KONINKLIJK NEDERLANDS METEOROLOGISCH INSTITUUT

MEDEDELINGEN EN VERHANDELINGEN

No. 95

I. CSIKÓS

ON THE THEORY OF THE ELECTROMAGNETIC SEISMOGRAPH

1975

Prijs f 22,—

STAATSDRUKKERIJ / 'S-GRAVENHAGE

ON THE THEORY OF THE ELECTROMAGNETIC SEISMOGRAPH

KONINKLIJK NEDERLANDS METEOROLOGISCH INSTITUUT

MEDEDELINGEN EN VERHANDELINGEN

No. 95

I. CSIKÓS

ON THE THEORY OF THE ELECTROMAGNETIC SEISMOGRAPH

1975

STAATSDRUKKERIJ/'S-GRAVENHAGE

PUBLIKATIENUMMER K.N.M.I. 102-95

PREFACE

The recordings of earthquakes by differing types of seismographs differ in appearance. Each seismograph has its specific characteristic magnification curve, indicating the wavelength interval that will be most pronounced in the seismographic record.

At the Royal Netherlands Meteorological Institute the traditional seismographs, recording a broad central region of the earthquake wave spectrum, have been supplemented in later years by seismographs with favourable characteristics for recording either the short waves or the long waves.

One may ask whether it is possible to design a seismograph having a certain prescribed magnification curve. This general problem is theoretically treated and solved in the present publication. In addition the author describes a practical method for determining the magnification curve of an existing seismograph from the record of an electric pulse supplied to the system.

This study has been accepted by the Faculty of Natural Sciences of the University of Utrecht as a thesis for the degree of Doctor.

The Director in Chief of the

Royal Netherlands Meteorological Institute

M. W. F. SCHREGARDUS

VOORWOORD

Aardbevingen worden door seismografen van verschillend type op verschillende wijze geregistreerd, waarbij het van de z.g. vergrotingskarakteristiek van een seismograaf afhangt welk golflengtegebied in het aardbevingsspectrum het duidelijkst op de registrering verschijnt.

Op het K.N.M.I. zijn naast de traditionele seismografen van het gebruikelijke type, die een breed midden-spectrum van aardbevingsgolven registreren, in latere jaren ook seismografen in gebruik genomen die hetzij de korte golven, hetzij de lange golven speciaal tot hun recht laten komen.

De vraag is in hoeverre het mogelijk is een seismograaf te ontwerpen die een bepaalde gewenste vergrotingskarakteristiek heeft. Dit algemene probleem wordt in de voorliggende publikatie theoretisch behandeld en opgelost. Tevens heeft de auteur een praktische methode uitgewerkt om de vergrotingskarakteristiek van een bestaande seismograaf te bepalen uit de registratie van een elektrische puls die aan het systeem wordt toegevoerd.

Deze studie werd door de Faculteit der Wiskunde en Natuurwetenschappen van de Rijksuniversiteit te Utrecht aanvaard als dissertatie.

De Hoofddirecteur van het

Koninklijk Nederlands Meteorologisch Instituut

M. W. F. SCHREGARDUS

CONTENTS

CHAPTER I. THEORY OF THE ELECTROMAGNETIC SEISMOGRAPH

The electromagnetic seismograph is a combination of two instruments, viz. a seismometer and a galvanometer. The seismometer generates an electromotoric force by a displacement of the ground. If the seismometer is electrically connected to a sensitive galvanometer, the movement of the ground can be recorded. In this chapter the theory of the seismometer-galvanometer system, the seismograph, is explained.

1 Free oscillation of the seismometer

The seismometer is an instrument consisting of a mass which is kept in a position of equilibrium by means of an elastic or quasi-elastic force. We will first study the free movement of the mass along a fixed axis (figure 1).

In this case the motion follows from the equation

$$K_S\ddot{\theta} = M_C + M_D \tag{1.1}$$

K_S is the moment of inertia of the mass with respect to the axis of rotation, θ is the angle of deviation and M_C and M_D are the movements of the elastic force and of the dissipative force, resp. The dissipative force causes loss of energy and damping of the movement of the seismometer.
The moment of the elastic or quasi-elastic force can, for small values of θ, be written as

$$M_C = -C\theta \tag{1.2}$$

The moment of the dissipative force is a function of the angular velocity of the mass. For small values of $\dot{\theta}$ the moment is

$$M_D = -B\dot{\theta} \tag{1.3}$$

Substitution of (1.2) and (1.3) into eq. (1.1) gives the equation of the free motion of the seismometer

$$\ddot{\theta} + 2\varepsilon_{so}\dot{\theta} + n_s^2\theta = 0 \tag{1.4}$$

where

$$2\varepsilon_{so} = \frac{\mathrm{B}}{K_s} \text{ and } n_s^2 = \frac{C}{K_s}$$

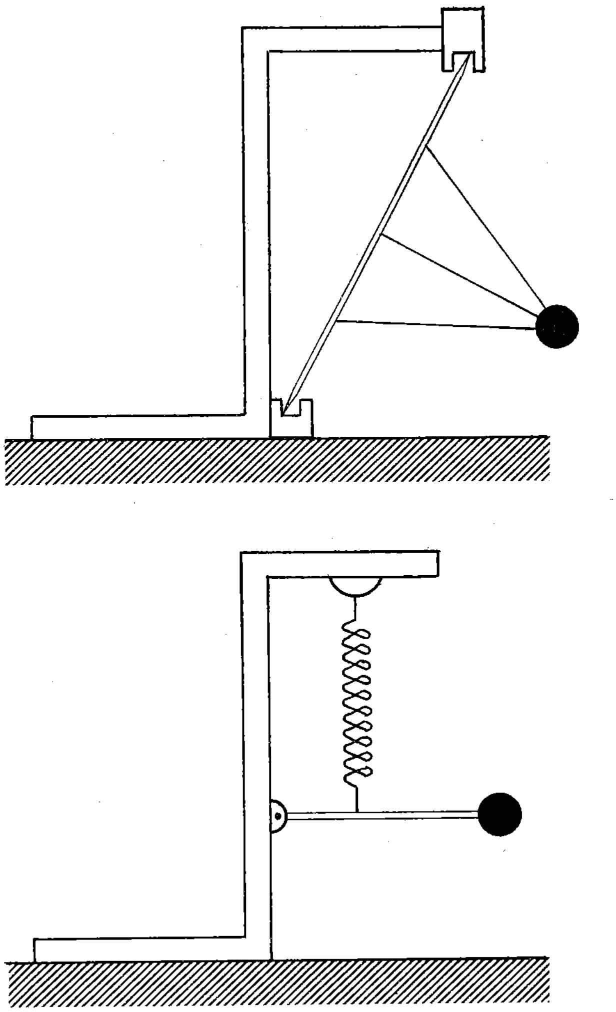

Fig. 1

The solution of eq. (1.4) is

$$\theta = a_1 e^{\alpha_1 t} + a_2 e^{\alpha_2 t}$$

where α_1 and α_2 are roots of the equation

$$\alpha^2 + 2\varepsilon_{so}\alpha + n_s^2 = 0$$

$$\alpha_1 = -\varepsilon_{so} + i\sqrt{n_s^2 - \varepsilon_{so}^2} \text{ and } \alpha_2 = -\varepsilon_{so} - i\sqrt{n_s^2 - \varepsilon_{so}^2}$$

The constants α_1 and α_2 can be found by means of the initial conditions, viz.:

for $t = 0$, $\theta = \theta_0$ aod $\dot{\theta} = \dot{\theta}_0$

a. weak damping

For $n_s > \varepsilon_{so}$ the solution is

$$\theta = \frac{\dot{\theta}_0 + \varepsilon_{so}\theta_0}{\sqrt{n_s^2 - \varepsilon_{so}^2}} e^{-\varepsilon_{so}t} \sin \sqrt{n_s^2 - \varepsilon_{so}^2}\, t + \theta_0 e^{-\varepsilon_{so}t} \cos \sqrt{n_s^2 - \varepsilon_{so}^2}\, t$$

or

$$\theta = a e^{-\varepsilon_{so}t} \sin (v_s t + \varphi) \tag{1.5}$$

where

$$a = \sqrt{\left(\frac{\dot{\theta}_0 + \varepsilon_{so}\theta_0}{v_s}\right)^2 + \theta_0^2}, \quad \varphi = \operatorname{arctg} \frac{v_s \theta_0}{\dot{\theta}_0 + \varepsilon_{so}\theta_0} \text{ and}$$

and

$$v_s = \sqrt{n_s^2 - \varepsilon_{so}^2} \tag{1.6}$$

For $n_s > \varepsilon_{so}$ the free motion of the seismometer is a damped harmonic oscillation with period

$$T' = \frac{2\pi}{v_s} = \frac{2\pi}{n_s \sqrt{1 - \left(\frac{\varepsilon_{so}}{n_s}\right)^2}} = \frac{T}{\sqrt{1 - \left(\frac{\varepsilon_{so}}{n_s}\right)^2}}$$

where T is the natural period of the seismometer if $\varepsilon_{so} = 0$, i.e. if no damping occurs. As $0 < \varepsilon_{so} < n_s$, $T' > T$.

If the ratio of two amplitudes of the free oscillation, separated by $\frac{1}{2}\, T'$, is determined one gets from (1.5.)

$$v = \frac{a_k}{a_{k+1}} = \exp \varepsilon_{so} \frac{T'}{2} \tag{1.7}$$

From (1.7) and (1.6) we can calculate the parameters ε_{so} and n_s, which determine the free motion of the seismometer.

b. *strong damping*

If $\varepsilon_{so} > n_s$, then $v_s = i\sqrt{\varepsilon_{so}^2 - n_s^2} = i\bar{v}_s$ and the general solution of (1.4.) is

$$\theta = e^{-\varepsilon_{so}t}\,(a_1\ \mathrm{sh}\bar{v}_s t + a_2\ \mathrm{ch}\bar{v}_s t)$$

where

$$\bar{v}_s = \sqrt{\varepsilon_{so}^2 - n_s^2} \qquad a_1 = \frac{\dot{\theta}_0 + \varepsilon_{so}\theta_0}{\bar{v}_s} \qquad \text{and } a_2 = \theta_0$$

In this case the free motion of the seismometer is aperiodic.

c. *critical damping*

If $\varepsilon_{so} = n_s$, then the general solution of (1.4) is

$$\theta = (a_1 + a_2 t)\mathrm{e}^{-n_s t}$$

where

$$a_1 = \theta_0 \text{ and } a_2 = \dot{\theta}_0 + n_s\theta_0$$

This motion is the transition between the periodic and the aperiodic movement.

2 Forced motion of the seismometer

We will now derive the differential equation of the seismometer standing on a moving support, the support being fixed to the earth. The motion of the earth is determined with respect to a coordinate system (X, Y, Z) which is in rest. The motion of the seismometer is measured in a coordinate system (x, y, z) which is connected with the support. The z-axis is the axis of rotation of the seismometer and the coordinate axes (x, y, z) are parallel with the axes (X, Y, Z). If the seismometer is in rest, the centre of gravity will be on the y-axis (see figure 2).

According to the laws of mechanics the equation of motion in a moving coordinate system is found by adding the moment of the inertial force to the equation for the fixed coordinate system. If we suppose that the movement of the earth takes place in the (X, Y) plane, we get for the equation of motion

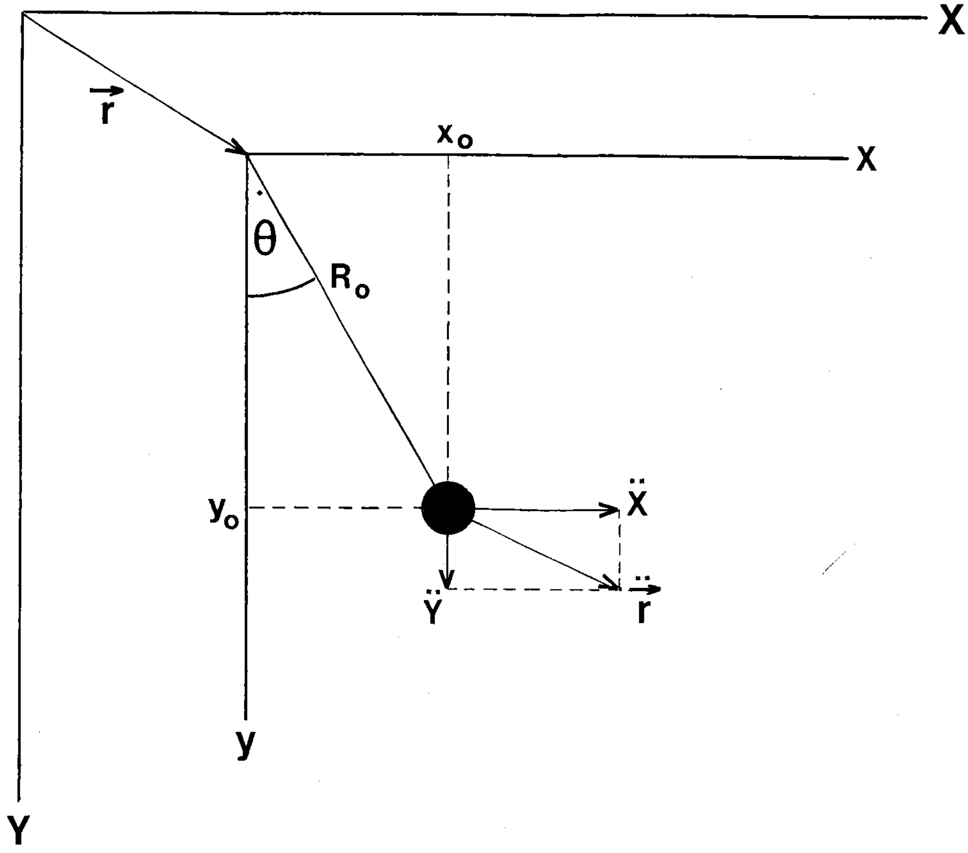

Fig. 2

$$K_s\ddot{\theta} + B\dot{\theta} + C\dot{\theta} = -M_z \quad (2.1)$$

where M_z is the movement of the inertial force with respect to the axis of rotation. As the inertial force acts in the centre of gravity of the system, we can write

$$M_z = M(\ddot{X}y_0 - \ddot{Y}x_0)$$

Furthermore

$$x_0 = R_0 \sin\theta \text{ and } y_0 = R_0 \cos\theta$$

For small values of θ, eq. (2.1) is transformed into

$$\ddot{\theta} + 2\varepsilon_{so}\dot{\theta} + n_s^2\theta = -\frac{MR_0}{K_s}(\ddot{X} - \ddot{Y}\theta) \tag{2.2}$$

From figure 3 we see that in the free motion of the seismometer, the gravity causes the moment

$$M_c = -MR_0 g \sin\theta$$

According to eq. (1.4) and (1.2)

$$n_s = \frac{C}{K_s} = \frac{\left(\frac{\partial M_c}{\partial \theta}\right)\theta = 0}{K_s} = \frac{MR_0 g}{K_s} = \frac{g}{l}$$

or

$$T_s = 2\pi\sqrt{\frac{l}{g}}$$

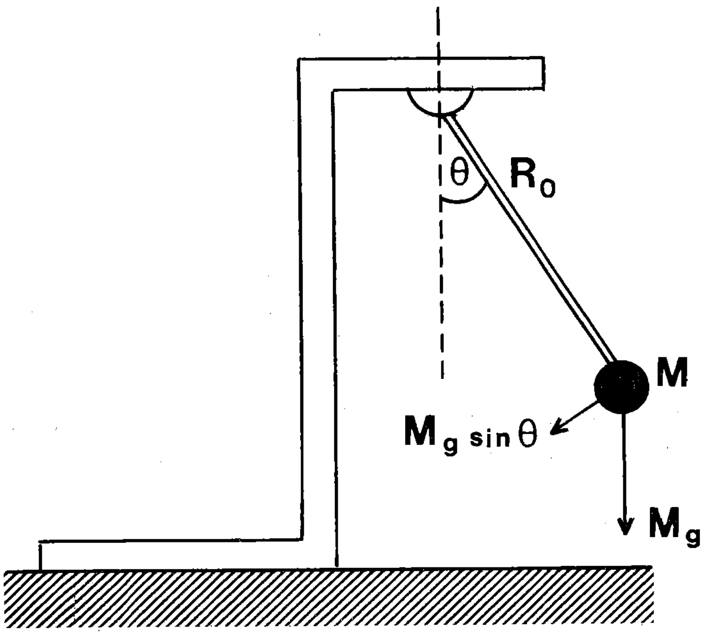

Fig. 3

The quantity

$$l = \frac{K_s}{MR_0}$$

is called the reduced pendulum length of the seismometer. For small θ one may neglect $\ddot{Y}\theta$ in (2.2), and one gets the following equation for the forced motion of the seismometer:

$$\ddot{\theta} + 2\varepsilon_{so}\dot{\theta} + n_s^2\theta = -\frac{\ddot{X}}{l}$$

3 Seismograph with galvanometric recording

In this case a coil is attached to the mass of the seismometer, which moves in a magnetic field. The coil is connected to a sensitive galvanometer.
When the mass is moving, the electromotoric force which is induced in the coil is

$$e_s = N_1 \frac{d\varphi_1}{dt} \tag{3.1}$$

where φ_1 is the magnetic flux and N_1 the number of turns of the coil. We suppose that only ohmic resistances are present in the seismometer-galvanometer network. R_s and R_g are the resistances of seismometer and galvanometer resp.; R_1 and R_2 are variable resistors; R_f is the resistance of a shunt or of a shunt galvanometer. By the movement of the seismometer an e.m.f. is generated in the coil and a current flows in the galvanometer. The deflection of the galvanometer causes an e.m.f. e_g, which counteracts the motion of the seismometer. Therefore the motion of the seismometer-galvanometer system must be described by coupled differential equations.
We will now derive the equations of motion for a seismometer-galvanometer system, in which two galvanometers (indicated by the indices f and g) are connected in parallel coupling to the seismometer (see figure 4).
Let the angles of deviation of the seismometer and hte galvanometers be θ, φ and Φ resp.; K_s, K_g, K_f are the moments of inertia of the seismometer and the galvanometers with respect to their axes of rotation. The equations of motions are

$$\begin{aligned} K_s\ddot{\theta} &= -B_s\dot{\theta} - C_s\theta - MR_0\ddot{X} + M_s \\ K_g\ddot{\varphi} &= -B_g\dot{\varphi} - C_g\varphi + M_g \\ K_f\ddot{\Phi} &= -B_f\dot{\Phi} - C_f\Phi + M_f \end{aligned} \tag{3.2}$$

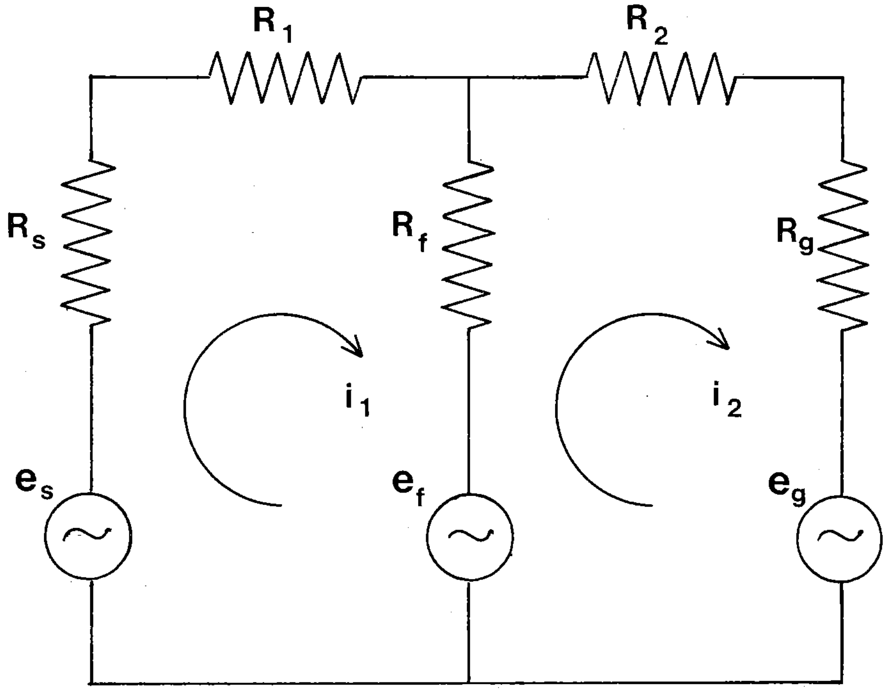

Fig. 4

Here B_s, B_g and B_f are the coefficients of the damping forces; C_s, C_g and C_f are the coefficients of the (quasi-)elastic forces, and $MR_0\ddot{X}$ is the moment of the driving inertial force. This force does not appear in the equations for the galvanometers, as the axis of rotation goes through the centre of gravity of the galvanometer coil. M_s is the moment working on the seismometer which is caused by the current i in the magnetic field of the seismometer. M_g and M_f are moments of forces deflecting the galvanometers, by the action of the currents in the coils of the galvanometers.

The force working on the coil of the seismometer, in which a current i_1 is flowing, is

$$F_s = N_s H_s a\, i_1$$

N_s is the number of turns, H_s is the strength of the magnetic field in the coil, a is the length of the coil.
Because M_s counteracts the motion of the seismometer

$$M_s = -F_s L = -N_s H_s a L i_1 = -G_s i_1 \tag{3.3}$$

L is the radius of the coil. The quantity $G_s = N_s H_s a L$ is the electrodynamic constant of the seismometer.

