IMPACT OF WAVES ON AIR-SEA EXCHANGE OF SENSIBLE HEAT AND MOMENTUM

V.K. MAKIN and C. MASTENBROEK*
Royal Netherlands Meteorological Institute (KNMI), De Bilt, The Netherlands

(Received in final form 9 February, 1996)

Abstract. The impact of sea waves on sensible heat and momentum fluxes is described. The approach is based on the conservation of heat and momentum in the marine atmospheric surface layer. The experimental fact that the drag coefficient above the sea increases considerably with increasing wind speed, while the exchange coefficient for sensible heat (Stanton number) remains virtually independent of wind speed, is explained by a different balance of the turbulent and the wave-induced parts in the total fluxes of momentum and sensible heat.

Organised motions induced by waves support the wave-induced stress which dominates the surface momentum flux. These organised motions do not contribute to the vertical flux of heat. The heat flux above waves is determined, in part, by the influence of waves upon the turbulence diffusivity.

The turbulence diffusivity is altered by waves in an indirect way. The wave-induced stress dominates the surface flux and decays rapidly with height. Therefore the turbulent stress above waves is no longer constant with height. That changes the balance of the turbulent kinetic energy and of the dissipation rate and, hence the diffusivity.

The dependence of the exchange coefficient for heat on wind speed is usually parameterized in terms of a constant Stanton number. However, an increase of the exchange coefficient with wind speed is not ruled out by field measurements and could be parametrized in terms of a constant temperature roughness length. Because of the large scatter, field data do not allow us to establish the actual dependence. The exchange coefficient for sensible heat, calculated from the model, is virtually independent of wind speed in the range of 3–10 ms$^{-1}$. For wind speeds above 10 ms$^{-1}$ an increase of 10% is obtained, which is smaller than that following from the ‘constant roughness length’ parameterization.

1. Introduction

Sea waves strongly affect the exchange of momentum and heat between the atmosphere and the ocean, and are thus an important link between the atmosphere and ocean.

1.1. Bulk Formulae

In bulk parametrization, the fluxes are related to the measured variables (wind speed $u$, potential temperature $T$) at the surface and at a certain height via exchange coefficients. For the momentum flux, $\tau$ the exchange coefficient is called the drag coefficient, $C_D$, for the sensible heat flux, $H$ – the Stanton number $C_H$. Thus

$$\frac{\tau}{\rho_a} = C_D (\Delta u)^2,$$

(1)

* The investigation was in part supported by the Netherlands Geosciences Foundation (GOA) with financial aid from the Netherlands Organization for Scientific Research (NWO).

\[
\frac{H}{\rho_a c_p} = C_H \Delta u \Delta T. \tag{2}
\]

In (1) and (2) \(\rho_a\) is the air density, \(c_p\) is the specific heat capacity of the air and \(\Delta\) denotes the difference between the variable at measured height \(h\), normally 10 metres, and its surface value. In a neutral stratiﬁed surface boundary layer the distribution of velocity and temperature with height is described by the logarithmic profile

\[
\Delta u(z) = \frac{u_*}{\kappa} \ln \frac{z}{z_0}, \quad \Delta T(z) = \frac{t_*}{\kappa} \Pr_t \ln \frac{z}{z_0}, \tag{3}
\]

\((u_* = \sqrt{\tau/\rho_a}\) is the friction velocity, \(\kappa\) is the von Karman constant, \(t_* = H/\rho_a c_p u_*\) is the scale temperature, \(\Pr_t = K/K_t\) is the turbulent Prandtl number; \(K, K_t\) are the turbulent diffusivities for momentum (eddy viscosity) and heat). The exchange coefficients can be now related to the roughness lengths for momentum \(z_0\) (sea roughness) and heat \(z_{0t}\)

\[
C_D = \frac{\kappa^2}{\ln^3 \frac{z}{z_0}}, \quad C_H = \frac{C_D^{1/2}}{\Pr_t \ln \frac{H}{z_0}}. \tag{4}
\]

At present it is well established that the drag coefficient depends strongly on wind speed (see reviews of Garratt, 1977, Geernaert, 1990, recent ﬁeld experiments of Smith et al., 1992, Anderson, 1993). In the wind speed range of \(5 < U_{10} < 20\) m s\(^{-1}\) the drag coeﬃcient increases roughly linearly with wind speed. The variation of \(C_D\) in this range is about 100%-150%. The increase of the drag coeﬃcient with the wind was explained by e.g. Phillips (1977) using the analogy between waves and roughness elements. With increasing wind speed waves grow and the sea surface becomes rougher. The increase of the sea roughness with wind speed is described by the famous Charnock relation (Charnock, 1955):

\[
z_0 = z_0^* \frac{u_*^2}{g} \tag{5}
\]

where \(z_0^*\) is the Charnock constant and \(g\) is the acceleration due to gravity. (The Charnock ‘constant’ can in principle be a function of the sea state (e.g. Smith et al., 1992). However, this relationship is not well established, see Donelan et al. (1993), Toba et al. (1990), the recent exchanges in the literature between Donelan et al. (1995) and Jones and Toba (1995), and the theoretical results of Janssen (1989) and Makin et al. (1995).

