A NOTE ON THE DRAG OF THE SEA SURFACE AT HURRICANE WINDS

V. K. MAKIN*
Royal Netherlands Meteorological Institute, De Bilt, The Netherlands

(Received in final form 14 September 2004)

Abstract. Based on the solution of the turbulent kinetic energy balance equation for the airflow in the regime of limited saturation by suspended sea-spray droplets, some experimental evidence, and simple arguments, a resistance law of the sea surface at hurricane winds is derived. It predicts the reduction of the drag coefficient for the wind speed exceeding hurricane values of 30–40 m s^{-1} in agreement with field data.

Keywords: Hurricane wind speeds, Sea drag, Sea-spray droplets.

1. Introduction

Knowledge of the air–sea interaction at hurricane wind speeds is of primary importance for severe weather forecasts, in risk assessments, wave and surge forecasts, marine engineering and transport safety. Up to now little was known on this subject, and the formulations of the surface stress (sea drag) for the above mentioned problems were, and still are, simply extrapolated from parameterizations obtained at much lower wind speeds. One of the few, if not the only, comprehensive field study of the air–sea interaction at high wind speeds in tropical cyclones was recently reported by Powell et al. (2003). The authors obtained quite an unexpected result – at wind speeds increasing above hurricane values of about 33 m s^{-1}, the drag coefficient levels off and starts to decrease with a further increase in the wind speed. This is contrary to the behaviour of the drag coefficient parameterizations that are currently used in ocean and atmosphere applications.

This result is illustrated in Figure 1, where the friction velocity u_* and the drag coefficient at 10-m height

$$C_{d10} = \frac{u_*^2}{u_{10}^2} \tag{1}$$

are shown (u_{10} is the wind speed at 10-m height); they were obtained as follows. Analysis of the measured wind profiles at hurricane wind speeds (Powell et al., 2003) reveals that, in the atmospheric layer 10–200 m above the ocean, the wind speed u is described by the logarithmic profile

---

* E-mail: makin@knmi.nl

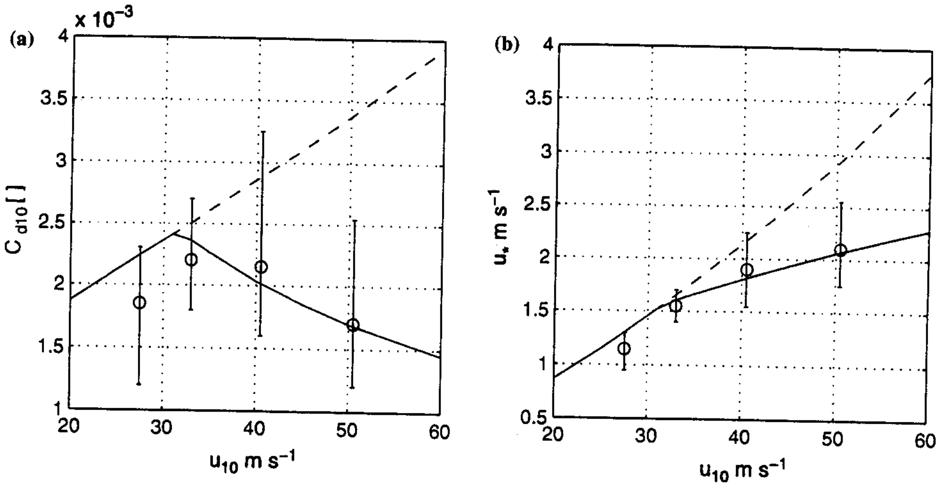

Figure 1. (a) Drag coefficient versus the wind speed, (b) Friction velocity versus the wind speed. Solid line, according to the resistance law (11); dashed line, according to the Charnock relation (3); open circles, data by Powell et al. (2003).

$$u(z) = \left(\frac{u_*}{\kappa}\right) \ln \frac{z}{z_0}, \tag{2}$$

where κ is the von Karman constant and z_0 is the aerodynamic roughness length. The original data (Figure 3 in Powell et al., 2003) contain four estimates of u_* obtained by fitting the logarithmic wind profile in the layers 10–100, 10–150, 20–100, and 20–150 m. In Figure 1 their average is shown with maximum and minimum estimates of the error bar. The drag coefficient and the friction velocity that follow from (1) and (2), with the roughness length traditionally defined by the Charnock relation (Charnock, 1955)

$$z_0 = c_{z_0} \frac{u_*^2}{g}, \tag{3}$$

are shown in Figure 1 as well. A typical value of the Charnock constant 0.01 (e.g., Smith, 1980) for developed seas is taken. The drag parameterization in terms of the Charnock relation is widely used in different ocean and atmosphere studies. From the diagram it is clear that the parameterization considerably overestimates the measured drag coefficient and the friction velocity when the wind speed exceeds 33 m s^{-1}. It is also clear that the observed drag coefficient levels off at about 33 m s^{-1} and decreases at higher wind speeds.