A current i_2 flowing in the coil of a galvanometer g will react with a force whose moment M_g is:

$$M_g = G_g i_2 \tag{3.4}$$

G_g is the electrodynamic constant of galvanometer g.
For the moment working on the galvanometer f, we get

$$M_f = G_f(i_1 - i_2) \tag{3.5}$$

The currents i_1 and i_2 follow from Kirchhoff's law (see figure 4)

$$\begin{aligned} i_1(R_s + R_1) + (i_1 - i_2)R_f &= e_s + e_f \\ (i_2 - i_1)R_f + i_2(R_2 + R_g) &= e_f + e_g \end{aligned} \tag{3.6}$$

e_s is the e.m.f. induced in the seismometer, e_g and e_f are the counteracting e.m.f.'s induced in the galvanometers

$$e_s = G_s\dot{\theta} \tag{3.7}$$

whereas

$$e_g = -G_g\dot{\varphi} \text{ and } e_f = -G_f\dot{\Phi}\dot{\theta} \qquad \text{(3.8) and (3.9)}$$

From (3.7), (3.8) and (3.9) we get

$$\begin{aligned} i_1 &= \frac{R_g + R_2 + R_f}{a} G_s\dot{\theta} - \frac{R_f}{a} G_g\dot{\varphi} - \frac{R_g + R_2 + 2R_f}{a} G_f\dot{\Phi} \\ i_2 &= \frac{R_f}{a} G_s\dot{\theta} - \frac{R_s + R_1 + R_f}{a} G_g\dot{\varphi} - \frac{R_s + R_1 + 2R_f}{a} G_f\dot{\Phi} \end{aligned} \tag{3.10}$$

By substituting

$$a = (R_s + R_1 + R_f)(R_g + R_2 + R_f) - R_f^2 \tag{3.10a}$$

and

$$A = \frac{R_g + R_2}{a} \qquad B = \frac{R_s + R_1}{a} \qquad C = \frac{R_f}{a} \tag{3.10b}$$

eq. (3.10) can be written as

$$i_1 = (A + C)\,G_s\dot{\theta} - CG_g\dot{\varphi} - (A + 2C)\,G_f\dot{\Phi}$$

$$i_2 = CG_s\dot{\theta} - (B + C)\,G_g\dot{\varphi} - (B + 2C)\,G_f\dot{\Phi}$$

By substituting the values of the currents i_1 and i_2 in (3.3), (3.4) and (3.5), we can express the moments M in the angular velocities $\dot{\theta}$, $\dot{\varphi}$ and $\dot{\Phi}$.

$$M_s = -(A + C)G_s^2\dot{\theta} + CG_gG_s\dot{\varphi} + (A + 2C)G_fG_s\dot{\Phi}$$

$$M_g = CG_sG_g\dot{\theta} - (B + C)G_g^2\dot{\varphi} - (B + 2C)G_fG_g\dot{\Phi}$$

$$M_f = AG_sG_f\dot{\theta} + BG_gG_f\dot{\varphi} - (S - B)G_f^2\dot{\Phi}$$

Introducing these moments in the equation of motion (3.2) we get for the equations of the seismometer and the galvanometers resp.:

$$\ddot{\theta} + \left[2\varepsilon_{so} + (A + C)\frac{G_s^2}{K_s}\right]\dot{\theta} + n_s^2\theta - C\,\frac{G_gG_s}{K_s}\,\dot{\varphi} - (A + 2C)\frac{G_fG_s}{K_s}\dot{\Phi} =$$
$$= -\frac{1}{1}\ddot{X}$$

$$\ddot{\varphi} + \left[2\varepsilon_{go} + (B + C)\frac{G_g^2}{K_g}\right]\dot{\varphi} + n_g^2\varphi - C\frac{G_sG_g}{K_g}\dot{\theta} + (B + 2C)\frac{G_fG_g}{K_g}\dot{\Phi} = 0$$

$$\ddot{\Phi} + \left[2\varepsilon_{fo} + (A - B)\frac{G_f^2}{K_f}\right]\dot{\Phi} + n_f^2\Phi - A\frac{G_sG_f}{K_f}\dot{\theta} - B\frac{G_gG_f}{K_f}\dot{\varphi} = 0$$

By using the following abbreviations

$$2\varepsilon_s = 2\varepsilon_{so} + (A + C)\frac{G_s^2}{K_s} \qquad \sigma_s = C\frac{G_gG_s}{K_s} \qquad \gamma_{sf} = (A + 2C)\frac{G_fG_s}{K_s}$$

$$2\varepsilon_g = 2\varepsilon_{go} + (B + C)\frac{G_g^2}{K_g} \qquad \sigma_g = C\frac{G_sG_g}{K_g} \qquad \gamma_{gf} = (B + 2C)\frac{G_fG_g}{K_g}$$

$$2\varepsilon_f = 2\varepsilon_{fo} + (A - B)\frac{G_f^2}{K_f} \qquad \gamma_{fs} = A\frac{G_sG_f}{K_f} \qquad \gamma_{fg} = B\frac{G_gG_f}{K_f} \tag{3.11}$$

the equations of motion finally become:

$$\ddot{\theta} + 2\varepsilon_s\dot{\theta} + n_s^2\theta - \sigma_s\dot{\varphi} - \gamma_{sf}\dot{\Phi} = -\frac{1}{l}\ddot{X} \tag{3.12}$$

$$\ddot{\varphi} + 2\varepsilon_g\dot{\varphi} + n_g^2\varphi - \sigma_g\dot{\theta} + \gamma_{gf}\dot{\Phi} = 0 \tag{3.13}$$

$$\ddot{\Phi} + 2\varepsilon_f\dot{\Phi} + n_f^2\Phi - \gamma_{fs}\dot{\theta} - \gamma_{fg}\dot{\varphi} = 0 \tag{3.14}$$

We see that the motion of the seismometer is determined not only by the displacement of the ground (X), but also by the motions of the galvanometers (φ and Φ). Likewise the deviation of one of the galvanometers is controlled by the motion of the seismometer and by the deflection of the other galvanometer.

If we suppose the movement $X = X(t)$ of the ground as known, we can calculate the reactions $\theta(t)$, $\varphi(t)$ and $\Phi(t)$ from the eq. (3.12), (3.13) and (3.14). The best way for solving the differential equations of a coupled elastic system is by making use of the Laplace transformation.

4 Laplace transformation

If $F(t)$ is a function which has not an infinite number of maxima, minima and discontinuities, it can be expressed by the integral

$$F(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{g(p)e^{pt}}{p}\,dp \tag{4.1}$$

where

$$g(p) = p\int_0^\infty e^{-pt}F(t)\,dt \tag{4.2}$$

The eq. (4.1) and (4.2) are the Fourier-Mellin equations.
Eq. (4.2) can be written as

$$g(p) = LF(t) \tag{4.3}$$

and eq. (4.1) as

$$F(t) = L^{-1} g(p) \tag{4.4}$$

$g(p)$ is the direct Laplace transform of $F(t)$, and $F(t)$ is the inverse Laplace transform of $g(p)$.

Below some formulas are given, which are needed for the solution of a second order differential equations with constant coefficients.

$$L k F(t) = k L F(t) \text{ where } k = \text{constant} \tag{4.5}$$

$$L \frac{d\,F(t)}{dt} = p L F(t) - p F(0) \tag{4.6}$$

$$L \frac{d^2 F(t)}{dt^2} = p^2 L F(t) - p^2 F(0) - p \left(\frac{d\,F(t)}{dt} \right) t = 0 \tag{4.7}$$

$$L \sin \omega t = \frac{\omega p}{p^2 + \omega^2} \tag{4.8}$$

5 The solution of the equations of motion

After having mentioned these formulas, we can start with solving the equations of the seismometer-galvanometer system (eq. 3.12, 3.13 and 3.14).
We suppose that the motion of the ground is a simple harmonic oscillation

$$X = A \sin \omega t \tag{5.1}$$

The initial conditions are

$$\theta = \dot{\theta} = \varphi = \dot{\varphi} = \Phi = \dot{\Phi} = 0 \text{ if } t = 0 \tag{5.2}$$

Furthermore we put

$$L\theta = z;\ L\varphi = y_1 \text{ and } L\Phi = y_2 \tag{5.3}$$

If we substitute the harmonic ground movement into the equations of motions (3.12), (3.13) and (3.14), and if we write down the Laplace transforms of these equations, we get

$$L\ddot{\theta} + 2\varepsilon_s L\dot{\theta} + n_s^2 L\theta - \sigma_s L\dot{\varphi} - \gamma_{sf} L\dot{\Phi} = \frac{1}{l} A\omega^2 L \sin\omega t$$

$$L\ddot{\varphi} + 2\varepsilon_g L\dot{\varphi} + n_g^2 L\varphi - \sigma_g L\dot{\theta} + \gamma_{gf} L\dot{\Phi} = 0 \tag{5.4}$$

$$L\ddot{\Phi} + 2\varepsilon_f L\dot{\Phi} + n_f^2 L\Phi - \gamma_{fs} L\dot{\theta} - \gamma_{fg} L\dot{\varphi} = 0$$

Now applying the formulas (4.6), (4.7) and (4.8), the equations of motion can be written in the following form:

$$\begin{aligned}
(p^2 + 2\varepsilon_s p + n_s^2)z \quad &- \quad \sigma_s p y_1 \quad - \quad \gamma_{sf} p y_2 = \frac{\frac{1}{l} A\omega^3 p}{p^2 + \omega^2} \\
-\sigma_g p z \quad &+ (p^2 + 2\varepsilon_g p + n_g^2) y_1 + \quad \gamma_{gf} p y_2 = 0 \\
-\gamma_{fs} p z \quad &- \quad \gamma_{fg} p y_1 \quad + (p^2 + 2\varepsilon_f p + n_f^2) y_2 = 0
\end{aligned} \tag{5.5}$$

As we want to know the reactions of the two galvanometers, we have to solve y_1 and y_2 from eq. (5.5).
Putting

$$\begin{vmatrix} (p^2 + 2\varepsilon_s p + n_s^2) & -\sigma_s p & -\gamma_{sf} p \\ -\sigma_g p & (p^2 + 2\varepsilon_g p + n_g^2) & \gamma_{gf} p \\ -\gamma_{fs} p & -\gamma_{fg} p & (p^2 + 2\varepsilon_f p + n_f^2) \end{vmatrix} = a \tag{5.6}$$

we find

$$y_1 = \frac{\sigma_g p(p^2 + 2\varepsilon_f p + n_f^2) - \gamma_{fs}\gamma_{gf} p^2}{a(p^2 + \omega^2)} \frac{1}{l} A\omega^3 p \tag{5.7}$$

$$y_2 = \frac{\gamma_{fs} p(p^2 + 2\varepsilon_g p + n_g^2) + \sigma_g \gamma_{fg} p^2}{a(p^2 + \omega^2)} \frac{1}{l} A\omega^3 p \tag{5.8}$$

According to (5.3) y_1 and y_2 are the direct Laplace transforms of φ and Φ resp., and therefore

$$\varphi = L^{-1} y_1 \text{ and } \Phi = L^{-1} y_2 \tag{5.9}$$

Therefore we can write the deflections of the galvanometers as follows:

$$\varphi = \frac{A}{l}\frac{\omega^3}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{\sigma_g p(p^2+2\varepsilon_f p+n_f^2)-\gamma_{fs}\gamma_{gf}p^2}{a(p^2+\omega^2)}e^{pt}\,dp \tag{5.10}$$

$$\Phi = \frac{A}{l}\frac{\omega^3}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{\gamma_{fs}p(p^2+2\varepsilon_g p+n_g^2)+\sigma_g\gamma_{fg}p^2}{a(p^2+\omega^2)}e^{pt}\,dp \tag{5.11}$$

The solutions (5.10 and (5.11) are the general solutions for the motions of two galvanometers connected to the seismometer. The integrals in (5.10) and (5.11) can be calculated by means of Cauchy's theorem:

$$F(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{g(p)e^{pt}}{p}\,dp = \sum \mathrm{Res}\,\frac{g(p)e^{pt}}{p}$$

If $\dfrac{g(p)e^{pt}}{p} = w(p) = \dfrac{F(p)}{G(p)}$ then

$$\mathrm{Res}\,w(p) = \frac{F(p_0)}{G'(p_0)} \quad \text{where } G'(p) = \frac{dG(p)}{dp}$$

and p_0 is a single root of the equation $G(p) = 0$.

First we will calculate the integral in (5.10); in this case

$$F(p) = [\sigma_g p(p^2+2\varepsilon_f p+n_f^2)-\gamma_{fs}\gamma_{gf}p^2]\,e^{pt} \tag{5.12}$$

and

$$G(p) = a(p^2+\omega^2) \tag{5.13}$$

The determinant $|\ a\ |$ can be written as

$$a = b_0 + b_2p^2 + b_4p^4 + p^6 + c_1p + c_3p^3 + c_5p^5$$

where

$$\begin{aligned}
b_0 &= n_s^2 n_g^2 n_f^2 \\
b_2 &= 4\varepsilon_s\varepsilon_g n_f^2 + 4\varepsilon_s\varepsilon_f n_g^2 + 4\varepsilon_g\varepsilon_f n_s^2 + \gamma_{fg}\gamma_{gf} n_s^2 - \gamma_{fs}\gamma_{sf} n_g^2 - \sigma_s\sigma_g n_f^2 + \\
&\quad + n_s^2 n_g^2 + n_s^2 n_f^2 + n_g^2 n_f^2 \\
b_4 &= 4\varepsilon_s\varepsilon_g + 4\varepsilon_s\varepsilon_f + 4\varepsilon_g\varepsilon_f + \gamma_{fg}\gamma_{gf} - \gamma_{fs}\gamma_{sf} - \sigma_s\sigma_g + n_s^2 + n_g^2 + n_f^2 \qquad (5.14) \\
c_1 &= 2\varepsilon_s n_g^2 n_f^2 + 2\varepsilon_g n_s^2 n_f^2 + 2\varepsilon_f n_s^2 n_g^2 \\
c_3 &= 2\varepsilon_s(n_g^2 + n_f^2) + 2\varepsilon_g(n_s^2 + n_f^2) + 2\varepsilon_f(n_s^2 + n_g^2) + 8\varepsilon_s\varepsilon_g\varepsilon_f + \\
&\quad + 2\varepsilon_s\gamma_{fg}\gamma_{gf} - 2\varepsilon_g\gamma_{fs}\gamma_{sf} - 2\varepsilon_f\sigma_s\sigma_g - \sigma_g\gamma_{fg}\gamma_{sf} + \sigma_s\gamma_{fs}\gamma_{gf} \\
c_5 &= 2\varepsilon_s + 2\varepsilon_g + 2\varepsilon_f
\end{aligned}$$

In order to calculate the residues, we have to put (5.13) equal to zero:

$$(p^6 + c_5p^5 + b_4p^4 + c_3p^3 + b_2p^2 + c_1p + b_0)(p^2 + \omega^2) = 0 \qquad (5.15)$$

The roots of (5.15) are calculated from the equations

$$p^6 + c_5p^5 + b_4p^4 + c_3p^3 + b_2p^2 + c_1p + b_0 = 0 \qquad (5.16)$$

and

$$p^2 + \omega^2 = 0 \qquad (5.17)$$

Solving eq. (5.16) can give three combinations of roots, viz.
a. negative real numbers $(-p_1, -p_2, \ldots, -p_6)$;
b. complex conjugated numbers $(-a_1 + ib_1, -a_1 - ib_1, \ldots)$;
c. a combination of negative real and complex conjugated numbers.

The roots of eq. (5.16) correspond with damped harmonic oscillations or with damped aperiodic deflections of the galvanometers. However, as the movement of the ground is supposed to be harmonic, we are interested in undamped solutions: these are given by eq. (5.17)

$$p_{1,2} = \pm i\omega$$

Now

$$\text{Res } w(p_1) = \frac{1}{2} \frac{\sigma_g(n_f^2 - \omega^2) + i\omega(2\varepsilon_f\sigma_g - \gamma_{fs}\gamma_{gf})}{(b_0 - b_2\omega^2 + b_4\omega^4 - \omega^6) + i(c_1\omega - c_3\omega^3 + c_5\omega^5)} e^{i\omega t}$$

$$\text{Res } w(p_2) = \frac{1}{2} \frac{\sigma_g(n_f^2 - \omega^2) - i\omega(2\varepsilon_f\sigma_g - \gamma_{fs}\gamma_{gf})}{(b_0 - b_2\omega^2 + b_4\omega^4 - \omega^6) - i(c_1\omega - c_3\omega^3 + c_5\omega^5)} e^{-i\omega t}$$

Let be

$$d_1 = \sigma_g(n_f^2 - \omega^2) \qquad f_1 = (2\varepsilon_f\sigma_g - \gamma_{fs}\gamma_{gf})\omega$$
$$b = b_0 - b_2\omega^2 + b_4\omega^4 - \omega^6 \tag{5.18}$$
$$c = c_1\omega - c_3\omega^3 + c_5\omega^5$$

then the solution for the undamped motion of the first galvanometer (g) is

$$\varphi = \tfrac{1}{2}\frac{A}{l}\omega^3 \left[\frac{d_1 + if_1}{b + ic}\mathrm{e}^{i\omega t} + \frac{d_1 - if_1}{b - ic}\mathrm{e}^{-i\omega t}\right]$$

or

$$\varphi = \frac{A}{l}\frac{\omega^3}{b^2 + c^2}\left[(bd_1 + cf_1)\cos\omega t - (bf_1 - cd_1)\sin\omega t\right] \tag{5.19}$$

6 Magnification of the harmonic ground motion

It is not the angular deflection $\varphi(t)$ of the galvanometer which is recorded, but the deviation $x(t)$ of a light spot (figure 5). If the distance between the galvanometer mirror and the recorder is λ, the deviation $x = 2\lambda\varphi$ for small values of φ.

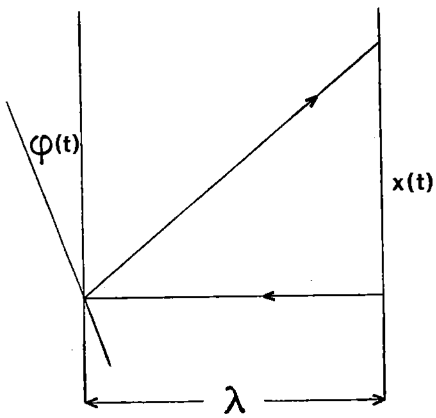

Fig. 5

The deviation x can according to (5.19) be written as

$$x(t) = \frac{2\lambda A}{l}\omega^3\sqrt{\frac{d_1^2+f_1^2}{b^2+c^2}}\sin(\omega_t+\alpha) \tag{6.1}$$

where

$$\alpha = \operatorname{arctg}\frac{bd_1 + cf_1}{b^2 + c^2}$$

The seismometer-galvanometer system is characterized by the magnification which is defined as

$$V = \frac{x_{\max}}{X_{\max}}$$

$X_{\max}$ = amplitude of the harmonic motion of the ground
$x_{\max}$ = amplitude of the deflection recorded by the galvanometer

According to (6.1) and (5.1) the magnification is

$$V = \frac{2\lambda}{l}\omega^3\sqrt{\frac{d_1^2+f_1^2}{b^2+c^2}} \tag{6.2}$$

Substituting (5.18) in (6.2) we get for the magnification of the galvanometer (g):

$$V_g = \frac{2\lambda}{l}\omega^3\sqrt{\frac{\sigma_g^2(n_f^2-\omega^2)^2+(2\varepsilon_f\sigma_g-\gamma_{fs}\gamma_{gf})^2\omega^2}{(b_0-b_2\omega^2+b_4\omega^4-\omega^6)^2+(c_1\omega-c_3\omega^3+c_5\omega^5)^2}} \tag{6.3}$$

and for the magnification of the galvanometer (f):

$$V_f = \frac{2\lambda}{l}\omega^3\sqrt{\frac{\gamma_{fs}^2(n_g^2-\omega^2)^2+(2\varepsilon_g\gamma_{fs}+\sigma_g\gamma_{fg})^2\omega^2}{(b_0-b_2\omega^2+b_4\omega^4-\omega^6)^2+(c_1\omega-c_3\omega^3+c_5\omega^5)^2}} \tag{6.4}$$

From (6.3) we see that the magnification $V_g(\omega)$ has a minimum value for frequencies about $\omega = n_f$. Therefore if two galvanometers are used it is possible to suppress the recording of ground motions for frequencies near the natural frequency of the shunt galvanometer.

The seismograph system with two galvanometers will be dealt with in chapter IV. We will first continue with the theory of the combination of a seismometer coupled

to one galvanometer. It is therefore supposed that the second galvanometer (f) is blocked.
In this case we can put $G_f = 0$, and according to (3.11)

$$\gamma_{fs} = \gamma_{sf} = \gamma_{gf} = \gamma_{fg} = 0 \tag{6.5}$$

By substituting (6.5) in eq. (5.14) the functions b and c in (5.18) are now

$$b = [(n_s^2 - \omega^2)(n_g^2 - \omega^2) - (4\varepsilon_s\varepsilon_g - \sigma_s\sigma_g)\omega^2](n_f^2 - \omega^2) - - 2\varepsilon_f\omega^2[2\varepsilon_s(n_g^2 - \omega^2) + 2\varepsilon_g(n_s^2 - \omega^2)] \tag{6.6}$$

$$c = \omega(n_f^2 - \omega^2)[2\varepsilon_s(n_g^2 - \omega^2) + 2\varepsilon_g(n_s^2 - \omega^2] + + 2\varepsilon_f\omega[(n_s^2 - \omega^2)(n_g^2 - \omega^2) - (4\varepsilon_s\varepsilon_g - \sigma_s\sigma_g)\omega^2] \tag{6.7}$$

Furthermore d_1 and f_1 in (5.18) are

$$d_1 = \sigma_g(n_f^2 - \omega^2) \qquad f_1 = 2\varepsilon_f\sigma_g\omega$$

If we put

$$x = [2\varepsilon_s(n_g^2 - \omega^2) + 2\varepsilon_g(n_s^2 - \omega^2)] \tag{6.8}$$

$$y = [(n_s^2 - \omega^2)(n_g^2 - \omega^2) - (4\varepsilon_s\varepsilon_g - \sigma_s\sigma_g)\omega^2] \tag{6.9}$$

the eq. (6.6) and (6.7) become

$$b = (n_f^2 - \omega^2)y - 2\varepsilon_f\omega^2 x$$

$$c = \omega(n_f^2 - \omega^2)x + 2\varepsilon_f\omega y$$

Now

$$(b^2 + c^2) = (\omega^2x^2 + y^2)[(n_f^2 - \omega^2)^2 + 4\varepsilon_f^2\omega^2]$$

$$d_1^2 + f_1^2 = \sigma_g^2[(n_f^2 - \omega^2)^2 + 4\varepsilon_f^2\omega^2]$$

and the equation of the magnification curve (6.2) will be

$$V = \frac{2\lambda}{l}\,\omega^3\sqrt{\frac{d_1^2 + f_1^2}{b^2 + c^2}} = \frac{2\lambda}{l}\,\sigma_g\omega^3\,\frac{1}{\sqrt{\omega^2x^2 + y^2}} \tag{6.10}$$

Substituting (6.8) and (6.9) in (6.10) we get

$$V = \frac{2\lambda}{l}\sigma_g\omega^3\{4\omega^2[\varepsilon_s(n_g^2 - \omega^2) + \varepsilon_g(n_s^2 - \omega^2)]^2 +$$

$$+ [(n_s^2 - \omega^2)(n_g^2 - \omega^2) - (4\varepsilon_s\varepsilon_g - \sigma_s\sigma_g)\omega^2]^2\}^{-\frac{1}{2}} \quad (6.11)$$

This is the well known equation of the magnification curve for a seismometer which is coupled with one galvanometer.

