ANALYTICAL CALCULATION OF STOKES PARAMETERS Q AND U OF ATMOSPHERIC RADIATION

L. G. Tilstra, N. A. J. Schutgens, P. Stammes

Koninklijk Nederlands Meteorologisch Instituut (KNMI), De Bilt, The Netherlands

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Abstract

For polarisation correction of the radiances measured by polarisation-sensitive satellite instruments like GOME, SCIAMACHY and GOME-2, both the Stokes parameters Q and U are of interest. Below a wavelength of about 300 nm, single Rayleigh scattering is the only mechanism contributing to the TOA reflectances. The single Rayleigh scattering polarisation therefore provides, next to these instruments' own polarisation measurements, an extra data point for their Polarisation Correction Algorithms. However, the Polarisation Correction Algorithm also needs the (single scattering) direction of polarisation for a proper correction of a large part of the visible spectrum. This makes the polarisation correction very sensitive to possible errors in the single scattering polarisation. This report presents a calculation of the single Rayleigh scattering Stokes parameters, and their verification using a radiative transfer code as well as POLDER data. The single scattering model is furthermore extended by including also Lambertian surface reflection.

1 Stokes parameters

The amount of circularly polarised light reflected by the Earth's atmosphere is negligible [1], and therefore only linearly polarised light needs to be considered. Linearly polarised light can be described by the Stokes parameters $\{I, Q, U\}$. They are defined as follows [2]

$$I = I_{0^\circ} + I_{90^\circ} \ , \tag{1}$$

$$Q = I_{0^\circ} - I_{90^\circ} \ , \tag{2}$$

$$U = I_{45^\circ} - I_{135^\circ} \ , \tag{3}$$

where I is the total intensity and Q and U contain all the information about the linear polarisation. In Equations (1) to (3) the angles denote the direction of the transmission axis of a linear polariser, relative to some reference plane. In this report, this is the local meridian plane, defined as the plane containing the local zenith and the spectrometer's viewing direction. The Stokes parameters Q and U can also be expressed in terms of the degree of linear polarisation P and the direction of polarisation χ in the following way [2]

$$Q/I = P \cdot \cos 2\chi \tag{4}$$

$$U/I = P \cdot \sin 2\chi \ . \tag{5}$$

For the degree of linear polarisation due to single Rayleigh scattering the following relation applies [3, 4]

$$P_{\rm ss} = \frac{1 - \cos^2\Theta}{1 + \Delta + \cos^2\Theta} \ , \tag{6}$$

where Θ is the scattering angle to be defined below and Δ is a correction factor for depolarisation due to molecular anisotropy [5]. It is related to the depolarisation factor $\rho_{\rm n}$ in the following way:

$$\Delta = \frac{2\rho_{\rm n}}{1 - \rho_{\rm n}} \ . \tag{7}$$

For a wavelength of 300 nm, ρ_n = 0.0318 and Δ accordingly amounts to 0.0656. Equation (6) is in fact very useful below 300 nm because in this regime only single scattering events contribute to the spectrometer's yield. In this report we will mainly focus on $\chi_{\rm ss}$, the single scattering direction of polarisation. In Section 5, the degree of polarisation P is also discussed.

2 Scattering geometry

Using the scattering geometry shown in Figure 1, the direction of polarisation for the single scattering case can be visualised in the following way. Consider the origin as the location of an imaginary volume element. The incident sunlight is described by the zenith angle θ_i and the azimuth angle ϕ_i, whereas the scattered light is characterized by the angles θ and ϕ. The position of the sun relative to the scattering volume is described by the zenith angle θ_0, which is given in the SCIAMACHY data product [6], while the azimuth angle ϕ_i is supplied as well. So we have the relations θ_i = $180^\circ - \theta_0$ and ϕ_i = ϕ_0, where the index '0' indicates that a variable is obtained from the SCIAMACHY data product. In this report, the azimuth angles are defined wrt a righthanded coordinate frame, using the positive zenith direction as the axis of rotation (cf. Figure 1).

The scattering angle Θ is, as always, defined as the angle between the directions of incident light and scattered light. Consequently, after some algebra, it can be found that it is given by

$$\cos\Theta = \cos\theta\cos\theta_i + \sin\theta\sin\theta_i\cos(\phi - \phi_i) \ , \tag{8}$$

or, alternatively, in terms of the incident and scattered light angles that are given in the data product of SCIAMACHY:

$$\cos\Theta = -\cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos(\phi - \phi_0) \ . \tag{9}$$

The incident light and the scattered light directions further define the scattering plane. Since we are only considering single Rayleigh scattering, the direction of polarisation of the scattered light lies in the polarisation

