{M_2}$	a_{M_4}/a_{MS_4}	$g_{MS_4}-g_{M_4}$	a_{M_6}/a_{2MS_6}	$g_{2MS_6}-g_{M_6}$
90°	44.5	290°	1.1	35°		
			1.1	64°		
47°			1.6	64°	1.0	43°
73°	55.0	259°	1.7	35°		°
56°		°	1.7	57°	1.6	51°
−23°	64.7	139°	2.2	49°	0.5	61°
68°		°	1.8	61°	0.7	46°
79°		°	1.1	61°	1.2	79°
58°		°	1.9	59°	1.1	49°
82°	54.0	131°	1.1	33°	1.1	68°
43°	57.0	291°			1.1	85°

This agreement of the values of g_a gives confidence that it is allowed to compare the results of these different analyses, although the other values clearly show some differences between the different periods. It is not to be supposed that there are important changes in the methods of observation that might have caused these variations, except of course the measurements at 10 m depth.

Variations of tidal currents over longer periods may be ascribed to one or more of the following causes:

1. Regional and local variations of the depth of the sea and, in the proximity of the coast, variations of the coastal configuration.
2. Variations of meteorological factors as wind, air pressure or sea temperature. These factors may be the cause of changes in the hydrodynamic behaviour of the sea area and may give some variations in the tidal currents.

3. Long-periodic variations of the tide-generating forces. The most important variation, the one that arises from the variation of the orbit of the moon with a period of 18.6 year, has already been accounted for.

TABLE VIII

		M_2 constituent tide				Residual current	
		a	b	α	g_a	R	α_R
Noord Hinder 1	1890–1894	63.1	12.2	19°	47°	3.2	21°
	1913*	77.8	13.3	27°	48°	—	—
	1910–1931	57.9	5.0	45°	42°	1.5	36°
Schouwen Bank 2	1898–1899	61.3	10.4	39°	58°	4.9	33°
	1910–1930	64.1	2.4	47°	61°	4.0	35°
	1930–1934	67.1	3.8	45°	64°	3.6	40°
Maas 1	1898–1899	56.6	5.0	36°	72°	6.2	11°
	1911–1930	68.7	4.1	57°	70°	9.8	36°
	1930–1939	76.1	2.6	55°	73°	10.8	19°
Haaks 1	1898–1899	68.8	8.5	12°	117°	7.0	358°
	1913*	58.6	7.6	22°	117°	—	—
	1910–1930	49.5	5.2	41°	116°	5.1	33°
	1930–1939	51.5	8.5	42°	115°	4.1	37°
Terschellinger Bank 4	1898–1899	72.1	11.8	55°	173°	5.8	54°
	1912*	67.8	9.7	61°	149°	—	—
	1910–1929	62.8	6.3	61°	171°	6.6	47°

* From current observations at 10 m depth.

It might be that other factors play also a role, but it is not possible to decide with certainty the importance of these various causes for the variations found. If we examine the different possibilities, we can say the following:

ad 1. Possible variations of the shoals near the lightvessels could have caused some variations of the tidal constants, especially of the speed and direction of the maximum current, a and α. Such variations of the bottom depth have been found in and in front of the delta area to the south-east of the Schouwen Bank and Maas lightvessels (Van Veen, 1956) and in the proximity of the inlets between the Wadden Islands (Gerritsen, 1954); the latter variations might have had some influence on the currents near the Haaks and Terschellinger Bank Lightvessels, but this influence cannot have been very important. For the shoals near Noord Hinder lightvessel it is probable that they are subject to only small changes, as is the case with the Flemish banks to the south-east, studied by van Veen (1936).

The closing of the Zuider Zee in 1932 has had an important influence on the tides in the Wadden Sea and in the inlets between the Wadden Islands. At Den Helder the amplitude of the vertical tide increased by about 10% (Gerritsen, 1954). But it seems improbable that this could also have given rise to important variations near the lightvessels Haaks and Terschellinger Bank.

ad 2. The influence of a variation of meteorological conditions on the tidal currents is difficult to discuss at this stage of the investigation. More will be said about the influence of the wind on the tidal currents in the second part of this study. Verploegh (1959) has given some data on the long term variations of sea temperature and wind at the lightvessels and it appears that significant variations of these elements do occur.
We can make, however, an estimate of the influence of climatological variations on the tidal currents by comparing the different data of Table IV, where the seasonal variations of the tidal currents are given. If variations in the meteorological conditions give rise to variations in the tidal currents, they should be most pronounced in the seasonal variations. As these seasonal variations in Table IV are smaller than the long term variations in Table VIII, this effect is not likely to play a major part.

