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Coupled sea surface–atmosphere model
1. Wind over waves coupling

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Abstract. A wind over waves coupling scheme to be used in a coupled wind waves–atmosphere model is described. The approach is based on the conservation of momentum in the marine atmospheric surface boundary layer and allows to relate the sea drag to the properties of the sea surface and the properties of the momentum exchange at the sea surface. Assumptions concerning the local balance of the turbulent kinetic energy production due to the mean and the wave-induced motions, and its dissipation, as well as the local balance between production and dissipation of the mean wave-induced energy allow to reduce the problem to two integral equations: the resistance law above waves and the coupling parameter, which are effectively solved by iterations. To calculate the wave-induced flux, the relation of *Plant* [1982] for the growth rate parameter is used. However, it is shown by numerical simulations that the local friction velocity rather than the total friction velocity has to be used in this relation, which makes the growth rate parameter dependent on the coupling parameter. It is shown that for light to moderate wind a significant part of the surface stress is supported by viscous drag. This is in good agreement with direct measurements under laboratory conditions. The short gravity and capillary-gravity waves play a significant role in extracting momentum and are strongly coupled with the atmosphere. This fact dictates the use of the coupled short waves–atmosphere model in the description of the energy balance of those waves.

1. Introduction

Oceans cover a significant part of the Earth surface. Winds blowing over the oceans excite the wind waves. As a matter of fact, these wind waves are responsible for the formation and regulation of the surface transfer of momentum, heat, moisture, and gases. The reliable estimates of the exchange coefficients are of primary importance for climate studies, weather predictions, and monitoring of the biological conditions of the marine ecosystems.

In the last decade an approach which allows to relate the momentum and heat fluxes directly to the sea state has been developed [*Janssen*, 1989; *Chalikov and Makin*, 1991; *Caudal*, 1993; *Makin et al.*, 1995; *Makin*, 1998]. The approach is based on conservation of momentum above waves. The total stress, which is constant over height, is split into two parts: the turbulent and the wave-induced stress. Close to the surface the wave-induced stress supports a significant part of the total stress. Relating the wave-induced stress at the surface (the form drag) to the geometrical properties of the surface, described statistically in terms of a directional wave variance spectrum, and the properties of the energy exchange between waves and wind at the surface, the momentum exchange coefficient (the sea drag) is thus related or coupled directly to waves or the sea state. To this end a traditional assumption in the wave study, namely, that the wave field is a superposition of sinusoidal waves with random phases and traveling at their linear phase velocity, is used. Whilst this assumption is often made in wave studies, there is increasing evidence, both from theory [*Belcher and Vassilicos*, 1997] and experiment [e.g., *Hara et al.*, 1997], that this assumption does not hold, particularly at the short wavelengths. This is an important notice, as it has been shown [*Caudal*, 1993; *Makin et al.*, 1995] that the short waves of the centimeter to meter wavelengths play a dominant role in supporting the form drag to waves, and thus determine the sea drag for moderate to high winds. However, given our current understanding of the problem, the approach taken in the previous and

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in the present papers is appropriate, because only further research will tell if the assumption of linear waves will lead to errors in the present physical problem.

We thus describe the sea state in terms of the directional wave number spectrum, which is defined in the whole range of wavenumbers. Introducing the statistical description of waves in terms of the directional wave spectrum, it is assumed that all undulations of the water surface can be considered as waves.

The fact that the short waves actually determine the sea drag dictates the reliable statistical description of these waves, especially in the range of short gravity (wavelengths from few centimeters to few meters) and capillary-gravity (wavelengths from few millimeters to few centimeters) waves. This is important not only for the atmospheric studies to calculate fluxes, but also of the relevance of short waves to interpret remote sensing data of the sea surface by radar instruments. Indeed, in the case of radar remote sensing, short gravity and capillary-gravity waves will serve as roughness elements on the sea surface which modulate the backscattered power of electromagnetic waves.

The evolution of the wave spectrum is defined by energy input from the wind, different dissipation processes and wave-wave nonlinear interactions. In turn, the wind input is determined by the friction velocity of the atmosphere which results from the interaction of wind and waves. We present here a coupled sea surface–atmosphere model which is designed mainly for use in remote sensing applications, as a part of electromagnetic models of the sea surface, and for coupled atmosphere-ocean studies. A physical model of the short wind waves coupled with the atmosphere is described in part 2 of the present paper.

A new scheme to couple waves with the atmosphere is described in part 1. This scheme is based on the ideas developed by *Makin et al.* [1995], although some important refinements are made. In *Makin et al.* [1995] the viscous stress was parameterized by introducing the viscous roughness length proportional to the surface turbulent stress and imposing the lower boundary condition on the instantaneous water surface. It is the principle assumption of our approach, and we discuss it in the present paper once again. The assumption allows to couple directly waves of all scales to the wind and allows to avoid the use of a Charnock-type relation for the "background" roughness, a serious drawback of earlier models by *Janssen* [1989] and *Chalikov and Makin* [1991]. This assumption follows the weak-in-the-mean formalism widely used in the description of sea waves [*Hasselmann*, 1968], meaning that breaking waves are not directly accounted for in the study.

The main refinement of the present model concerns the treatment of the eddy viscosity coefficient, which is used to parameterize the turbulent stress. In *Makin et al.* [1995] (as well as in *Janssen* [1989], and *Chalikov and Makin* [1991]) an erroneous expression for the eddy viscosity is used, which results from an erroneous treatment of the local balance between the turbulent kinetic energy (TKE) production and its dissipation. Above waves, both mean and wave-induced motions support the turbulent motion. Not accounting for the second source of the TKE in the TKE balance equation leads to a significant underestimation of the eddy viscosity above waves, as was shown by *Makin and Mastenbroek* [1996] by constructing a differential one-dimensional model of the boundary layer above waves based on full balance equations for the TKE and the dissipation rate. It was further shown that the diffusive transport of the TKE in the balance equation of the TKE above waves can still be neglected [see *Chalikov and Belevich*, 1993].

