

# HIRLAM singular vectors

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## 1 Abstract

As a first step towards a sensitivity analysis of the Hirlam model code has been developed for calculating singular vectors. Preliminary tests for two case studies indicate that the fast growing structures are identified. We briefly discuss some basic properties of the leading singular vector for one case.

## 2 Background

Weather forecasting has to deal with many uncertainties, which may stem from various sources. For example, the parameter settings in physics packages, which are not truly known, or the uncertainty in the analysis itself. Here we shall focus on the latter: the uncertainty in the initial condition of the model. We will use adjoint techniques to determine sensitive geographical areas of the analysis, where uncertainties in the analysis may have the largest impact on the Hirlam forecast. As such, the computations of these fast-growing perturbations or Hirlam singular vectors is a natural component of the Hirlam-Aladin initiative GLAMEPS (see this Newsletter) to define a limited-area model ensemble for Europe. For an extensive discussion of the use of singular vectors in ensemble forecasting we refer to [4].

## 3 Uncertainty for initial conditions

Assume that the Hirlam forecast model is written in the form

$$\dot{x} = F(x, \theta) \quad x(0) = x_0 \quad (1)$$

where  $x$  denotes the state vector,  $\theta$  is a vector with model parameters,  $x_0$  is a vector with initial conditions and  $F$  is a differentiable vector valued function.

For a given initial condition and model parameters the nonlinear model can be integrated in time to give a reference trajectory  $x(t)$ . If we add a small perturbation  $\epsilon_0$  to our initial condition, i.e.  $z_0 = x_0 + \epsilon_0$ , and again integrate the model, we obtain a new trajectory  $z(t) = x(t) + \epsilon(t)$ . Substituting the solution  $z$  in equation (1) and Taylor expanding  $F$  around the trajectory  $x(t)$  gives

$$\dot{x} + \dot{\epsilon} = F(x, \theta) + \left. \frac{\partial F}{\partial x} \right|_{x(t)} \epsilon + \text{h.o.t.} \quad (2)$$

Since  $x(t)$  is a solution of (1) we conclude that up to first order the time evolution of  $\epsilon_0$  is given by the tangent linear model

$$\dot{\epsilon} = \left. \frac{\partial F}{\partial x} \right|_{x(t)} \epsilon \quad (3)$$

where the Jacobian  $\frac{\partial F}{\partial x}$  is evaluated along the nonlinear trajectory  $x(t)$ . In [3] it is shown that even for asymptotically stable tangent linear models (for which  $\epsilon(t) \rightarrow 0$  as  $t \rightarrow \infty$ ) large error growth can occur during short forecasts. Equation 3 is linear, which allows us to write

$$\epsilon(T) = \mathbf{M}(T)\epsilon(0) \quad (4)$$

The operator  $\mathbf{M}$  is called the resolvent or propagator of 3. In order to be able to determine linearly fast-growing perturbations  $\epsilon(0)$ , we still have to define a norm at initial time and at the optimization time during which the perturbation is allowed to grow. Here we use the total energy norm given by:

$$\iint \left[ u^2 + v^2 + \frac{c_p}{T_r} T^2 \right] d\Sigma \frac{\partial p_r}{\partial \eta} d\eta + \frac{1}{2} \int \left[ R \frac{T_r}{p_r} (\ln p_s)^2 \right] d\Sigma \quad (5)$$

where  $u, v, T$  and  $\ln p_s$  are perturbations in the  $x$  and  $y$  component of the wind, temperature and logarithm of the surface pressure respectively. At the moment no humidity perturbations  $q$  are used in the total energy norm. The integration is carried out over the complete integration domain.

For a given unit norm  $\epsilon(0)$  we are interested in those  $\epsilon(0)$  that produce the largest perturbation growth in terms of total energy. In other words we are looking for unit norm  $\epsilon(0)$ , which maximize

$$(\mathbf{M}\epsilon(0), \mathbf{M}\epsilon(0)) = (\epsilon(0), \mathbf{M}^*\mathbf{M}\epsilon(0)) \quad (6)$$

So the fast-growing perturbations  $\epsilon(0)$ , or singular vectors, are given by the leading eigenvectors of the operator

$$\mathbf{M}^*\mathbf{M} \quad (7)$$

Here  $\mathbf{M}^*$  is the propagator of the adjoint model of Hirlam. In the above derivation we silently assumed that this adjoint is determined w.r.t. the inner product induced by the total energy norm. In fact the Hirlam adjoint is derived w.r.t. an inner product almost similar to the Euclidean inner product, which results in a slightly more complicated operator in equation (7). Also a projection operator can be added at optimization time to compute singular vectors which grow in a pre-selected geographical area.