ad 3. Long-periodic variations of the tide-generating forces might be expected to have roughly the same effect on the currents in a larger area, as for instance the whole southern North Sea. The tendency of the variation is, however, different for the northern and for the southern lightvessels. Therefore, it is not certain whether an influence of such variations plays a major role.
We can only say that for a further study of these changes of the currents the present data are not sufficient. It may be mentioned that a variation of the tidal currents over the period 1880–1940 has also been observed at the Danish lightvessels "Horns Rev" and "Vyl" (Thomsen, 1939). The possibility of a period of 84 to 93 years has been mentioned by Thomsen as an explanation for these variations. The present data of the Netherlands lightvessels do not clearly show such a period.
Special mention should be made of the variation of the direction of the long axis of the current ellipse. Compared with the data of Van der Stok the directions for the period 1920–1939 are more easterly for all lightvessels. This is not only the case for the M_2 tide, but also for the other constituent tides calculated here, as far as they can be compared with the data of Van der Stok. Furthermore these differences between the values of α for the various constituent tides (M_2 and S_2, for Noord Hinder also K_1 and K_2) are nearly the same for each lightvessel. The differences are:

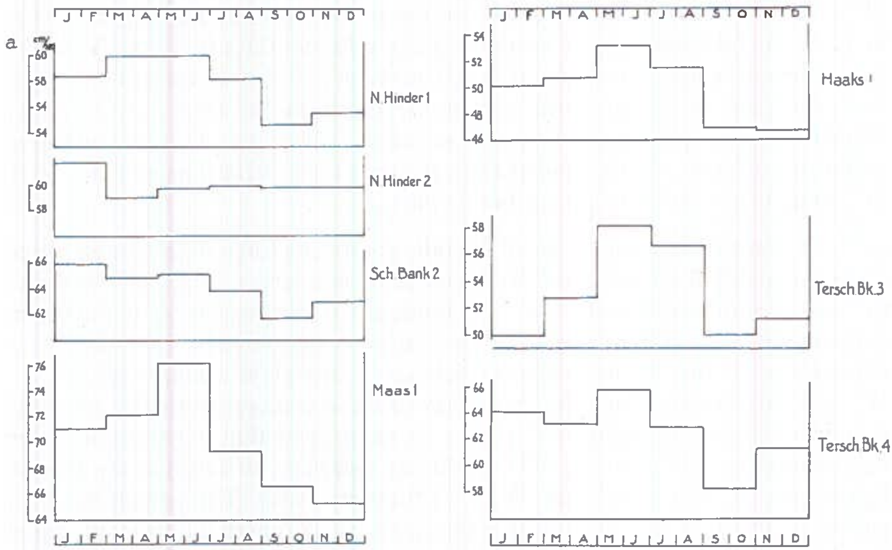
Noord Hinder 1	25° to 31°
Schouwen Bank 2	4° to 8°
Maas 1	18° to 21°

Haaks 1	29° to 30°
Terschellinger Bank 4	5° to 6°

As all directions are true, an influence of the magnetic variation can be ruled out.

6.3 Annual variation of the M_2 tide

In the elaboration of the tidal ellipses for 2-monthly periods given in Table IV a systematic variation of the maximum tidal current *a* in the course of the year catches the eye. This variation is shown in greater detail in fig. 2. From



this it can be seen that generally *a* attains higher values in May-June and lower values in September–October or November–December. The relative variations lie between ±2.3% and ±7.8% of the annual mean of *a* (Noord Hinder 2 having the minimum, Maas the maximum variation).

The direction of the maximum current shows no greater variations in the course of the year than 1° or 2°. The variations of g_a are also small, but have a tendency towards lower values in the winter and higher values in the summer. The maximum current thus occurs somewhat earlier in the winter.

Such seasonal variations of the tidal current are probably the result of the variations of the meteorological influences on the tide, as already mentioned in par. 6.2.