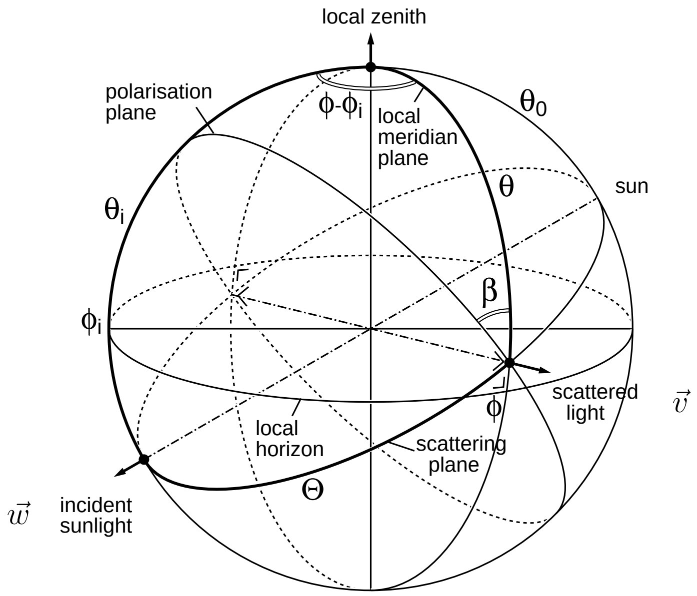

Figure 1: Unit sphere showing the incident light and scattered light directions and the angles involved. The imaginary volume element that is responsible for the scattering of light is located in the origin. In this report we make use of a righthanded coordinate frame, meaning that $\phi - \phi_i$ as sketched lies in the interval $(0°, 180°)$.

plane, which is perpendicular to the scattering plane and contains the direction of the scattered light. The local meridian plane, which is the plane containing the local zenith and the scanning direction, is also the reference plane of SCIAMACHY. Now, the single Rayleigh scattering direction of polarisation $\chi_{\rm ss}$ is usually defined as 'one of the angles' between the polarisation plane and the local meridian plane. In the following we discuss a more precise definition of χ.

3 Direction of polarisation for single Rayleigh scattering

3.1 Definition of χ wrt the local meridian plane

From a physical point of view the most natural approach would be to define the Stokes parameters with respect to the (single) scattering plane. This is the plane containing both the viewing direction and the direction of the incident sunlight. In this case the single scattering calculation is extremely simple, because Rayleigh scattering produces a (linear) polarisation vector perpendicular to the scattering plane. We then have $U_{\rm ss} = 0$ and, by definition, for the direction of polarisation we find $\chi_{\rm ss} = 90°$. For most satellite instruments, however, the Stokes parameters are commonly defined with respect to the local meridian plane. Thus, the reference plane is the plane containing the local zenith and the instrument's viewing direction.

Unfortunately, this definition of the reference frame does not uniquely define Stokes parameter U. For Q, the definition is both trivial and unique, but for U, the so-called $45°$-direction is not specified by the definition that was given. This leaves us with two possible definitions for U (and therefore also for χ). Both conventions are being used in the literature, usually without much care about the proper definition. Figure 2 should illustrate the situation more clearly, using the additional information that the light is supposed to be travelling into the paper. The definition in the left frame, called 'Type 1' from now on, is used by among others Van de Hulst [2], Chandrasekhar [3] and Hovenier & de Haan [7, 8]. On the other hand, Slijkhuis [9] and Aben *et al.* [10] use the rightmost definition, referred to as 'Type 2' in the remainder of this report.

The whole point is that if one calculates the single scattering Stokes parameters in the analysis of data from a spectrometer like SCIAMACHY, one is not free to choose the definition. The proper definition should be prescribed by the actual reference plane inside the spectrometer itself plus the proper direction of rotation

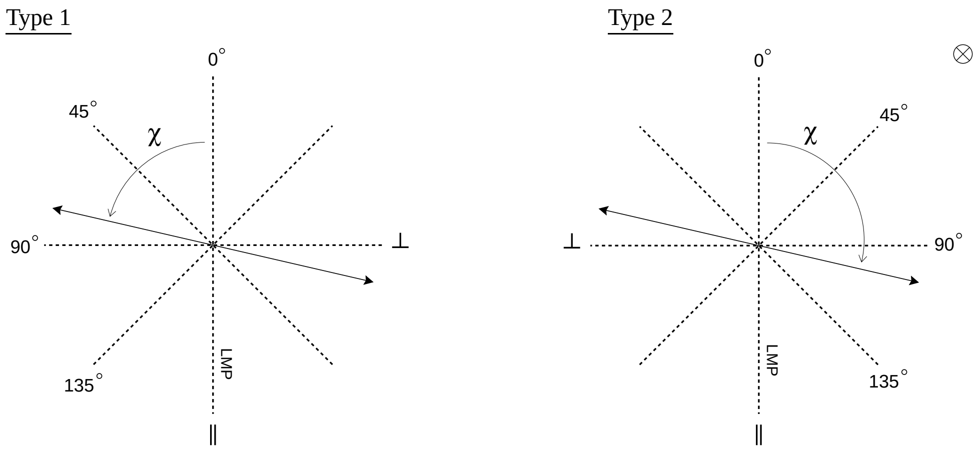

Figure 2: Two different definitions can exist that define the Stokes parameters Q and U with respect to the local meridian plane. In both figures (left and right), the light is travelling perpendicular to and into the paper ($\otimes$). The difference between both conventions is the sign of U. However, Q is the same for both conventions.