The formula of the magnification curve was derived for a harmonic ground movement. This is of course never realized, but any ground motion can be composed of harmonic vibrations. Therefore eq. (6.11) gives in principle the reaction of a seismometer-galvanometer system to an arbitrary ground displacement.

REFERENCES

SAWARENSKI, E. F. und KIRNOS, D. P.: 'Elemente der Seismologie und Seismometrie', (Akademie-Verlag, Berlin, 1960).

SUTTON, GEORGE and OLIVER, JACK: 'Seismographs of High Magnification at long Periods', (Annales de Géophysique, no. 4, 1959, p. 423–432).

CHAPTER II. THE DETERMINATION OF THE PARAMETERS OF A SEISMOMETER-GALVANOMETER SYSTEM WITH A GIVEN MAGNIFICATION CURVE

1 Development of an approximated magnification function

An important problem in seismology is the question, how to choose a seismometer-galvanometer combination for obtaining a certain desired magnification of the ground displacement.
Looking at eq. (I.6.11) we see that the magnification V as a function of the frequency (ω) or the period (T) of the ground motion, is determined by the following parameters:

n_s and n_g (own frequencies of seismometer and galvanometer)

ε_s and ε_g (damping constants of seismometer and galvanometer)

$\sigma_s\sigma_g$ (coupling constant of seismometer and galvanometer)

The question is now, how to choose these five parameters in order to approximate an ideal magnification curve (figure 6) as well as possible.

To obtain this, we must write eq. (I.6.11) in an other form.
If we put

$$\begin{aligned} b_0 &= n_s^2 n_g^2 \\ b_2 &= 4\varepsilon_s\varepsilon_g - \sigma_s\sigma_g + n_s^2 + n_g^2 \\ c_1 &= 2\varepsilon_s n_g^2 + 2\varepsilon_g n_s^2 \\ c_3 &= 2\varepsilon_s + 2\varepsilon_g \end{aligned} \tag{1.1}$$

then eq. (I.6.11) becomes

$$V = \frac{2\lambda}{l}\sigma_g\omega^3\{b_0^2 + (c_1^2 - 2b_0b_2)\omega^2 + (b_2^2 + 2b_0 - 2c_1c_3)\omega^4 + + (c_3^2 - 2b_2)\omega^6 + \omega^8\}^{-\frac{1}{2}}$$

or

$$V = \frac{2\lambda}{l}\sigma_g\omega^3\{a_0 + a_2\omega^2 + a_4\omega^4 + a_6\omega^6 + \omega^8\}^{-\frac{1}{2}} \tag{1.2}$$

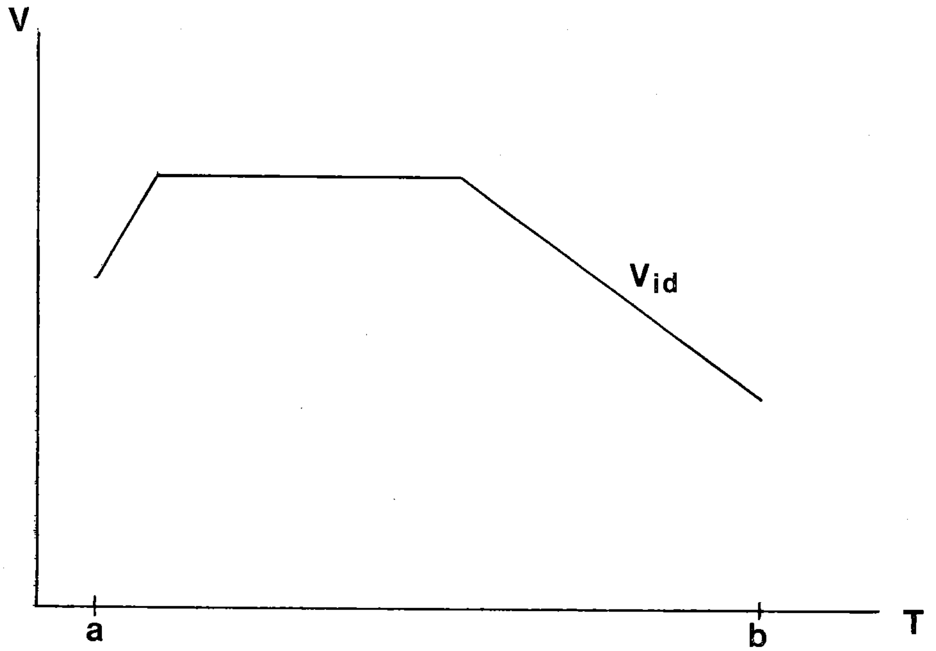

Fig. 6

where

$$a_0 = b_0^2$$
$$a_2 = c_1^2 - 2b_0b_2$$
$$a_4 = 2b_0 + b_2^2 - 2c_1c_3$$
$$a_6 = c_3^2 - 2b_2 \tag{1.3}$$

By taking the reciprocal of (1.2) and by squaring, we get

$$V^{-2} = \left(\frac{2\lambda}{l}\sigma_g\right)^{-2}(a_0\omega^{-6} + a_2\omega^{-4} + a_4\omega^{-2} + a_6 + \omega^2)$$

and if

$$\left(\frac{2\lambda}{l}\sigma_g\right)^{-2} = c \tag{1.4}$$

then

$$V^{-2} = a_0c\omega^{-6} + a_2c\omega^{-4} + a_4c\omega^{-2} + a_6c + c\omega^2 \tag{1.5}$$

If we would succeed in determining the coefficients (a_0, a_2, . . ., and c) in eq. (1.5) in such a way that the difference between the ideal curve V_{id} and the curve which follows from (1.5) is minimal, we would have reached our goal. From the coefficients a_0, a_2, a_4, a_6 we can find b_0, b_2, c_1, c_3 by means of (1.3), and then the parameters of the seismograph can be determined from eq. (1.1).
Usually the magnification is presented as a function of the period T; therefore we write instead of (1.5)

$$V^{-2} = (2\pi)^2 cT^{-2} + a_6 c + \frac{a_4 c}{(2\pi)^2} T^2 + \frac{a_2 c}{(2\pi)^4} T^4 + \frac{a_0 c}{(2\pi)^6} T^6 \qquad (1.6)$$

Now it is required that

$$M = \int_a^b [V_{id}^{-2} - V^{-2}(T)]^2 dT \qquad (1.7)$$

is minimal; a and b are appropriate limits of the magnification curve (see figure 6). We must, therefore, investigate for what values of a_0, a_2, a_4, a_6 and c the formula (1.7) is minimal. This is the case when

$$\frac{\partial M}{\partial a_0} = 0, \frac{\partial M}{\partial a_2} = 0, \ldots, \frac{\partial M}{\partial c} = 0 \qquad (1.8)$$

However, when the conditions (1.8) are applied, it appears that we have to calculate the coefficients of (1.6) from a set of non-linear equations, and this is an almost impossible task.
In order to avoid this difficulty we introduce the following functions:

$$\begin{aligned}
\Phi_{-2} &= b_{-2,-2}T^{-2} + b_{-2,0} \\
\Phi_0 &= +1 \\
\Phi_2 &= b_{2,-2}T^{-2} + b_{2,0} + b_{2,2}T^2 \\
\Phi_4 &= b_{4,-2}T^{-2} + b_{4,0} + b_{4,2}T^2 + b_{4,4}T^4 \\
\Phi_6 &= b_{6,-2}T^{-2} + b_{6,0} + b_{6,2}T^2 + b_{6,4}T^4 + b_{6,6}T^6
\end{aligned} \qquad (1.9)$$

Instead of eq. (1.6) we write

$$V^{-2} = \beta_{-2}\Phi_{-2}(T) + \beta_0\Phi_0(T) + \beta_2\Phi_2(T) + \beta_4\Phi_4(T) + \beta_6\Phi_6(T) \qquad (1.10)$$

by which V^{-2} is written as a linear combination of polynomials of powers of T. Now it is clear that we can determine the coefficients β_{2k} so that (1.10) is identical with (1.6). If we substitute the functions of (1.9) in (1.10) we find that the coefficients of eq. (1.6) must be

$$
\begin{aligned}
c &= \frac{1}{(2\pi)^2}(\beta_{-2}b_{-2,-2} + \beta_2 b_{2,-2} + \beta_4 b_{4,-2} + \beta_6 b_{6,-2}) \\
a_6 c &= (\beta_{-2}b_{-2,0} + \beta_0 + \beta_2 b_{2,0} + \beta_4 b_{4,0} + \beta_6 b_{6,0}) \\
a_4 c &= (2\pi)^2 \ (\beta_2 b_{2,2} + \beta_4 b_{4,2} + \beta_6 b_{6,2}) \\
a_2 c &= (2\pi)^4 \ (\beta_4 b_{4,4} + \beta_6 b_{6,4}) \\
a_0 c &= (2\pi)^6 \ \beta_6 b_{6,6}
\end{aligned}
\qquad (1.11)
$$

Now we require that Φ_{-2}, Φ_0, Φ_2, Φ_4 and Φ_6 have to satisfy the following conditions of normalized orthogonal functions

$$
\begin{aligned}
&\int_a^b \Phi_{2i}(T)\,\Phi_{2j}(T)\,dT = 0 \\
&\qquad\qquad\qquad\qquad (i, j = -1, 0, 1, 2, 3) \\
&\Phi_{2i}(T_0) = 1
\end{aligned}
\qquad (1.12)
$$

Where T_0 is an arbitrary value of the period T.
By means of the conditions (1.12) we can calculate the coefficients (1.9). According to (1.12) in case of $i = 0$ and $j = -1$

$$\int_a^b \Phi_0(T)\,\Phi_{-2}(T)\,dT = \int_a^b (b_{-2,-2}T^{-2} + b_{-2,0})\,dT = 0$$

or

$$[\int_a^b T^{-2}dt]b_{-2,-2} + [\int_a^b dT]b_{-2,0} = 0 \qquad (1.13)$$

and

$$\frac{1}{T_0^2}b_{-2,-2} + b_{-2,0} = 1$$

Eq. (1.13) gives the two coefficients of the first polynomial Φ_{-2} of (1.9). Likewise it is required that in case $i = 1, j = -1$ and $i = 1, j = 0$

$$\int_a^b \Phi_2\Phi_{-2}dT = 0 \quad \int_a^b \Phi_2\Phi_0 dT = 0 \text{ and } \Phi_2(T_0) = 1$$

or

$$[b_{-2,-2}\int_a^b T^{-4}dT + b_{-2,0}\int_a^b T^{-2}dT]\,b_{2,-2} +$$

$$+ [b_{-2,-2}\int_a^b T^{-2}dT + b_{-2,0}\int_a^b dT]b_{2,0} +$$

$$+ [b_{-2,-2}\int_a^b dT + b_{-2,0}\int_a^b T^2dT]b_{2,2} = 0$$

$$[\int_a^b T^{-2}dT]b_{2,-2} + [\int_a^b dT]b_{2,0} + [\int_a^b T^2dT]b_{2,2} = 0$$

$$\frac{1}{T_0^2}b_{2,-2} + b_{2,0} + T_0^2 b_{2,2} = 1 \tag{1.14}$$

Eq. (1.14) gives the three coefficients of the third polynomial Φ_2 of (1.9). Proceeding along these lines, we get for the fourth polynomial four equations for the four unknown coefficients, and for the fifth polynomial Φ_6 five equations for the five unknown coefficients.

The condition for the best approximation of the ideal magnification curve was according to (1.7) and (1.10)

$$M = \int_a^b [V_{id}^{-2} - \sum_{-1}^{+3} \beta_{2k}\Phi_{2k}]^2 dT$$

and

$$\frac{\partial M}{\partial \beta_{2k}} = -2\int_a^b [V_{id}^{-2} - \sum_{-1}^{+3} \beta_{2j}\Phi_{2j}]\,\Phi_{2k}dT = 0$$

Taking into account the orthogonality relation (1.12) of the polynomials, we can write

$$\int_a^b V_{id}^{-2}\Phi_{2k}dT = \beta_{2k}\int_a^b \Phi_{2k}^2 dT$$

$$\beta_{2k} = \frac{\int_a^b V_{id}^{-2}\,\Phi_{2k}\,dT}{\int_a^b \Phi_{2k}^2 dT} \tag{1.15}$$

Eq. (1.15) gives us the coefficients of the approximated magnification curve (1.10).

2 Calculation of the coefficients of the approximated magnification function

We will calculate the coefficients of the orthogonal polynomials of eq. (1.9) as indicated in (1.13) and (1.14). We choose the following values for the limits of integration a and b and for the period T_0:

$$a = 5 \text{ sec} \quad b = 100 \text{ sec} \quad \text{and} \quad T_0 = 100 \text{ sec}$$

The result of the calculation is:

$$\begin{array}{lll}
b_{-2,-2} = -526 \cdot 10^{0} & b_{2,-2} = +416 \cdot 10^{-1} & b_{4,-2} = -347 \cdot 10^{-1} \\
b_{-2,0} = +105 \cdot 10^{-2} & b_{2,0} = -666 \cdot 10^{-3} & b_{4,0} = +628 \cdot 10^{-3} \\
 & b_{2,2} = +166 \cdot 10^{-6} & b_{4,2} = -454 \cdot 10^{-6} \\
 & & b_{4,4} = +492 \cdot 10^{-10}
\end{array}$$

$$\begin{array}{l}
b_{6,-2} = +320 \cdot 10^{-1} \\
b_{6,0} = -645 \cdot 10^{-3} \\
b_{6,2} = +869 \cdot 10^{-6} \\
b_{6,4} = -234 \cdot 10^{-9} \\
b_{6,6} = +163 \cdot 10^{-13}
\end{array} \tag{2.1}$$

The orthogonal polynomials of (1.9) are shown in figure 7.

The integrals in the denominator of eq. (1.15) can be calculated by means of eq. (1.9).

$$\begin{array}{ll}
\int_5^{100} \Phi_{-2}^2(T)dT = 633 \cdot 10^{0} & \int_5^{100} \Phi_4^2(T)dT = 108 \cdot 10^{-1} \\
\int_5^{100} \Phi_0^2(T)dT = 950 \cdot 10^{-1} & \int_5^{100} \Phi_6^2(T)dT = 753 \cdot 10^{-2} \\
\int_5^{100} \Phi_2^2(T)dT = 190 \cdot 10^{-1} &
\end{array} \tag{2.2}$$

The integrals in the numerator of eq. (1.15) must be calculated numerically or, if possible, analytically. When the β's have been calculated by means of (1.15), we can find the coefficients of (1.6) from (1.11), and we can calculate the magnification as a function of the period of the ground motion.

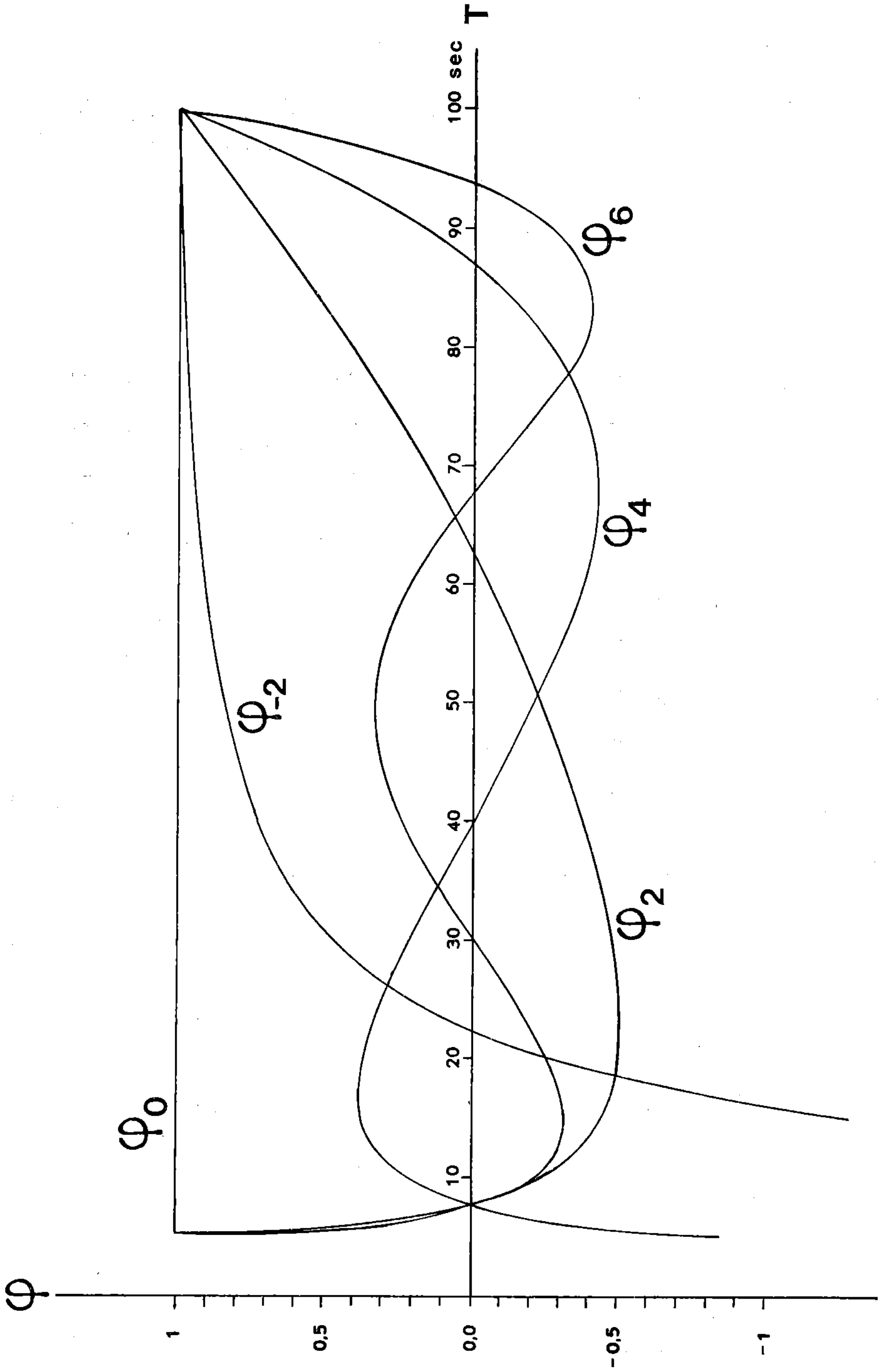

Fig. 7 Polynomials to approximate the ideal magnification curve.

Now that c, a_0, a_2, a_4 and a_6 are known, we are able to determine, making use of eq. (1.3) and (1.1), the constants of the seismometer and the galvanometer for a chosen magnification curve. This goes as follows.

Starting from (1.3) we put

$$a_4 - 2b_0 = a'_4$$

As $b_0 = \sqrt{a_0}$, a'_4 is known. Elimination of c_1 and c_3 in eq. (1.3) leads to

$$b_2^4 - (2a'_4 + 16b_0)b_2^2 - 8(a_2 + b_0a_6)b_2 + (a_4'^2 - 4a_2a_6) = 0 \tag{2.3}$$

With the aid of (1.1), (1.3) and (2.3) we can calculate b_0, b_2, c_1 and c_3. Eq. (2.3) has four roots for b_2, but only the roots which are positive real numbers, are usable. The reason is that b_2 must be positive, as

$$b_2 = 4\varepsilon_s\varepsilon_g - \sigma_s\sigma_g + n_s^2 + n_g^2$$

and after eq. (I.3.11)

$$2\varepsilon_s = 2\varepsilon_{so} + (A + C)\frac{G^2}{K_s} \text{ and } 2\varepsilon_g = 2\varepsilon_{go} + (B + C)\frac{G^2}{K_g}$$

$$\sigma_s = C\frac{G_gG_s}{K_s} \quad \sigma_g = C\frac{G_gG_s}{K_g}$$

from which follows: $4\varepsilon_s\varepsilon_g > \sigma_s\sigma_g$, so that $b_2 > 0$.

If $R_f = \infty$ (see figure 4) it follows from (I.3.10) that $A = B = 0$, and if we neglect the mechanical damping ($\varepsilon_{so} = \varepsilon_{go} = 0$) we get from (I.3.11)

$$4\varepsilon_s\varepsilon_g = \sigma_s\sigma_g \tag{2.4}$$

This is the condition for a seismometer-galvanometer system with maximal coupling. If $R_f = 0$, i.e. if the shunt resistance can be neglected, $C = 0$, and it follows from (I.3.11) that

$$\sigma_s\sigma_g = 0 \tag{2.5}$$

We see that in all possible cases b_2 must be positive. Furthermore eq. (1.1) shows that c_1 and c_3 must be positive.

The conditions (2.4) and (2.5) represent the two limits between which a seismograph system can operate. It is possible to determine from eq. (1.1), (2.4) and (2.5) the free periods of the seismometer and the galvanometer for the two limits, between which one has to choose the constants of seismometer and galvanometer in order to obtain a prescribed magnification curve.

If $4\varepsilon_s\varepsilon_g = \sigma_s\sigma_g$, than according to (1.1)

$$n_s^2 n_g^2 = n_s^2(b_2 - n_s^2) = b_0$$

or

$$n_s^4 - b_2 n_s^2 + b_0 = 0$$

with the solution

$$n_s^2 = \frac{b_2 \pm \sqrt{(b_2^2 - 4b_0)}}{2}$$

and

$$n_g^2 = \frac{b_0}{n_s^2} \tag{2.6}$$

Eq. (2.6) gives us the free periods of seismometer and galvanometer with maximal coupling.