Powell et al. (2003) speculate that increased foam coverage resulting from intensively breaking waves could progressively form a 'slip' surface at the air–sea interface that leads to the reduction of the sea drag at wind speeds above 40 m s^{-1}. In addition sea spray is hypothesized to significantly influence the transfer of momentum. Their evidence is cited: As the wind approaches 50 m s^{-1}, the sea becomes completely covered by a layer of foam and it is difficult to discern individual wave-breaking elements in the reduced visibility from spray and rain.

The present study is based on the experimental evidence and results of Powell et al. (2003). An attempt is made to explain the reduction of the sea drag at hurricane wind speeds, and to that end a resistance law of the sea surface at hurricane wind speeds is derived. It is obvious that at hurricane wind speeds sea-spray droplets originating from actively breaking waves form a deep suspension layer above the sea surface. It is further assumed that a thin region adjacent to the sea surface part of the suspension layer is characterized by a regime of limiting saturation. In this regime, for the case of an unrestricted supply of droplets by actively breaking waves, which is expected under hurricane winds, the airflow absorbs the maximum possible amount of droplets for the given friction velocity and other parameters. The description of the suspension layer in the regime of limiting saturation is based on the balance equation of the turbulent kinetic energy (TKE) for an airflow with suspended particles, in this case sea-spray droplets. Introducing an additional assumption that some droplet parameters in the regime of limiting saturation reach their critical value, a reduction of the drag coefficient for wind speeds exceeding hurricane values of about 30–40 m s^{-1} is obtained in agreement with the experimental data. It should be made clear that this note explores a well known and well established TKE approach and its solution for a flow containing suspended particles in the regime of limiting saturation (Barenblatt, 1953, 1979), and attempts to explain a phenomenon to which the approach was not applied before.

2. Suspension Layer in the Regime of Limiting Saturation

At very high wind speeds a deep part of the marine atmospheric surface layer is filled with spray droplets, resulting from intensive wave breaking, that form the so-called suspension layer. A particle remains in suspension at some average height due to the balance between the downward gravitational force and the upward force due to the drag exerted on the particle by the turbulent stress (e.g., Barenblatt, 1979). In the suspension layer the heaviest particles remain closer to the surface, so that the particle concentration should monotonically decrease with height. In other words particles, the spray droplets over the ocean, form a very stable boundary layer close to the surface. It is assumed that under hurricane wind speeds a thin region adjacent to the sea surface part of the suspension layer could be characterized by the regime of limiting saturation.

The solution for a flow containing suspended particles in the regime of limiting saturation is based on the TKE equation and is given in detail by Barenblatt (1979). The advantage of using this solution is that the determination of the regime of limiting saturation does not require prescribing any boundary condition for the particle concentration. The profile of the wind velocity is given by (Barenblatt, 1979)

$$u(z) = \left(\frac{u_*}{\kappa\omega}\right) \ln z + c, \tag{4}$$

where c is an integration constant, and the function ω is positive and satisfies the condition

$$\omega = \frac{a}{\kappa u_*} < 1. \tag{5}$$

In Equation (5) a is the terminal fall velocity of the droplets. A simple physical consideration for the condition (5) is given by Barenblatt (1979): the friction velocity is proportional to the mean square of the turbulent velocity fluctuation, and the latter should be large enough to carry the particle into the airflow and keep it sustained. In terms of the flux Richardson number Rf, which is a ratio of the turbulent flux of suspended particles to the production of TKE due to the shear stress and characterizes the impact of particles on the flow dynamics, function ω could be written as (Barenblatt, 1979)

$$\omega = (1 - Rf)^{1/4}\Phi(Rf), \tag{6}$$

where Φ is a universal function satisfying the condition $\Phi(0) = 1$ and being a non-increasing function of its argument Rf. When the impact of suspended particles on the flow is negligible $Rf = 0$, $\omega = 1$ and (4) describes a logarithmic wind profile in the flow without particles. With Rf increasing particles start playing an important role in the flow dynamics, ω is decreasing, and (4) describes the acceleration of the airflow in the regime of limiting saturation. So, under the same external forcing, the particles accelerate the flow in comparison with the flow without particles.