for χ. On the basis of this technical information, one should select the expressions for either 'Type 1' or 'Type 2'. When by mistake the wrong definition is applied, this will result in a sign-error in U, and instead of finding $\chi_{\rm ss}$ one actually calculates $180° - \chi_{\rm ss}$. From Figures 1 and 2 it is clear that we should, in any case, calculate the auxiliary angle β in order to proceed.

3.2 Calculation of β

We will calculate the auxiliary angle β shown in Figure 1 using a vector approach. The scattered light and the incident sunlight are described by two vectors $\vec{v}$ and $\vec{w}$ for which we can write down [11]

$$\vec{v} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta) \qquad (10)$$

$$\vec{w} = (\cos\phi_i\sin\theta_i, \sin\phi_i\sin\theta_i, \cos\theta_i)\ , \qquad (11)$$

which are the usual Cartesian vectors in terms of spherical coordinates. Both $\vec{v}$ and $\vec{w}$ are normalised in the sense that $|\vec{v}| = |\vec{w}| = 1$, so that we are operating on the unit sphere sketched in Figure 1. Next we calculate the normal vector of the scattering plane that is made up by $\vec{v}$ and $\vec{w}$. It is given by $\vec{n}_s = \vec{w} \times \vec{v}$, or, more explicitly, by

$$\begin{aligned}\vec{n}_s = (&\sin\phi_i\sin\theta_i\cos\theta - \sin\phi\sin\theta\cos\theta_i,\\ &\cos\phi\sin\theta\cos\theta_i - \cos\phi_i\sin\theta_i\cos\theta,\\ &\cos\phi_i\sin\theta_i\sin\phi\sin\theta - \cos\phi\sin\theta\sin\phi_i\sin\theta_i)\ .\end{aligned} \qquad (12)$$

Notice that we have $|\vec{n}_s| = \sin\Theta$ because of the definition of the vector cross product. This will be used further on in the calculation. Also notice that $\vec{n}_s$, if drawn, would point upward in Figure 1 since $\phi - \phi_i$ was assumed to lie in the interval $(0°, 180°)$. This is not always the case, and we have to keep this in mind. To proceed we must calculate the normal to the local meridian plane, which is made up by the local zenith $\vec{e}_z$ and the scattered light vector $\vec{v}$. The normal is defined as $\vec{n}_m = \vec{v} \times \vec{e}_z$, and reads, after performing the appropriate normalisation,

$$\vec{n}_m = (\sin\phi, -\cos\phi, 0)\ . \qquad (13)$$

The next step in the calculation of the single-scattering direction of polarisation is to calculate the angle α between the normals in Equations (12) and (13) by means of their inner product, recalling that $|\vec{n}_m| = 1$ and $|\vec{n}_s| = \sin\Theta$. The need for this procedure is more or less explained in Figure 3, which shows the relevant geometry for two different situations that can occur, and the relation between α and β. The left frame relates to a geometry with $\sin(\phi - \phi_0) > 0$ and the second does so for an almost identical situation, but with ϕ and ϕ_0 interchanged to arrive at $\sin(\phi - \phi_0) < 0$. Both situations are in fact visualising what is known as the