In the present paper, to calculate the eddy viscosity above waves, we assume a local balance between TKE production (by both mean, and wave-induced motions) and its dissipation and express the latter in terms of the eddy viscosity and the mixing length. These assumptions considerably simplify the model and allow to derive an explicit relation for the resistance law (drag coefficient) above waves, which is found to be a function of the coupling parameter (ratio of the form drag to the total stress). The coupling parameter is calculated from the wave spectrum and the growth rate parameter (dimensionless energy flux from wind to waves). Two integral equations are then effectively solved by iterations. The solution is compared to the differential model [*Makin and Mastenbroek*, 1996] based on full balance equations of the TKE and its dissipation, and a good comparison is found. Since this coupled scheme is aimed to be used as a module in electromagnetic models of the sea surface and in coupled atmosphere-ocean models, this simplification is viewed important.

Another important refinement of the present model concerns the parameterization of the growth rate parameter, particularly for short slowly moving waves. It was already mentioned that these waves extract a considerable part of the wave-induced stress, and an accurate estimate of the growth rate parameter for these waves is required. *Makin et al.* [1995] used numerical calculations of *Burgers and Makin* [1993] to support a parameterization of the growth rate parameter in terms of the friction velocity and the wind speed at a certain height. Those calculations were based on a two-dimensional nonlinear model of the boundary layer above waves (2-D WBL model), where a mixing length turbulence closure scheme was used. Recent analytical theory of the wave growth, based on a rapid distortion theory of turbulence above waves [*Belcher and Hunt*, 1993], has revealed that the use of the mixing length approach in the description of the wave-induced turbulence above waves leads to erroneous estimations of the growth rate parameter. This conclusion was supported by the numerical study by *Mastenbroek* [1996], and *Mastenbroek et al.* [1996], based on the same 2-D WBL model, but with a second-order turbulent stress closure scheme, which accounts for the rapid distortion effects. Both analytical and numerical studies suggest that the growth rate parameter depends on the square of the friction velocity, which supports the empirical parameterization of the growth rate parameter by *Plant* [1982].

In the present paper we use the numerical model of *Mastenbroek et al.* [1996] to show that the local turbulent stress (difference between the total stress and the wave-induced stress near the surface) rather than the total stress (square of the friction velocity) has to be used in the parameterization of the growth rate. As close to the surface the wave-induced stress supports a considerable part of the total stress, a parameterization of the growth rate in terms of the local turbulent stress quenches the energy flux to short waves and thus provides the necessary feedback mechanism between waves and the atmosphere: an increase in the spectral density level of short waves leads to an increase of the wave-induced stress, a decrease in the local turbulent stress and thus in the growth rate parameter, and further to a decrease of the growth of those waves.

We further treat a constant in the growth rate parameterization as a tuning parameter. With the use of open sea measurements of the drag coefficient [*Large and Pond*, 1982; *Anderson*, 1993] and an empirical wavenumber spectrum of *Elfouhaily et al.* [1997], this constant is shown to match a value of Plant's original parameterization.

It is shown that for light to moderate winds (wind speed less than $10\,\mathrm{m\,s^{-1}}$) a significant part of the surface stress is supported by viscous drag which is in a good agreement with first, though limited, direct measurements under laboratory conditions by *Banner and Peirson* [1998]. This conclusion is supported by the field measurements of *Snyder et al.* [1981].

We show that short gravity and gravity-capillary waves play an important role in extracting momentum from the atmosphere and are strongly coupled to the atmosphere. This fact dictates the use of the coupled sea surface–atmosphere model in the description of the energy balance of these waves.

2. Conservation of Momentum Above Waves

The approach is based on the conservation of momentum in the marine surface boundary layer

$$\frac{\partial u}{\partial t} = \frac{\partial \tau}{\partial z}, \qquad (1)$$

where u is the mean wind velocity, and τ is the total stress. It is assumed that the wind is spatially homogeneous, its direction coincides with the mean direction of waves propagation, and the wave field (wave spectrum) is symmetrical relative to that direction.

The total stress is supported by the mean turbulent stress $\tau^t = -\overline{u'w'}$, the wave-induced stress τ^w due to the organized wave motions in the atmosphere induced by waves, and the viscous stress τ^ν. If the wind is statistically steady the total stress τ is independent of height and by definition equals the square of the friction velocity u_*^2

$$\tau = \tau^t(z) + \tau^w(z) + \tau^\nu(z) = \text{const} = u_*^2. \qquad (2)$$

Equation (2) was apparently first introduced by *Phillips* [1966]. The viscous stress is important only very near the water surface. Far above the water the wave-induced stress vanishes and $\tau = \tau^t = u_*^2$. As was stated by *Phillips* [1966, p.93]: "the interchange between τ^t and τ^w (we add also τ^ν) as the surface is approached, and the relation between τ^w and the momentum flux to the waves are the central questions in this whole problem" (i.e., the coupling between wind and waves).

The explicit description of fluxes as the wave surface is approached dictates the use of a vertical coordinate system which follows the instantaneous water surface $z = \eta(\vec{x}, t)$, e.g., $\hat{z} = h(z-\eta)/(h-\eta)$ and hereinafter the circumflex above z is dropped. Here h denotes the height of the boundary layer where the wave-induced flux is negligible. This height scales with the wavelength of the energy containing waves; $h = 10$ m is a good estimate [*Makin et al.*, 1995]. The wind velocity at that height u_{10} is assumed to be known.

2.1. Viscous Stress

Except very near the water surface, in the viscous sublayer, the viscous stress can be neglected. However, if we want to relate the wave-induced stress directly to waves of all scales up to capillaries, the viscous stress has to be accounted for in the description of the momentum flux near the surface.