Equation (4) could be rewritten as

$$u(z) = \left(\frac{u_*}{\kappa\omega}\right) \ln\frac{z}{z_0^l}, \tag{7}$$

where the boundary condition $u(z = z_0^l) = 0$ is used, and z_0^l is the local roughness length characterizing the wind profile inside the suspension layer in the regime of limiting saturation. For $\omega = 1$ the roughness length $z_0^l = z_0$ as described by the Charnock relation (3). It is assumed that in the regime of limiting saturation the local roughness length z_0^l still could be described by the Charnock relation (3). It was shown by Kudryavtsev and Makin (2001) that the separation of the airflow from short steep breaking waves at high wind speeds is responsible for the formation of the surface stress and is well described by the Charnock relation (see their Figure 1b). The local roughness length reflects the impact of waves on the airflow dynamics. At hurricane wind speeds all short waves are actively breaking and the use of the Charnock relation inside the thin suspension layer in the regime of limiting saturation could be justified. It is important to note that, inside the suspension layer in the regime of limiting saturation, the wind profile (7) is characterized by the slope $u_*/\kappa\omega$ and the local roughness length z_0^l, while above that layer the wind profile is described by the logarithmic form (2) with the slope u_*/κ and the effective roughness length z_0. The effective roughness length z_0 now reflects the impact of both waves and sea-spray droplets on the flow dynamics and cannot be described by the Charnock relation.

3. Resistance Law of the Sea Surface at Hurricane Winds

To proceed further in the derivation of the resistance law function ω should be defined. As the universal function Φ in (6) is not known, the condition (5) and the data depicted in Figure 1 are used here to determine the terminal velocity a. It is first noted that for wind speeds up to 33 m s^{-1} the distribution of the drag coefficient is well defined by the logarithmic resistance law (2), where the roughness length is described by the Charnock relation (3). This means that the impact of the sea droplets on the dynamics of the airflow at this regime is still small, so that function ω should be equal to 1. At a wind speed of about 33 m s^{-1} the drag coefficient starts levelling off, implying that the spray droplets begin to influence the dynamics of the airflow. The corresponding measured friction velocity for this regime is $u_{*\mathrm{cr}} = 1.55$ m s^{-1} (Figure 1b), and from (5) the critical value of the terminal velocity could be estimated as $a_{\mathrm{cr}} = \kappa u_{*\mathrm{cr}} = 0.64$ m s^{-1}. This estimate of the terminal velocity corresponds to a droplet radius of about 80 μm. This is a plausible value. Though the production rate of the sea spray is quite poorly known, several of the empirical forms available have a peak in the volume production rate in the radius range 80–200 μm (e.g., Andreas, 1998), with a very sharp drop-off at larger radii. These are initial radii; the equilibrium radius of the droplet is roughly half of this in an ambient humidity of 80%. Thus, the estimated value of 80 μm agrees well with empirical estimates of the maximum droplet production. It is further assumed that the critical value a_{cr} remains the same with increasing the wind speed. So, for function ω the relation

$$\omega = \min\left(1, \frac{a_{\mathrm{cr}}}{\kappa u_*}\right) \tag{8}$$

is used.

Assume now a two-layer model for the marine atmospheric surface layer in the presence of sea-spray droplets: a thin region adjacent to the sea-surface suspension layer in the range of limiting saturation of height h_l, $z_0^l \leq z \leq h_l$, and the logarithmic boundary layer above: $h_l \leq z \leq H$, where H is its height. According to Powell et al. (2003) at hurricane wind speeds H is about 200 m.

In the suspension layer in the regime of limiting saturation the velocity distribution is described by (7), and in the layer above by (2). By overlapping profiles at height $z = h_l$, a resistance law of the sea surface at hurricane wind speeds is obtained

$$C_{dz} = \frac{u_*^2}{u_z^2} = \kappa^2 \left[\ln \frac{z}{h_l} + \omega^{-1} \ln \frac{h_l}{z_0^l} \right]^{-2}, \tag{9}$$

where C_{dz} is the drag coefficient at height $h_l \leq z \leq H$.

To evaluate (9) the height of the suspension layer in the regime of limiting saturation h_l should be defined. Kudryavtsev and Makin (2001) have shown that at high wind speeds the sea drag is formed by the separation stress resulting from the breaking of steep short gravity waves. In the extreme case when all the short waves break, which is expected at hurricane wind speeds, the roughness length (and thus the sea drag) is shown to be proportional to the characteristic height of the breaking waves h_b (their Equation (12)) and scales with u_*^2/g . The proportionality coefficient was estimated in the range between 0.01 and 0.1. Arguing again that the impact of the spray droplets on the dynamics of the air flow is small for wind speeds not exceeding 33 m s^{-1}, and using the measured value of the roughness parameter 0.0025 m (estimated from Powell et al., 2003, their Figure 3b) and $u_{*\mathrm{cr}} = 1.55$ m s^{-1} the proportionality coefficient c_b in $h_b = c_b u_*^2/g$ is found to be in the range of 0.1–1. The height h_b is much smaller than the significant wave height H_S. An empirical relation for the significant wave height is (e.g., Donelan, 1990) $H_S = 0.24 u_{10}^2/g$. Using a typical value of the drag coefficient C_{d10} of about 2×10^{-3} this becomes $H_S = c_S u_*^2/g$, where c_S is of about 100.