If $\sigma_s\sigma_g = 0$, than according to (1.1)

$$b_2 = 4\varepsilon_s\varepsilon_g + n_s^2 + n_g^2 \tag{2.7}$$

From the first, second and third formula of (1.1) it follows that

$$4\varepsilon_s\varepsilon_g = \frac{c_1 c_3(n_s^2 + n_g^2) - (c_1^2 + b_0 c_3^2)}{(n_s^2 + n_g^2)^2 - 4b_0} \tag{2.8}$$

Taking

$$z = (n_s - n_g)^2 \tag{2.9}$$

then

$$n_s^2 + n_g^2 = z + 2\sqrt{b_0} \tag{2.10}$$

Substituting (2.8) in (2.7) we get

$$\frac{c_1c_3(n_s^2+n_g^2)-(c_1^2+b_0c_3^2)}{(n_s^2+n_g^2)^2-4b_0}+(n_s^2+n_g^2)=b_2$$

and substituting (2.10)

$$z^3+(6\sqrt{b_0}-b_2)z^2+(c_1c_3+8b_0-4b_2\sqrt{b_0})z+$$
$$+(2c_1c_3\sqrt{b_0}-c_1^2-b_0c_3^2)=0 \qquad (2.11)$$

Eq. (2.11) gives values for $z=(n_s-n_g)^2$. These values, combined with $b_0=n_s^2n_g^2$, give the free period of seismometer and galvanometer for the case of minimal coupling.

3 Application of a given magnification curve

We will apply the theory to an ideal magnification curve, consisting of rectilinear segments (see figure 6). In this case the function V can have the following forms

$$V=mT+b \quad \text{or} \quad V=c \qquad (3.1)$$

Now the integrals in the orthogonality relation (1.15) can be calculated analytically. Taking into account the polynomials of (1.9), the following integrals appear:

$$\int_{T_1}^{T_2}\frac{dT}{(mT+b)^2T^2}=\frac{1}{b^2}\left[m\left(\frac{1}{V_1}-\frac{1}{V_2}\right)+\frac{2m}{b}\ ln\frac{V_2}{V_1}+\right.$$
$$\left.+\left(\frac{1}{T_1}-\frac{1}{T_2}\right)-\frac{2m}{b}\ ln\frac{T_2}{T_1}\right]$$

$$\int_{T_1}^{T_2}\frac{dT}{(mT+b)^2}=\frac{1}{m}\left(\frac{1}{V_1}-\frac{1}{V_2}\right)$$

$$\int_{T_1}^{T_2}\frac{T^2dT}{(mT+b)^2}=\frac{b^2}{m^3}\left[\left(\frac{1}{V_1}-\frac{1}{V_2}\right)-\frac{2}{b}\ ln\frac{V_2}{V_1}+\frac{1}{b^2}(V_2-V_1)\right]$$

$$\int_{T_1}^{T_2}\frac{T^4dT}{(mT+b)^2}=\frac{b^4}{m^5}\left[\left(\frac{1}{V_1}-\frac{1}{V_2}\right)-\frac{4}{b}\ ln\frac{V_2}{V_1}+\frac{6}{b^2}(V_2-V_1)-\right.$$

$$- \frac{2}{b^3}(V_2^2 - V_1^2) + \frac{1}{3}\ \frac{1}{b^4}(V_2^3 - V_1^3)\Bigg]$$

$$\int_{T_1}^{T_2} \frac{T^6 dT}{(mT+b)^2} = \frac{b^6}{m^7}\Bigg[\left(\frac{1}{V_1} - \frac{1}{V_2}\right) - \frac{6}{b}\ ln\ \frac{V_2}{V_2} + \frac{15}{b^2}(V_2 - V_1) -$$

$$- \frac{10}{b^3}(V_2^2 - V_1^2) + \frac{5}{b^4}(V_2^3 - V_1^3) - \frac{1}{2b^5}(V_2^4 - V_1^4) +$$

$$+ \frac{1}{5b^6}(V_2^5 - V_1^5)\Bigg]$$

or if $V = c$

$$\frac{1}{c^2}\int_{T_1}^{T_2} \frac{dT}{T^2} = \frac{1}{c^2}\left(\frac{1}{T_1} - \frac{1}{T_2}\right); \qquad \frac{1}{c^2}\int_{T_1}^{T_2} dT = \frac{1}{c^2}(T_2 - T_1)$$

$$\frac{1}{c^2}\int_{T_1}^{T_2} T^2 dT = \frac{1}{3c_2^2}(T_2^3 - T_1^3); \qquad \frac{1}{c^2}\int_{T_1}^{T_2} T^4 dT = \frac{1}{5c^2}(T_2^5 - T_1^5)$$

We will compose the ideal curve from four segments:

a. $T_1 = 5$ sec $\quad V_1 = 340$
$\quad T_2 = 10$ sec $\quad V_2 = 475 \quad V = 27T + 205 \quad$ for $5 \leqslant T \leqslant 10$ sec

b. $T_1 = 10$ sec $\quad V_1 = 475$
$\quad T_2 = 20$ sec $\quad V_2 = 565 \quad V = 9T + 385 \quad$ for $10 \leqslant T \leqslant 20$ sec

c. $T_1 = 20$ sec
$\quad T_2 = 65$ sec $\quad V = 565 \quad$ for $20 \leqslant T \leqslant 65$ sec

d. $T_1 = 65$ sec
$\quad T_2 = 100$ sec $\quad V = -\frac{37}{7}T + \frac{6360}{7}$ for $65 \leqslant T \leqslant 100$ sec

Using the above mentioned integrals we get

$$\int_5^{100} V_{id}^{-2}\, T\, dT = 994 \cdot 10^{-9} \qquad \int_5^{100} V_{id}^{-2}\, T^4\, dT = 985 \cdot 10^1$$

$$\int_5^{100} V_{id}^{-2}\,dT = 372\cdot 10^{-6} \qquad \int_5^{100} V_{id}^{-2}T^6\,dT = 752\cdot 10^5$$

$$\int_5^{100} V_{id}^{-2}T^2\,dT = 148\cdot 10^{-2}$$

Now it is a simple procedure to calculate the integrals in (1.15); the coefficients of eq. (1.10) appear to be:

$$\beta_{-2} = -207\cdot 10^{-9}$$
$$\beta_0 = +392\cdot 10^{-8}$$
$$\beta_2 = +204\cdot 10^{-8}$$
$$\beta_4 = +117\cdot 10^{-8}$$
$$\beta_6 = +121\cdot 10^{-9}$$

Eq. (1.11) gives the coefficients of (1.6)

$$c = +399\cdot 10^{-8} \qquad a_0 = +306\cdot 10^{-10}$$
$$a_2 = +113\cdot 10^{-7}$$
$$a_4 = -837\cdot 10^{-6}$$
$$a_6 = +751\cdot 10^{-3}$$

We know from (1.1) and (1.3) that

$a_0 = b_0^2$ whereas $b_0 = n_s^2 n_g^2$ so that

$b_0 = 175\cdot 10^{-6}$ and $n_s n_g = 132\cdot 10^{-4}$ or $T_s T_g = 2990$

Eq. (2.3) becomes

$$b_2^4 - 424\cdot 10^{-6}\, b_2^2 - 114\cdot 10^{-5}\, b_2 - 326\cdot 10^{-7} = 0$$

The four roots are

$$b_2 = +114\cdot 10^{-3}$$
$$b_2 = -283\cdot 10^{-4}$$
$$b_2 = -428\cdot 10^{-4} \pm 910\cdot 10^{-4} i$$

We know from (1.1) that b_2 must be real and positive, so we can only use the first root for further calculations.
For the first limiting case ($\sigma_s\sigma_g = 4\varepsilon_s\varepsilon_g$) we get from (2.6) for the free periods of the seismometer and the galvanometer

$$n_s = 335 \cdot 10^{-3} \quad \text{or} \quad T_s = 18{,}8 \text{ sec}$$

$$n_g = 395 \cdot 10^{-4} \quad \text{or} \quad T_g = 159 \text{ sec}$$

From (1.3) it follows that

$$c_1 = 715 \cdot 10^{-5}$$

$$c_3 = 989 \cdot 10^{-3}$$

So that eq. (2.11) becomes

$$z^3 - 345 \cdot 10^{-4} z^2 + 245 \cdot 10^{-5} z - 351 \cdot 10^{-7} = 0$$

whose three roots are

$$z = +163 \cdot 10^{-4} \quad \text{and} \quad z = +909 \cdot 10^{-5} \pm 455 \cdot 10^{-4} i$$

According to (2.9) only the positive real root is usable. Now we can calculate from $z = (n_s - n_g)^2$ and $b_0 = n_s^2 n_g^2$ the free periods of seismometer and galvanometer for the second limiting case, viz. $\sigma_s\sigma_g = 0$.
They are

$$n_s = 195 \cdot 10^{-3} \quad \text{or} \quad T_s = 32{,}2 \text{ sec}$$

$$n_g = 677 \cdot 10^{-4} \quad \text{or} \quad T_g = 92{,}8 \text{ sec}$$

In all possible combinations of a seismometer and a galvanometer the periods must satisfy the conditions:

$$T_s \cdot T_g = 2990 \qquad 18{,}8 \text{ sec} < T_s < 32{,}2 \text{ sec}$$

$$92{,}8 \text{ sec} < T_g < 159 \text{ sec}$$

Furthermore

$$b_0 = 175 \cdot 10^{-6} \qquad c_1 = 715 \cdot 10^{-5}$$

$$b_2 = 114 \cdot 10^{-3} \qquad c_3 = 989 \cdot 10^{-3}$$

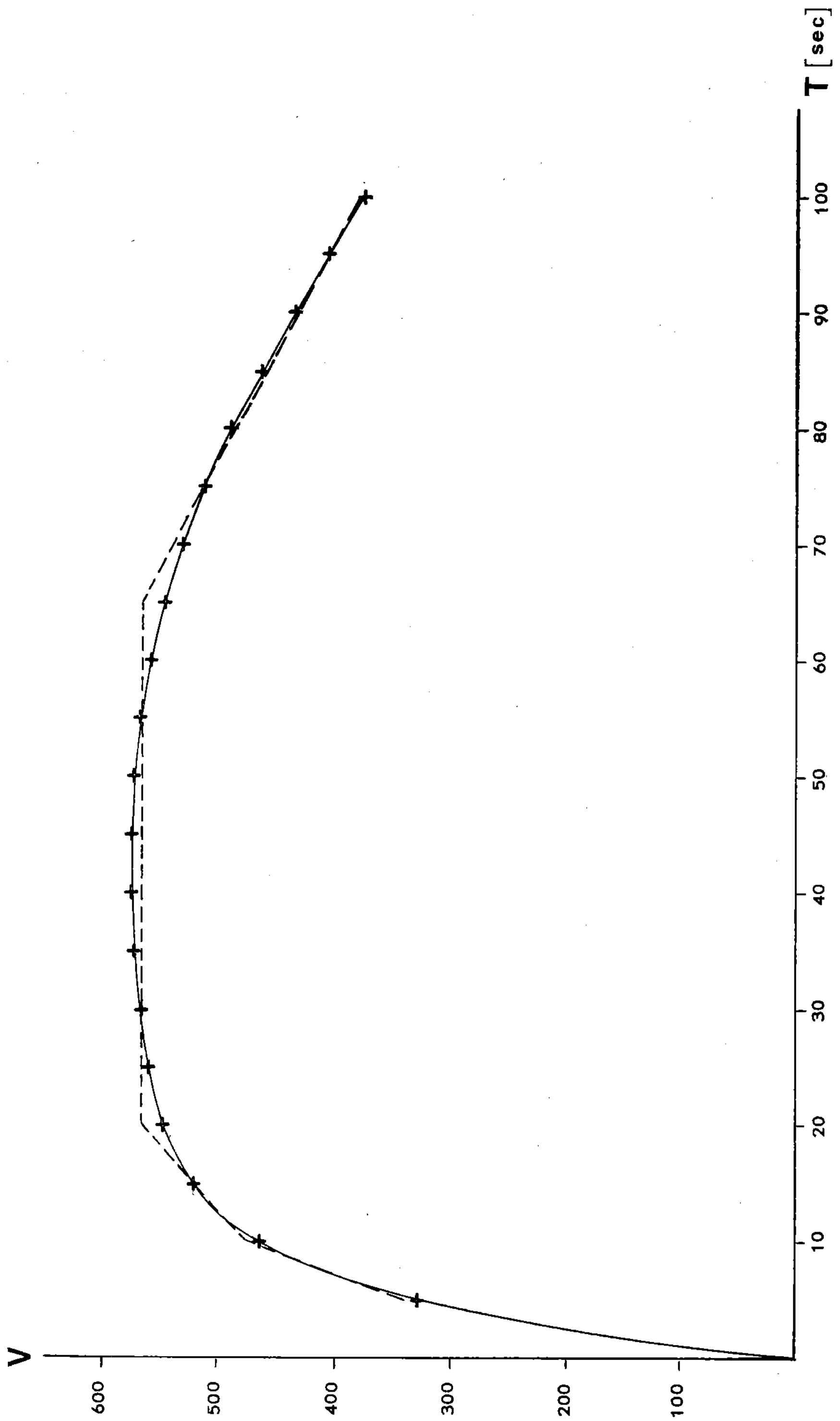

Fig. 8 Ideal magnification curve (broken line) and calculated curve (full line).

If we have for instance a galvanometer with period $T_g = 100$ sec, then the seismometer must be adjusted to a period $T_s = 29{,}9$ sec. The other parameters in (1.1) must have the following values

$$\varepsilon_s = 0{,}454 \qquad \varepsilon_g = 0{,}040 \qquad \sigma_s \sigma_g = 0{,}007$$

Supposing a favourable choice of seismometer and galvanometer, these parameters can be attained by regulating the resistances in the seismometer-galvanometer chain. Figure 8 shows the agreement between the ideal curve and the magnification curve calculated from the chosen parameters of the seismometer-galvanometer system.

CHAPTER III. THEORY OF THE GALITZIN AND BENIOFF SEISMOGRAPHS

The Galitzin and Benioff seismographs belong to the widely used instruments; they are based on some special relations between the parameters. In the Galitzin system the seismometer and the galvanometer have the same periods and both are critically damped. In the Benioff system the seismometer and the galvanometer are also critically damped, but the periods of galvanometer and seismometer differ greatly, so that the magnification curve has a broad maximum.

1 The Galitzin seismograph

The conditions for the Galitzin seismograph can be formulated as follows:

$$b_0 = c_0^4 \quad b_2 = 6c_0^2 \quad c_1 = 4c_0^3 \quad c_3 = 4c_0 \tag{1.1}$$

where c_0 is a positive constant.

The coefficients of the equation (II.2.11) for the periods of seismometer and galvanometer in case of minimal coupling ($\sigma_s\sigma_g = 0$) now get the values

$$6\sqrt{b_0} - b_2 = 0; c_1c_3 + 8b_0 - 4b_2\sqrt{b_0} = 0; 2c_1c_3\sqrt{b_0} - c_1^2 - b_0c_3^2 = 0$$

and the equation (II.2.11) is reduced to $z^3 = 0$.
As $z = (n_s - n_g)^2$, it follows that $n_s = n_g$, and after (II.1.1) and (1.1)

$$n_s = n_g = \varepsilon_s = \varepsilon_g = c_0 \tag{1.2}$$

These are the well known Galitzin conditions (equal periods of galvanometer and seismometer, both critically damped) for the limit of zero coupling ($\sigma_s\sigma_g = 0$).

Now we consider the other limit, viz. maximal coupling ($4\varepsilon_s\varepsilon_g = \sigma_s\sigma_g$). From (II.2.6) and (II.1.1) we get

$$n_s = \sqrt{3 + 2\sqrt{2}}\ c_0 \qquad n_g = \sqrt{3 - 2\sqrt{2}}\ c_0$$

$$\varepsilon_s = \left(1 + \frac{\sqrt{2}}{2}\right)c_0 \qquad \varepsilon_g = \left(1 - \frac{\sqrt{2}}{2}\right)c_0 \tag{1.3}$$

We see that if the coupling is different from zero, the periods of seismometer and galvanometer are no longer the same, and the damping is no longer critical. Seismographs with these properties are called 'false' Galitzins.

Taking for example $c_0 = \frac{2\pi}{12}$, then we get for the case $\sigma_s\sigma_g = 0$ the values $T_s = 12$ sec and $T_g = 12$ sec, and for the case $4\varepsilon_s\varepsilon_g = \sigma_s\sigma_g$ the values $T_s = 28{,}97$ sec and $T_g = 4{,}97$ sec.

If we choose the periods of the seismometer and the galvanometer between these limits and if we calculate the other parameters from eq. (II.1.1), taking into account the Galitzin conditions (1.1), we will get identical magnification curves for all combinations of seismometers and galvanometers which can be realised.

Taking for example $T_s = 16$ sec, then $T_g = 9$ sec according to eq. (II.1.1) and condition (1.1). The other parameters appear to be

$$\varepsilon_s = 0{,}38 \qquad \varepsilon_g = 0{,}67 \qquad \sigma_s\sigma_g = 0{,}73 \cdot 10^{-2}$$

so that the damping constants and the coupling are determined.

We see, that an infinite number of 'false' Galitzins are possible and that the theory enables us to choose the limits between which we have to take the parameters in order to construct instruments which have the same magnification function as the 'true' Galitzin.

2 The Benioff seismograph

The conditions for the Benioff seismograph can be formulated as

$$\begin{aligned} b_0 &= c_{1,0}^2 c_{2,0}^2 & b_2 &= 4c_{1,0}c_{2,0} + c_{1,0}^2 + c_{2,0}^2 \\ c_1 &= 2c_{1,0}c_{2,0}^2 + 2c_{2,0}c_{1,0}^2 & c_3 &= 2c_{1,0} + 2c_{2,0} \end{aligned} \qquad (2.1)$$

where $c_{1,0}$ and $c_{2,0}$ are arbitrary positive constants.

First we want to find the parameters for the case $\sigma_s\sigma_g = 0$. The coefficients of eq. (II.2.11) are now

$$6\sqrt{b_0} - b_2 = -(c_{1,0} - c_{2,0})^2 \qquad c_1c_3 + 8b_0 - 4b_2\sqrt{b_0} = 0$$
$$2c_1c_3\sqrt{b_0} - c_1^2 - b_0c_3^2 = 0$$

and the equation (II.2.11) gets the form

$$z^3 - (c_{1,0} - c_{2,0})^2 z^2 = 0 \tag{2.2}$$

Eq. (2.2) has two real solutions

$$z = (c_{1,0} - c_{2,0})^2 \quad \text{and} \quad z = 0 \tag{2.3}$$

Using eq. (II.1.1) and (II.2.9) we get from the first solution

$$n_s = \varepsilon_s = c_{1,0}; \quad n_g = \varepsilon_g = c_{2,0}; \quad \sigma_s \sigma_g = 0 \tag{2.4}$$

These are the well known Benioff conditions for the case of zero coupling. The second solution of eq. (2.3) is $z = 0$. In this case we get from eq (II.1.1)

$$n_s = n_g = \sqrt{c_{1,0} c_{2,0}}; \quad \varepsilon_s = \varepsilon_g = \frac{c_{1,0} + c_{2,0}}{2}; \quad \sigma_s \sigma_g = 0 \tag{2.5}$$

The combination (2.5) is called 'false' Benioff in the literature (see: Willmore, 1960). However, it can be shown that the combination (2.5) is indeed false, and cannot be realised. For a real seismograph $\sigma_s \sigma_g$ must be positive, whereas the coupling near the second solution appears to be negative.

To show this, we write $\sigma_s \sigma_g$ as a function of z. It follows from (II.1.1) and (II.2.11) that

$$z^3 - (c_{1,0} - c_{2,0})^2 z^2 - \sigma_s \sigma_g z^2 - 4 c_{1,0} c_{2,0} \sigma_s \sigma_g z = 0$$

or

$$\sigma_s \sigma_g = \frac{z^2 - (c_{1,0} - c_{2,0})^2 z}{z + 4 c_{1,0} c_{2,0}} \quad \text{where } z = (n_s - n_g)^2$$

It is obvious that the function $\sigma_s \sigma_g$ has a maximum for $z = 0$. As for this value $\sigma_s \sigma_g = 0$, it follows that near the point $z = 0$, $\sigma_s \sigma_g$ must be negative and this means that the parameter combination (2.5) cannot be realised.

For the case of maximal coupling ($\sigma_s \sigma_g = 4 \varepsilon_s \varepsilon_g$) eq. (II.1.1) gives the following values for the periods of seismometer and galvanometer

$$n_g = \frac{(c_{1,0} + c_{2,0}) + \sqrt{[(c_{1,0} + c_{2,0})^2 + 4 c_{1,0} c_{2,0}]}}{2}$$

$$n_s = \frac{-(c_{1,0} + c_{2,0}) + \sqrt{[(c_{1,0} + c_{2,0})^2 + 4 c_{1,0} c_{2,0}]}}{2} \tag{2.6}$$

The damping constants, belonging to these periods follow from eq. (II.1.1). Just as in the case of the Galitzin seismograph, an infinite number of true 'false' Benioff seismographs are possible, provided the parameters are chosen between the limits defined by the conditions $\sigma_s\sigma_g = 0$ and $\sigma_s\sigma_g = 4\varepsilon_s\varepsilon_g$.

We will now calculate the parameters for a special case. Taking for example

$$c_1 = \frac{2\pi}{16} \quad \text{and} \quad c_{2,0} = 2\pi$$

we find for the case of zero coupling

$$n_s = \varepsilon_s = \frac{2\pi}{16} \quad n_g = \varepsilon_g = 2\pi \quad \text{or} \quad T_s = 16 \text{ sec and } T_g = 1 \text{ sec}$$

In the case of maximal coupling we find

$$n_g = \frac{2\pi}{0{,}894} \quad n_s = \frac{2\pi}{17{,}9} \quad \text{or} \quad T_s = 17{,}9 \text{ sec and } T_g = 0{,}89 \text{ sec}$$

The damping constants are

$$\varepsilon_s = 0{,}318 \qquad \varepsilon_g = 6{,}35$$

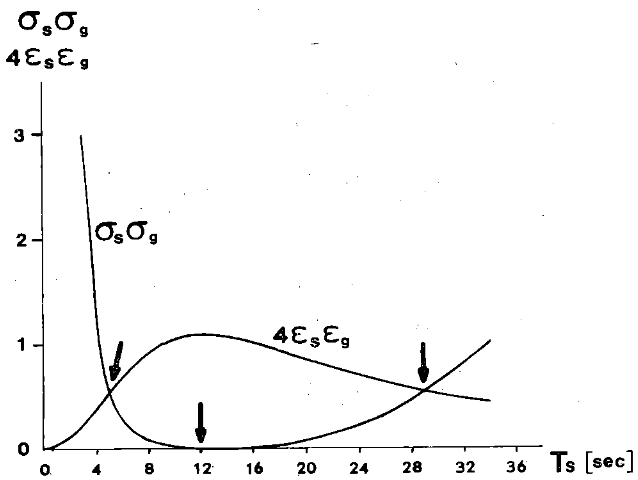

Fig. 9 Values of $\sigma_s\sigma_g$ and $4\varepsilon_s\varepsilon_g$ for a Galitzin seismograph. The arrows indicate the periods between which the seismograph can operate.