6.4 Meteorological tides

In the foregoing paragraph the possible influence of meteorological factors on the M_2 tide has been mentioned. In par. 6.6 the seasonal variations of the residual current will be discussed. These variations can be expressed as an annual and a semi-annual tidal constituent of mainly meteorological origine. This has not been done here, because it was thought better to give directly the values of *R* for the different seasons and months as has been done in the Tables V and VI.
S_2 may be partly due to meteorological influences, but compared with the astronomical influence these are thought to be negligible. This is confirmed by the fact that the S_1 tide, which can be considered as to be mainly of meteorological origine, is only important for the lightvessel Maas. The reason is not clear why only for this lightvessel an important contribution of S_1 has been found. The data given by Verploegh (1958) on the daily variation of the wind do not give an indication of a greater variation at Maas than at the other lightvessels. It might be that the proximity of the coast or possibly the greater difference in density between surface and bottom water and therefore the greater influence of the wind on the surface layer plays a role here.
For the sake of completeness it should be mentioned here that the constituent tides dealt with in par. 4.8 might also be partly due to some meteorological influence. As can be seen from formula (21), the K_1 constituent for instance can be considered as a current with a diurnal variation having a phase-lag that shows an annual variation. It is possible to think of an annual variation of the diurnal variation of the wind that can cause such variations of the current. However, as the importance of the S_1 constituent is already very small, this influence, if existing, will certainly be unimportant.

6.5 Variation of the residual current over a longer period

Table VIII also gives the values of the mean residual current calculated over different periods. The residual current calculated from the observations of 1912 and 1913 is not given here, as, on account of the short measuring period, these values are not comparable with those calculated over periods of several years.
We see that the direction of the residual current in the period 1910–1930 is more easterly, compared with the period 1890–1899, except at Terschellinger Bank. Between the periods 1910–1930 and 1930–1939 the change in direction is less systematic, while the variations in the current strength cannot be reduced to a general tendency over the whole area.
That such fairly large variations of the mean residual current over longer periods are no exception is evident from the measurements of Carruthers

(1935) at the Varne lightvessel, where during the period 1926–1935 the direction of the residual current changed from about north-east at the beginning of this period to about north at the end. Later observations (Carruthers, 1939; Carruthers, Lawford and Veley, 1950) indicate again a more north-easterly direction of the residual current for the years 1938 and 1939.

Long term variations of the wind near the lightvessels (Verploegh, 1959) or of the wind field over the whole North Sea may be responsable for these variations. The influence of the wind on the residual current will be dealt with in Part II of this study.

Variations of the position of shoals and of the coastal configuration were discussed in par. 6.2 in connection with the possible influence they may have on the tidal currents. It is obvious that such variations also affect the residual current.

The S_6 tide is included in the residual current over the years 1910–1939 (see par. 3.2). It is conceivable that variations of S_6 in this period occur and that these are partly responsible for the variations of R. The difference with the values of R found by Van der Stock (1890–1899) might also be ascribed to this residue of the S_6 tide. Finally the shift of the moment of observation from Amsterdam Time (= G.M.T. +20 minutes) to G.M.T. in 1930 perhaps could have affected the difference of R before and after this date because of a resulting change in the residue of S_6.

In par. 4.11 the contribution of the S_6 tide to R has been estimated to be at most 0.5 cm/sec. The variations shown in Table VIII are larger, so that an important influence seems to be improbable. In order to explain the difference between the values of Van der Stok and the present data entirely by means of the influence of the S_6 tide an amplitude between 1.1 and 4.1 cm/sec would be necessary; in order to explain the difference of the values before and after 1930 S_6 should have an amplitude between 8.7 and 18.2 cm/sec. As such amplitudes of S_6 lie far beyond the expectations, the influence of the S_6 residue in R on the long term variations is of only secondary importance.