If waves are not actively involved in breaking, the instantaneous water surface is a smooth surface and the viscous sublayer always covers the undulating water surface, that is, waves. It means that the tangential stress at the surface $z = 0$ is supported by molecular viscosity, and the total stress at the surface is $\tau = \tau^w(0) + \tau^\nu(0)$. To avoid the explicit description of the stress within the viscous layer, we shall use a classical parameterization of the viscous layer by introducing the viscous roughness scale [*Townsend*, 1956; *Monin and Yaglom*, 1971]

$$z_0^\nu = 0.1\frac{\nu}{u_{*0}^l}, \qquad (3)$$

where ν is the kinematic viscosity of the air, and u_{*0}^l is the friction velocity at the top of the viscous sublayer. The viscous stress in (2) can be now neglected

$$\tau^t(z) + \tau^w(z) = u_*^2, \qquad (4)$$

and the surface momentum is related to the height $z = z_0^\nu$. The friction velocity u_{*0}^l is calculated from the local turbulent stress

$$\tau^t(z_0^\nu) = u_*^2 - \tau^w(z_0^\nu), \qquad (5)$$

as friction is not supported by the form drag at the surface $\tau^w(0) = \tau^w(z_0^\nu)$

$$u^l_{*0} = u_* \sqrt{1 - \frac{\tau^w(0)}{u_*^2}}. \quad (6)$$

The ratio

$$\alpha_c = \frac{\tau^w(0)}{u_*^2} \quad (7)$$

is called the coupling parameter. The coupling parameter shows which part of the total stress at the surface is supported by the form drag. It is bounded by the condition $\alpha_c < 1$.

The parameterization of the viscous stress above the wave surface is possible if the very short waves with lengths $\lambda < \delta_\nu \sim \nu/u^l_{*0}$ do not contribute to the wave-induced flux τ^w, or, in other words, if the wave-induced stress is constant within the viscous sublayer δ_ν. These scales are so small that capillaries are already damped by viscous dissipation [*Donelan and Pierson*, 1987; *Jähne and Riemer*, 1990; *Elfouhaily et al.*, 1997].

One may always say that, except at low wind speeds, breaking always occurs in the ocean. And that is true. Breaking is a highly nonlinear local process which disrupts the water surface. However, if the process is weak-in-the-mean, as in the formalism widely used in the description of sea waves [*Hasselmann*, 1968], the instantaneous water surface can be regarded in the mean as being smooth. Notice that in the above description the instantaneous water surface is distinguished from the sea surface. The latter, being viewed from a distance far above the waves, is a rough surface due to the presence of waves.

2.2. Turbulent Stress

The local turbulence closure relates the turbulent flux to the gradient of the velocity field via the eddy viscosity K

$$\tau^t(z) = K\frac{\partial u}{\partial z}. \quad (8)$$

From (4) and (8) follows

$$K\frac{\partial u}{\partial z} = u_*^2 - \tau^w(z). \quad (9)$$

The surface boundary condition is $u = 0$ at $z = z_0^\nu$, and the velocity profile above waves follows from (9)

$$u(z) = u_*^2 \int_{z_0^\nu}^{z} \left[1 - \frac{\tau^w(z)}{u_*^2}\right] K^{-1} dz. \quad (10)$$

To proceed further, the eddy viscosity $K(z)$ above waves should be estimated.

2.3. Balance Equation of the TKE

The diffusive transport of the TKE in the balance equation above waves can be neglected, and the balance equation reduces to [*Chalikov and Belevich*, 1993; *Makin and Mastenbroek*, 1996]

$$P = \varepsilon, \quad (11)$$

which says that the TKE production P equals its dissipation to heat ε. The TKE production P above waves has two contributions: production by interaction of the turbulent stress and the mean velocity shear

$$P^t = \tau^t\frac{\partial u}{\partial z}, \quad (12)$$

and by interaction of the wave-induced turbulent stress $\tilde{\tau}^t$ and the wave-induced velocity shear $P^w = \langle \tilde{\tau}^t \partial\tilde{u}/\partial z\rangle$ where the tilde denotes a wave-induced fluctuation and angle brackets denote the mean average [e.g., *Makin and Mastenbroek*, 1996]

$$P = P^t + P^w. \quad (13)$$

The production terms P^t and P^w represent the transfer of kinetic energy from the mean and wave-induced motions to the turbulent motion. They appear with the opposite sign as sink terms in the equations for the kinetic energy of the mean and wave-induced motions: $E = u^2/2$ and $E_w = \langle \tilde{u}^2 + \tilde{w}^2\rangle/2$, respectively.

The balance equation of E can be derived from (1) by multiplying it by u (the viscous stress is already neglected)

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial z}u(\tau^t + \tau^w) - \tau^t\frac{\partial u}{\partial z} - \tau^w\frac{\partial u}{\partial z}. \quad (14)$$

The first term on the right-hand side describes the transport of E, the second term is the sink of the mean energy into the TKE P^t, the third term describes the mutual transfer of energy between the mean and the wave-induced motions. It appears with the opposite sign in the balance equation of the mean wave-induced energy E_w, where the vertical flux of the wave-induced energy is neglected [*Makin and Mastenbroek*, 1996]

$$\frac{\partial E_w}{\partial t} = \tau^w\frac{\partial u}{\partial z} - P^w. \quad (15)$$

For a growing wind wave field $\tau^w > 0$, and this term describes the sink of the mean energy into the mean wave-induced energy.

In stationary conditions the TKE production above waves is thus

$$P = P^t + P^w = (\tau^t + \tau^w)\frac{\partial u}{\partial z} = u_*^2\frac{\partial u}{\partial z}. \quad (16)$$

Using the mixing length theory and expressing the dissipation in terms of K and the mixing length $l = \kappa z$, $\varepsilon = K^3 l^{-4}$, the equation for the eddy viscosity is found from (9), (13), and (16)

$$K = l u_* \left(1 - \frac{\tau^w(z)}{u_*^2}\right)^{1/4}. \quad (17)$$

If an erroneous balance between the production due to the mean motion only and the dissipation is assumed [*Janssen*, 1989; *Chalikov and Makin*, 1991; *Makin et al.*, 1995], that is,

$$P^t = \varepsilon, \tag{18}$$

it follows from (18), (12), and (8) that

$$K = l^2 \frac{\partial u}{\partial z}, \tag{19}$$

and further accounting for (9),

$$K = l u_* \left(1 - \frac{\tau^w(z)}{u_*^2}\right)^{1/2}. \tag{20}$$

Near the surface relation (20) considerably underestimates the eddy viscosity compared to relation (17) as $\tau^w(z)/u_*^2$ approaches 1 under strong winds. In *Makin et al.* [1995] this underestimation of K was compensated by increasing the tuning constant in the relation for the growth rate parameter to get the right values for the drag coefficients. However, for some applications, for example, calculation of the heat flux at the surface, the wrong functional relation (20) gives an erroneous wind speed dependence of the heat exchange coefficient [*Makin and Mastenbroek*, 1996], which cannot be "cured" by tuning.