1.2. THE STANTON NUMBER DEPENDENCE ON WIND SPEED

Measurements of sensible heat ﬂux show clearly that the Stanton number \(C_H\) is much less dependent on the wind speed than the drag coeﬃcient. These measurements include open ocean measurements of Smith (1980, 1988), Large and Pond
IMPACT OF WAVES ON AIR-SEA EXCHANGE OF SENSIBLE HEAT AND MOMENTUM

(1982), Geernaert (1990), Anderson (1993), the earlier measurements reviewed in Friese and Schmitt (1976); and coastal measurements obtained during the Humidity Exchange over the Sea Main Experiment (HEXMAX) and reported by Smith and Anderson (1988), DeCosmo (1991), DeCosmo et al. (1996). However the difficulties in heat flux measurements result in a considerable scatter of the data (30%-60% of the mean). This obscures the actual dependence of the Stanton number on wind speed.

The most common parameterization of the field data is to assign a constant value to the Stanton number (Pond et al., 1971; Smith, 1980 and 1988; Large and Pond, 1982; DeCosmo, 1991; Anderson, 1993). However a dependence on wind speed is not ruled out by the field measurements. Large and Pond (1982) argue that for wind speeds above 10 m s^{-1} the parameterization of the Stanton number in terms of a constant temperature roughness length is more appropriate, though the statistical improvement of such a fit to their data is not significant compared to the constant Stanton number parameterization. However such a choice makes a difference. What immediately follows from (4) is that \( C_H \sim C_D^{1/2} \) which could, for high wind speed, increase the Stanton number up to 50% compared to the constant value of \( C_H \), and increase estimates of heat exchange between the atmosphere and the ocean.

In the bulk parameterization, the water surface is regarded as a 'solid' boundary, with the principal problem being to determine the characteristic roughness lengths of the water surface. The bulk parameterization is based on empirical relationships for the roughness lengths (exchange coefficients) and does not explain the physics which determine those relations.

In such situations, when the number of field measurements is low, and data show such a large scatter that the actual dependence of the exchange coefficient of sensible heat on wind speed is obscured, it is appealing to use a simple model to establish theoretically the dependence of the exchange coefficient upon wind speed. Such a theory will provide independent support for this or that empirical relationship between the exchange coefficient of sensible heat and wind speed. To construct such a theory will be the main goal of the present paper.

1.3. THE INFLUENCE OF WAVES

Already in the early sixties Stewart (1961) showed, on general theoretical considerations and some experimental knowledge, that the interaction of wind and waves has to be taken into account to understand the transfer of momentum and heat above sea.

Kitaigorodskii and Donelan (1984) proposed a semiempirical mixing length model for heat/mass transfer. The surface mixing length for contaminants was allowed to depend on Prandtl number and roughness Reynolds number. The latter depends on the sea state via the surface roughness. They obtained fairly near-constant heat and mass transfer coefficients with wind speed variation. However
they remark that ‘precise parameterization of gas transfer depends on knowing not only the effective surface roughness, but also the way in which energy and momentum are transferred through the boundary layer’. This idea was not explored at that time.

The first attempt to obtain theoretically the drag coefficient (or the sea roughness) by explicitly taking into account the sea state and the impact of waves on the atmospheric boundary layer was made by Janssen (1989) and later by Chalikov and Makin (1991). However both had to use a Charnock-type relation for the ‘background’ roughness parameter entering their theories. That ensures proper values of the drag coefficient when seas are fully developed. Caudal (1993) bypassed this problem by assuming that the total surface stress is supported only by waves. This assumption does not hold in general. A consistent theory to calculate the sea drag, which accounts for balance between the wave-induced and the turbulent stress at the surface, and which avoids the use of a Charnock-type relation for the ‘background’ roughness was introduced by Makin et al. (1995).

It has been shown (Janssen, 1989; Makin et al., 1995) that for typical sea conditions the wave-induced stress at the surface (the form drag) contributes a considerable fraction to the total surface stress. The wave-induced flux decays rapidly with height. Therefore the turbulent stress above waves is no longer constant with height. That changes the balance of the turbulent kinetic energy (TKE) and of the dissipation rate and, hence, the diffusivity. In this way the organized wave-induced stress alters the turbulence diffusivity above waves. This effect was not taken into account in Janssen (1989), Chalikov and Makin (1991) and Makin et al. (1995) as they used a mixing-length theory to parameterize the turbulent stress. Chalikov and Belevich (1993) use a one-equation eddy viscosity model to account for a proper balance of the TKE, but needed to specify the mixing length.

Here we introduce a model which explicitly accounts for the impact of waves on the fluxes of sensible heat and momentum. The model is based on Makin et al. (1995). The proper balance of the TKE and of the dissipation rate is accounted for by using a two-equation eddy viscosity scheme to parameterize the turbulent stresses in the balance equations of momentum and heat.

There is no counterpart to the effective exchange mechanism of momentum due to the form drag in the sensible heat exchange at the sea surface. The heat flux is determined by turbulence diffusivity, which is affected by waves. Therefore, as was remarked by Kitaigorodskii and Donelan (1984), it is important to properly account for momentum and energy transfer above waves in order to describe the air-sea exchange of heat.