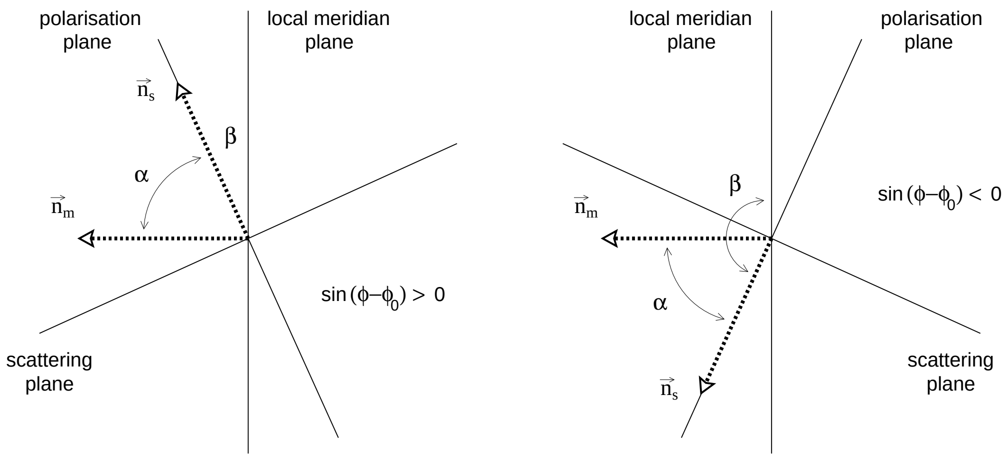

Figure 3: The relevant planes, and their normal vectors, as seen by the satellite instrument, looking into the direction of the scattering volume element (cf. Figure 1). The left-hand frame shows a particular situation with $\sin(\phi - \phi_0) > 0$. The right-hand frame shows an almost identical situation, but with ϕ and ϕ_0 numerically interchanged so we have $\sin(\phi - \phi_0) < 0$. The two fi gures are illustrating what is known as the mirror property (that is, for a homogeneous atmosphere $U \rightarrow -U$ when $\phi - \phi_0 \rightarrow \phi_0 - \phi$).

mirror property: $U \rightarrow -U$ when $\phi - \phi_0 \rightarrow \phi_0 - \phi$; see [7]. The result for α is of course applicable to either of the situations and reads

$$\cos\alpha = -\frac{\sin\theta\cos\theta_i - \sin\theta_i\cos\theta\cos(\phi - \phi_i)}{\sin\Theta}\,. \tag{14}$$

According to Figure 3, depending on the situation, we then have either $\beta = 90^\circ - \alpha$ or $\beta = 90^\circ + \alpha$, where the $-$ sign applies for $\sin(\phi - \phi_0) > 0$ and the $+$ sign for all other situations. The formula can also be applied for exact nadir view. For exact back- and forward scattering β does not exist, which is correct as χ cannot be defined, or at least has no meaning, for this particular scattering geometry.

3.3 Final recipe for χ_{ss}

We are now in a position to give the final recipe for the calculation of the single scattering direction of polarisation. Using Figures 1 and 2, we find that χ_{ss} is equal to $180^\circ - \beta$ for Type 1. For Type 2, χ_{ss} is equal to β. The end result is therefore given by the following equation:

$$\chi_{ss} = \begin{cases} -(90^\circ \pm \alpha) \mod 180^\circ & \text{Type 1} \\ 90^\circ \pm \alpha \mod 180^\circ & \text{Type 2} \end{cases} \tag{15}$$

where, in both cases,

$$\cos\alpha = \frac{\sin\theta\cos\theta_0 + \sin\theta_0\cos\theta\cos(\phi - \phi_0)}{\sin\Theta}\,. \tag{16}$$

As for the $\pm$, the $-$ sign should be applied when $\sin(\phi - \phi_0) > 0$, and the $+$ sign for all other situations. For the definition of Types 1 and 2, see Section 3.1. Note that χ_{ss} is defined modulo 180 degrees. From χ_{ss} in combination with Equations (4) to (6) we then calculate $(Q/I)_{ss}$ and $(U/I)_{ss}$.

4 Verification

In this section we verify the correctness of Equations (15) and (16). This is done in three different ways. First of all, a comparison is made with a polarised radiative transfer code. After that, we compare the results of our calculation with those of other known expressions in the literature. We end the verification by comparing our result with real-life POLDER polarisation data.

4.1 Comparison with a polarised radiative transfer code

As a means of comparison, we used the polarised radiative transfer code DAK ('Doubling-Adding KNMI') [12, 13, 14] to calculate the TOA Stokes fractions Q/I and U/I directly. This was done for a wavelength of 298 nm, for which the results should be similar to those of single scattering theory because of the strong ozone absorption at this wavelength [16]. The surface albedo was set to zero, and clouds and aerosol were removed, so only Rayleigh scattering and ozone absorption were allowed. Figure 4 presents the ratio Q/I as determined from the calculation performed in Section 3, while Figure 5 represents the Q/I calculated by the DAK code. Figures 6 and 7 do basically the same but now for the fractional Stokes parameter U/I.

The plots were made by selecting several θ, θ_0 and $\phi - \phi_0$ values covering the complete range relevant to SCIAMACHY, which were fed into the vector calculation type '1' described in Section 3, in combination with Equations (4) to (6). For Q/I, the agreement is excellent. For U/I we also find complete agreement, including the proper sign, which is a direct consequence of the fact that DAK is (also) of type '1' since it uses the same definition of χ as References [12, 13] do. All this proves that (i) the direction of polarisation by single Rayleigh scattering alone given by Equation (15) was calculated correctly, (ii) expression (6) is seen to hold. The last argument is important because it confirms that at the wavelength used, there are no other scattering processes but single Rayleigh scattering ones that contribute to the TOA Stokes vector. We can therefore conclude that it was well-justified to perform the comparison done.