2.4. Wave-Induced Stress

If the sea surface is assumed to be a superposition of random waves of all scales, characterized by a directional wave spectrum $S(k,\phi)$ (k is the wavenumber which satisfies the gravity-capillary dispersion relation, ϕ is the propagation direction of the k wave component relative to the wind direction), the wave-induced stress can be written

$$\tau^w(z) = \tau^w(0) f(z). \tag{21}$$

Here $\tau^w(0)$ is the form drag

$$\tau^w(0) = \int_0^\infty \mathcal{T}(k) d(\ln k), \tag{22}$$

$\mathcal{T}$ is the omnidirectional spectrum of the momentum flux to waves

$$\mathcal{T}(k) = \int_{-\pi}^{\pi} c^2 B(k,\phi)\beta(k,\phi)\cos\phi d\phi, \tag{23}$$

$B(k,\phi) = k^4 S(k,\phi)$ is a saturation spectrum, c is the phase velocity, $\beta(k,\phi)$ is the growth rate parameter describing the energy flux to waves from the atmosphere, and $f(z)$ is a dimensionless function describing the vertical decay of the mean wave-induced stress. The function $f(z)$ is defined as

$$f(z) = \frac{1}{\tau^w(0)} \int_0^\infty \mathcal{T}(k) F(k,z) d(\ln k) \tag{24}$$

where $F(k,z)$ is another dimensionless function describing the vertical distribution of the individual wave components. The function $F(k,z)$ decays with height and satisfies the condition $F(k, z_0^\nu) = 1$ at the surface.

2.4.1. Growth rate parameter.

Makin et al. [1995] showed that the short gravity-capillary and gravity waves (in the centimeter to meter wavelength range) play a dominant role in supporting the form drag (and thus the sea drag at high winds) at the surface. Waves longer than 10 meter support only 10% of the form drag. Thus an accurate estimation of the growth rate parameter for short slowly moving waves is required.

Recent analytical theory of the wave growth, based on a rapid distortion theory of turbulence above waves [*Belcher and Hunt*, 1993, 1998], and numerical solution of the two-dimensional nonlinear Reynolds equations above waves with a second-order turbulence closure scheme [*Mastenbroek et al.*, 1996; *Mastenbroek*, 1996] show that the growth rate parameter is proportional to $(u_*/c)^2$ for waves traveling slower than the wind. These theoretical studies support thus a parameterization of the growth rate parameter for short waves of *Plant* [1982], which is based on several laboratory and field measurements

$$\beta = c_\beta \left(\frac{u_*}{c}\right)^2 \tag{25}$$

where $c_\beta = 32 \pm 16$ is an empirical constant.

In relation (25) the friction velocity u_* is calculated from the total stress. In the case of linear analytical theories the total stress and thus the friction velocity do not change with height, while in numerical studies, usually done for waves with a small amplitude, this change with height is negligible ($< 3\%$).

Sea waves extract a considerable part of the total stress, and the turbulent stress changes with height as does the local friction velocity

$$u_*^l(z) = \tau^t(z)^{1/2}. \tag{26}$$

Note that the total friction velocity u_*, equations (2) and (4), is constant over height. According to *Makin et al.* [1995], the coupling parameter α_c reaches a value of about 0.8 for high wind speeds. From (6) it means that the local friction velocity near the water surface is only 50% of the total friction velocity. Which friction velocity, the total or the local one, has to be used in scaling the growth rate β in (25)?

2.4.1.1. Growth rate parameter for monochromatic waves: To answer this question, we use a two-dimensional nonlinear boundary layer model (2-D WBL) above monochromatic Stokes waves, based on a second-order turbulent stress closure [*Mastenbroek et al.*, 1996]. The solution of this model depends on three dimensionless parameters: the inverse wave age u_λ/c (velocity is related to the height $z = \lambda = 2\pi/k$), the dimensionless roughness kz_0 and the wave slope ak. Here a, k, and z_0 are the amplitude, the wavenumber, and the background roughness of a monochromatic wave, respectively.

The following numerical experiment was designed. The air flow is simulated above a slow ($u_\lambda/c = 5$ and, $kz_0 = 6.6 \times 10^{-3}$) wave of which the slope is varied from

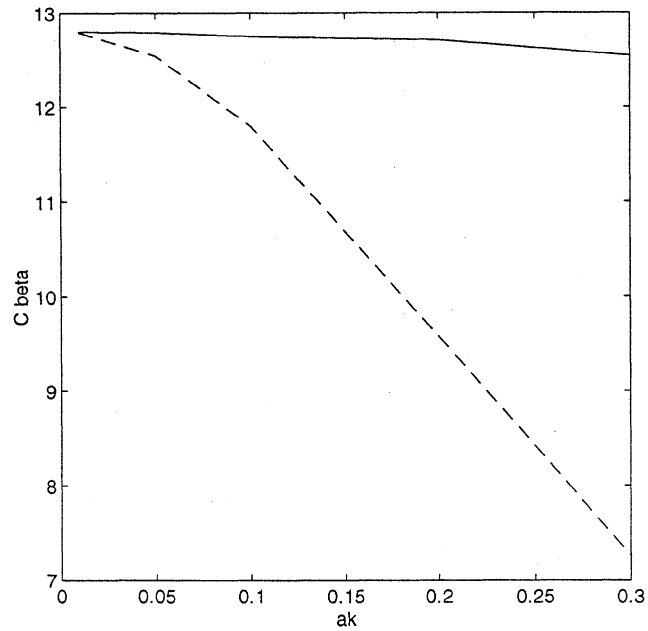

Figure 1. Variable c_β as a function of the wave slope ak of a monochromatic wave ($u_\lambda/c = 5$ and $kz_0 = 6.6 \times 10^{-3}$). Dashed line, scaling with the total friction velocity, relation (25); solid line, scaling with the local friction velocity, relation (27).