The Stanton number and the drag coefficient follow from the solution of the balance equations for momentum and sensible heat, the TKE and the dissipation rate for a given wind speed and sea state. Sea state is defined in terms of a wave spectrum. To calculate the form drag the energy flux to waves from the atmosphere is defined in terms of the growth rate parameter. The dependence of $C_H$ on wind speed is then established.
It follows that in the range of wind speeds 3–10 ms\(^{-1}\) the Stanton number is virtually independent of wind speed and increases by about 10% in the range 10–20 ms\(^{-1}\). The increase is smaller than that following from the 'constant roughness lengths' parameterization of the Stanton number. While the exchange coefficient for heat remains virtually independent of wind speed in the range of 5–20 ms\(^{-1}\), the calculated drag coefficient increases up to 100% in the same range of wind speed. This is in agreement with field measurements.

In the literature it is argued that for wind speeds higher than 15 ms\(^{-1}\), the effect of sea spray evaporation becomes important and increases the exchange coefficients for heat and moisture (Andreas et al., 1995; Fairall et al., 1994). This effect is not accounted for by the present model.

2. **One dimensional model of the WBL**

The lowest part of the marine atmospheric surface layer where the wave-induced motion can be detected is considered. The depth of this surface 'wave boundary layer' or WBL scales with the wavelength of the energy containing waves. The \(h = 10\) metre height is a good estimate for the upper boundary of the WBL (Makin et al., 1995).

Stationary and spatially homogeneous conditions are assumed. The wind direction coincides with the mean direction of wave propagation and the wave field (wave spectrum) is symmetric relative to that direction. Waves propagating against the wind are not considered.

The wave-induced fluxes are concentrated in the vicinity of the water surface. Explicit description of this region dictates the use of a vertical coordinate system which follows the wave surface \(z = \eta\), e.g. \(\tilde{z} = h(z - \eta)/(h - \eta)\). The mean, spatially averaged variable \(f\), is defined by the following procedure

\[
f(x, \tilde{z}) = \langle f \rangle (\tilde{z}) + \tilde{f}(x, \tilde{z}), \quad \langle f(\tilde{z}) \rangle = \frac{1}{L} \int_0^L f(x, \tilde{z}) dx,
\]

where \(f\) is any variable, \(\langle f \rangle\) is the mean, \(\tilde{f}\) is the wave fluctuation, \(L\) is the length of averaging. Hereafter the brackets indicating the averaging are kept only for the second moments, i.e. \(\langle \tilde{f}\tilde{g} \rangle\) and dropped elsewhere.

The procedure (6) is defined in the domain \(0 < \tilde{z} < h\), \((\eta < z < h)\). Hereafter the top hat 'above \(z\) is dropped.

2.1. **Balance Equation of Momentum**

The balance equation of momentum above the waves reads

\[
\frac{\partial}{\partial z}(\tau^t + \tau^w) = 0,
\]

(7)
where the mean turbulent stress \( \tau^t \) is

\[
\tau^t = -\bar{u}'w' \tag{8}
\]

and stresses are scaled with the density of air.

Expressions for the two main components of the mean wave-induced stress \( \tau^w = \tau^w_p + \tau^w_a \) (Makin, 1990) are

\[
\tau^w_p = \left( \frac{\partial \eta}{\partial x} (\bar{p} + \bar{u}'w')(1 - \frac{z}{h}) \right), \quad \tau^w_a = -\langle \bar{u} \bar{W} \rangle, \tag{9}
\]

where \( \bar{p} \) and \( \bar{u}'w' \) are the wave-induced pressure and the normal turbulent stress, \( W = w - (1 - z/h)\left((\partial \eta/\partial x)u + \partial \eta/\partial \theta \right) \) is a modified vertical velocity.

In a stationary WBL the total stress \( \tau = \tau^t + \tau^w \) is constant with height, so

\[
\tau = \text{Const.} = u_*^2 \tag{10}
\]

and by definition equals the square of the friction velocity. In the upper part of the WBL (far enough from the waves) the wave-induced stress \( \tau^w = 0 \) and the friction velocity relates to the turbulent stress \( \tau^t \). In this part of the WBL the bulk formalism can be applied: by measuring the turbulent stress and the wind velocity the roughness length follows from (3). Near the wave surface the wave-induced stress \( \tau^w \) supports a considerable part of the total stress \( \tau \) and the local friction velocity related to the local turbulent stress \( u_*^*(z) = \sqrt{\tau^t(z)} \) varies with height. If turbulent stress is measured in the layer of the WBL affected by the waves, no bulk formalism can be applied for estimation of the sea roughness.

The 2D numerical simulations of the air flow above unidirectional sea waves (Makin, 1987) and monochromatic waves (Makin, 1990; Makin and Mastenbroek, 1996) have shown that near the surface the wave-induced stress is dominated by the first term \( \tau^w_p \) in (9). At the surface \( z = 0 \), where \( \tau^w_a = 0 \), and the term \( \tau^w_p \) is called the form drag.