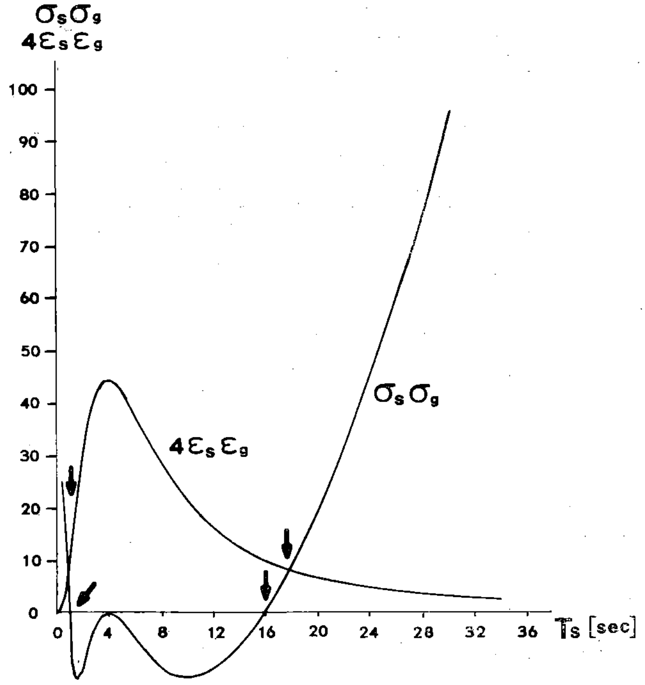

Fig. 10 Values of $\sigma_s\sigma_g$ and $4\varepsilon_s\varepsilon_g$ for a Benioff seismograph. The arrows indicate the periods between which the seismograph can operate.

Figures 9 and 10 show the values of $\sigma_s\sigma_g$ and $4\varepsilon_s\varepsilon_g$ as functions of T_s, taking into account the conditions for the Galitzin and Benioff seismographs ((1.1) and (2.1) resp.). The arrows mark the limits between which the instruments can be realised. From figure 10 it is clear that solution (2.5) is 'false' indeed.

As all formulas for the periods of seismometer and galvanometer are symmetrical with respect to n_s and n_g, we can substitute n_s by n_g and conversely.

REFERENCES

GRENET, G. and COULOMB, J.: 'Nouveaux principes de construction des séismographes electromagnétiques', Ann. Phys., Paris, sér. 11, **3**, 321 (1935).

BENIOFF, H. 'A new vertical seismograph', Bull. Seismol. Soc. Amer. **22**, 155 (1932).

WILLMORE, P. L. 'The detection of earth movements', in Methodes and Techniques in Geophysics, ed. S. K. Runcorn, Intern. Publ. Inc., New York, London (1960).

CHAPTER IV. THE MAGNIFICATION OF A SEISMOGRAPH SYSTEM WITH TWO GALVANOMETERS

Generally a seismograph system consists of a seismometer connected to one galvanometer. However, the connection with two galvanometers can be important, if it is required to minimize the magnification in a part of the seismic spectrum. This can be necessary if the seismograph is set up in a region with strong microseismic activity. The second galvanometer can in this case be used as a filter which suppresses the magnification of the microseisms.
Below the magnification of a system of a seismometer connected to two galvanometers in series is discussed (figure 11).

1 The equation of the magnification curve

According to Kirchoff's law we can write

$$i_1 R_s + (i_1 - i_2)R_1 = e_s$$

$$(i_2 - i_1)R_1 + i_2(R_f + R_g) = e_f + e_g$$

Hence

$$i_1 = (A + C)\,e_s + Ce_g + Ce_f$$

$$i_2 = Ce_s + (B + C)\,e_g + (B + C)\,e_f$$

where

$$A = \frac{R_f + R_g}{a} \qquad B = \frac{R_s}{a} \qquad C = \frac{R_1}{a} \tag{1.1}$$

$$a = (R_s + R_1)(R_f + R_g + R_1) - R_1^2$$

According to eq. (I.3.7), (I.3.8) and (I.3.9)

$$e_s = G_s\dot{\theta} \qquad e_g = -G_g\dot{\varphi} \qquad e_f = -G_f\dot{\Phi}$$

and according to (I.3.3), (I.3.4) and (I.3.5)

$$M_s = -G_s i_1 \qquad M_g = G_g i_2 \qquad M_f = G_f i_2$$

or

$$M_s = -(A + C)G_s^2\dot{\theta} + C\quad {}_gG_s\dot{\varphi} + CG_fG_s\dot{\Phi}$$

$$M_g = CG_sG_g\dot{\theta} - (B + C)G_g^2\dot{\varphi} - (B + C)G_fG_g\dot{\Phi}$$

$$M_f = +CG_sG_f\dot{\theta} - (B + C)G_gG_f\dot{\varphi} - (B + C)G_f^2\dot{\Phi}$$

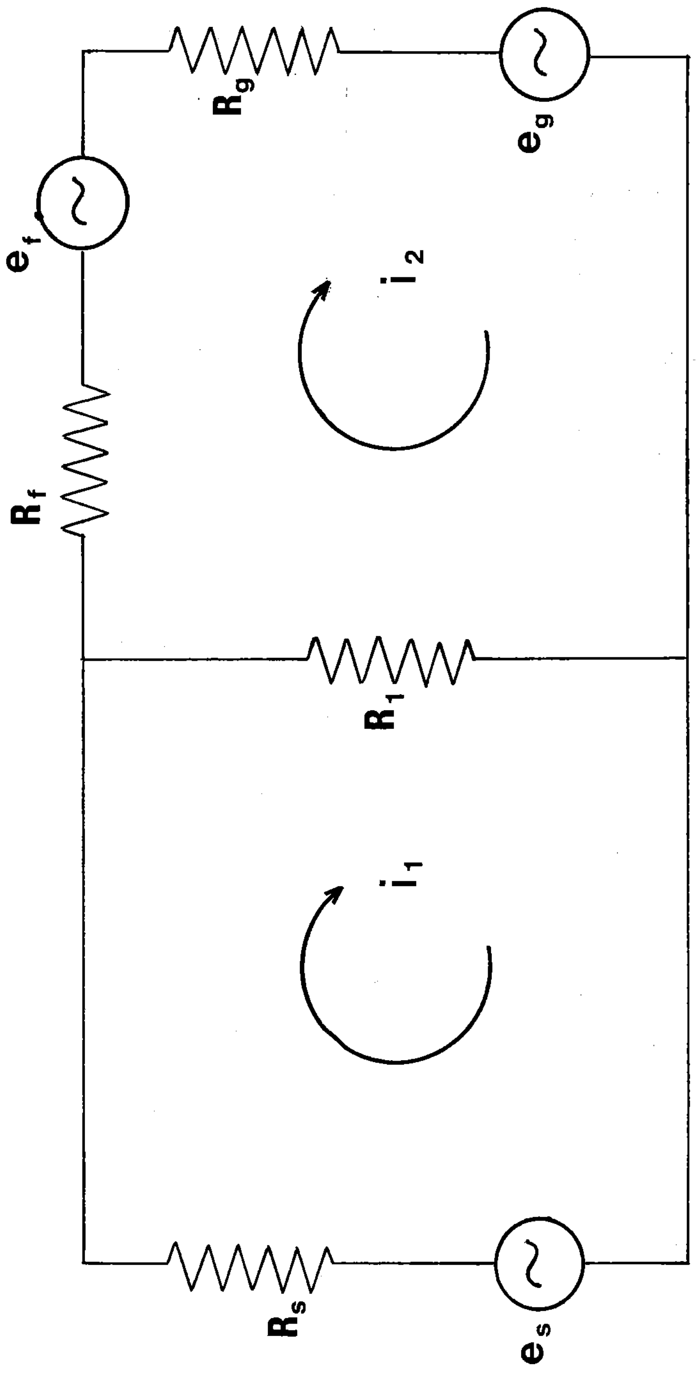

Fig. 11

Substituting these moments into eq. (I.3.2) we get the equations of motion:

$$\ddot{\theta} + 2\varepsilon_s\dot{\theta} + n_s^2\theta - \gamma_{sg}\dot{\varphi} - \gamma_{sf}\dot{\Phi} = -\frac{1}{l}\ddot{x}$$

$$\ddot{\varphi} + 2\varepsilon_g\dot{\varphi} + n_g^2\varphi - \gamma_{gs}\dot{\theta} + \gamma_{gf}\dot{\Phi} = 0 \qquad (1.2)$$

$$\ddot{\Phi} + 2\varepsilon_f\dot{\Phi} + n_f^2\Phi - \gamma_{fs}\dot{\theta} + \gamma_{fg}\dot{\varphi} = 0$$

where

$$2\varepsilon_s = 2\varepsilon_{so} + (A + C)\frac{G_s^2}{K_s} \quad \gamma_{sg} = C\frac{G_sG_g}{K_s} \quad \gamma_{sf} = C\frac{G_sG_f}{K_s}$$

$$2\varepsilon_g = 2\varepsilon_{go} + (B + C)\frac{G_g^2}{K_g} \quad \gamma_{gs} = C\frac{G_gG_s}{K_g} \quad \gamma_{gf} = (B + C)\frac{G_gG_f}{K_g} \qquad (1.3)$$

$$2\varepsilon_f = 2\varepsilon_{fo} + (B + C)\frac{G_f}{K_f} \quad \gamma_{fs} = C\frac{G_fG_s}{K_f} \quad \gamma_{fg} = (B + C)\frac{G_fG_s}{K_f}$$

Comparing eq. (1.2) and (1.3) with (I.3.11), (I.3.12), (I.3.13) and (I.3.14) and supposing that the mechanical damping of the filter galvanometer can be neglected ($\varepsilon_{fo} = 0$) we can write down the equation of the magnification of the galvanometer (g) with the aid of (I.6.3)

$$V = \frac{2\lambda}{l}\,\omega^3 \sqrt{\frac{\gamma_{gs}^2(n_f^2 - \omega^2)^2}{(b_0 - b_2\omega^2 + b_4\omega^4 - \omega^6) + (c_1\omega - c_3\omega^3 + c_5\omega^5)^2}} \qquad (1.4)$$

because $2\varepsilon_f\sigma_g - \gamma_{fs}\gamma_{gf} = 2\varepsilon_f\gamma_{gs} - \gamma_{fs}\gamma_{gf} = 0$

For $\omega = n_f$, $V = 0$; the system will prevent recording of that frequency of the ground motion which is equal to the frequency of the filter galvanometer.
In analogy with eq. (I.5.14) we can write for the parameters in (1.4):

$$b_0 = n_s^2 n_g^2 n_f^2$$

$$b_2 = (4\varepsilon_s\varepsilon_g - \gamma_{sg}\gamma_{gs})n_f^2 + (4\varepsilon_s\varepsilon_f - \gamma_{sf}\gamma_{fs})n_g^2 + (4\varepsilon_g\varepsilon_f - \gamma_{gf}\gamma_{fg})n_s^2 + \\ + n_s^2n_g^2 + n_s^2n_f^2 + n_g^2n_f^2$$

$$b_4 = (4\varepsilon_s\varepsilon_g - \gamma_{sg}\gamma_{gs}) + (4\varepsilon_s\varepsilon_f - \gamma_{sf}\gamma_{fs}) + (4\varepsilon_g\varepsilon_f - \gamma_{gf}\gamma_{fg}) + n_s^2 + n_g^2 + n_f^2$$

$$c_1 = 2\varepsilon_s n_g^2 n_f^2 + 2\varepsilon_g n_s^2 n_f^2 + 2\varepsilon_f n_s^2 n_g^2$$

$$c_3 = 2\varepsilon_s(n_g^2 + n_f^2 - \gamma_{gf}\gamma_{fg}) + 2\varepsilon_g(n_s^2 + n_f^2 - \gamma_{sf}\gamma_{fs}) + $$
$$+ 2\varepsilon_f(n_s^2 + n_g^2 - \gamma_{sg}\gamma_{gs}) + 8\varepsilon_s\varepsilon_g\varepsilon_f + 2\gamma_{sf}\gamma_{gs}\gamma_{fg}$$
$$c_5 = 2\varepsilon_s + 2\varepsilon_g + 2\varepsilon_f \tag{1.5}$$

2 Development of an approximated magnification function

We are now in a position to apply the method developed in chapter II to the system with two galvanometers.
It follows from eq. (1.4) that

$$(n_f^2 - \omega^2)^2 V^{-2} = ca_2\omega^{-6} + ca_4\omega^{-4} + ca_4\omega^{-2} + ca_6 + ca_8\omega^2 + $$
$$+ ca_{10}\omega^4 + c\omega^6 \tag{2.1}$$

where

$$a_o = b_0^2 \qquad c = \left(\frac{2\lambda}{l}\gamma_{gs}\right)^{-2}$$
$$a_2 = c_1^2 - 2b_0b_2$$
$$a_4 = 2b_0b_4 + b_2^2 - 2c_1c_3$$
$$a_6 = 2c_1c_5 + c_3^2 - 2b_0 - 2b_2b_4$$
$$a_8 = 2b_2 + b_4^2 - 2c_3c_5$$
$$a_{10} = c_5^2 - 2b_4 \tag{2.2}$$

In order to determine the coefficients ($a_0, a_2, \ldots, a_{10}$ and c) of eq. (2.1) by the methods of least squares, we introduce the following functions

$$\Phi_{-6} = b_{-6,-6}\omega^{-6} + b_{-6,-4}\omega^{-4} + b_{-6,-2}\omega^{-2} + 1$$
$$\Phi_{-4} = b_{-4,-4}\omega^{-4} + b_{-4,-2}\omega^{-2} + 1$$
$$\Phi_{-2} = b_{-2,-2}\omega^{-2} + 1$$
$$\Phi_0 = 1$$
$$\Phi_2 = b_{2,-6}\omega^{-6} + b_{2,-4}\omega^{-4} + b_{2,-2}\omega^{-2} + 1 + b_{2,2}\omega^2$$
$$\Phi_4 = b_{4,-6}\omega^{-6} + b_{4,-4}\omega^{-4} + b_{4,-2}\omega^{-2} + 1 + b_{4,2}\omega^2 + b_{4,4}\omega^4$$
$$\Phi_6 = b_{6,-6}\omega^{-6} + b_{6,-4}\omega^{-4} + b_{6,-2}\omega^{-2} + 1 + b_{6,2}\omega^2 + b_{6,4}\omega^4 + $$
$$+ b_{6,6}\omega^6 \tag{2.3}$$

for which we require that

$$\int_{\omega_1}^{\omega_2} \frac{\Phi_{2j}(\omega)\Phi_{2k}(\omega)}{(n_f^2-\omega^2)^2}\,d\omega = 0 \quad \text{if } i \neq j \qquad (i, j = -3, \ldots)\ -2,\ -1,\ 0,\ 1,\ 2,\ 3) \tag{2.4}$$

Instead of (2.1) we can write

$$\begin{aligned}(n_f^2-\omega^2)^2 V^{-2} = \beta_{-6}\Phi_{-6}(\omega) + \beta_{-4}\Phi_{-4}(\omega) + \beta_{-2}\Phi_{-2}(\omega) + \beta_0\Phi_0(\omega) + \\ + \beta_2\Phi_2(\omega) + \beta_4\Phi_4(\omega) + \beta_6\Phi_6(\omega)\end{aligned} \tag{2.5}$$

It follows from (2.1), (2.3) and (2.5) that the coefficients (a_0, a_2, ..., a_{10} and c) in eq. (2.1) and eq. (2.2) must satisfy the equations:

$$\begin{aligned}
ca_0 &= (\beta_{-6}b_{-6,-6} + \beta_2 b_{2,-6} + \beta_4 b_{4,-6} + \beta_6 b_{6,-6}) \\
ca_2 &= (\beta_{-6}b_{-6,-4} + \beta_{-4}b_{-4,-4} + \beta_2 b_{2,-4} + \beta_4 b_{4,-4} + \beta_6 b_{6,-4}) \\
ca_4 &= (\beta_{-6}b_{-6,-2} + \beta_{-4}b_{-4,-2} + \beta_{-2}b_{-2,-2} + \beta_2 b_{2,-2} + \beta_4 b_{4,-2} + \\
&\quad + \beta_6 b_{6,-2}) \\
ca_6 &= (\beta_{-6} + \beta_{-4} + \beta_{-2} + \beta_0 + \beta_2 + \beta_4 + \beta_6) \\
ca_8 &= (\beta_2 b_{2,2} + \beta_4 b_{4,2} + \beta_6 b_{6,2}) \\
ca_{10} &= (\beta_4 b_{4,4} + \beta_6 b_{6,4}) \\
c &= \beta_6 b_{6,6}
\end{aligned} \tag{2.6}$$

Now the question is, how to choose the seven parameters (β_{-6}, β_{-4}, β_{-2}, β_0, β_2, β_4 and β_6) of eq. (2.5) in order to approximate a given ideal magnification curve (figure 12) as well as possible.

It is required that

$$Z = \int_{\omega_1}^{\omega_2} \frac{[(n_f^2-\omega^2)^2 V_{id}^{-2} - (n_f^2-\omega^2)^2 V^{-2}]^2}{(n_f^2-\omega^2)^2}\,d\omega \tag{2.7}$$

is minimal, so according to (2.5)

$$Z = \int_{\omega_1}^{\omega_2} \frac{[(n_f^2-\omega^2)^2 V_{id}^{-2} - \sum \beta_{2k}\Phi_{2k}]^2}{(n_f^2-\omega^2)^2}\,d\omega$$

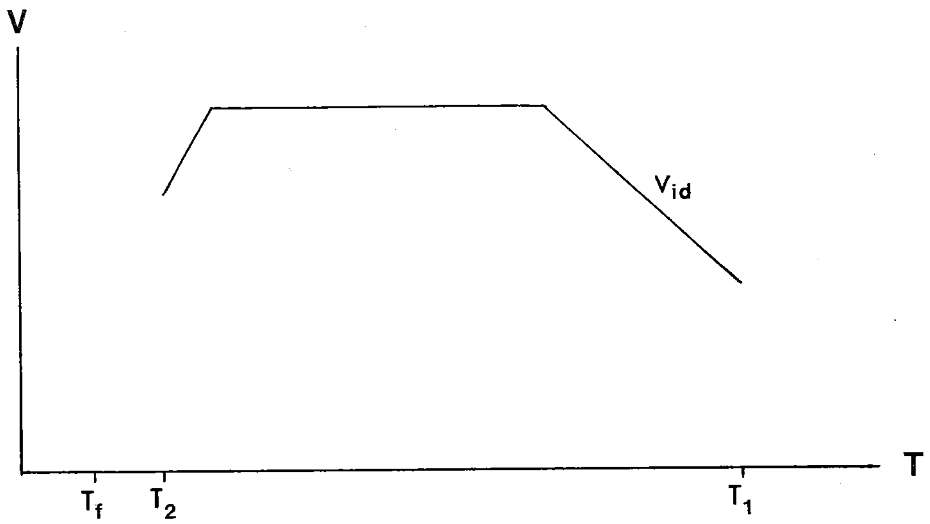

Fig. 12

and

$$\frac{\partial Z}{\partial \beta_{2k}} = -2 \int_{\omega_1}^{\omega_2} \frac{[(n_f^2 - \omega^2)^2 V_{id}^{-2} - \sum \beta_{2j} \Phi_{2j}]}{(n_f^2 - \omega^2)^2} \Phi_{2k} d\omega = 0$$

According to (2.4) we get for the parameters of eq. (2.5)

$$\beta_{2k} = \frac{\int_{\omega_1}^{\omega_2} V_{id}^{-2} \Phi_{2k} d\omega}{\int_{\omega_1}^{\omega_2} \frac{\Phi_{2k}^2 \, d\omega}{(n_f^2 - \omega^2)^2}} \quad (k = -3, -2, -1, 0, 1, 2, 3) \tag{2.8}$$

The coefficients of (2.3) can be calculated using the orthogonality condition (2.4) in the same way as described in chapter II.