The singular vector calculation is based on the Hirlam 4D-VAR code and uses the minimization algorithm MCGLSV to determine the eigenspectrum of equation (7). We point out that during this computation the operator  $\mathbf{M}^*\mathbf{M}$  is not explicitly known because of its large dimension. The square root of the eigenvalues of  $\mathbf{M}^*\mathbf{M}$  are usually called singular values.

## 4 First results

To test the singular vector calculation two meteorological cases were considered. During the development phase of the code we compared ARPACK routines and MCGLSV, using Hirlam 7.1alpha3 (linux) and 7.1rc1 (ECMWF). Both approaches gave identical results. Initial reported problems, such as incorrect behaviour near the boundaries, appeared to be due to the MPI environment and could easily be solved.

We present some results for the case with starting date 5 March 2006 at 3 UTC. The domain is given by  $NLON \times NLAT \times NLEV = 54 \times 50 \times 40$  with a  $0.5 \times 0.5$  degree resolution. Figure 1 shows the wind field present in the nonlinear reference trajectory at model level 19 (500 hPa) at 3 UTC and the 5 hour forecast. In the SV computation the optimization time is set to 6 hours.

Figure 2 shows the rms of the wind field at model level 19 (500 hPa) for the leading singular vector (SV) at initial and final (optimization) time. The 6 hour time evolution is obtained by integrating the tangent linear

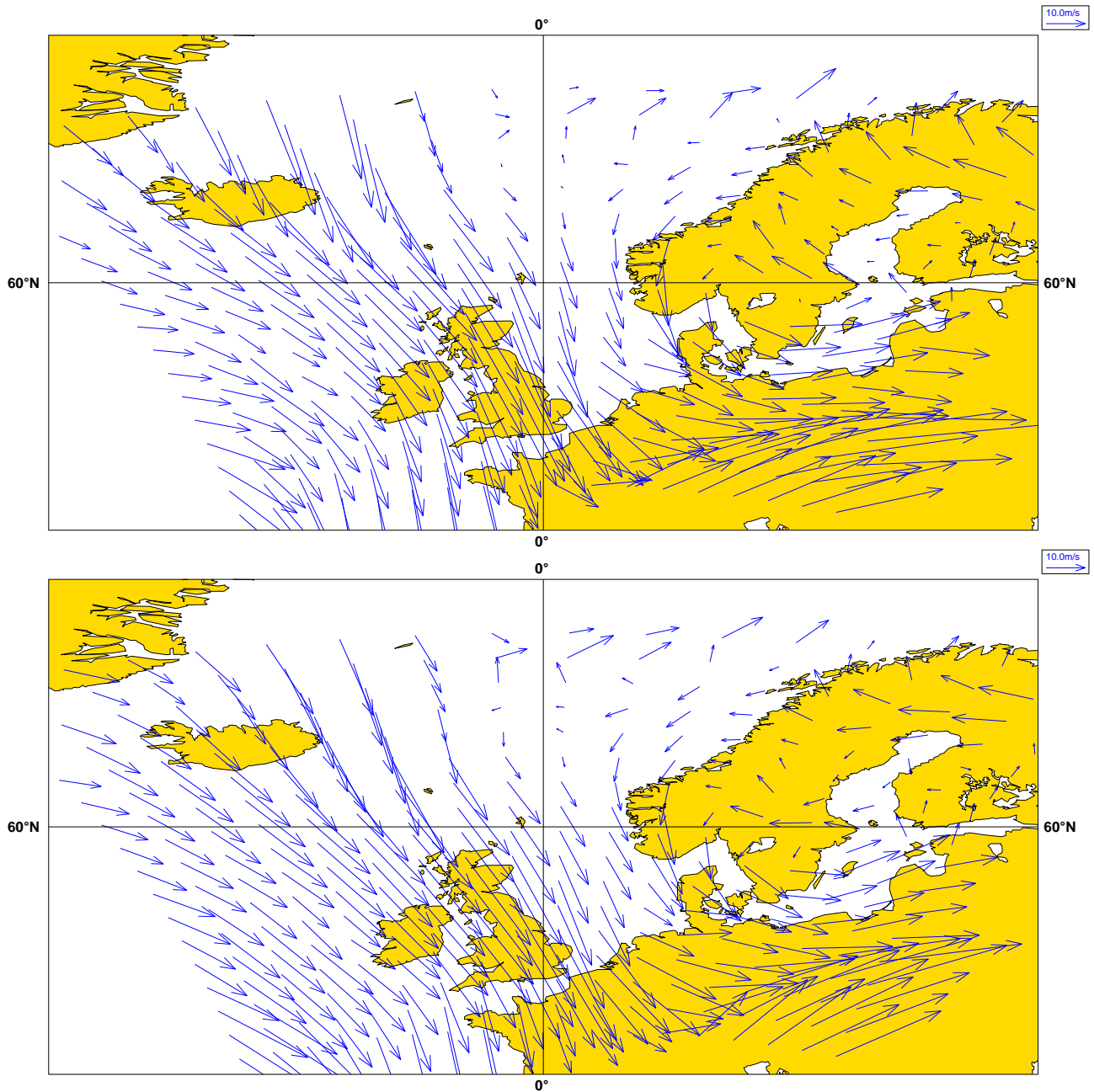


Figure 1: Wind field at model level 19 (500 hPa) for 5 March 2006 at 3 UTC (top) and the 5 hour forecast (bottom)