The wave-induced stress decays rapidly with height. For each wave component the vertical decay can be approximated by \( \exp(-z/r(k)) \), where \( r \) is the decay length and \( k \) is the wave number. The 2D numerical calculations of Makin (1989), based on a mixing-length theory to parameterize the turbulent stress, have shown that the decay length can be approximated by \( r(k) = 1/(2k) \). This decay length was adopted to calculate the wave-induced flux above the sea waves in Makin et al. (1995). Kitaigorodskii and Donelan (1984) use the same decay length to calculate the wave-induced flux above waves. However the rapid distortion theory of the turbulent air flow above waves (Belcher and Hunt, 1993) indicates that the wave-induced flux decays much faster due to the smearing of the wave fluctuations of turbulent stresses in the outer region. Mastenbroek et al. (1996) have used the 2nd order LRR (Launder et al., 1975) closure scheme, which accounts for the rapid distortion effects, to model turbulence above waves. Their calculations show that the wave-induced flux indeed decays faster and that the decay length can be related
to the height of the inner region. For short waves which support most of the surface stress (Makin et al., 1995), the height of the inner region and hence the decay length can be approximated by \( r = 1/(5k) \) (Mastenbroek et al., 1996). [In Belcher and Hunt (1993) two main regions are distinguished. In the inner region—a thin region adjacent to the wave surface—the wave-induced turbulence is in local equilibrium with the wave-induced velocity shear. The wave-induced motion here is viscous and rotational. The growth of waves is related to the change of wave fluctuating turbulent stress \( \tau^t \) across this inner layer. Outside the inner region—in the outer region the turbulent eddies are rapidly distorted and the wave-induced turbulence is not in local equilibrium with shear. The wave fluctuation of the turbulent stress \( \tau^t \) is suppressed and the wave-induced motion is inviscid and potential. The mean wave-induced flux \( \tau^w \), the one we consider in the present study, is thus suppressed too in the outer region.]

The form drag at the sea surface \( \tau^w_0 = \tau^w_0 \) and the distribution of the wave-induced stress \( \tau^w(z) \) with height can be calculated according to:

\[
\tau^w(z) = \int_0^\infty \int_{-\pi/2}^{\pi/2} \omega^2 S \beta \cos \theta k d\theta e^{-z/r} dk.
\]

Equation (11) presumes that: 1) all undulations of the sea surface are considered as waves, which can then be statistically described by a directional wave spectrum \( S(k, \theta) \), where the wave number \( k \) satisfies the dispersion relation

\[
\omega^2 = gk + \mathcal{T}k^3.
\]

(\( \mathcal{T} \) is the dynamical surface water tension and \( \theta \) is the propagation direction of the \( k \)-wave component, \( \omega \) is the wave frequency); 2) the energy input to waves from the atmosphere is known and can be described in terms of the growth rate parameter \( \beta(k, \theta) \); 3) the decay rate of the wave-induced flux of the individual wave component is known. For present calculations the decay length \( r = 1/(5k) \) is chosen. (The \( 1/(2k) \) decay length was also assessed, which results in about 3% increase in the modelled drag coefficient, compared to one using the \( 1/(5k) \) dependence. This increase is much less than a typical uncertainty of about 20% in flux measurements.)

It has been shown (Janssen, 1989; Makin et al., 1995) that for typical sea conditions the form drag contributes a considerable fraction of the total surface stress. The value of the coupling coefficient—the ratio of the form drag to the total stress

\[
\alpha = \frac{\tau^w_0}{u^2_0}
\]

is about 0.5 for a wind speed of 6 ms\(^{-1}\) and increases to about 0.95 for a wind speed of 20 ms\(^{-1}\) (see figure 3 in Makin et al., 1995). What immediately follows from (10) and (11) is that the turbulent stress is significantly smaller near the sea
surface than at the top of the WBL. Due to the fact that the wave-induced stress decreases rapidly with height the turbulent stress will increase rapidly with height compared to its surface value in order to satisfy the conservation of momentum (7). Such a distribution of $\tau^f$ will change the balance of the TKE and the dissipation rate of turbulence $\varepsilon$, compared to that in the logarithmic boundary layer, and hence will influence the turbulence diffusivity near the wave surface. In this indirect way the wave-induced stress has an impact on the turbulence diffusivity above waves. In order to calculate properly the turbulence diffusivity above waves the balance equations for the TKE and the dissipation rate are to be considered in the description of the WBL.

2.2. BALANCE EQUATION OF THE TKE

The balance equations of the TKE, $e$ and the dissipation rate, $\varepsilon$ read:

$$- \frac{\partial}{\partial z} (e' + p') w' + P - \varepsilon = 0$$
(14)

and

$$- \frac{\partial e' w'}{\partial z} + \frac{\varepsilon}{e} (c_{1e} P - c_{2e} \varepsilon) = 0,$$
(15)

and constants in (15) have standard values (e.g. Rodi, 1984): $c_{1e} = 1.44, c_{2e} = 1.92$.

In Equations (14), (15) the first terms on the left hand side are the diffusive transport of the TKE and $\varepsilon$. $P$ is the production of the TKE, which has two contributions: production by interaction of the turbulent stress and mean velocity shear $P^t$ and production $P^w$ due to dissipation of the mean wave-induced energy

$$P = P^t + P^w.$$  
(16)

The production terms $P^t$ and $P^w$ represent the transfer of kinetic energy from the mean and mean wave-induced motions to the turbulent motion. They appear with opposite sign as sink terms in the equations for the kinetic energy of the mean $(E = u^2/2)$ and mean wave-induced $(E_w = \langle (\tilde{u}^2 + \tilde{w}^2) \rangle/2)$ motions.

