How much would be gained from ocean-data assimilation if El Niño were a stochastic oscillator?

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Abstract

Assimilation of sub-surface ocean data is one of the factors that have contributed to the increase in skill of ENSO forecasting systems, but it is not well known how much.

Here this question will be addressed in the context of about the simplest ENSO model possible: the stochastic oscillator, a noise-driven two-variable system. The first variable is a Niño SST index, the second is an indicator of the sub-surface ocean. Measuring the second variable stands for ocean-data assimilation in this system.

It is found that the impact of ocean data assimilation depends critically on the noise terms. That is, different choices exist that have the same properties of the Niño index, and the same predictability if the second variable is not measured, yet very different increases in predictability if sub-surface ocean data are assimilated. This illustrates the need for a proper representation of small-scale processes if one wishes to assess the predictability of the ENSO system.

1 Introduction

The assimilation of sub-surface ocean data has been an important factor in successfully forecasting the 1997/1998 El Niño event by the major operational centres. However, it is not well known how much the forecast skill of a perfect model increases by assimilating sub-surface ocean data, as theoretical studies have given very different, and often rather optimistic, estimates of the predictability of ENSO (El Niño – Southern Oscillation) (Latif et al. 1998).

Here it will be shown that this question is non-trivial even in a very simple model of ENSO, the stochastic oscillator, which is the leading POP mode approximation to the ENSO system.

2 The stochastic oscillator

ENSO is dominated in space by a single pattern and in time by a single time scale. It is widely accepted that this irregularly oscillating mode is connected to the delayed-oscillator mechanism (Suarez and Schopf 1988, Battisti and Hirst 1989). It is still an open question whether the ENSO mode is unstable or stable, and what the role of weather noise is (Neelin et al., 1998). Here we assume that the ENSO mode can be described by a stochastic oscillator (Burgers 1999). In a stochastic oscillator, noise is responsible for sustaining an irregular cycle. Alternatively, one can view the stochastic oscillator as a leading POP (Principal Oscillation Pattern, Hasselmann 1988) approximation to the system. The stochastic oscillator concept has been applied to a variety of problems by (Griffies and Tziperman 1995, Griffies and Bryan 1997, Jin 1997, Rivin and Tziperman 1997, Chang et al. 1997, Münnich et al. 1998).

The two-parameter, discrete form of the stochastic oscillator is

$$\begin{aligned} x_{i+1} &= a x_i - b y_i + \xi_i \\ y_{i+1} &= b x_i + a y_i + \eta_i \; . \end{aligned} \tag{1}$$

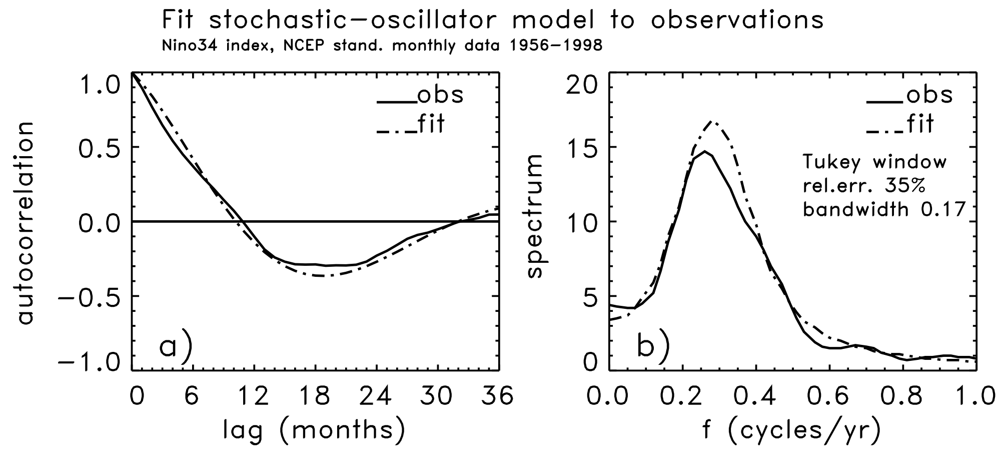

Fig. 1. Fit of a stochastic-oscillator model to observed data. In (a) the autocorrelation and in (b) the spectrum (normalized to 2π) of the observed Niño34 index over the period 1956–1998 (solid lines) is compared to the autocorrelation and the spectrum of a stochastic-oscillator model fitted to the observed series (dash-dotted lines).

Here x and y are the variables of the two degrees of freedom of the oscillator, a and b constants and ξ and η noise terms, which may be correlated. The index i labels the time. It is assumed that the noise between different timesteps is not correlated.