4.2 Other results found in literature

Another thing worth checking is that Equation (15) obeys what is known as the mirror property [7]. This property can best be summarised as $U \rightarrow -U$ when $\phi - \phi_0 \rightarrow \phi_0 - \phi$. The vector calculation indeed does not break the property. The calculation presented in Reference [15], however, does violate the mirror property. This was not a problem at the time as Reference [15] was written for application with GOME, which is presumably not sensitive to U. So the sign of U was not relevant. But strictly speaking, the equations presented there are not correct as the mirror properly should be followed in all cases.

As for the calculation by Slijkhuis [9] and Aben *et al.* [10], they do obey the mirror property, but the results are presented without the division in types '1' or '2'. In fact, both are of type '2'. These calculations agree with each other, at least numerically, and they also agree with the vector calculation (of 'Type 2') presented in this report. At the same time, the expression Hovenier uses for $\chi_{\rm ss}$ [8] completely agrees with the vector calculation 'Type 1'. Neither of the four analytic expressions under consideration are the same but the only numerical difference between the four approaches takes place at the backscattering condition (Θ=$180°$), which is allowed because polarisation is absent for this geometry because of symmetry (P=0). Here the direction of polarisation cannot be defined in a physically meaningful way.

Reference [7] presents several analytical results for a homogeneous, plane-parallel atmosphere. For exact nadir view (θ = 0°), the ratio U/Q, which by definition equals $\tan 2\chi$, was found to be equal to

$$U/Q = \tan 2\chi = \tan 2(\phi - \phi_0)\ . \tag{17}$$

This expression is valid only for type '1' because this was the convention used by Hovenier & de Haan. For type '2', a minus sign needs to be placed in front of the last term. With little calculus, it can be found that expression (15) indeed obeys Equation (17) for type '1'. The calculations performed by Slijkhuis [9] and Aben *et al.* [10] differ only by a minus sign, as expression (15) for type '2'. The result presented in Reference [15] violates Equation (17). In conclusion, the result given in Equations (15) and (16) presented in this report does appear to be valid on the grounds of analytical considerations also.

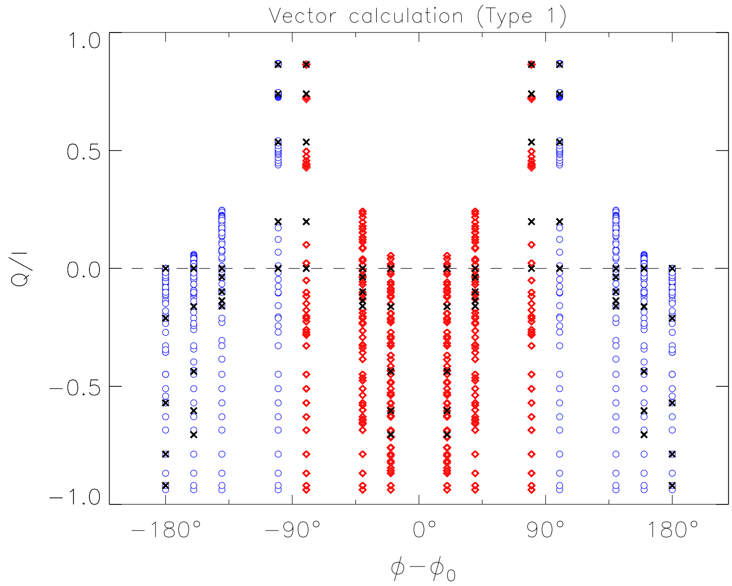

Figure 4: Fractional Stokes parameter Q/I, calculated for the analytical vector approach Type 1, as a function of $\phi - \phi_0$ for a set of combinations of θ, θ_0 and $\phi - \phi_0$. Red diamonds relate to east pixels, for which $|\phi - \phi_0| < 90°$, blue circles relate to west pixels, for which $|\phi - \phi_0| > 90°$. Exact nadir pixels, that have θ = 0°, are indicated by the black crosses.

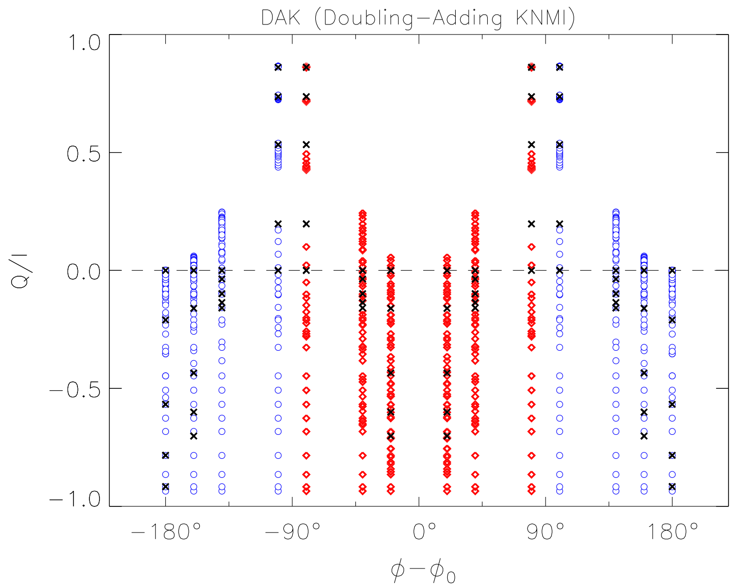

Figure 5: The same plot, but now calculated numerically by the multiple scattering DAK code for a clear Midlatitude Summer (MLS) atmosphere with surface albedo zero at 298 nm. Notice that the picture agrees with the previous one, in all aspects. The numerical agreement between the analytical result and the DAK result is very high.