The balance equation of $E$ can be derived from (7) by multiplying it by $u$:

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial z} u (\tau^t + \tau^w) - \tau^t \frac{\partial u}{\partial z} - \tau^w \frac{\partial u}{\partial z}.$$  
(17)

The first term on the right-hand side describes the transport of $E$, the second term is the sink of the mean energy into the TKE, so that

$$P^t = \tau^t \frac{\partial u}{\partial z};$$  
(18)

the third term describes the mutual transfer of energy between the mean and the wave-induced motions

$$D = \tau^w \frac{\partial u}{\partial z}.$$  
(19)
For a growing wind-wave field \( \tau^w > 0 \), and this term describes the sink of the mean energy into the mean wave-induced energy and further supports the energy flux to waves (which is why waves grow). The term \( D \) appears with the opposite sign in the balance equation of the mean wave-induced energy \( E_w \):

\[
\frac{\partial E_w}{\partial t} = \frac{\partial \Pi}{\partial z} + D - P^w.
\]

(The exact form of this equation can be found in Panchenko and Chalikov, 1984; Makin and Chalikov, 1986.) The first term on the right-hand side of (20) describes the transport of \( E_w \) by the wave-induced flux. At the surface \( \Pi_0 = -\langle \hat{w}(\bar{p} + \bar{w}^w \bar{w}) + \bar{\tau}^w \rangle \) is the energy input to waves. Assuming that the wave-induced flux of energy decays on the same length as the wave-induced stress, the term \( \Pi(z) \) reads

\[
\Pi(z) = \int_{-\infty}^{\infty} \int_{0}^{\infty} \omega^2 c S \beta k d\theta e^{-z/\tau} dk,
\]

where \( c = \omega/k \) is the phase speed of component \( k \). Once the term \( \Pi(z) \) is known the sink of the mean wave-induced energy into the TKE can be calculated from the balance Equation (20) in the stationary WBL. In Chalikov and Belevich (1993) the transport term \( \Pi \) in Equation (20) was neglected, and the local balance \( D = P^w \) assumed.

In the very vicinity of the wave surface \( \partial \Pi/\partial z \approx 0 \) and \( D = P^w \). From (18), (19) and (13) it follows that

\[
\frac{P^w}{\bar{p}^t + P^w} = \alpha.
\]

For high wind speeds the coupling parameter \( \alpha \) is close to 1 and most of the TKE production is due to the dissipation of the wave-induced energy.

2.3. BALANCE EQUATION OF HEAT

The balance equation of sensible heat above waves reads

\[
\frac{\partial}{\partial z} (\sigma^t + \sigma^w) = 0
\]

(fluxes are scaled with the density of air and \( c_p \)).

In the stationary WBL the total heat flux \( \sigma = \sigma^t + \sigma^w \) is constant with height

\[
\sigma = \text{Const.} = \Im \tau^w.
\]

The mean turbulent flux \( \sigma^t \) is

\[
\sigma^t = -\overline{T^w w^t}.
\]
The only contributions to the wave-induced flux $\sigma^w$ are due to advection

$$\sigma^w = -\langle T\bar{W} \rangle.$$  \hfill (26)

The wave-induced flux $\sigma^w = 0$ at the wave surface due to the kinematic boundary condition $\bar{W} = 0$. So, there is no counterpart to the effective ‘form drag’ momentum exchange mechanism in the exchange of heat at the wave surface. Above waves the wave-induced flux $\sigma^w$ appears to be much smaller than the mean turbulent flux $\sigma^T$ (Makin and Mastenbroek, 1996) and in the first approximation can be neglected in the balance Equation (23).

The heat flux above waves is thus determined mainly by the turbulence diffusivity in the air flow. On qualitative grounds this immediately explains why the air-sea exchange coefficient of heat $C_H$ is less dependent on wind speed than the drag coefficient $C_D$. With increasing wind the form drag increases and supports a larger fraction of the total stress. The heat flux is influenced by waves only indirectly via the turbulence diffusivity.

2.4. TURBULENT CLOSURE SCHEME

Local closure, via $K$-theory, which relates the turbulent flux to the gradient of the associated mean variable, is used here.

The turbulent fluxes which appear in the balance equations of momentum, heat, TKE and dissipation rate can be written: for the turbulent stress

$$\tau^i(z) = K \frac{\partial u}{\partial z};$$  \hfill (27)

for the turbulent diffusive flux of the TKE

$$-\langle (e' + p')w' \rangle = K \frac{\partial e}{\sigma_e \partial z};$$  \hfill (28)

for the turbulent diffusive flux of the dissipation rate

$$-\langle \bar{e}'w' \rangle = K \frac{\partial \bar{e}}{\sigma_e \partial z};$$  \hfill (29)

for the turbulent flux of heat

$$-\langle \bar{T}'w' \rangle = K \frac{\partial \bar{T}}{Pr \partial z}.$$  \hfill (30)

The eddy viscosity $K$ is calculated from the TKE and the dissipation rate:

$$K = c_\mu \frac{\bar{e}^2}{\bar{e}}.$$  \hfill (31)

Values of constants which appear in (27)-(31) are (e.g. Rodi, 1984): $c_\mu = 0.09$, $\sigma_e = 1.30$, $\sigma_e = 1$, $Pr = 1$. 
2.5. LOCAL ROUGHNESS LENGTH

The local roughness reflects the local dynamical properties of the instantaneous water surface which, if not actively involved in breaking, is a smooth surface. Except at low wind speeds breaking always occurs in the ocean. It is a highly nonlinear local process which disrupts the water surface. However, if the process is weak-in-the-mean, as in the formalism used in the description of the sea waves (Hasselmann, 1968; Komen et al., 1994), the water surface can be regarded as a smooth one. Notice, that we distinguish between the ‘instantaneous’ water surface and the sea surface. The latter, due to the presence of waves, is a rough surface.