$ak = 0.01$ to $ak = 0.3$. As a wave becomes steeper, a pronounced variation with height (up to 30%) of the friction velocity is developed [*Makin*, 1990]. The calculated value of the growth rate parameter is scaled with the total friction velocity u_* and with the local friction velocity u_*^l taken in the vicinity of the wave surface. Results are presented in Figure 1. The conclusion is clear: the parameter c_β remains constant being scaled with the local friction velocity. The physical explanation of that fact follows from a wave growth theory by *Belcher and Hunt* [1993]. According to this theory the growth of waves results from an asymmetric pressure perturbation at the wave surface, caused by the action of the Reynolds shear stress inside the inner region, a thin region adjacent to the wave surface. When the mean Reynolds stress varies with height (in the case of a monochromatic wave this variation is caused by the large wave slope), it is logical to use the value of the friction velocity inside the inner region as a principle parameter in the parameterization of the wave growth. With the height of the inner surface layer (lower part of the inner region) as a reference, expression (25) for the growth rate β can be written in the form

$$\beta = c_\beta \left(\frac{u_*^l(\delta)}{c} \right)^2 . \quad (27)$$

The height of the inner surface layer δ is proportional to the square root of the height of the inner layer $\mathcal{L}$ and the roughness length z_0, $\delta \sim (\mathcal{L} z_0)^{1/2}$. The height of the inner region of the wave component k is defined by $k\mathcal{L} = (2\kappa u_*)/|u_\mathcal{L} - c|$ [see *Belcher and Hunt*, 1993]. Rough estimates for these heights are $k\mathcal{L} \sim 0.1$ and $k\delta \sim 0.01$ [*Mastenbroek et al.*, 1996]. For short slow waves $u_*^l(\delta) \to u_*(1-\alpha_c)^{1/2}$, while for faster waves $u_*^l(\delta) \to u_*$.

Notice that calculations of the growth rate parameter above monochromatic waves, described in this paragraph, are done only to establish the functional dependence (27) of the growth rate parameter on the local friction velocity. This functional relation will be used in the next paragraph to establish the growth rate parameter for sea waves, which is further used in our wind over sea waves model. The 2-D WBL model is used also in section 2.4.2 to establish the functional relation for the decay function $F(k,z)$ in (24).

2.4.1.2. Growth rate parameter for sea waves: Relation (27) is valid for the growth rate of monochromatic waves. If the sea surface is presented as a sum of many wave spectral components the local turbulent stress (and the local friction velocity $u_*^l(z)$) is supported by all waves. It means that for a certain wave component k the friction velocity $u_*^l(\delta)$, associated with the height of the inner surface layer of the same wave component $\delta(k)$, does not have to be constant within $\delta(k)$. In this case the friction velocity $u_*^l(\delta)$ should be replaced by the friction velocity averaged over the surface boundary layer from z_0^l to $\delta(k)$. We define this averaged friction velocity $(\bar{u}_*^l)^2$ as

$$(\bar{u}_*^l)^2 = \int_0^\infty (u_*^l)^2 e^{-z/\delta} d(z/\delta) = u_*^2(1 - \alpha_c \bar{f}(k)), \quad (28)$$

where $\bar{f}(k)$ is the dimensionless wave-induced stress averaged over the surface boundary layer

$$\bar{f}(k) = \int_0^\infty f(z) e^{-z/\delta} d(z/\delta), \quad (29)$$

and $f(z)$ is defined by (24). Hence we rewrite relation (27)

$$\beta = c_\beta \left(\frac{u_*}{c} \right)^2 \left[1 - \alpha_c \bar{f}(k)\right] \cos\phi |\cos\phi|. \quad (30)$$

Notice that $\bar{f}(k) \to 1$ for very short waves, while $\bar{f}(k) \to 0$ for longer waves. To account for waves traveling at an angle to the wind, an angular dependence of β is introduced in (30). This angular dependence follows from the numerical study of *Mastenbroek* [1996].

It is well known that for fast moving waves the growth rate parameter reduces to zero (experiment of *Snyder et al.* [1981], numerical simulations of *Mastenbroek et al.* [1996], analytical theory of *Belcher and Hunt* [1998], and *Cohen* [1997]), so that parametrization (30) becomes invalid in the vicinity of $u_{10} \sim c_p$, where c_p is the phase speed at the peak of the wave spectrum.

To account for the fact that the growth rate for fast moving waves tends to zero, a relaxation function R is introduced, so that $c_\beta = m_\beta R$, where m_β is a constant and

$$R = 1 - 1.3\left(\frac{c}{u_{10}}\right)^5. \quad (31)$$

For slowly moving waves, $R \sim 1$ and parameterization (30) with $c_\beta = m_\beta$ is recovered, for fast moving waves $R \to 0$ and $\beta \to 0$. For waves moving faster than the wind ($R < 0$) we simply assume $\beta = 0$, as the negative energy flux from fast moving waves to the wind is small [*Mastenbroek et al.* [1996]] and can be neglected in this study. The final parameterization of β used throughout the study has the form (30) with $c_\beta = m_\beta R$ and (31), where m_β will be treated as a tuning constant of the wind over waves coupled model.

2.4.2. Vertical decay function. The vertical decay function $F(k,z)$ in (24) defines the vertical distribution of the wave-induced flux for a wave component k. It seems that the decay for the second wave-induced moment should be at least $F(k,z) = \exp(-2kz)$, as the first moment (velocity components) decays as $\exp(-kz)$ in the potential flow. However, the rapid distortion theory of turbulence above waves [*Belcher and Hunt*, 1993] suggests another decay length, the height of the inner region $\mathcal{L}$, as a result of smearing of the wave fluctuations of turbulent stresses in the outer region. In the outer region the turbulent eddies are rapidly distorted, and the wave-induced turbulence is not in local equilibrium with shear. The wave-induced turbulent stress is suppressed, and the wave-induced motion is close to inviscid and potential. The mean wave-induced flux is thus suppressed in the outer region as well. The vertical decay of the wave-induced stress can then be approximated by

$$F(k,z) = e^{-z/\mathcal{L}}. \quad (32)$$

In Figure 2 the decay function $F(k,z)$ for a slowly moving wave is shown. The numerical result shows that the height of the inner region is indeed the decay length of the wave-induced flux. The vertical oscillatory decay behavior of the flux reflects the nature of the oscillatory flow caused by the orbital velocities in the viscous fluid. The numerical solution is approximated by

$$F(k,z) = e^{-z/\mathcal{L}} \cos\left(\frac{\pi}{2}\frac{z}{\mathcal{L}}\right), \quad (33)$$

and this relation is used in the model.