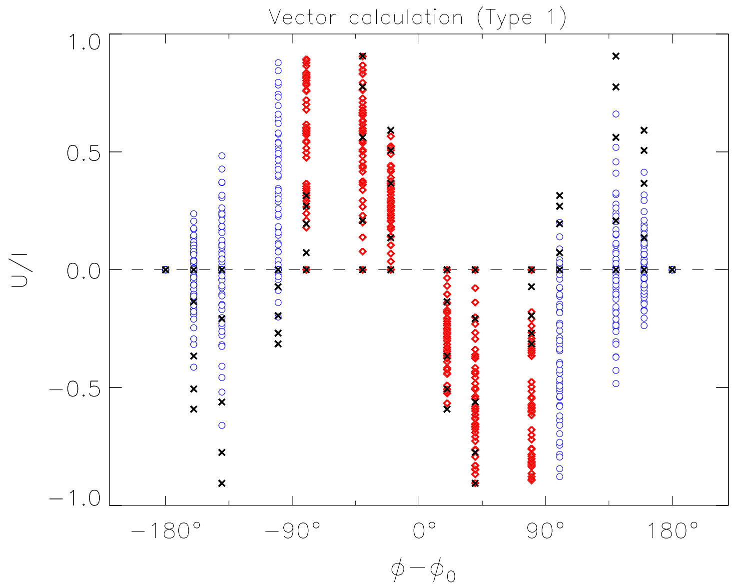

Figure 6: Fractional Stokes parameter U/I, calculated for the analytical vector approach (according to definition '1'). The colours and symbols used in the plot have the same meaning as in Figure 4.

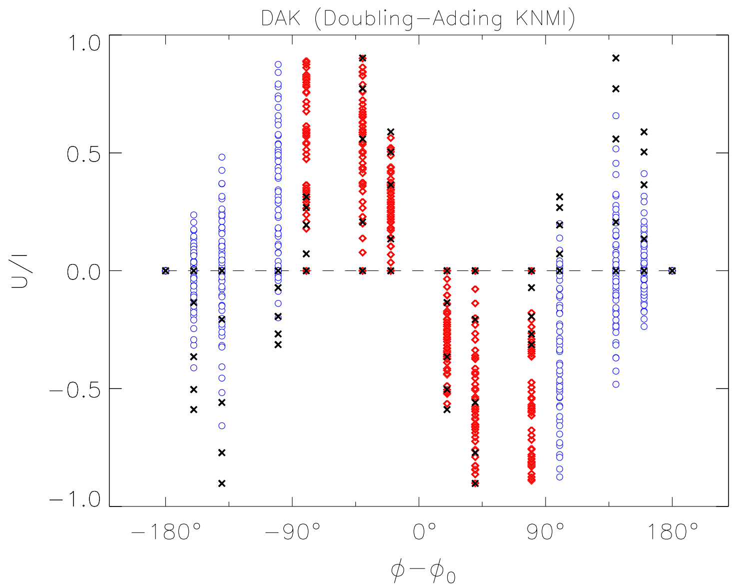

Figure 7: The same plot, but now calculated numerically by the DAK code. Notice that the picture again agrees with the previous one, including the overall sign of U/I. This is because DAK follows the same definition of χ.

4.3 Comparison with POLDER data

We also compared χ_{ss} found from Equation (15) with real atmospheric data obtained by the spaceborne satellite instrument POLDER, which was part of the ADEOS polar orbiting satellite launched in August 1996. POLDER (POLarization and Directionality of the Earth Reflectances) was designed to measure all three relevant components of the Stokes vector (I, Q, and U) for a number of wavelengths (443–670–865 nm), and it did so perfectly for a period of 8 months after which the ADEOS platform died unexpectedly.

Figure 8 presents the direction of polarisation at 443 nm, obtained from the POLDER Stokes parameters Q and U, as a function of the calculated χ_{ss}. Different surface types are indicated by the colours green and blue (land/water) and the possible presence of clouds can be found from the symbols used. It should be noted that, apparently, the POLDER data product presents its Stokes parameters Q and U with respect to reference frame '2', because on the horizontal axis, χ_{ss} was calculated for 'Type 2'. For all scenery types, the agreement is very good, which is the reason why the single scattering direction of polarisation can be used to force a fixed relationship between Q and U in the SCIAMACHY data processor [6] for a large part of the visible spectrum.