In Makin et al. (1995) it was assumed that the water surface can be treated as smooth. The dynamical and temperature local roughness lengths are thus related to the scale of the molecular sublayer:

\[
\begin{align*}
z_0' &= \frac{\nu}{u_*}, \\
z_\text{th}' &= \frac{\nu}{u_*^w} \tag{32}
\end{align*}
\]

where \( \nu = 1.4 \cdot 10^{-5} \text{ m}^2 \text{ s}^{-1} \) is the kinematic viscosity of the air. The local friction velocity \( u_*' \) is:

\[
u_*' = u_* \sqrt{1 - \frac{\tau_0'}{u_*^w}} \tag{33}
\]

The surface turbulent stress \( \tau_0' = (u_*')^2 \) is only a part of the total stress \( \tau = u_*^2 \), and the form drag dominates the surface momentum flux for wind speeds above 6 m s\(^{-1}\) (Janssen, 1989; Makin et al., 1995).

For the dynamical roughness, \( z_0' = 0.1 \) is adopted (e.g. Monin and Yaglom, 1971).

Kader and Yaglom (1972) suggested that

\[
c_0' = \exp(-\frac{F(\text{Pr})\kappa}{\text{Pr}_t}) \tag{34}
\]

where \( F \) is a function of the Prandtl number \( \text{Pr} = \nu/\chi_H \) (\( \chi_H \) is the molecular diffusivity of heat):

\[
F(\text{Pr}) = 12.5\text{Pr}^{2/3} + 2.12 \ln \text{Pr} - 5.3 \tag{35}
\]

In the air \( \text{Pr} \approx 0.70 \) and given \( \text{Pr}_t = 1 \) results in \( c_0' = 0.21 \). Liu et al. (1979) recommend a value of \( c_0' = 0.18 \). Results for both mentioned values of \( c_0' \) differ by no more than 1%, with the tendency for an increase in the exchange coefficient with increasing \( c_0' \).

Note, that the local roughness lengths follow from the solution of the problem, as local friction velocity (33) depends on the form drag \( \tau_0' \) and friction velocity \( u_*' \).
2.6. BOUNDARY CONDITIONS AND METHOD OF SOLUTION

Each of the balance Equations (7), (14), (15), (23) requires two boundary conditions. The upper boundary condition at the top of the WBL $h = 10$ m is:

$$
z = h : u = U_{10}, \quad \hat{T} = 1, \quad \frac{\partial e}{\partial z} = 0, \quad \frac{\partial \varepsilon z}{\partial z} = 0, \quad (36)
$$

where for convenience temperature is scaled by the difference $\Delta T = T_h - T_0$, $\hat{T} = (T - T_0)/\Delta T$.

The surface boundary condition is:

$$
z = z_0 : u = 0, \quad \hat{T} = 0, \quad \frac{\partial e}{\partial z} = 0, \quad \frac{\partial \varepsilon z}{\partial z} = 0. \quad (37)
$$

A relaxation method (Press et al., 1992, pg.753) is used to solve the balance Equations (7), (14), (15), (23) with boundary conditions (36) and (37).

The turbulent fluxes are calculated from (27) - (30). The wave-induced flux in the balance equation for momentum is calculated due to (11). For that a wave spectrum $S(k, \theta)$ and the growth rate parameter $\beta(k, \theta)$ need to be defined.

We use the same model for the wave spectrum $S(k, \theta)$ as was adopted in Makin et al. (1995). The model is based on Donelan et al. (1985) and Donelan and Pierson (1987). The wave spectrum consists of two parts: the energy containing part (Donelan et al. 1985) and the tail (Donelan and Pierson, 1987), which is patched to the former. The energy containing part of the spectrum is proportional to the inverse wave-age parameter $u_{10}/C_p$ to the power of 0.55 ($C_p$ is the phase velocity of the wave component in the peak of the spectrum). The fact that the tail of the adopted spectrum has no dependence on wave age results in the sea drag virtually independent of wave age (Makin et al., 1995). For the present calculations we use $u_{10}/C_p = 0.83$.

Experiments (Plant, 1982), recent analytical theory of wave growth (Belcher and Hunt, 1993), and numerical calculations based on the second-order turbulent stress closure scheme (Mastenbroek et al., 1996) show that the growth rate parameter for waves travelling slower than the wind is roughly proportional to $(u_*/c)^2$.

Therefore we adopt the following relation for the growth rate parameter $\beta(k, \theta)$ specifying the energy flux to the k-component of the wave field in (11):

$$
\beta = 16 \left( \frac{u_* r}{c} \right)^2 \cos^2 \theta \quad (38)
$$

where $u_*$ is the friction velocity at the top of the inner region $r(k) = 1/(5k)$. For waves travelling with the speed of the wind, parameterization (38) overpredicts the growth rate (Mastenbroek et al., 1996). Mastenbroek et al. (1996) have shown that for fast waves the energy flux is supported to a large extent by the work of tangential stress on the orbital velocity and is negative, meaning that waves lose
their energy to the atmosphere. We therefore take, for waves travelling with the
phase speed \( c > 28u_\ast \cos \theta \),

\[
\beta = -4 \frac{u_{2r} u_{2r}}{c} \cos^2 \theta,
\]

(39)

where \( u_r \) is the wind velocity at the top of the inner region.