3 Calculation of the magnification

We choose for the limits of integration ω_1 and ω_2 and for n_f the values

$$n_f = 1 \; (T_f = 2\pi); \; \omega_2 = 1/2 \; (T_2 = 4\pi); \; \omega_1 = 1/16 \; (T_1 = 32\pi)$$

and we obtain for the coefficients the following values

$$
\begin{array}{lll}
b_{-6,0} = +1 & b_{-4,0} = +1 & b_{-2,0} = +1 \\
b_{-6,-2} = -112015\cdot 10^{-6} & b_{-4,-2} = -756026\cdot 10^{-7} & b_{-2,-2} = -362774\cdot 10^{-7} \\
b_{-6,-4} = +160811\cdot 10^{-8} & b_{-4,-4} = +424612\cdot 10^{-9} & \\
b_{-6,-6} = -517844\cdot 10^{-11} & & \\
 & & \\
b_{6,6} = -279908\cdot 10^{-3} & & \\
b_{6,4} = +128013\cdot 10^{-3} & b_{4,4} = +350739\cdot 10^{-4} & \\
b_{6,2} = -186915\cdot 10^{-4} & b_{4,2} = -115531\cdot 10^{-4} & b_{2,2} = -499249\cdot 10^{-5} \\
b_{6,0} = +1 & b_{4,0} = +1 & b_{2,0} = +1 \\
b_{6,-2} = -190994\cdot 10^{-7} & b_{4,-2} = -246093\cdot 10^{-7} & b_{2,-2} = -348386\cdot 10^{-7} \\
b_{6,-4} = +132406\cdot 10^{-9} & b_{4,-4} = +193975\cdot 10^{-9} & b_{2,-4} = +318694\cdot 10^{-9} \\
b_{6,-6} = -290423\cdot 10^{-12} & b_{4,-6} = -457600\cdot 10^{-12} & b_{2,-6} = -813050\cdot 10^{-12}
\end{array}
\tag{3.1}
$$

Now the orthogonal polynomials of (2.3) are known, so we can calculate the integrals in the denominator of (2.8).

$$
\begin{array}{ll}
\displaystyle\int_{\omega_1}^{\omega_2} \frac{\Phi_6^2}{(1-\omega^2)^2}\,d\omega = 825419\cdot 10^{-9} & \displaystyle\int_{\omega_1}^{\omega_2} \frac{\Phi_{-2}^2}{(1-\omega^2)^2}\,d\omega = 128686\cdot 10^{-5} \\
\displaystyle\int_{\omega_1}^{\omega_2} \frac{\Phi_4^2}{(1-\omega^2)^2}\,d\omega = 338399\cdot 10^{-8} & \displaystyle\int_{\omega_1}^{\omega_2} \frac{\Phi_{-4}^2}{(1-\omega^2)^2}\,d\omega = 760531\cdot 10^{-6} \\
\displaystyle\int_{\omega_1}^{\omega_2} \frac{\Phi_2^2}{(1-\omega^2)^2}\,d\omega = 176752\cdot 10^{-7} & \displaystyle\int_{\omega_1}^{\omega_2} \frac{\Phi_{-6}^2}{(1-\omega^2)^2}\,d\omega = 460378\cdot 10^{-6} \\
\displaystyle\int_{\omega 1}^{\omega_2} \frac{\Phi_0^2}{(1-\omega^2)^2}\,d\omega = 545323\cdot 10^{-6} &
\end{array}
\tag{3.2}
$$

If the ideal magnification curve is composed of rectilinear segments, the integrals in the numerator of (2.8) can be calculated analytically.
The following integrals are of importance.

a. if $V = \dfrac{a + c\omega}{\omega}$ (second and fourth segment in fig. 13), then

$$\int V^{-2}\omega^6 d\omega = \frac{1}{c^2}\left[\frac{\omega^7}{7} - 2\left(\frac{a}{c}\right)\frac{\omega^6}{6} + 3\left(\frac{a}{c}\right)^2\frac{\omega^5}{5} - 4\left(\frac{a}{c}\right)^3\frac{\omega^4}{4} + \right.$$

$$+ 5\left(\frac{a}{c}\right)^4\frac{\omega^3}{3} - 6\left(\frac{a}{c}\right)^5\frac{\omega^2}{2} + 7\left(\frac{a}{c}\right)^6\omega - 8\left(\frac{a}{c}\right)^7 \ln(a + c\omega) -$$

$$\left. - \left(\frac{a}{c}\right)^7\frac{a}{(a + c\omega)}\right]$$

$$\int V^{-2}\omega^4 d\omega = \frac{1}{c^2}\left[\frac{\omega^5}{5} - 2\left(\frac{a}{c}\right)\frac{\omega^4}{4} + 3\left(\frac{a}{c}\right)^2\frac{\omega^3}{3} - 4\left(\frac{a}{c}\right)^3\frac{\omega^2}{2} + \right.$$

$$\left. + 5\left(\frac{a}{c}\right)^4\omega - 6\left(\frac{a}{c}\right)^5 \ln(a + c\omega) - \left(\frac{a}{c}\right)^5\frac{a}{(a + c\omega)}\right]$$

$$\int V^{-2}\omega^2 d\omega = \frac{1}{c^2}\left[\frac{\omega^3}{3} - 2\left(\frac{a}{c}\right)\frac{\omega^2}{2} + 3\left(\frac{a}{c}\right)^2\omega - \right.$$

$$\left. - 4\left(\frac{a}{c}\right)^3 \ln(a + c\omega) - \left(\frac{a}{c}\right)^3\frac{a}{(a + c\omega)}\right]$$

$$\int V^{-2} d\omega = \frac{1}{c^2}\left[\omega - 2\left(\frac{a}{c}\right) \ln(a + c\omega) - \left(\frac{a}{c}\right)\frac{a}{(a + c\omega)}\right]$$

$$\int V^{-2}\omega^{-2} d\omega = -\frac{1}{c}\,\frac{1}{(a + c\omega)}$$

$$\int V^{-2}\omega^{-4} d\omega = -\frac{c}{a^2}\,\frac{1}{(a + c\omega)} + \frac{2c}{a^3} \ln\frac{(a + c\omega)}{\omega} - \frac{1}{a^2}\,\frac{1}{\omega}$$

$$\int V^{-2}\omega^{-6} d\omega = -\frac{c^3}{a^4}\,\frac{1}{(a + c\omega)} + \frac{4c^3}{a^5} \ln\frac{(a + c\omega)}{\omega} - \frac{3c^2}{a^4}\,\frac{1}{\omega} +$$

$$+ \frac{c}{a^3}\,\frac{1}{\omega^2} - \frac{1}{3a^2}\,\frac{1}{\omega^3} \qquad (3.3)$$

b. if $V = V_0$ (third segment in fig. 13), then

$$\int V^{-2}\omega^6 d\omega = V_0^{-2}\,\frac{\omega^7}{7};\ \int V^{-2}\omega^4 d\omega = V_0^{-2}\,\frac{\omega^5}{5};\ \int V^{-2}\omega^2 d\omega = V_0^{-2}\,\frac{\omega^3}{3}$$

$$\int V^{-2}d\omega = V_0^{-2}\omega; \int V^{-2}\omega^{-2}d\omega = -V_0^{-2}\frac{1}{\omega}; \int V^{-2}\omega^{-4}d\omega = -V_0^{-2}\frac{1}{3\omega^3}$$

$$\int V^{-2}\omega^{-6}d\omega = -V_0^{-2}\frac{1}{5\omega^5} \tag{3.4}$$

c. if $V = \dfrac{a}{\omega}$ (first segment in fig. 13), than

$$\int V^{-2}\omega^6 d\omega = \frac{1}{a^2}\,\frac{\omega^9}{9};\quad \int V^{-2}\omega^4 d\omega = \frac{1}{a^2}\,\frac{\omega^7}{7};$$

$$\int V^{-2}\omega^2 d\omega = \frac{1}{a^2}\,\frac{\omega^5}{5}$$

$$\int V^{-2}d\omega = \frac{1}{a^2}\,\frac{\omega^3}{3};\quad \int V^{-2}\omega^{-2}d\omega = \frac{1}{a^2}\omega;\quad \int V^{-2}\omega^{-4}d\omega = -\frac{1}{a^2}\,\frac{1}{\omega}$$

$$\int V^{-2}\omega^{-6}d\omega = -\frac{1}{a^2}\,\frac{1}{3\omega^3} \tag{3.5}$$

We will define the four segments as follows:

1. $V^{-2} = \dfrac{1}{(0{,}38)^2}\omega^2$ for $4\pi \leqslant T \leqslant 5\pi$

2. $V^{-2} = \dfrac{\omega^2}{(\frac{1}{30} + \frac{13}{15}\omega)^2}$ for $5\pi \leqslant T \leqslant 8\pi$

3. $V^{-2} = 1$ for $8\pi \leqslant T \leqslant 20\pi$

4. $V^{-2} = \dfrac{\omega^2}{(-\frac{7}{80} + \frac{45}{24}\omega)^2}$ for $20\pi \leqslant T \leqslant 32\pi$

Eq. (3.3), (3.4) and (3.5) give the following values for the integrals

$$\int_{1/16}^{1/2} V^{-2}\omega^6 d\omega = 155365\cdot 10^{-8} \qquad \int_{1/16}^{1/2} V^{-2}\omega^{-2} d\omega = 209030\cdot 10^{-4}$$

$$\int_{1/16}^{1/2} V^{-2}\omega^4 d\omega = 829776\cdot 10^{-8} \qquad \int_{1/16}^{1/2} V^{-2}\omega^{-4} d\omega = 274906\cdot 10^{-2}$$

$$\int_{1/16}^{1/2} V^{-2}\omega^2 d\omega = 516357\cdot 10^{-7} \qquad \int_{1/16}^{1/2} V^{-2}\omega^{-6} d\omega = 507373\cdot 10^{0}$$

$$\int_{1/16}^{1/2} V^{-2}\, d\omega = 520388\cdot 10^{-6}$$

These values, together with (2.3), (3.1), (3.2) and (2.8) give the coefficients of eq. (2.5)

$$\begin{aligned}
\beta_6 &= -14002 \cdot 10^{6-} \\
\beta_4 &= +453906 \cdot 10^{-6} \\
\beta_2 &= -115800 \cdot 10^{-6} \\
\beta_0 &= +954275 \cdot 10^{-6} \\
\beta_{-2} &= -184884 \cdot 10^{-6} \\
\beta_{-4} &= +141150 \cdot 10^{-6} \\
\beta_{-6} &= -60069 \cdot 10^{-6}
\end{aligned} \tag{3.6}$$

According to (2.1) and (2.6) we can make use of (3.6) and (3.1) for calculating the magnification curve which forms the best approximation of the ideal curve (figure 13).

$$\begin{aligned}
(1-\omega^2)^2 V^{-2} = &+201573 \cdot 10^{-12} \omega^{-6} \\
&+126240 \cdot 10^{-10} \omega^{-4} \\
&-410416 \cdot 10^{-8}\ \omega^{-2} \\
&+117458 \cdot 10^{-5} \\
&-440417 \cdot 10^{-5}\ \omega^{2} \\
&+141278 \cdot 10^{-4}\ \omega^{4} \\
&+391929 \cdot 10^{-5}\ \omega^{6}
\end{aligned} \tag{3.7}$$

From these values we find by eq. (2.1)

$$\begin{aligned}
c &= +391929 \cdot 10^{-5} & a_{10} &+ +360468 \cdot 10^{-5} & a_4 &= -104717 \cdot 10^{-8} \\
& & a_8 &= -112372 \cdot 10^{-5} & a_2 &= +322100 \cdot 10^{-11} \\
& & a_6 &= +299691 \cdot 10^{-6} & a_0 &= +514310 \cdot 10^{-13}
\end{aligned}$$

The set of equations (2.2) can now be solved; the solutions are:

$$\begin{aligned}
b_0 &= 226784 \cdot 10^{-9} & c_1 &= 664053 \cdot 10^{-8} \\
b_2 &= 901202 \cdot 10^{-7} & c_3 &= 745328 \cdot 10^{-6} \\
b_4 &= 160930 \cdot 10^{-5} & c_5 &= 261214 \cdot 10^{-5}
\end{aligned} \tag{3.8}$$

According to (1.5), the values of (3.8) are connected with the parameters of the seismometer-two galvanometers system, and the next step must be to calculate the parameters from eq. (1.5).

For this purpose we write eq. (1.5) in a more conveniently arranged form. We see in (1.5) that this set of equations enables to compute six unknown parameters only. Now we suppose for example:

$$R_s = R_f = R_g = 1 \tag{3.9}$$

Furthermore we suppose that the mechanical dampings of the seismometer and the galvanometers can be neglected:

$$\varepsilon_{so} = \varepsilon_{go} = \varepsilon_{fo} = 0 \tag{3.10}$$

This last assumption is not essential; if we would like to make other estimates for the damping, it would not influence the further calculations fundamentally.

Putting

$$\frac{G_s^2}{K_s} = x \quad \frac{G_g^2}{K_g} = y \quad \frac{G_f^2}{K_f} = z \tag{3.11}$$

we are able to calculate from (1.1) and (1.3) all parameters in (1.5), taking into account (3.9), (3.10) and (3.11). As $n_f = 1$, we find

$$\begin{aligned}
b_0 &= n_s^2 n_g^2 \\
b_2 &= \frac{1}{3R_1 + 2}(xy + n_g^2 xz) + n_s^2 n_g^2 + n_s^2 + n_g^2 \\
b_4 &= \frac{1}{3R_1 + 2}(xy + xz) + n_s^2 + n_g^2 + 1 \\
c_1 &= \frac{R_1 + 2}{3R_1 + 2} n_g^2 x + \frac{R_1 + 1}{3R_1 + 2} n_s^2 y + \frac{R_1 + 1}{3R_1 + 2} n_s^2 n_g^2 z \\
c_3 &= \frac{R_1 + 2}{3R_1 + 2}(n_g^2 + 1)x + \frac{R_1 + 1}{3R_1 + 2}(n_s^2 + 1)y + \frac{R_1 + 1}{3R_1 + 2}(n_s^2 + n_g^2)z \\
c_5 &= \frac{R_1 + 2}{3R_1 + 2}x + \frac{R_1 + 1}{3R_1 + 2}y + \frac{R_1 + 1}{3R_1 + 2}z
\end{aligned} \tag{3.12}$$

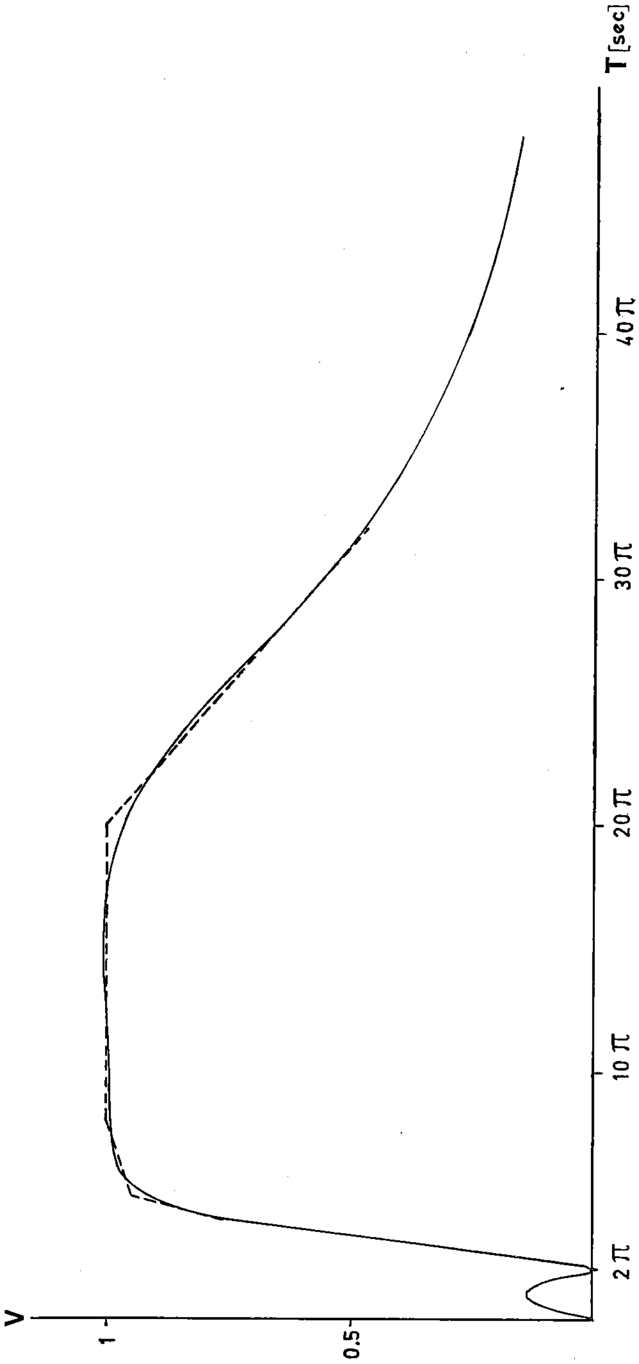

Fig. 13 Ideal magnification curve (broken line) and calculated magnification curve (full line) for a seismograph with a filter galvanometer.

The parameters n_s, n_g, x, y, z and R_1 can now be solved from eq. (3.12). The values are

$x = 1{,}22$ $\qquad n_s^2 = 681 \cdot 10^{-4}$ $\qquad R_1 = 3{,}72$

$y = 0{,}181$ $\qquad n_g^2 = 333 \cdot 10^{-5}$

$z = 5{,}62$

If we start with other values for the resistances of the seismometer and the galvanometers, for example:

$$R_s = R_f = R_g = 500\Omega$$

the solutions are

$n_s^2 = 681 \cdot 10^{-4}$ $\qquad T_s = 24{,}1$ sec $\qquad T_f = 2\pi$ sec

$n_g^2 = 333 \cdot 10^{-5}$ $\qquad T_g = 108{,}8$ sec

$$\frac{G_s^2}{K_s} = 610 \quad \frac{G_g^2}{K_g} = 90 \quad \frac{G_f^2}{K_f} = 2812 \quad R_1 = 1861\,\Omega$$

The magnification curve can be calculated using eq. (3.7) or eq. (1.4). The result is shown in figure 13.

CHAPTER V. DETERMINATION OF THE PARAMETERS OF AN ELECTROMAGNETIC SEISMOGRAPH BY APPLYING A FORCE

When dealing with problems, such as the determination of the magnification of a seismograph, or the calculation of the energy of P- and S-waves, it is often necessary to develop a seismic signal in a Fourier series. Generally this is a procedure which is time consuming, as it requires much numerical integrating.

In this chapter a fast method is developed which is based on approximating the recorded signals by polynomials. The result can be used for solving the following problem: how to determine the parameters of a seismograph from the galvanometric recording of a push applied to the seismometer.

1 Approximating a seismic signal by Jacobi polynomials

A seismic signal $f(t)$ has some marked characteristics. At $t = 0, f(t) = 0$; furthermore the first and the second derivative may be zero at $t = 0$. Finally any seismic record $f(t)$ becomes zero at large values of t, so $f(t) = 0$ at $t = \infty$.
Therefore we can distinguish three cases:

a. $f(t) = 0$, at $t = 0$; and $f(t) = 0$, at $t = \infty$

b. $f(t) = f'(t) = 0$ at $t = 0$; and $f(t) = 0$, at $t = \infty$ (1.1)

c. $f(t) = f'(t) = f''(t) = 0$, at $t = 0$; and $f(t) = 0$, at $t = \infty$

If we approximate the recorded signal by polynomials which consist of functions e^{-nt}, the last condition will be fulfilled in all three cases. We will first consider case c. as this can be applied to the galvanometric record of a seismic signal.
We develop $f(t)$ in the following series:

$$f(t) = c_0\varphi_0(t) + c_1\varphi_1(t) + \ldots + c_n\varphi_n(t) \tag{1.2}$$

where the functions $\varphi_n(t)$ are:

$$\varphi_n(t) = e^{-t}(1 - e^{-2t})^3(1 + a_{n,1}e^{-2t} + a_{n,2}e^{-4t} + \ldots + a_{n,n}e^{-2nt}) \tag{1.3}$$

All conditions in (1.1c) are satisfied; furthermore, if

$$\int_0^\infty \varphi_n(t)\,\varphi_m(t)\,dt = 0 \text{ for } n \neq m \tag{1.4}$$

the functions $\varphi_n(t)$ are orthogonal, which facilitates the calculation of the coefficients

$c_0, c_1, \ldots, c_n$

Let $e^{-2t} = x$, then eq. (1.3) can be written as (1.5)

$$\varphi_n(x) = \sqrt{x}(1 - x)^3 S_n(x) \tag{1.6}$$

where $S_n(x) = 1 + a_{n,1}x + a_{n,2}x^2 + \ldots + a_{n,n}x^n$

The orthogonality condition (1.4) becomes

$$\int_0^1 (1 - x)^6 S_n(x) S_m(x) dx = 0 \text{ for } n \neq m \tag{1.7}$$

This condition is satisfied by the Jacobi polynomials

$$J_n(\alpha;\gamma;x) = 1 + \sum_{k=1}^{n} (-1)^k \binom{n}{k} \frac{(\alpha + n)(\alpha + n + 1)\ldots(\alpha + n + k - 1)}{\gamma(\gamma + 1)\ldots(\gamma + k - 1)} x^k \tag{1.8}$$

which fulfil the integral relations

$$\int_0^1 x^{\gamma-1}(1 - x)^{\alpha-\gamma} J_n(\alpha;\gamma;x) J_m(\alpha;\gamma;x)\, dx = 0 \text{ for } n \neq m \tag{1.9}$$

$$\int_0^1 x^{\gamma-1}(1 - x)^{\alpha-\gamma} J_n^2(\alpha;\gamma;x) dx =$$

$$= \frac{\Gamma(\gamma)\Gamma(\alpha + 1 - \gamma)}{\Gamma(\alpha)} \frac{(\alpha + 1 - \gamma)(\alpha + 2 - \gamma)\ldots(\alpha + n - \gamma)}{\alpha(\alpha + 1)\ldots(\alpha + n - 1)\gamma(\gamma + 1)\ldots(\gamma + n - 1)} \frac{n!}{\alpha + 2n} \tag{1.10}$$

Now according to (1.7) and (1.9)

$\gamma - 1 = 0$ and $\alpha - \gamma = 6$ or $\alpha = 7$ and $\gamma = 1$

so $S_n(x) = J_n(7;\, 1;\, x)$

It follows from (1.8) that

$$S_n(x) = 1 + \sum_{k=1}^{n} (-1)^k \binom{n}{k} \frac{(7 + n)(8 + n)\ldots(6 + k + n)}{k!} x^k \tag{1.11}$$

and therefore eq. (1.3) can be written as

$$\varphi_n(t) = e^{-t}(1 - e^{-2t})^3 \left\{ 1 + \sum_{k=1}^{n} (-1)^k \frac{(7+n)(8+n)\dots(6+k+n)}{k!} e^{-2kt} \right\} \tag{1.12}$$

where

$$\int_0^\infty \varphi_n(t)\varphi_m(t)dt = \tfrac{1}{2}\int_0^1 (1-x)^6 S_n(x)S_m(x)dx = 0 \text{ for } n \neq m$$

$$\int_0^\infty \varphi_n^2(t)dt = \tfrac{1}{2}\int_0^1 (1-x)^6 S_n^2(x)dx = \frac{1}{14 + 4n} \tag{1.13}$$

As in eq. (1.2) the functions $\varphi_0(t)$, $\varphi_1(t)$, ..., $\varphi_n(t)$ satisfy the orthogonality conditions (1.13), the coefficients c_0, c_1, ..., c_n can be determined from the relation

$$c_n = (14 + 4n)\int_0^\infty f(t)\varphi_n(t)dt \tag{1.14}$$

where $f(t)$ is the seismic signal and $\varphi_n(t)$ the orthogonal functions which are determined by eq. (1.12).