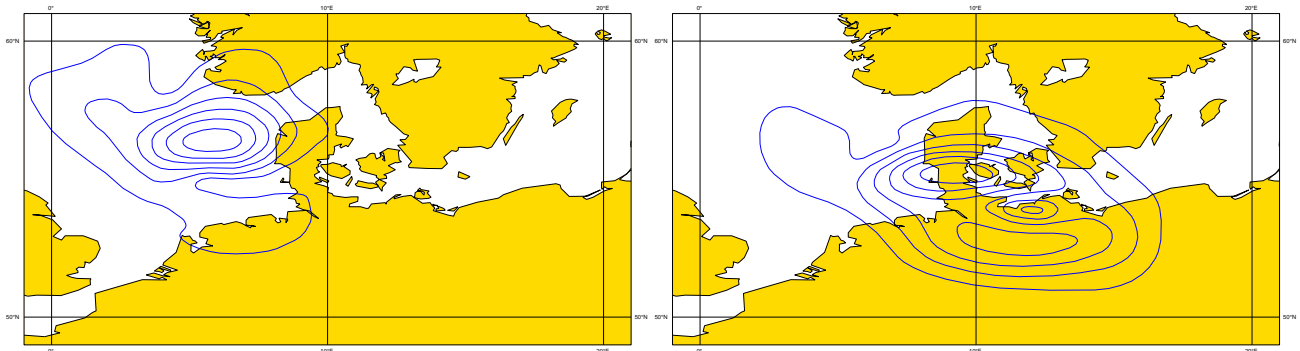


Figure 2: Rms of the wind field for the leading singular vector at model level 19 (500 hPa) at initial time (left) and final time (right) using the same contour interval

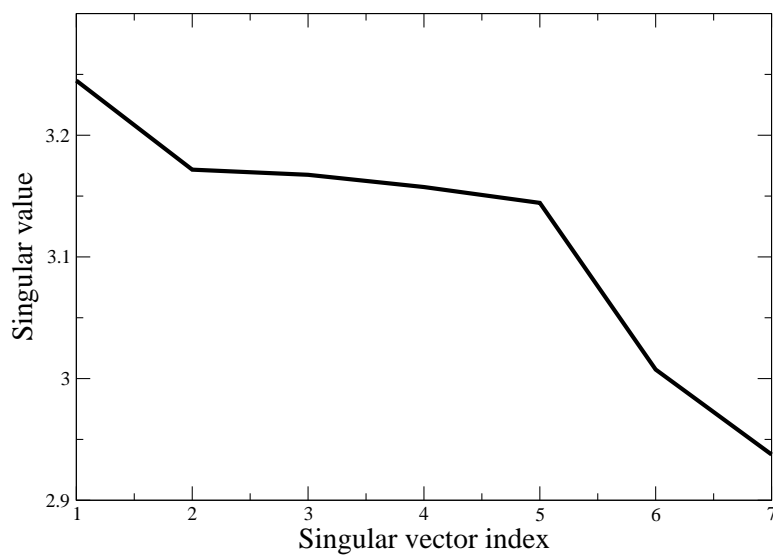


Figure 3: Singular values of the leading singular vectors for March the 5th 2006 at 03 UTC with an optimization time of 6 hours

model (3). Initially the perturbation is located over the North Sea and moves towards Denmark and the north of Germany at final time. Although the amplification of the leading SV is not very large, it shows that the SV algorithm is still capable to find growing structures, even for this short optimization time of 6 hours.

In figure 3 the singular values of the leading SV's are given.

## 5 Outlook

In the near future we will perform computations with an optimization time of 12 hours and Hirlam SV's will be compared with ALADIN SV's. Also we plan to extend the current SV calculation so that it allows the computation of so-called forcing singular vectors [2] and the study of parameter sensitivity. Another area of application of HIRLAM SV's will be the Hirlam-Aladin initiative to define an ensemble for the European area: GLAMEPS [1].

## References

- [1] GLAMEPS. Hirlam Newsletter 52, this issue.
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- [4] M. Leutbecher and T.N. Palmer. *Ensemble Forecasting*. ECMWF Tech. Memo 514, 2007.