The production term \( P \) in the balance Equations (14) and (15) is calculated from (16), (18) - (21). The momentum and sensible heat flux, as well as the drag coefficient \( C_D \) and the Stanton number \( C_H \), follow from the model.

3. Results

The impact of waves on the height dependence of the momentum flux is shown in Figure 1 for a wind speed of 15 m s\(^{-1}\). The total stress \( \tau = u^2_z \) is constant with height (as stress is normalized on \( u^2_z \) it equals 1). The total stress is supported by the wave-induced stress \( \tau^w \) and the turbulent stress \( \tau^t \). In this example, at the surface more than 60% of the stress \( u^2_z \) is supported by waves. The wave-induced flux decays rapidly with height, supporting only 20% of the total stress at 0.1 m height. Correspondingly the turbulent stress \( \tau^t \) increases rapidly with height. At the top of the WBL where the wave-induced stress is negligible the turbulent stress equals \( u^2_z \), a value which is almost three times larger than the surface value of \( \tau^t \).

The TKE, which is roughly proportional to the turbulent stress, is no longer constant with height as it would be in a logarithmic boundary layer. Figure 2 shows that in the layer of about 1 m above the waves the TKE experiences a 40% increase over its surface value. The dissipation rate, normalized by \( u^2_z/(\kappa z) \), is constant with height in the logarithmic boundary layer. In the WBL it increases to twice the surface value.

Such behaviour of the TKE and the dissipation rate induces a flux of TKE (Equation (28)) and changes the vertical distribution of the dissipation rate flux (Equation (29)), Figure 3. (In the logarithmic boundary layer the former equals zero and the latter, scaled with \( u^2_z/(\kappa z) \), equals 1). Our calculations show that the transport of TKE (the first term in Equation (14)) and the transport of mean wave-induced energy (the first term on the right-hand side of Equation (20) which enters the balance equation of the TKE (14) via the production term), and contribute to the balance no more than 10%, are self-cancelling and can be ignored in the balance equation of the TKE. This is in agreement with estimates of Chalikov and Belevich (1993).

However, there is a drastic difference in the balance of \( P \) and \( \epsilon \) compared to the logarithmic boundary layer. Near the surface a considerable part of the TKE production is due to the dissipation of the wave-induced motion \( P^w = D = \tau^w \partial u/\partial z \). (Note, that according to Equations (16), (18), (19) and \( P^w = D \) Figure 1 gives the balance \( P^t/P + P^w/P = 1 \)). That changes considerably the balance
of the dissipation rate, Equation (15). The transport of $\varepsilon$ (the first term in (15) which, being scaled with $u_*^4/(\sigma_e z^2)$, equals 1 in the logarithmic boundary layer), is reduced more than twice near the surface (Figure 3). The changes in the balance equations of the TKE and the dissipation rate due to the presence of wave-induced stress change the turbulence diffusivity near the surface.

In the mixing length model the eddy viscosity $K$ is derived from the assumption that the dissipation rate $\varepsilon = K^2/l^4$ ($l$ is the mixing length) equals the TKE production due to the interaction of the turbulent stress and mean velocity gradient $P^t = \tau^t \partial u/\partial z = K(\partial u/\partial z)^2$. An equation for $K$ then follows: $K = l^2|\partial u/\partial z|$. Such calculation of the eddy viscosity above waves leads to an underestimation of $K$ near the surface. The reason is that the production of the TKE due to the dissipation of the mean motion is only a part (which is small for high wind speed) of the total production of the TKE above waves.

The distribution of the eddy viscosity is shown in Figure 4. The eddy viscosity which follows from the mixing length theory is underestimated compared to the $e - e$ model. The details of the $K$ distribution are important as the heat flux is determined by the turbulence diffusivity above waves. The mixing length $l$ is
shown in Figure 5. It is calculated from $l = (ec_\mu^{1/2})^{1/2}/\epsilon$. In the layer of about 1 m above waves the mixing length exceeds its equilibrium value $l = \kappa z$.

Just above the surface the turbulence diffusivity is enhanced due to the presence of waves. That enhances the turbulent mixing of sensible heat above waves and the exchange coefficient of heat.

The dependence of the Stanton number on wind speed is assessed in the following way. The open ocean data set of recent simultaneous measurements of wind stress and sensible heat flux (Anderson, 1993) have been chosen. Wind stresses and sensible heat fluxes in the experiment were determined by the inertial-dissipation method.

Anderson reports the drag coefficient and the Stanton number in the wind speed range of 5-20 m s$^{-1}$. His data show a clear 100% increase of the drag coefficient with wind. The heat flux coefficient is much less dependent on wind speed. However within the scatter his data do not resolve whether $C_H$ or the temperature roughness length $z_{0t}$ is independent of wind speed. If the latter is true the Stanton number would increase by more than 30% in the given wind speed range and for the reported drag coefficients.
Figure 3. Flux of the TKE \(- (e' + p') v / (\kappa / u_1^2)\) – solid; flux of the dissipation rate \(\bar{e} w' / (\sigma_e z / u_1^2)\) – dashed; transport of the dissipation rate \(- \partial \bar{e} w' / \partial z (\sigma_e z^2 / u_1^4)\) – dashed-dotted.