Assuming that most of the spray at hurricane wind speeds is produced by mechanical tearing by the wind from steep short waves it is not unreasonable to assume that the height of the suspension layer in the regime of limiting saturation h_l is proportional to, and larger than, the height of the breaking waves h_b, but smaller than the significant wave height H_S :

$$h_l = c_l \frac{u_*^2}{g}, \tag{10}$$

where $c_b < c_l < c_S$, so that $1 < c_l < 100$. The value $c_l = 10$ is finally chosen.

With (10) and $z_0^l = c_{z_0} u_*^2/g$ the resistance law (9) can be rewritten as

$$C_{dz} = \frac{u_*^2}{u_z^2} = \kappa^2 \left[\ln \frac{z}{z_0} \right]^{-2}, \tag{11a}$$

$$z_0 = c_l^{(1-1/\omega)} c_{z_0}^{1/\omega} \frac{u_*^2}{g}, \tag{11b}$$

where z_0 is the effective roughness length reflecting the impact of waves and spray droplets on the airflow dynamics. With $\omega = 1$ (no spray effect) z_0 is described by the Charnock relation (3). With increasing impact of the droplets on the airflow dynamics ω decreases, the effective roughness length decreases, and the drag coefficient reduces.

The value of c_l in (11) is defined rather poorly. Relating the relative error in the drag coefficient $\delta C_{dz}/C_{dz}$ to the relative error in the c_l parameter $\delta c_l/c_l$ by $\delta C_{dz}/C_{dz} = R\delta c_l/c_l$, where $R = 2(1 - 1/\omega)\ln(z/z_0)^{-1}$, and using typical values of the parameters entering R for the problem considered, it is found that R is smaller than 0.1. This means that the possible mistake made in the choice of c_l is reduced in the solution for C_{dz} for more than 100% or, in different words, the solution is not sensitive to the exact choice of c_l (this result is confirmed by the direct solution of (11)). The lack of sensitivity to a quite poorly supported assumption argues for increased confidence in the overall result.

The resistance law (11) is solved by iterations given the wind speed u_z at height z. Traditionally the height is taken at $z = 10$ m. Results for the friction velocity u_* and C_{d10} are given in Figure 1. First, it is noted that for moderate to high wind speeds ($u_{10} < 33$ m s^{-1}) the resistance of the sea surface is well described by the Charnock relation (3). The physical mechanism of air–sea interaction involved at this stage, viz the form drag formed by the non-separated airflow sheltering and by separation of the airflow from steep short waves, is described by Kudryavtsev and Makin (2001). At this stage the suspension layer in the regime of limiting saturation is not yet formed and the concentration of spray droplets is not enough to influence the dynamics of the flow. At wind speeds exceeding 33 m s^{-1} the suspension layer in the regime of limiting saturation is formed, spray droplets influence the airflow dynamics, and the resistance law (11) predicts a reduction of drag coefficient and a levelling off of the friction velocity with an increase of the wind speed, in agreement with data by Powell et al. (2003).

Acknowledgements

Numerous discussions with Vladimir Kudryavtsev and valuable comments by Dr. Jeff Kepert are gratefully acknowledged. The support by the Office of Naval Research (ONR) - grant N00014-03-1-0619 (PR NO.: 03PR09880-00), and EU INTAS grant INTAS-2001-2132 is acknowledged.

References

Andreas, E. L.: 1998, 'A New Sea Spray Generation Function for Wind Speeds up to 32 m s^{-1}', *J. Phys. Oceanog.* **28**, 2175–2184.

Barenblatt, G. I.: 1953, 'On the Motion of Suspended Particles in a Turbulent Flow', *Prikl. Mat. Mekh.* **17**, 261–274.

Barenblatt, G. I.: 1979, *Similarity, Self-Similarity, and Intermediate Asymptotics*, Consultants Bureau, Plenum Press, New York and London, 218 pp.

Charnock, H.: 1955, 'Wind Stress on a Water Surface', *Quart. J. Roy. Meteorol. Soc.* **81**, 639–640.

Donelan, M. A.: 1990, 'Air–Sea Interaction', in B. Le Mehaute and D. M. Haines (eds.), *The Sea: Ocean Engineering Science*, Vol. 9, pp. 239–292.

Kudryavtsev, V. N. and Makin, V. K.: 2001, 'The Impact of Air-Flow Separation on the Drag of the Sea Surface', *Boundary-Layer Meteorol.* **98**, 155–171.

Powell, M. D., Vickery, P. J., and Reinhold, T. A.: 2003, 'Reduced Drag Coefficient for High Wind Speeds in Tropical Cyclones', *Nature* **422**, 279–283.

Smith, S. D.: 1980, 'Wind Stress and Heat Flux over the Ocean in Gale Force Winds', *J. Phys. Oceanog.* **10**, 709–726.