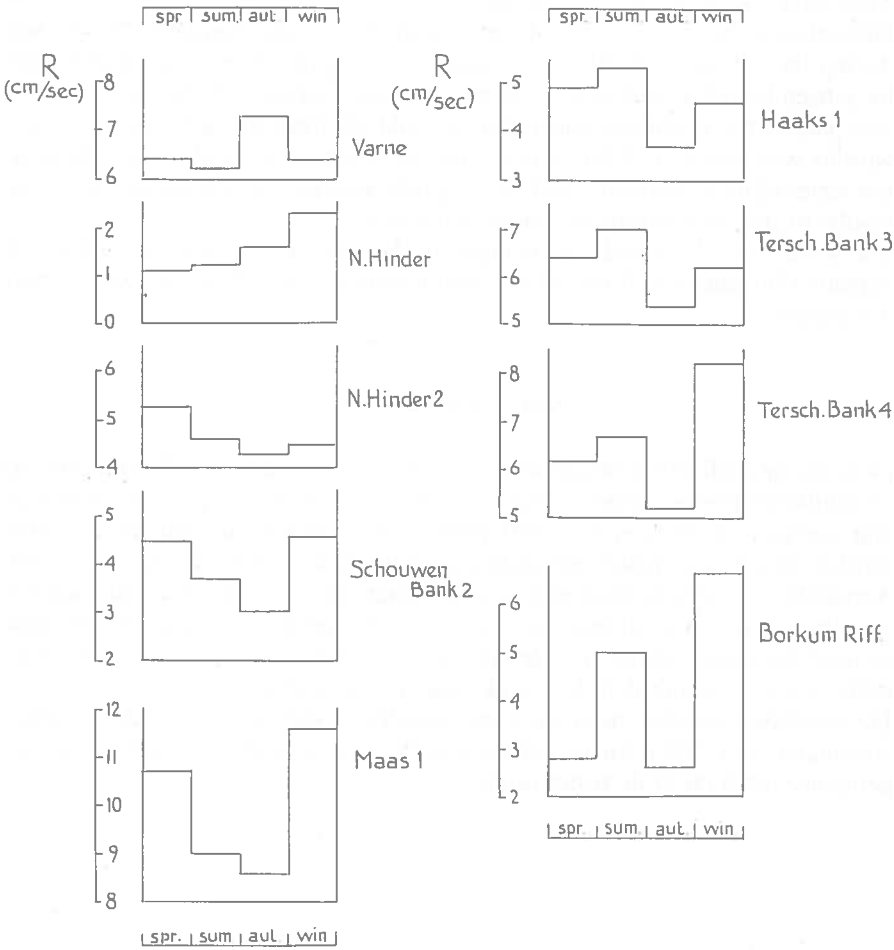
6.6 Annual variation of the residual current

The annual variation of the residual current is presented in Tables V and VI as well as in fig. 3 for the lightvessels Noord Hinder 1 and 2, Schouwen Bank 2, Maas 1, Haaks 1 and Terschellinger Bank 3 and 4. This figure also shows the annual variation of the residual current as calculated from the data of Carruthers (1935) and Mandelbaum (1934) for the lightvessels Varne and Borkum Riff, neighbours of the lightvessels Noord Hinder and Terschellinger Bank, respectively. The lightvessels can be divided into two groups, according to the variation in current strength, as indicated in fig. 3:

1. The southern group: Noord Hinder 2, Schouwen Bank 2 and Maas 1, with

a gradual decrease of the residual current from spring till autumn and a sudden increase in winter (for Noord Hinder 2 in spring).

2. The northern group: Haaks 1, Terschellinger Bank 3 and 4 and Borkum Riff, also with a minimum residual current strength in autumn, but with a second maximum of the current strength in summer.



The lightvessels Noord Hinder 1 and Varne do not fit in well with this scheme. These variations may be ascribed to the annual variation of the mean wind field over the North Sea and the adjoining sea areas, and for lightvessels close to river-mouths, such as the Maas lightvessel, probably also to the variations in the discharge of river water.

SUMMARY

This study presents a detailed analysis of current observations made by Netherlands light-vessels at different positions in the Southern North Sea during the period 1910–1939. The tidal constants of various constituent tides have been found, as well as the values of the residual current. Special attention was paid to the problems arising for the analysis from the fact that the observations were made at 4 hours intervals and from occasional interruptions in the series of measurements, and their significance for the interpretation of the results of the calculations has been evaluated.
The results are discussed and compared with the results of other studies. It appears that time variations of the tidal constants and of the residual current are present.

SAMENVATTING

Deze studie geeft een gedetailleerde analyse van stroomwaarnemingen, die op verschillende posities in de zuidelijke Noordzee door Nederlandse lichtschepen zijn verricht in de periode 1910–1939. Van verscheidene partiële getijden werden de getijconstanten gevonden, evenals de waarden van de reststroom. Aandacht werd geschonken aan de problemen die bij de analyse ontstaan ten gevolge van het interval van vier uur tussen de opeenvolgende waarnemingen en door de onderbrekingen in de meetreeksen, en hun betekenis voor de interpretatie van de resultaten der berekeningen is nagegaan.
De resultaten worden besproken en vergeleken met die van andere onderzoekingen. Het blijkt dat er variaties in de loop van de tijd optreden in de getijconstanten en in de reststroom.

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