We need to be able to reproduce the experimental dependence of the drag coefficient \(C_D\) on wind speed, which is more or less well established. Comparison of model results and data is shown in Figure 6. The regression line of Anderson is for all data in the range \(4.5 < u_{10} < 21 \text{ ms}^{-1}\): \(10^2 C_D = 0.40 + 0.079 u_{10}\). For data in the range of wind speed \(u_{10} > 10 \text{ ms}^{-1}\) his regression line is \(10^2 C_D = 0.59 + 0.065 u_{10}\), which is in close agreement with Large and Pond’s (1982) regression: \(10^2 C_D = 0.49 + 0.065 u_{10}\). However in the range of \(4 < u_{10} < 10 \text{ ms}^{-1}\) Large and Pond approximate their data with the constant value \(10^2 C_D = 1.14\) (standard deviation 0.20). For wind speeds higher than \(10 \text{ ms}^{-1}\) the agreement of model results with data is reasonable. In the range of \(5–10 \text{ ms}^{-1}\) the model gives somewhat higher values of the drag coefficient. The model results in this range correspond better with the measurements of Large and Pond (1982), obtained with the same method as Anderson. Taking into account that conditions of low wind speed introduce more uncertainty into measurements and that the model results do not contradict the other data sets (Large and Pond, 1982) we conclude the overall reasonable agreement of model results with data. The overall increase of the drag coefficient in the wind speed range 5-20 \text{ ms}^{-1} is 80\%. 
The simultaneously calculated values of the Stanton number should then fall in the cloud of experimental values of $C_H$. We then argue that the model results establish the dependence of the Stanton number on wind speed. Modelled and measured values of $C_H$ are shown in Figure 7. Considering the scatter of data it is not too surprising that the model results are well within the cloud of data.

In the range of wind speed 3–10 ms$^{-1}$ we obtain a Stanton number which is virtually independent of wind speed $C_H = 1.0 \cdot 10^{-3}$. It increases by about 10% in the range of 10–20 ms$^{-1}$. The increase is smaller than that following from the ‘constant roughness lengths’ parameterization. Assuming $C_H = 0.029C_D^{1/2}$ which gives $C_H = 10^{-3}$ for a wind speed of 5 ms$^{-1}$ we obtain an overall increase in $C_H$ of 30% in the range of wind speed 5–20 ms$^{-1}$. The increase is three times larger than the model results in the same range of wind speed and the difference increases for higher wind speeds.

Thus our theory agrees better with the ‘constant Stanton number’ parameterization than the ‘constant roughness lengths’ parameterization of sensible heat flux.
Most of the experimental data sets are fitted to constant Stanton number. Pond et al. (1971), Friese and Schmitt (1976), Smith (1980, 1988), Large and Pond (1982), DeCosmo (1991) and DeCosmo et al. (1996) all give constant values for $C_H$ between $0.9 \times 10^{-3}$ and $1.1 \times 10^{-3}$. (These values refer to the overall mean including coefficients for stable and unstable conditions.)

Using a mixing length scheme to calculate the eddy viscosity results in too low values of the exchange coefficient for high wind speeds with a trend to decrease with the increase of wind speed. This is due to the fact that a mixing length scheme underestimates the turbulent diffusivity near the wave surface.

In the literature it is argued that for wind speed higher than 15 ms$^{-1}$ effects of the sea spray evaporation become important and increase the exchange coefficients for heat and moisture (Andreas et al., 1995; Fairall et al., 1994). The model results of Fairall et al. (1994) show that the contribution of spray to air-sea fluxes becomes comparable to the direct fluxes at wind speeds about 35 ms$^{-1}$ for sensible heat. Andreas et al. (1995) estimate the spray contribution to the heat fluxes to be about 10% at 15 ms$^{-1}$ wind speed. However, experimental evidence for such an increase is lacking, and few data are available at high wind speeds. The effect of sea spray is not accounted for by the present model.
4. Conclusions

The different wind-speed dependences of the drag coefficient and the exchange coefficient for sensible heat (the Stanton number) can be explained by the difference in exchange mechanisms of momentum and heat at the sea surface. Momentum to a large extent is transported by the organised wave-induced motions correlated with the waves (the form drag). Heat is transported only by molecular processes.

The form drag dominates the surface stress and determines the vertical structure of turbulence in the WBL. The sensible heat flux above waves is determined by the turbulence diffusivity which is affected by waves.

To describe the structure of the WBL, a two-equation eddy viscosity model rather than a mixing length model is appropriate. The production of TKE due to dissipation of the wave-induced motion, and the change in the diffusive transport of the dissipation rate due to the presence of wave-induced stress, both affect the turbulence diffusivity near the sea surface. The former model accounts for these processes.
The exchange coefficients for sensible heat and momentum can be computed from the wind speed and the sea state using an approach which is based on the conservation of momentum and heat in the turbulent boundary layer above sea waves.

We find that for winds between 3–10 ms$^{-1}$ the Stanton number is virtually independent of wind speed. It increases by about 10% in the range of wind speed 10–20 ms$^{-1}$. The increase is smaller than that following from the ‘constant roughness lengths’ parameterization. Our theory favours the ‘constant Stanton number’ parametrization.

**Acknowledgments**

Our acknowledgements are for G. Burgers, G. Komen and J. Onvlee for helpful comments.
References