3. Resistance Law Above Waves

From (10) and (17) the wind profile above waves can now be written

$$u(z) = \frac{u_*}{\kappa}\int_{z_0^\nu}^{z}\left[1 - \frac{\tau^w(z)}{u_*^2}\right]^{3/4} d(\ln z). \quad (34)$$

The resistance law in terms of the drag coefficient $C_D = u_*^2/u_{10}^2$ follows from (34) and (7),

$$C_D^{1/2} = \frac{u_*}{u_{10}} = \kappa\left(\int_{z_0^\nu}^{10}\left[1 - \alpha_c f(z)\right]^{3/4} d(\ln z)\right)^{-1}, \quad (35)$$

and the coupling parameter is expressed via (22),

$$\alpha_c = \frac{1}{u_*^2}\int_0^\infty \mathcal{T}(k)\, d(\ln k). \quad (36)$$

Relation (35) relates the drag of the sea surface directly to the properties of the momentum exchange at the surface via the coupling parameter (36) and the vertical distribution of the wave-induced stress $f(z)$ (equation (24)). The coupling parameter is determined by the geometrical properties of the surface via the directional wavenumber wave spectrum $S(k,\phi)$, and by peculiarities of energy exchange between wind and waves, expressed via the growth rate parameter. The growth rate parameter (30) in turn depends on the drag coefficient (the friction velocity) and the coupling parameter.

The wind over waves coupled model, described above, is in fact a model of the marine atmospheric surface boundary layer where the momentum flux is related directly to the geometrical properties of the sea surface via the wavenumber directional wave spectrum and properties of momentum exchange at the sea surface. Equations (35) and (36) are solved by iterations after the form of a wave spectrum $S(k,\phi)$ is given by a functional relation or is specified by a physical model based on the solution of the wave energy balance equation.

4. Results

The wind over waves coupled model is first tested using an empirical wave spectrum model suggested by *Elfouhaily et al.* [1997]. This spectrum is known to

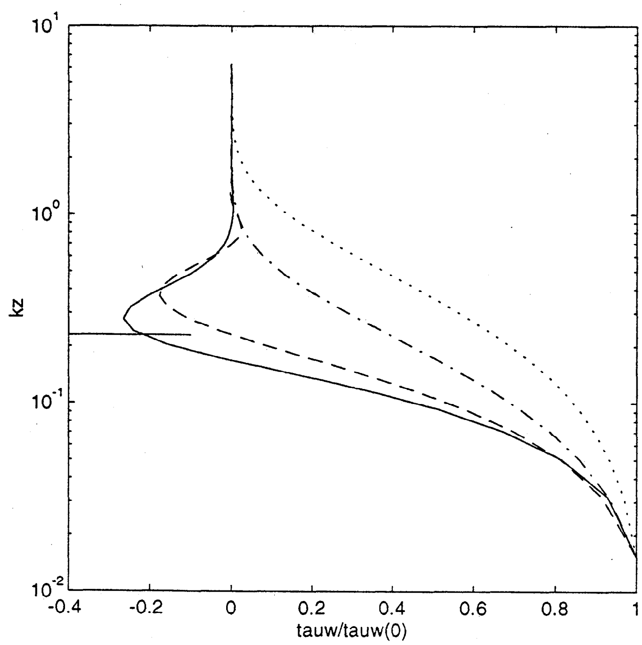

Figure 2. The vertical decay function $F(k,z)$ of the wave-induced flux for a slowly moving wave ($u_\lambda/c = 10$ and $kz_0 = 6.6 \times 10^{-3}$). Dotted line, potential decay $\exp(-2kz)$; dashed- dotted line, decay at the height of the inner region $\exp(-z/\mathcal{L})$; solid line, numerical calculation, dashed line, approximation (33). Solid horizontal line shows the height of the inner region $\mathcal{L}$.

reproduce correctly the wind speed dependence of the total mean squared slope parameters, as measured by *Cox and Munk* [1954], and the capillary-gravity spectrum range, as measured in controlled laboratory conditions [*Jähne and Riemer*, 1990].

The tuning constant m_β in (30) with $c_\beta = m_\beta R$ is chosen by comparing the model results for the drag coefficient C_D, (35), with the data obtained in the open ocean by *Anderson* [1993] and *Large and Pond* [1982]. The drag coefficient C_D for a fully developed sea is shown in Figure 3. The regression line of Large and Pond is $10^3 C_D = 0.49 + 0.065 U_{10}$ in the wind speed range $10 < U_{10} < 20\,\mathrm{m\,s^{-1}}$, and $10^3 C_D = 1.14 \pm 0.20$ in the wind speed range $5 < U_{10} < 10\,\mathrm{m\,s^{-1}}$. A value of the tuning constant $m_\beta = 36$ gives a reasonable agreement between model results and measurements of C_D in the full range of the wind speed. For waves that are not too long ($R \to 1$), and not too short ($\bar{f}(k) \to 0$), this value of m_β agrees with empirical estimates of c_β in the parameterization of *Plant* [1982] (relation (25)). For shorter waves ($\bar{f}(k) \to 1$) the dependence of the proportionality coefficient in (30) on the coupling parameter can partly explain the scatter in empirical estimates of c_β. The fact that the tuning parameter m_β is close to empirical estimates gives additional evidence for the self-consistency of the model.