2 Recurrence relations

The calculation of the polynomials is much facilitated if a recurrence relation can be derived for eq. (1.11) or (1.12). Supposing that this relation has the form which holds for orthogonal polynomials like the polynomials of Legendre, Laguerre, Hermite or Tchebycheff, we put

$$S_n(x) = aS_{n-2}(x) + bS_{n-1}(x) + cxS_{n-1}(x) \tag{2.1}$$

We must find three equations for the constants a, b and c. If we start from the first, second and last terms of the nth polynomial, we get after (1.11) and (2.1)

$$a + b = 1$$

$$(n-2)(n+5)a + (n-1)(n+6)b - c = n(n+7)$$

$$(n+6)n\,c = -(2n+5)(2n+6) \tag{2.2}$$

The constants can be computed from (2.2) and we get for $S_n(x)$ the recurrence relation:

$$n(n+6)(n+2)S_n(x) = -(n-1)(n+3)(n+5)S_{n-2}(x) +$$
$$+ (2n^3 + 15n^2 + 19n - 15)S_{n-1}(x) - (n+2)(2n+5)(2n+6)xS_{n-1}(x) \quad (2.3)$$

Making use of (2.3) and starting from $S_0(x)$ and $S_1(x)$ we find the following series of polynomials:

$$S_0(x) = 1$$
$$S_1(x) = 1 - 8x$$
$$S_2(x) = 1 - 18x + 45x^2$$
$$S_3(x) = 1 - 30x + 165x^2 - 220x^3$$
$$S_4(x) = 1 - 44x + 396x^2 - 1144x^3 + 1001x^4$$
$$S_5(x) = 1 - 60x + 780x^2 - 3640x^3 + 6825x^4 - 4368x^5$$

.

.

.

Returning to eq. (1.1) we want also to calculate the series of polynomials for the cases a. and b.

Case a. $f(t) = 0$ for $t = 0$ and $f(t) = 0$ for $t = \infty$
In this case the seismic signal can be approximated by

$$f(t) = c_0\varphi_0(t) + c_1\varphi_1(t) + \ldots + c_n\varphi_n(t) \quad (2.4)$$

and the polynomials $\varphi_0(t)$, $\varphi_1(t)$, ..., $\varphi_n(t)$ must be

$$\varphi_n(t) = \sqrt{x}(1-x)S_n(x) \text{ with } x = e^{-2t}$$

The first six functions $S_n(x)$ are

$$S_0(x) = 1$$
$$S_4(x) = 1 - 4x$$
$$S_2(x) = 1 - 10x + 15x^2$$
$$S_3(x) = 1 - 18x + 63x^2 - 56x^3$$

$$S_5(x) = 1 - 28x + 168x^2 - 336x^3 + 210x^4$$

$$S_5(x) = 1 - 40x + 360x^2 - 1200x^3 + 1650x^4 - 792x^5$$

with the recurrence relation

$$n^2(n+2)S_n(x) = -(n-1)(n+1)^2 S_{n-2}(x) +$$
$$+ (2n^3 + 3n^2 - n - 1)S_{n-1}(x) - 2n(n+1)(2n+1)xS_{n-1}(x)$$

and with the following formula for the coefficients of (2.4)

$$c_n = (6 + 4n)\int_0^\infty f(t)\varphi_n(t)dt$$

The orthogonality condition for these polynomials is

$$\int_0^\infty \varphi_n(t)\varphi_m(t)dt = \tfrac{1}{2}\int_0^1 (1-x)^2 S_n(x)S_m(x)dx = 0 \text{ for } n \neq m$$

Case b. $f(t) = f'(t) = 0$ for $t = 0$, and $f(t) = 0$ for $t = \infty$

In this case the polynomials must be

$$\varphi_n(t) = \sqrt{x}(1-x)^2 S_n(x) \text{ with } x = e^{-2t}$$

The first six functions $S_n(x)$ are

$$S_0(x) = 1$$

$$S_1(x) = 1 - 6x$$

$$S_2(x) = 1 - 14x + 28x^2$$

$$S_3(x) = 1 - 24x + 108x^2 - 120x^3$$

$$S_4(x) = 1 - 36x + 270x^2 - 660x^3 + 495x^4$$

$$S_5(x) = 1 - 50x + 550x^2 - 2200x^3 + 3575x^4 - 2002x^5$$

The recurrence relation is for this case

$$n(n+1)(n+4)S_n(x) = -(n-1)(n+2)(n+3)S_{n-2}(x) +$$
$$+ (2n^3 + 9n^2 + 5n - 6)S_{n-1}(x) - (n+1)(2n+3)(2n+4)xS_{n-1}(x)$$

The coefficients c_n of the series by which the seismic signal is approximated are:

$$c_n = (10 + 4n)\int_0^\infty f(t)\varphi_n(t)dt \tag{2.5}$$

The orthogonality relation is for case b.:

$$\int_0^\infty \varphi_n(t)\varphi_m(t)dt = \tfrac{1}{2}\int_0^1 (1-x)^4 S_n(x)S_m(x)dx = 0 \text{ for } n \neq m$$

3 Application of the theory

We want to apply the above developed theory to the seismic signal which is shown in figure (14). It is the deflection of the galvanometer of a Press-Ewing seismograph caused by a force applied to the seismometer. The deflection was read every five seconds and is indicated by crosses.

Whereas generally case c. fits the galvanometric record of a seismic signal, a good approximation is already obtained by the initial conditions according to case b. Moreover the amount of numerical calculations is considerably less than in case c., so that it is worth while to start with case b.

We must calculate the coefficients c_n from

$$c_n = (10 + 4n)\int_0^\infty f(t)\varphi_n(t)dt \tag{3.1}$$

For the numerical integration the method of Gauss is used (see e.g. Kosten, 1963).

We write the integral

$$\int_0^\infty f(t)\varphi_n(t)dt = \int_0^\infty e^{-t}(1-e^{-2t})^2 S_n(t)f(t)dt$$

in the form (substituting $e^{-2t} = x$)

$$\int_0^\infty f(t)\varphi_n(t)dt = \tfrac{1}{2}\int_0^1 (1-x)^4 S_n(x)\frac{f(x)}{\sqrt{x}(1-x)^2}dx = \int_0^1 w(x)g(x)dx = \sum_{j=1}^{n} C_j g(x_j) \tag{3.2}$$

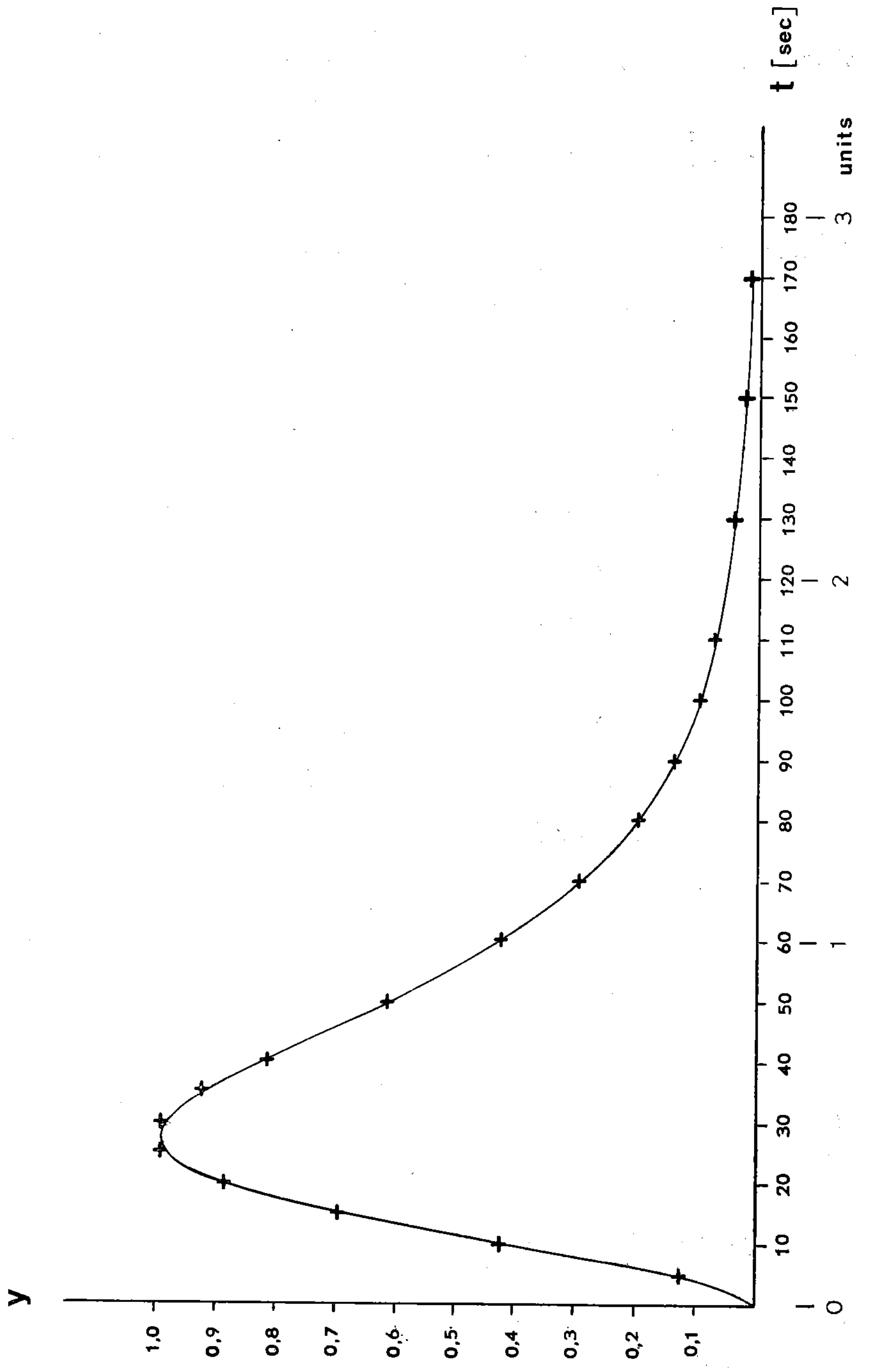

Fig. 14 Record of a push applied to the seismometer. The readings (indicated by crosses) are approximated by a series of four polynomials.

where $w(x) = (1 - x)^4$ and $g(x) = \frac{f(x)}{2\sqrt{x}(1 - x)^2} S_n(x)$

$$C_j = \int_0^1 w(x) l_j(x) dx \text{ and } l_j(x) = \prod_{k=1, \neq j}^{n} \frac{x - x_k}{x_j - x_k} \tag{3.3}$$

The values x_j ($x_1, x_2, \ldots, x_n$) are the zero values of S_n in the interval (0.1).

We choose a 10 points integration. In this case we must use the polynomial S_{10} which is

$S_{10} =$		and the zero values of S_{10} are:	j	x_j
+	1			
−	$150x$		1	0,90995892
+	$5400x^2$		2	0,81517755
−	$81600x^3$		3	0,70217685
+	$642600x^4$		4	0,57752731
−	$2930256x^5$		5	0,44892658
+	$8139600x^6$		6	0,32440510
−	$13953600x^7$		7	0,21176419
+	$14389650x^8$		8	0,11806934
−	$8171900x^9$		9	0,04919931
+	$1961256x^{10}$		10	0,00946152

The values t_j belonging to x_j are shown in table 1, first column. Furthermore, this table gives the values of the first 10 polynomials for these t_j.

The factors C_j in (3.2) can be computed by means of eq. (3.3), and now we can write for the integral (3.2):

$$\int_0^\infty f(t)\varphi_n(t)dt = \sum_{j=1}^{10} \left[\frac{C_j}{2\sqrt{x_j}(1 - x_j)} S_n(x_j)\right] f(t_j) \tag{3.4}$$

The values of the function between brackets are presented as $C'_j S_n(x_j)$ in table 2 for the first 10 polynomials.
Now the numerical integration is reduced to an elementary operation. The factor

$C'_j S_n(x_j)$ must be multiplied by the values $f(t_j)$ belonging to the ten values $t_1, t_2, \ldots, t_{10}$.
The seismic signal $y = f(t)$ in figure (14) has been obtained by reading the deflection of the galvanometer as a function of the time in seconds. In order to cover the record of the signal by the values of t_j beloning to the zero values of S_{10}, we have to change the time scale; this is done by putting 1 unit = 60 seconds (see figure 14). The vertical scale is chosen so that $y_{\max} = 1$.

From eq. (3.4) and (3.1) it follows that

$$c_0 = +1{,}884$$
$$c_1 = -1{,}649$$
$$c_2 = -0{,}135$$
$$c_3 = +0{,}133$$

It is not necessary to calculate the other coefficients, as the first four terms give already a very good approximation.

We can therefore approximate the seismic signal in a fast converging series as follows:

$$\begin{aligned} f(t) &= c_0\varphi_0(t) + c_1\varphi_1(t) + c_2\varphi_2(t) + c_3\varphi_3(t) \\ &= \varphi_0(t)\left[c_0 S_0(t) + c_1 S_1(t) + c_2 S_2(t) + c_3 S_3(t)\right] \\ &= +0{,}233e^{-t} + 8{,}125e^{-3t} - 6{,}369e^{-5t} - 28{,}519e^{-7t} + 42{,}481e^{-9t} - \\ &\quad - 15{,}950e^{-11t} \end{aligned} \tag{3.5}$$

Figure (14) shows that this approximation (full line) is indeed very good.

Now we can proceed to the original problem, viz. how to carry out the Fourier analysis of the signal. The two Fourier integrals which must be calculated, are:

$$S(\omega) = \int_0^\infty f(t) \sin \omega t \, dt \text{ and } C(\omega) = \int_0^\infty f(t) \cos \omega t \, \mathrm{d}t \tag{3.6}$$

Substituting (3.5) into (3.6) we see that we have to calculate the following type of integrals

$$\int_0^\infty e^{-kt} \sin \omega t \, dt = \frac{\omega}{k^2 + \omega^2}$$

$$\int_0^\infty e^{-kt} \cos \omega t \, dt = \frac{k}{k^2 + \omega^2}$$

The result is:

$$S(\omega) = +0{,}233\frac{\omega}{1 + \omega^2} + 8{,}125\frac{\omega}{9 + \omega^2} - 6{,}369\frac{\omega}{25 + \omega^2} -$$

$$- 28{,}519\frac{\omega}{49 + \omega^2} + 42{,}481\frac{\omega}{81 + \omega^2} - 15{,}950\frac{\omega}{121 + \omega^2} \qquad (3.7)$$

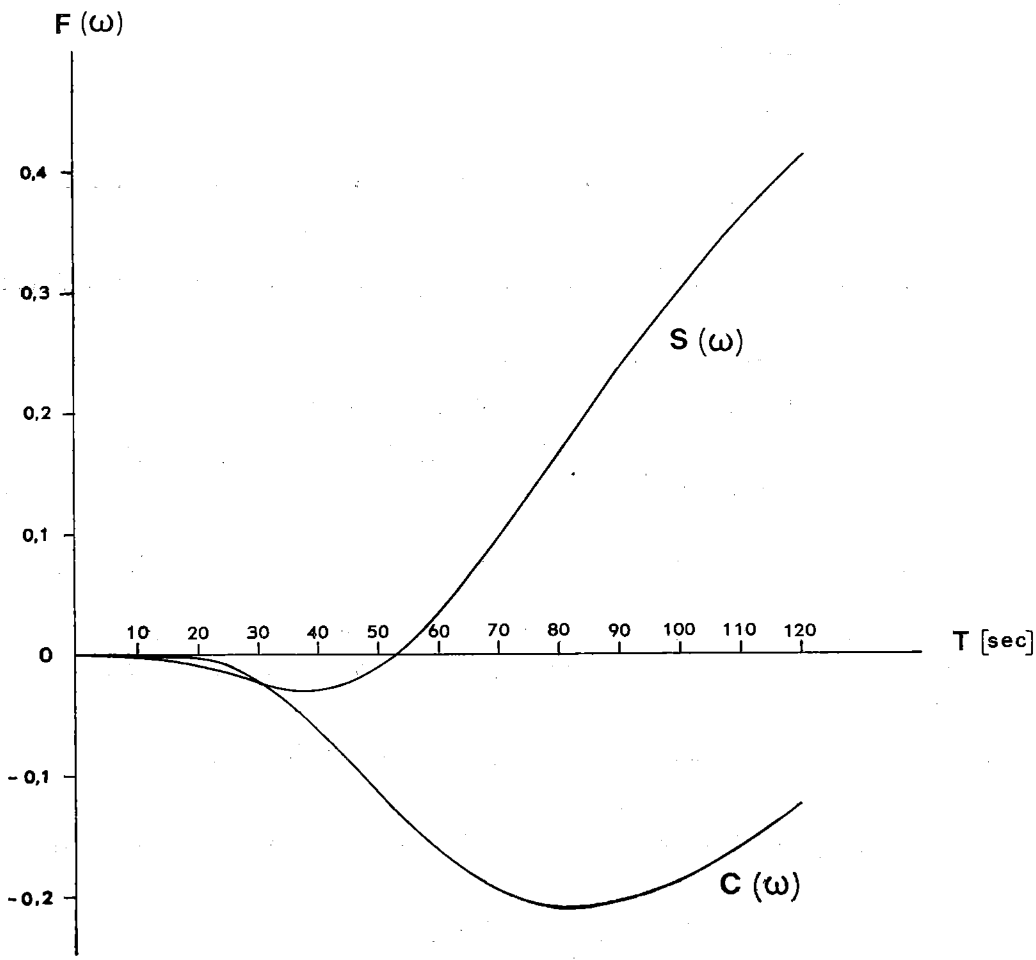

Fig. 15 Fourier integrals of the seismic signal of fig. 14.

and

$$C(\omega) = +0{,}233\frac{1}{1+\omega^2} + 8{,}125\frac{3}{9+\omega^2} - 6{,}369\frac{5}{25+\omega^2} -$$

$$- 28{,}519\frac{7}{49+\omega^2} + 42{,}481\frac{9}{81+\omega^2} - 15{,}950\frac{11}{121+\omega^2}$$

The Fourier integrals of the seismic signal (see figure 14) are shown in figure (15) as functions of the period *T*.

4 Determination of the magnification of a seismograph

Willmore (1959) has developed a method for calibrating a seismometer-galvanometer combination by means of a balanced bridge circuit (see figure 16). The seismometer is taken up in the bridge, and the galvanometer is connected to the points B and D. The bridge is designed so that the resistance R_1 is much smaller than the resistance of the seismometer R_s. The ressitance R_2 is much greater than R_s; in equilibrium $R_s/R_1 = R_2/R_3$. The bridge is balanced when the seismometer is clamped, so that no current flows in the galvanometer. A current passing through the bridge with the seismometer in unclamped position will in that case only act on the seismometer. The movement of the seismometer will cause a deflection of the galvanometer.

Willmore (1959) has applied this principle to a steady-state method. A harmonic alternating current is sent through the seismometer; the seismometer behaves as under the influence of a harmonic ground movement.

Espinosa, Sutton and Miller (1962) have used a transient method for determining the magnification of the seismograph. They calculated magnification curves from supposed values of the parameters of the seismograph. For each magnification function they calculated the response of the galvanometer to a step function of the current. Next they compared the observed response of the galvanometer with the theoretical responses, and in that way they found the right magnification function.

However, this is an indirect method. It will be shown that it is possible to derive the magnification function directly from the observed response of the galvanometer. Let the current through the seismometer be i, then the moment acting on the coil of the seismometer is $G_s i$. This moment is proportional to an acceleration of the ground

$$\ddot{x} = -\frac{G_s i}{l M} \tag{4.1}$$

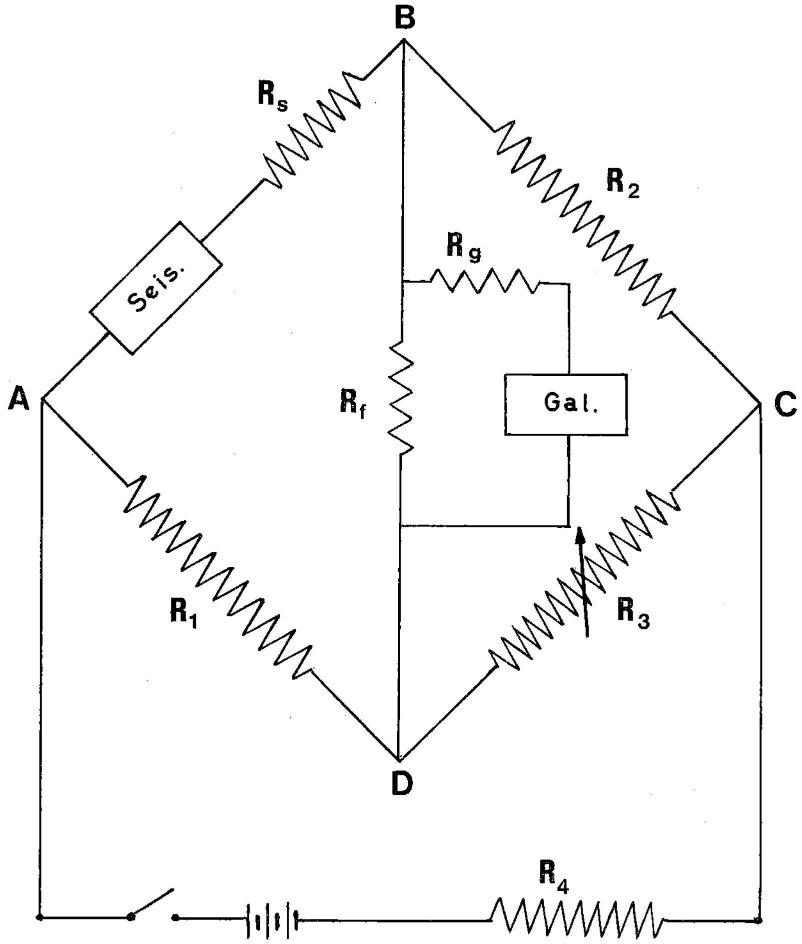

Fig. 16 Willmore bridge for calibrating a seismograph.

where G_s is the electrodynamic constant, l the reduced pendulum length and M is the mass of the seismometer.