The variable x stands for an Niño SST index, e.g. the standardized Niño3.4 index, and y for another, independent index. Probably the thermocline depth in the Western Pacific is a second important ENSO variable (Jin 1997, Li 1997). It would be related to y by a linear transformation, necessary to achieve the symmetry in the deterministic part of (1).

The autocorrelation function of x that corresponds to (1) is an exponentially decaying cosine that depends on three parameters ω, γ and α: $\rho_k = e^{-k\gamma\Delta t}\cos(k\omega\Delta t + \alpha)/\cos\alpha$, where Δt (chosen here to be 1 month) is the timestep, $e^{-\gamma\Delta t}\, e^{i\omega\Delta t} = a + ib$, and the phase shift α is a function of ω, γ and the noise covariances.

Considering x only, (1) reduces to

$$x_{i+1} = 2ax_i - (a^2 + b^2)x_{i-1} + \epsilon_i - k\epsilon_{i-1} \ . \tag{2}$$

Estimates for the stochastic oscillator parameters can be found from a fit of observations to (2) that minimizes the variance of ϵ. A fit to the standarized Niño3.4 index over the period 1956-1998 gives $2\pi/\omega = 44 \pm 5$ months, $1/\gamma = 21 \pm 6$ months, and $\alpha = 5^\circ \pm 6^\circ$. In Figure 1, the autocorrelation and the spectrum of the stochastic-oscillator fit are compared to the autocorrelation and the spectrum of the observed timeseries.

3 Forecasts with and without using sub-surface data

Assuming perfect observations of x and no observations of y, it is straightforward to make a Kalman Filter based on (1) that can be used to make El Niño forecasts. In Burgers (1998, 1999) it is shown that the stochastic-oscillator has substantial skill in predicting Niño3.4 index observations and thus may serve as a baseline for the skill of El Niño forecasting systems.

However, in this contribution the focus is on comparing the forecast skill for the case that only the SST index x is measured to the case that both x and the sub-surface ocean variable y are measured. This is done for a theoretical system that is a stochastic-oscillator with parameters $2\pi/\omega = 45$ months, $1/\gamma = 20$ months and $k = 0.90$ ($\alpha = 7^\circ$). For the case only x is measured, the forecast skill is shown in Figure 2a. The anomaly correlation drops to 0.6 after about half a year.

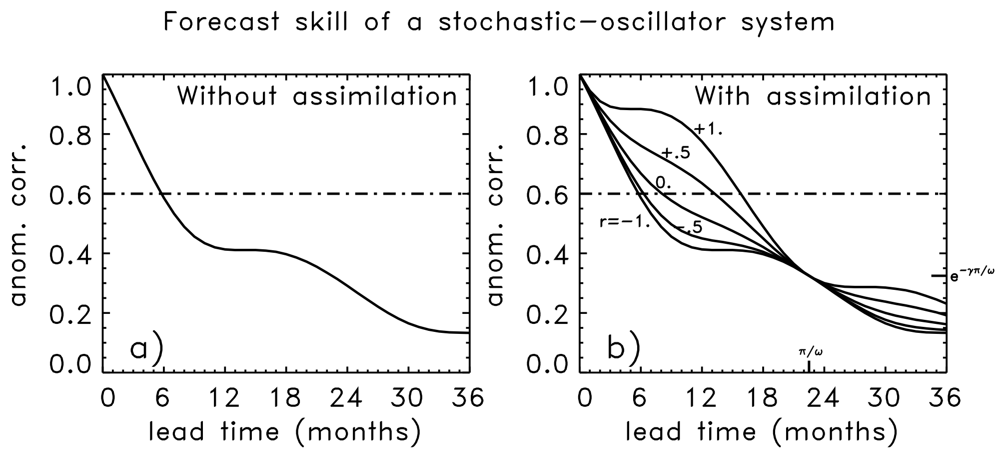

Fig. 2. Forecast skill of a stochastic oscillator system with $2\pi/\omega = 45$ months, $1/\gamma = 20$ months and $\alpha = 7°$. In (a) the anomaly correlation of Kalman-filter forecasts is shown if only the first variable (the Niño SST index) is measusred. and in (b) if both the first and the second variable (representing sub-suface information) of the stochastic oscillator are measured. In (b), the anomaly correlation depends on the noise correlation r between ξ and η in (1); shown are lines for $r = -1, -0.5, 0, +0.5$ and $+1$. The horizontal dash-dotted line indicates a forecast-skill level of 0.6.