The dashed-dotted line depicted in the same figure shows the model calculations of the drag coefficient based on a differential model of the wind over waves

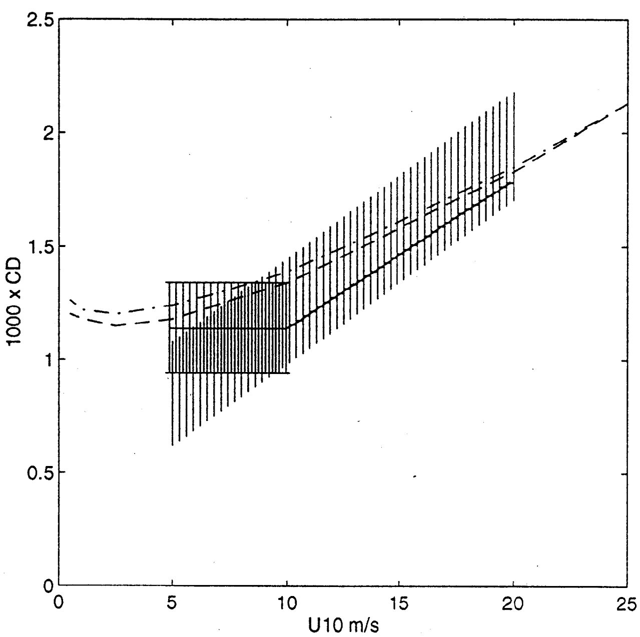

Figure 3. Drag coefficient versus u_{10}. Model results: dashed-dotted line, present approach; dashed line, calculations with a differential model of *Makin and Mastenbroek* [1996]. Data of *Anderson* [1993] fall inside the shaded area. Regression line from *Large and Pond* [1982], solid line; in the wind speed range 5-10 $\mathrm{m\,s^{-1}}$ the standard deviation is shown.

coupling by *Makin and Mastenbroek* [1996] for the same functional relations of the growth rate parameter, the wave spectrum, the vertical decay of the wave-induced flux, and the tuning constant m_β. This model solves the full balance equations of the TKE, the dissipation rate, and the mean wave-induced energy. A good correspondence in results of the present approach and the differential model justifies the assumptions of the approach concerning the balance equation of the TKE (11), the equation of the mean wave-induced energy (15), and the turbulence closure scheme.

It is emphasized, that the drag of the sea surface is calculated from basic assumptions concerning the local properties of the sea surface, and from known functional relations for the wave spectrum and the growth rate parameter. The sea roughness results in this approach from the dynamical coupling of waves with the atmosphere.

The coupling parameter α_c, the principle parameter of the wind over waves coupling, (36), which characterizes the degree of dynamical coupling of waves with the atmosphere is shown in Figure 4b for a fully developed sea. For low wind speeds $u_{10} < 5\ \mathrm{m\,s^{-1}}$ the stress at the surface is dominated by the viscous drag, so most of the momentum is transferred directly from the air flow to the water flow. This is in agreement with observations that the sea surface is aerodynamically smooth for wind below 2 - 3 $\mathrm{m\,s^{-1}}$ [*Geernaert*, 1990]. Analysis of the field experiment of *Snyder et al.* [1981] by *Harris et al.* [1996] suggests that the water surface is more likely to be transitional or even smooth, than rough for the conditions of the experiment (the range of wind speeds is 2 - 7 $\mathrm{m\,s^{-1}}$). Results presented in Figure 4b support this conclusion, that for light winds a significant part of the momentum is transferred by viscous stress. Recent direct laboratory measurements of the surface fluxes [*Banner and Peirson*, 1998], though limited to two experimental points, have shown, that for a wind speed of 7 $\mathrm{m\,s^{-1}}$ the coupling parameter is about 0.38, and for a wind speed of 13 $\mathrm{m\,s^{-1}}$ should be about or less than 0.67, which is in a good agreement with the present model results. For wind speeds of about 15 $\mathrm{m\,s^{-1}}$ and higher, most of the stress goes through waves and the sea becomes aerodynamically rough.

To assess which waves contribute most to the wave-induced stress at the surface, the surface flux spectrum $\mathcal{T}(k)$ (equation (23)) and the cumulative spectrum $Cu(k)$ are shown in Figures 4c and 4d. The corresponding omnidirectional wave curvature spectrum is shown in Figure 4a for a wind speed of 10 $\mathrm{m\,s^{-1}}$, and a fully developed sea $u_{10}/c_p = 0.83$. The cumulative spectrum is

$$Cu(k) = \frac{1}{\tau^w(0)} \int_k^\infty \mathcal{T}(k) d(\ln k). \qquad (37)$$

It is shown as a function of the wavelength $\lambda = 2\pi/k$ and shows an increase in the wave-induced stress since more and more longer waves are taken into account in