It follows from (4.1) that the seismometer-galvanometer system will behave as if the ground movement $x(t)$ is:

$$x(t) = \tfrac{1}{2}\frac{G_s i}{l M}\, t^2 = \tfrac{1}{2}ct^2 \tag{4.2}$$

A function $f(t)$ can be written as a Fourier integral:

$$f(t) = \frac{1}{\pi}\int_0^\infty F(\omega)\sin(\omega t - \psi)d\omega \tag{4.3}$$

where

$$F(\omega) = \sqrt{C^2(\omega) + S^2(\omega)}$$

and $C(\omega)$ and $S(\omega)$ can be calculated by means of (3.6).
$F(\omega)$ is the amplitude spectrum of $f(t)$ and $\psi(\omega)$ is the phase spectrum.

The values of $C(\omega)$ and $S(\omega)$ have already been calculated for the response of the galvanometer (see eq. 3.7).
For calculating the magnification of the seismograph we have to derive the amplitude spectrum of the ground motion.

The function $S(\omega)$ and $C(\omega)$ are in the case of a motion according to eq. (4.2)

$$S(\omega) = \tfrac{1}{2}c\int_0^\infty t^2 \sin \omega t \, dt = \frac{c}{\omega^3}$$

$$C(\omega) = \tfrac{1}{2}c\int_0^\infty t^2 \cos \omega t \, dt = 0$$

The magnification of the seismograph is therefore

$$V = \frac{1}{c}\,\omega^3\sqrt{C^2(\omega) + S^2(\omega)}$$

$S(\omega)$ and $C(\omega)$ are now the Fourier integrals of the reaction of the galvanometer, as given by (3.7).

For small values of T the integrals $S(\omega)$ and $C(\omega)$ are very small; as ω^3 is very large, the magnification V is inaccurate for these small values of T.
We can, therefore, improve the result by applying case c. ($f(t) = f'(t) = f''(t) = 0$ for $t = 0$).

It might seem a more direct method to start with case c. from the beginning. However, a quicker method is to find an approximate function $f(t)$ by starting from the initial conditions of case b., and then to calculate the magnification curve using the analytical form (3.5) of $f(t)$.

The coefficients c_n in eq. (1.14) can easily be calculated, making use of the analytical form of $f(t)$. The result is

$$c_0 = +1{,}923$$

$$c_1 = -2{,}209$$

$$c_2 = +0{,}581$$

$$c_3 = -0{,}089$$

$$c_4 = +0{,}041$$

$$c_5 = -0{,}021$$

Now the magnification function can be calculated in better approximation; the values of V are shown in figure (17) (crosses). The values have been normalized by taking $V_{max} = 1$.

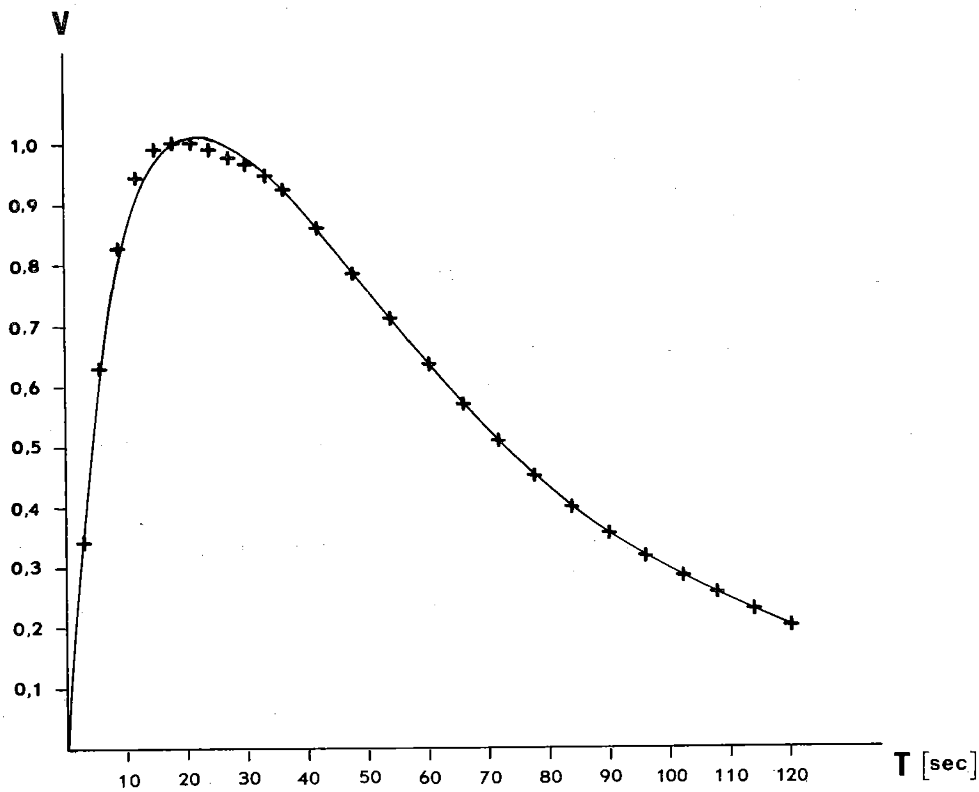

Fig. 17 Magnification curve calculated from Fourier analysis of a seismic signal.

There are two possibilities to find the right scale for the magnification function. One possibility is to calculate $c = G_s i/l\ M$ from the electro-dynamical constant G_s, the reduced pendulum length l and the mass M of the seismograph. Of course, the scale of the functions $C(\omega)$ and $S(\omega)$ must be multiplied by the real value y_{max} of the galvanometer deflection. The other possibility is to send an alternating current of known amplitude i_0 and period T in the seismometer coil. In this case the constant c in eq. (4.2) is

$$c = \frac{G_s i_0 T^2}{4\pi^2 l\ M}$$

The scale of the magnification function can be determined by calculating V for this special period T (Willmore, 1959).

5 Determination of the parameters of a Press-Ewing seismograph

Now we can apply the theory described in chapter II, making use of the values of the magnification shown in figure (17). It is possible to bring these values in an analytical form by numerical integration according to eq. (II.1.15).

The equation of the magnification curve appears to be:

$$V^{-2} = 63{,}2T^{-2} + 0{,}725 + 0{,}218 \cdot 10^{-3} T^2 + 0{,}656 \cdot 10^{-7} T^4 + 0{,}206 \cdot 10^{-11} T^6$$

The values of b_0, b_2, c_1, and c_3 are for the Press-Ewing seismograph, which recorded the signal of figure (14):

$$b_0 = n_s^2 n_g^2 = 282 \cdot 10^{-6}$$

$$b_2 = 4\varepsilon_s\varepsilon_g - \sigma_s\sigma_g + n_s^2 + n_g^2 = 164 \cdot 10^{-3}$$

$$c_1 = 2\varepsilon_s n_g^2 + 2\varepsilon_g n_s^2 = 125 \cdot 10^{-4}$$

$$c_3 = 2\varepsilon_s + 2\varepsilon_g = 884 \cdot 10^{-3} \qquad (5.1)$$

The period of the galvanometer of this Press-Ewing seismograph could easily be determined; it was $T_g = 87{,}3$ sec with $\varepsilon_{go} = 0{,}024$. It follows from b_0 that $T_s = 27{,}0$ sec; $\varepsilon_{so} = 0$.

The parameters ε_s, ε_g and $\sigma_s\sigma_g$ can be calculated from the equation for b_2, c_1 and c_3. The values are: $\varepsilon_s = 0{,}361$, $\varepsilon_g = 0{,}081$ and $\sigma_s\sigma_g = 0{,}012$. Putting $\sigma_s\sigma_g = 0$ or $\sigma_s\sigma_g =$

$= 4\varepsilon_s\varepsilon_g$, and solving n_s and n_g from eq. (5.1), we get the limits of the periods of the seismometer and galvanometer:

$$15{,}6 \text{ sec} < T_s < 40{,}0 \text{ sec}$$

$$58{,}8 \text{ sec} < T_g < 151 \quad \text{sec}$$

We see that the periods of seismometer and galvanometer of the Press-Ewing seismograph in question indeed satisfy these conditions.
Other parameters of this seismograph can be calculated from the resistances:

$$R_s = R_g = 500\Omega; \quad R_f = 300\Omega; \quad R_1 = R_2 = 0.$$

It follows from eq. (I.3.10a), (I.3.10b) and (I.3.11) that:

$$a = 550 \cdot 10^3 \quad A = 909 \cdot 10^{-6} \quad \frac{G_s^2}{K_s} = 496{,}6$$

$$B = 909 \cdot 10^{-6}$$

$$C = 545 \cdot 10^{-6} \quad \frac{G_g^2}{K_g} = 80{,}54$$

The constants G_s^2/K_s and G_g^2/K_g are necessary if it would be required to change the magnification curve into a desired form.

REFERENCES

KOSTEN, L.: Numerieke analyse, in: Handboek der Wiskunde ed. Kuipers en Timman, 1963.

WILLMORE, P. L.: The application of the Maxwell impedance bridge to the calibration of electromagnetic seismographs, Bull. Seism. Soc. Am. **49**, 99–114, 1959.

ESPINOSA, A. F., SUTTON, G. H. and MILLER, H. J.: A transient technique for seismograph calibration, Bull, Seism. Soc. Am. **52**, 767–779, 1962.

TABLE I.

n	t	S_0	S_1	S_2	S_3
1	0,047	+1	— 4,459754	+11,445282	—21,828562
2	0,102	+1	— 3,891065	+ 8,193919	—11,800372
3	0,176	+1	— 3,213061	+ 4,974989	— 4,147785
4	0,274	+1	— 2,465164	+ 2,253676	+ 0,046164
5	0,400	+1	— 1,693559	+ 0,358010	+ 1,134616
6	0,563	+1	— 0,946431	— 0,594989	+ 0,483269
7	0,776	+1	— 0,270585	— 0,709065	— 0,378745
8	1,068	+1	+ 0,291584	— 0,262640	— 0,525616
9	1,506	+1	+ 0,704804	+ 0,378986	+ 0,066347
10	2,330	+1	+ 0,943231	+ 0,870045	+ 0,782490

n	S_4	S_5	S_6	S_7
1	+33,903552	—44,633941	+50,508419	—48,736128
2	+12,134142	— 8,014619	+ 0,848363	+ 5,869556
3	+ 0,682004	+ 2,752106	— 3,203382	+ 0,514836
4	— 1,802228	+ 0,873894	+ 1,148922	— 1,246883
5	— 0,354914	— 0,946269	+ 0,246076	+ 0,852303
6	+ 0,685696	— 0,046016	— 0,613486	— 0,320041
7	+ 0,212226	+ 0,520748	+ 0,344092	— 0,100341
8	— 0,476715	— 0,208502	+ 0,119865	+ 0,351478
9	— 0,193320	— 0,370281	— 0,449091	— 0,430371
10	+ 0,683001	+ 0,574325	+ 0,459439	+ 0,341452

n	S_8	S_9	S_{10}	φ_0
1	+38,360432	—20,864937	0,000000	+ 0,007734
2	— 8,595096	+ 6,016436	0,000000	+ 0,030842
3	+ 2,397564	— 2,550875	0,000000	+ 0,074326
4	— 0,531602	+ 1,326299	0,000000	+ 0,135639
5	— 0,121969	— 0,785099	0,000000	+ 0,203473
6	+ 0,352444	+ 0,507930	0,000000	+ 0,259966
7	— 0,412313	— 0,349112	0,000000	+ 0,285916
8	+ 0,393626	+ 0,248150	0,000000	+ 0,267262
9	— 0,329875	— 0,175085	0,000000	+ 0,200520
10	+ 0,223513	+ 0,108714	0,000000	+ 0,095438

TABLE II.

j	$S_0C'_j$	$S_1C'_j$	$S_2C'_j$	$S_3C'_j$
1	+0,00035843	—0,00159851	+0,00410233	—0,00782401
2	+0,00322777	—0,01255946	+0,02644808	—0,03808889
3	+0,00635453	—0,02041749	+0,03161372	—0,02635722
4	+0,01503077	—0,03705331	+0,03387448	+0,00069388
5	+0,02898143	—0,04908178	+0,01037564	+0,03288279
6	+0,04802100	—0,04544854	—0,02857195	+0,02320656
7	+0,07037951	—0,01904365	—0,04990362	—0,02665589
8	+0,09296700	+0,02710769	—0,02441689	—0,04886494
9	+0,11182214	+0,07881270	+0,04237899	+0,00741911
10	+0,12208253	+0,11515201	+0,10621733	+0,09552836
	$S_4C'_j$	$S_5C'_j$	$S_6C'_j$	$S_7C'_j$
1	+0,01215205	—0,01599814	+0,01810373	—0,01746849
2	+0,03916622	—0,02586935	+0,00273832	+0,01894558
3	+0,00433382	+0,01748834	—0,02035598	+0,00327154
4	—0,02708888	+0,01313530	+0,01726919	—0,01874161
5	—0,01028591	—0,02742424	+0,00713164	+0,02470096
6	+0,03292779	—0,00220971	—0,02946020	—0,01536869
7	+0,01493635	+0,03664999	+0,02421706	—0,00706196
8	—0,04431879	—0,01938384	+0,01114352	+0,03267586
9	—0,02161748	—0,04140556	—0,05021827	—0,04812506
10	+0,08338245	+0,07011508	+0,05608946	+0,04168530
j	$S_8C'_j$	$S_9C'_j$		
1	+0,01374953	—0,00747862		
2	—0,02774299	+0,01941967		
3	+0,01523540	—0,01620961		
4	—0,00799039	+0,01993530		
5	—0,00353483	—0,02275330		
6	+0,01692471	+0,02439129		
7	—0,02901837	—0,02457036		
8	+0,03659424	+0,02306977		
9	—0,03688736	—0,01957834		
10	+0,02728706	+0,01327226		

SUMMARY

The basic question dealt with in this treatise, is how to choose the instrumental parameters of a seismometer-galvanometer system in order to get a desired magnification of the earth's movement. This 'inverse problem' is treated for the coupling of the seismometer with one galvanometer as a recording element. The theory is also developed for the combination of a seismometer with two galvanometers. In this case a filter galvanometer is added to the system, in series with the recording galvanometer.

The method used is based on the conversion of the classical equation of the magnification curve into a polynomial, which is possible after an elementary transformation. Applying the method of least squares, the coefficients of the polynomial development are determined, so that the best approximation is obtained to a desired magnification curve. As the coefficients of the polynomials are functionally related to the instrumental parameters, it is possible to calculate the values of the parameters needed for obtaining the desired magnification curve.

This method of developing the seismograph theory has some advantages. Firstly the right choice of instruments is facilitated, starting from the desired magnification characteristic. Furthermore the theory determines the limits between which the instrumental parameters must be chosen in order to obtain instruments with the desired magnification characteristic, which can physically be realized. The so called 'false' Galitzin and 'false' Benioff seismographs as known from the literature, are studied to find out which solutions are suited to a physical realisation, and which solutions are indeed false.

Secondly, a method is developed to calculate the magnification function of a seismograph system from the response of the seismograph to a push. The seismic record of the push is approximated by polynomials of Jacobi. In these polynomials certain properties of the system are accounted for, such as the right conditions for $t = 0$, and the limiting value for large t. The conditions ensure the right analytical form of the response and furthermore a fast convergence of the approximation. Using Fourier integrals the amplitude spectrum of the response can be calculated. As the magnification characteristic is proportional to the amplitude content, the magnification characteristic of the instrument can be calculated as well in a fast and simple way.

Van de reeks MEDEDELINGEN EN VERHANDELINGEN *zijn bij het Staatsdrukkerij- en Uitgeverij-bedrijf nog verkrijgbaar de volgende nummers:*

23, 25, 27, 29*b*, 30, 31, 34*b*, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 59.

alsmede

60. C. Kramer, J. J. Post en J. P. M. Woudenberg. Nauwkeurigheid en betrouwbaarheid van temperatuur- en vochtigheidsbepalingen in buitenlucht met behulp van kwikthermometers, 1954. (60 blz. met 11 fig.) 3,60

62. C. Levert. Regens. Een statistische studie. 1954. (246 blz. met 67 fig. en 143 tab.) 10,30

63. P. Groen. On the behaviour of gravity waves in a turbulent medium, with application to the decay and apparent period increase of swell. 1954. (23 blz.) 1,55

64. H. M. de Jong. Theoretical aspects of aeronavigation and its application in aviation meteorology. 1956. (124 blz. met 80 fig., 9 krt. en 3 tab.) 4,60

65. J. G. J. Scholte. On seismic waves in a spherical earth. 1956. (55 blz. met 24 fig.) 5,15

66. G. Verploegh. The equivalent velocities for the Beaufort estimates of the wind force at sea. 1956. (38 blz. met 17 tab.) 1,80

67. G. Verploegh. Klimatologische gegevens van de Nederlandse lichtschepen over de periode 1910—1940.

Deel I: Stormstatistieken. — Climatological data of the Netherlands light-vessels over the period 1910—1940. P. I: Statistics of gales. 1956. (68 blz. met tabellen) 3,60

Deel II: Luchtdruk en wind; zeegang. — Climatological data of the Netherlands light-vessels over the period 1910—1940. P. II: Air pressure and wind: state of the sea. 1958. (91 blz. met tabellen.) 7,70

Deel III: Temperaturen en hydrometeoren; onweer. — Climatological data of the Netherlands light-vessels over the period 1910—1940. P. III: temperatures and hydrometeors; thunderstorms. 1953. (146 blz. met tabellen.) 8,25

68. F. H. Schmidt. On the diffusion of stack gases in the atmosphere. 1957. (60 blz., 12 fig. en tab.) 5,15

69. H. P. Berlage. Fluctuations of the general atmospheric circulation of more than one year; their nature and prognostic values. 1957 7,70

70. C. Kramer. Berekening van de gemiddelde grootte van de verdamping voor verschillende delen van Nederland volgens de methode van Penman. 1957. (85 blz., fig. en tab.) 7,20

71. H. C. Bijvoet. A new overlay for the determination of the surface wind over sea from surface weather charts. 1957. (35 blz., fig. en tab.) 2,60

72. J. G. J. Scholte. Rayleigh waves in isotropic and anisotropic elastic media. 1958. (43 blz., fig. en tab.) 3,10

73. M. P. H. Weenink. A theory and method of calculation of wind effects on sea levels in a partly-enclosed sea, with special application to the southern coast of the North Sea. 1958. (111 blz. met 28 fig. en tab.) 8,25

74. H. M. de Jong. Geostrophic flow. Geostrophic approximation in the upper air flow with application to aeronavigation and air trajectories. 1959. (100 blz. met 17 fig., 14 krt. en 2 tab.) 5,15

75. S. W. Visser. A new analysis of some features of the 11-year and 27-day cycles in solar activity and their reflection in geophysical phenomena. 1959. (65 blz. met 16 fig. en 12 tab.) 3,60

76. A. R. Ritsema and J. Veldkamp. Fault plane mechanisms of South East Asian earthquakes. 1960. (63 blz. met 26 fig. en 11 tab.) 4,10

77. G. Verploegh. On the annual variation of climatic elements of the Indian Ocean. P. I: text. P. II: charts. 1960. (64 blz., 15 fig., 28 krt.) 6,15

78. J. A. As. Instruments and measuring methods in paleomagnetic research. 1960. (56 blz., 20 fig.) 2,55

79. D. J. Bouman. Consistency of approximations in discontinuous fields of motion in the atmosphere with an introduction to the use of generalized functions or distributions in meteorology. 1961. (94 blz., 6 fig.) 6,70

80. H. Timmerman. The influence of topography and orography on the precipitation patterns in the Netherlands. 1963. (49 blz., 37 fig. en 5 tab.) 6,70

81. A. W. Hanssen & W. J. A. Kuipers: On the relationship between the frequency of rain and various meteorological parameters (with reference to the problem of objective forecasting). 1965. (77 blz., 18 fig. en 12 tab.) 10,25

82. G. A. de Weille: Forecasting crop infection by the potato blight fungus. A fundamental approach to the ecology of a parasite — host relationship. 1964. (144 blz., 37 fig. en 37 tab.) 14,90

83. H. J. de Fluiter, P. H. van de Pol, J. P. M. Woudenberg (redactie) e.a. Fenologisch en faunistisch onderzoek over boomgaardinsekten. Phenological and faunistic investigations on orchard insects. 1964. (226 blz., 84 fig. en 59 tab.) 9,50

84. D. J. Bouman & H. M. de Jong: Generalized theory of adjustment of observations with applications in meteorology. 1964. (89 blz., 8 fig. en 1 tab.) 11,30

85. L. Otto: Results of current observations at the Netherlands lightvessels over the period 1910—1939. P. I: Tidal analysis and the mean residual currents. 1964. (56 blz. en 8 tab.) 6,40

86. F. H. Schmidt: An analysis of dust measurements in three cities in the Netherlands. 1964. (68 blz., 14 fig. en 22 tab.) 5,65

87. Commissie Meteorologische Voorlichting van Straalvliegtuigen: Climatology of Amsterdam Airport (Schiphol). 1966. (145 blz., 6 fign., 10 tab.) 17,00

88. H. P. Berlage: The southern oscillation and world weather. 1966. (152 blz. 46 fig., 34 tab.) 15,95

89. G. Verploegh: Observation and analysis of the surface wind over the ocean. 1967. (67 blz., 14 tab., 4 krt.) 6,70

90. R. Dorrestein: Wind and wave data of Netherlands lightvessels since 1949. 1967. (123 blz., 22 tab.) 15,95

91. P. J. Rijkoort: The increase of mean wind speed with height in the surface friction layer. 1968. (115 blz., 31 fig., 16 tab.) 15,95

92. C. J. E. Schuurmans: The influence of solar flares on the tropospheric circulation. Statistical indications, tentative explanation and related anomalies of weather and climate in Western Europe. 1969. (123 blz., 40 fig., 19 tab.) 19,00

93. H. M. de Jong: Optimal track selection and 3-dimensional flight planning. Theory and practice of the optimization problem in air navigation under space-time varying meteorological conditions. 1974. (140 blz., 41 fig., 14 tab., 4 fig. in annex) . 55,00

94. S. J. Bijlsma: On minimal-time ship routing. 40,00

96. L. Otto: Oceanography of the Ría de Arosa (N.W. Spain). 1975. (210 blz., 79 fig., 19 tab., 3 fig. in annex, 5 tab. in annex) 60,00

427257*-316