However, we do not want to comment on the validity of this approximation, as we are only interested in validating the main result of this report, namely Equation (15). A concise, systematic report on the validity of the assumption that $\chi \approx \chi_{ss}$ can be found in Reference [17]. In summary, real atmospheric data confirm that Equation (15) is the proper expression for the single scattering direction of polarisation.

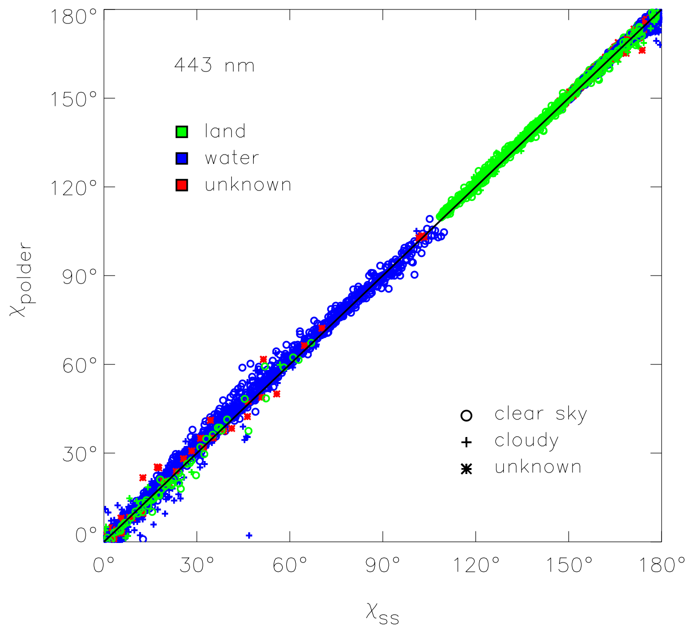

Figure 8: Direction of polarisation χ_{polder} as measured by the POLDER instrument for various scenes at a wavelength of 443 nm, as a function of the calculated single Rayleigh scattering direction of polarisation χ_{ss}. Here χ_{ss} is calculated wrt the local meridian plane, as given by Equations (15) and (16). Scenery types (land/water) and cloud presence are indicated by the use of colours and symbols, respectively. The overall agreement is very good.

5 Including Lambertian surface reflection

In what follows, we will attempt to extend the single scattering model by including also (Lambertian) surface reflection into the model. This results in a valid approximation of the TOA Stokes parameters only if the scene to be modelled is not contaminated with clouds, and then only for wavelengths sufficiently large to ensure that multiple scattering events do not contribute significantly to the TOA signal.

5.1 Single scattering and surface reflection

Surface reflection, assuming it is Lambertian, does not contribute to the Stokes parameters Q and U. Hence, the direction of polarisation is insensitive to surface reflection and therefore Equation (15) is still applicable to the situation at hand. For I, and thus for P, the surface type does play a significant role, especially at the larger wavelengths, where the optical thickness of the (Rayleigh) atmosphere is low. Calculating the effects of Lambertian surface reflection on P using a model atmosphere is relatively simple. For the intensity of the light that is being reflected back into space we can write down straightaway

$$\begin{aligned} I &= \frac{3}{4}\Delta'(1+\Delta+\cos^2\Theta)\int_0^{\tau_\mathrm{s}} d\tau\, e^{-\tau/\mu}\, e^{-\tau/\mu_0}\, \mu_0 E/\pi \\ &\quad + A\, e^{-\tau_\mathrm{s}/\mu}\, e^{-\tau_\mathrm{s}/\mu_0}\, \mu_0 E/\pi\ , \end{aligned} \tag{18}$$

where τ_s denotes the scattering optical thickness of the atmosphere, τ is the optical depth variable, A refers to the Lambertian surface albedo, $\mu = |\cos\theta|$, $\mu_0 = |\cos\theta_0|$, and E is the solar irradiance, perpendicular to the beam. The correction factors Δ and Δ' are due to molecular anisotropy [5]. We have neglected the effects of absorption altogether, because this does not contribute much at the wavelengths under consideration and it facilitates the calculation. Next we calculate the Stokes parameter Q. This is done with respect to the single scattering plane, because by definition it coincides with the physical reference plane of molecular (Rayleigh) scattering. The proper expression is then found to be

$$Q = \frac{3}{4}\Delta'(-1+\cos^2\Theta)\int_0^{\tau_\mathrm{s}} d\tau\, e^{-\tau/\mu}\, e^{-\tau/\mu_0}\, \mu_0 E/\pi\ . \tag{19}$$

For Rayleigh scattering, Stokes parameter U wrt the scattering plane is zero by definition, and this is not altered by the (Lambertian) surface reflection. The integrals in Equations (18) and (19) can be dealt with in an analytical way. Using $P = -Q/I$, we conclude that the degree of linear polarisation due to the sum of single Rayleigh scattering of incident sunlight by molecules in the Earth's atmosphere and Lambertian surface reflection is given by the rather simple expression

$$P_\mathrm{sr}(M,\Theta) = \frac{1-\cos^2\Theta}{1+\Delta+\cos^2\Theta+\dfrac{4}{3}\dfrac{AM}{\Delta'}\left\{\dfrac{\exp(-M\tau_\mathrm{s})}{1-\exp(-M\tau_\mathrm{s})}\right\}}\ . \tag{20}$$

In this equation, $M = 1/\mu + 1/\mu_0$ is the geometrical airmass. Equation (20) represents a simple model, but may in fact be very useful for many situations in practice.