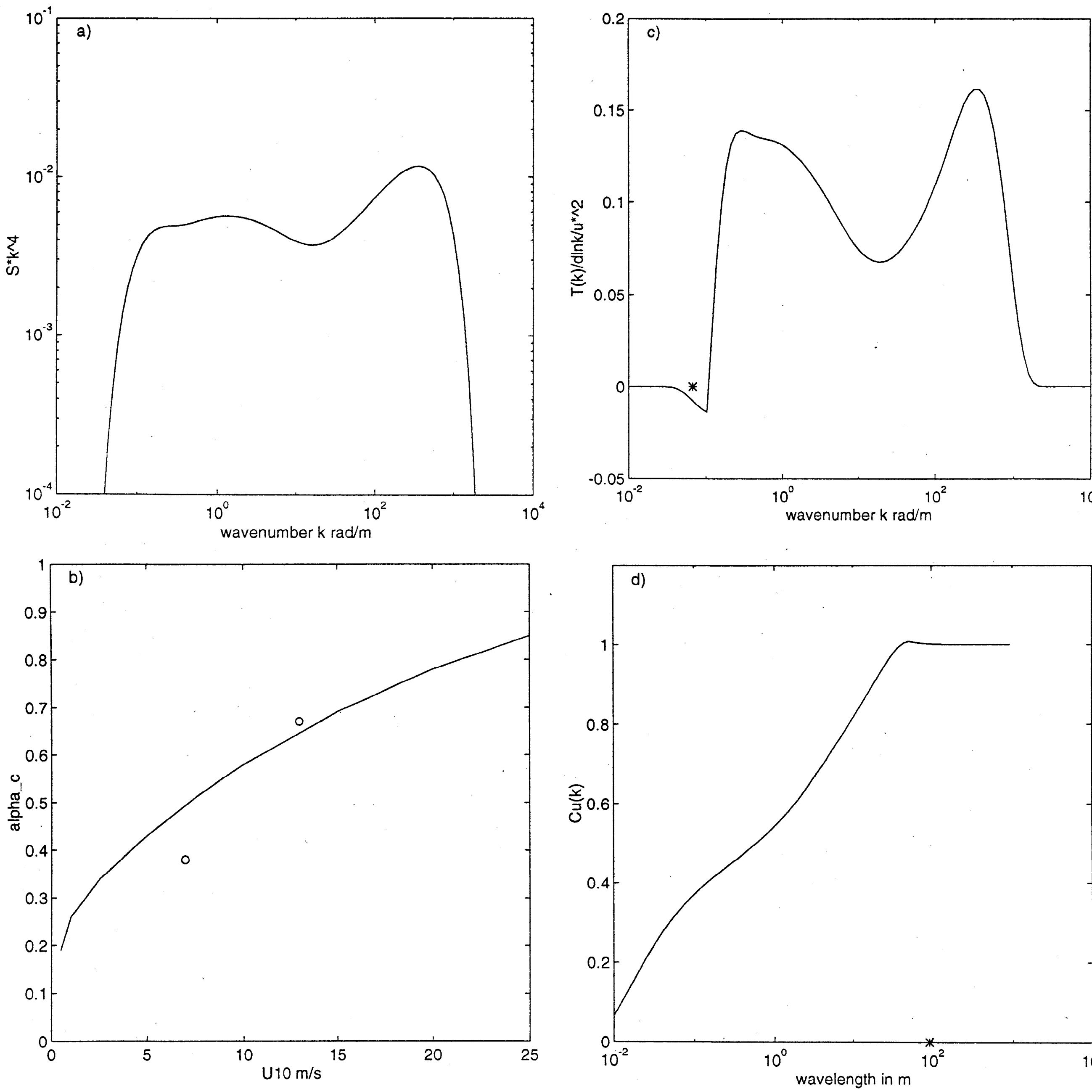

Figure 4. (a) Omnidirectional wave curvature spectrum $k^4 S(k)$. (b) Coupling parameter α_c; circles, experiment of *Banner and Peirson* [1998]. (c) Surface flux spectrum $\mathcal{T}(k)/d(lnk)/u_*^2$. (d) Cumulative spectrum of the surface flux $Cu(k)$. Wind speed $u_{10} = 10$ m s^{-1} and $u_{10}/c_p = 0.83$. The asterisk indicates the peak of the wave spectrum.

the integral (37). The wavenumber of the peak of the spectrum is 0.068 rad m^{-1} which corresponds to a wavelength of 93 m.

Most (80%) of the form drag is supported by waves shorter than 10 m, and 50% of the form drag is supported by waves shorter that 1 m. About 10% of this is supported by gravity-capillary waves with lengths $\lambda < 0.017$ m. Thus the short gravity and gravity-capillary waves in the range $0.01 < \lambda < 1$ m play an important role in extracting momentum, and they are strongly coupled with the atmosphere. The above result shows that most of the wave-induced momentum is transferred to centimeter to meter waves, so that the sea drag depends crucially on the form of the wave spectrum in the corresponding wavenumber range.

5. Conclusions

A new scheme to couple waves with the atmosphere is described in part 1 of the present paper. The main motivations to revise our original approach [*Makin et al.*, 1995] were to introduce the correct description of turbulence in the boundary layer above waves and to develop much simpler model based on explicit (integral) equations as an alternative to the differential model by *Makin and Mastenbroek* [1996]. When this scheme is to be used as a module in models for different applications, for example, in a coupled sea surface–atmosphere model, described in part 2, in atmospheric circulation models to calculate surface fluxes above sea, or in wave prognostic models to calculate the sea drag, such a simplification becomes a matter of principle.

The simplification was achieved by assuming a local balance between turbulent kinetic energy production and its dissipation, and between production and dissipation of the mean wave-induced energy. The problem of the boundary layer above waves can then be reduced to the solution of two integral equations. The sea drag is expressed via the coupling parameter and the decay function of the wave-induced stress, while the coupling parameter is calculated through the wavenumber wave spectrum and the growth rate parameter. The growth rate parameter in turn depends on the sea drag and the coupling parameter. We show the validity of the above mentioned assumptions by comparing results of the present approach with results of the one-dimensional differential model by *Makin and Mastenbroek* [1996] from which a good agreement in the sea drag coefficients was found.

In achieving the main goals we also show that the production of the TKE above waves consists of two sources: production due to the mean motion, and production due to the wave-induced motion. Neglecting the second source leads to an erroneous estimation of the eddy viscosity coefficient above waves.

The growth rate parameter used in the paper has the general form of the *Plant* [1982] relation, that is, proportional to the square of the friction velocity (the total stress). However, by using the 2-D WBL model [*Mastenbroek et al.*, 1996] we show that the local turbulent stress (the difference between the total and the wave-induced stress inside the inner region) rather that the total stress has to be used to parameterize the growth rate. This fact makes the proportionality coefficient in Plant's relation dependent on the coupling parameter. When the coupling parameter grows with increasing wind speed, the coefficient tends to zero and quenches the energy flux to short waves. As mainly the short waves support the wave-induced flux, a further growth of the coupling parameter is prevented as well. This property is of great importance as it always secures the adjustment of the airflow to waves.

The coupling parameter is shown to strongly vary with the wind speed which is in a good agreement with direct, though limited, measurements by *Banner and Peirson* [1998]. Establishing the correct functional wind speed dependence of the coupling parameter is of primary importance to estimate heat and gas exchanges at the sea surface. We show that viscous stress is dominant for wind speeds less than 10 m s^{-1} and still plays a role for strong winds supporting about 20% of the total stress. This means that the coupling parameter never comes very close to 1. Finally, the wind-generated short gravity waves (typical wavelength from a few centimeters to a few decimeters) and capillary-gravity waves (lengths of about a centimeter) are shown to be strongly coupled to the atmosphere.

The study of these waves is motivated because of their relevance to interpretation of remotely sensed data of the sea surface by microwave radar instruments. In the case of radar remote sensing, the short gravity and capillary-gravity waves serve as roughness elements on the ocean surface to modulate the scattering of radar waves. To gain further understanding and, for example, to better interpret high-resolution radar image contrasts from synthetic aperture radar instruments, physical models of the short gravity and capillary-gravity wave spectrum are needed, as they describe the physical properties of the sea surface under the joint action of wind and surface currents.

Such a physical model of short waves, which is based on the solution of the wave energy balance equation and accounts for the balance between the wind input, different dissipation processes and nonlinear interactions, is described in part 2 of the present paper. The wave spectrum model is coupled to the atmosphere by the wind over waves coupling scheme presented here, and the evolution of the coupled system sea surface–atmosphere is assessed.

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