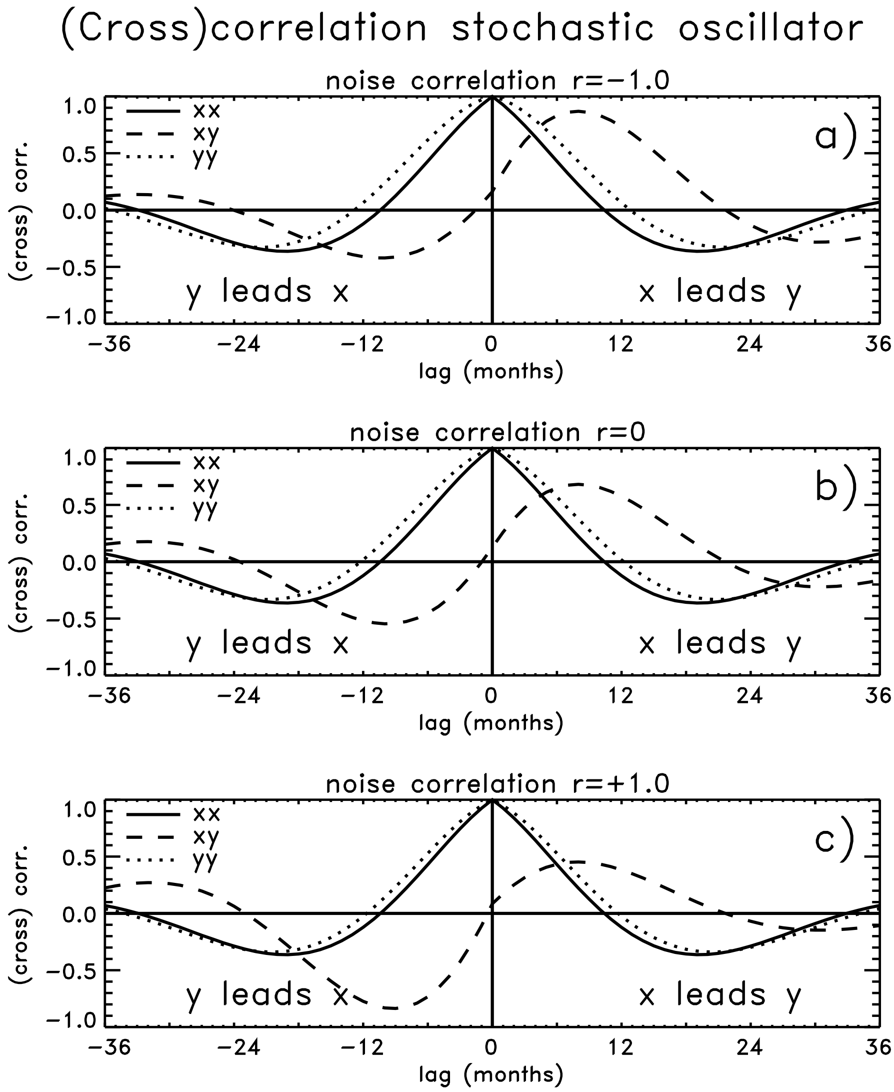

Fig. 3. (Cross)correlations of a stochastic oscillator with $2\pi/\omega = 45$ months, $1/\gamma = 20$ months and $\alpha = 7°$, for a noise correlation r between ξ and η in (1) that has a value of $r = -1$ in (a), $r = 0$ in (b) and $r = -1$ in (c). The solid line is the autocorrelation of the variable x of (1), the dashed line the cross-correlation of x and y, and the dotted line the autocorrelation of y.

Next we consider the case that both x and y are measured, and examine how much the forecast skill increases by measuring y. This depends, because a stochastic-oscillator fit to a timeseries of x does not specify completely the parameters in (1). Different combinations of noise covariances exist that yield the same autocorrelation of x but a different variance of y, depending on the *noise correlation* r between ξ and η in (1).

Figure 2b shows that the forecast skill depends on the noise correlation r as well. The increase in forecast skill (as measured by the lead time at which the anomaly correlation drops to 0.6) is quite substantial if $r = 1$, but if $r = -1$ measuring x was already enough to specify y and there is no increase in forecast skill at all! The difference in predictability is connected to a difference in cross-correlation between x and y, as shown in Figure 3: the larger the predictability, the larger the magnitude of the extreme in the cross-correlation function for y leading x.

4 Conclusions

Although the stochastic oscillator is about the simplest model possible for ENSO, knowledge of the full statistics of a Niño index does *not* tell how much forecast skill would be gained from measuring the "second" variable of the oscillator. The gain depends on the "noise terms" in the normal form of the stochastic oscillator and varies from zero to about a year in extreme cases. This illustrates that small-scale processes must be represented well before one can assess the predictability of a model.

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