5.2 Comparison with POLDER data

To test the simple polarisation model of the previous subsection, we restricted ourselves to POLDER data of cloud-free scenes over the Atlantic ocean. The surface albedo of seawater does not vary as much as that of land, and the POLDER ground pixels are small enough (6×7 km^2) to actually find cloud-free scenes, making a reliable comparison possible. For the wavelength we resorted to 443 nm, where we expect the model to have more difficulties in agreeing with the data than at, for instance, the also available wavelength of 865 nm. At the latter wavelength, however, the Q and U Stokes parameters are much smaller and therefore much less reliable because of the low optical thickness of the (clear-sky) atmosphere. Figure 9 presents some of the results. On the vertical axis we plotted P_polder (in blue) and on the horizontal axis P_ss according to Equation (6). The black circles present the model values P_sr of Equation (20).

For the model, we used reasonable values for the surface albedo A and optical thickness τ_s. At the wavelength used, however, multiple scattering may not always be ignored and therefore it cannot be excluded that both parameters may effectively contain multiple scattering contributions. Nevertheless, the agreement of the model with the actual data is very satisfactory. The use of red dots in part of the data has not been addressed yet. They relate to scattering geometries around the rainbow scattering angle of 140° that yield an increase in the polarised radiance when the atmosphere contains water droplets with a radius larger than

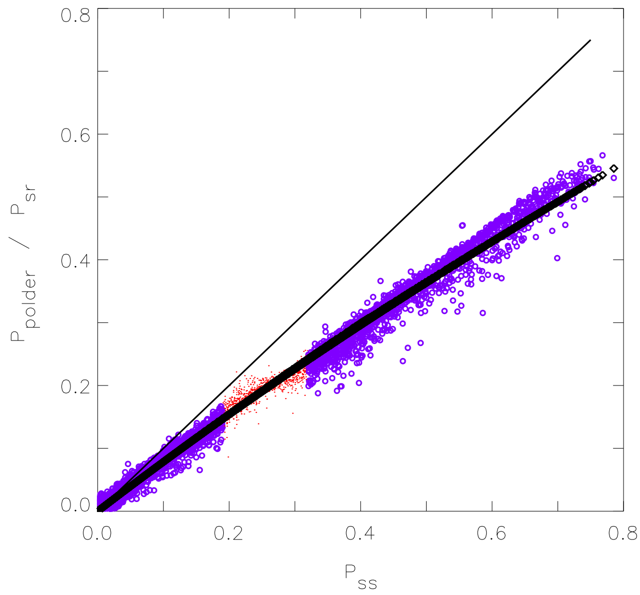

Figure 9: Degree of polarisation measured by the POLDER instrument, as a function of the single scattering degree of polarisation P_{ss}. The data was taken from a scene over the Atlantic ocean and only clear-sky pixels are shown (data points are indicated by blue circles, the red dots are explained in the text). The model calculation P_{sr} of Equation (20) is presented as well, by the black points. The agreement of the model calculations with the POLDER data is good.

the wavelength [1, 2]. Although scenes containing clouds were taken out of our data set, we do see a small increase in the red-dot region, indicating that some of the pixels may still have a small cloud fraction even though the POLDER cloud mask classified the pixel as 'unclouded'.

In conclusion, the simple Lambertian-single scattering model is able to describe the data quite well, even for a wavelength of 443 nm. At the longer wavelengths (from, say, 500 nm and upward), the model should be quite accurate in describing the degree of polarisation, at least for clear-sky situations. Of course, an accurate lookup table for Rayleigh optical thickness and surface albedo is then needed.

6 Conclusion

We have calculated the single Rayleigh scattering direction of polarisation defined with respect to the local meridian plane. The calculation was performed using a simple vector approach and validated with multiple scattering calculations and POLDER polarisation measurements. The resulting equation is relevant for among others the polarisation correction algorithm of SCIAMACHY. Other expressions in the literature do exist, but they either differ numerically or make no distinction between the two possible definitions of the direction of polarisation with respect to the local meridian plane mentioned in this report. We also present an extension of the atmospheric single scattering model by including Lambertian surface reflection into the model. The model appears to be quite successful in determining the Stokes parameters Q and U for cloud-free observations made by the POLDER satellite instrument.

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