ADM-Aeolus, Ocean Albedo

Technical note on Ocean Albedo as a function of local wind and wave conditions and its effect on the zero-wind calibration. Name code: AE-TN-KNMI-L1B-001 Authors: **Jos de Kloe**, Ad Stoffelen, KNMI

> issue number: 1.0 issue date: 15-April-2008



Change log

Version	Date	Comment
0.1	07-Nov-2007	A first try.
0.2	12-Dec-2007	full description of the reflectivity and wave model
0.3	08-Jan-2008	added calculation results for LOS surface velocities
0.4	11-Jan-2008	incorporated internal KNMI comments and comments from O. Le Rille
1.0	15-Apr-2008	added stokes drift results and upwind/downwind effect

Contents

Cl	hange log	2									
Tε	Table of contents 5										
\mathbf{Li}	List of figures 4										
\mathbf{Li}	st of tables	4									
1	Introduction	5									
2	Documents and acronyms 2.1 Applicable documents 2.2 Reference documents 2.3 Literature 2.4 acronyms 2.5 document preparation	6 6 6 7 7									
3	Lidar equation 3.1 Atmospheric scattering 3.2 Ground backscatter	7 7 8									
4	Reflection4.1Specular reflection4.2Diffusive reflection4.3Sub-surface scattering4.4Albedo4.5White-cap occurrence4.6The Menzies and Tratt model	8 8 10 10 10 11 11									
5	Wave-shape5.1Trochoid definition5.2Stokes drift5.3Hydrodynamic modulation5.4Calculation of LOS surface velocity5.5Numerical issues5.6Examples	 13 15 15 15 15 16 									
6	Connecting wave shape and reflectivity	16									
7	Results 7.1 Results specular reflection	21 21 22 23									
8	Conclusion	25									

List of Figures

1	White-cap coverage as a function of wind speed at 10 m above the sea surface, according to	
0	the expression by Monahan and O'Muircheartaigh.	11
2	Lidar backscatter against incidence angle calculated using the model by Menzies et al	13
3 4	Example of a wave share composed a single trachoid wave with wave length $I = 6$ [m] and	14
4	amplitude $A=0.335$ [m] (without stokes drift) The blue line gives the wave surface. The	
	areen line gives the wave slope s. The black arrows indicate the surface movement of the water.	16
5	Example of a wave-shape composed of two superimposed trochoid waves. The first one has a wave-length $L=6$ m, and amplitude $A=0.30$ [m] and the second one has a wave-length $L=1$ [m], and amplitude $A=0.050$ [m]. The blue line gives the wave surface. The green line gives	10
	the wave slope s. The black arrows indicate the surface movement of the water	17
6	Example of a wave-shape composed of 3 superimposed trochoid waves. The first one has wave- length $L=6$ m, and amplitude $A=0.269$ [m], the second one has wave-length $L=1$ [m], and amplitude $A=0.045$ [m], and the third one has wave-length 0.05 [m] and amplitude 0.0045 [m]. The blue wave gives the wave surface. The green line gives the wave slope s. The black	
7	arrows indicate the surface movement of the water	17
	<i>m.</i>	19
8	Histogram of slopes for three trochoid waves superimposed. Wavelengths are $6 m$, $1.01m$ and	
	0.102 m.	20
9	LOS surface velocity against surface slopes for three trochoid waves superimposed. Wave-	
10	lengths are $b [m]$, $1.01 [m]$ and $0.102 [m]$, for nadir geometry	20
10	LOS surface velocity against surface slopes for three trochold waves superimposed. Wave- lengths are 6 $[m]$, 1.01 $[m]$ and 0.102 $[m]$, for off-nadir geometry	21
List	of Tables	
$\frac{1}{2}$	slope variance for different wind speeds	12
3	geometry (case without Stokes drift)	18
	trom comparing the LOS surface velocity results to the slope values, for nadir and off-nadir	10
4	geometry, when Stokes drift is taken into account.	19
4 5	water surface LOS velocities for specular reflection only when Stokes drift is taken into account	$\frac{44}{22}$
6	water surface LOS velocities for specular reflection only, when stokes unit is taken into account.	
0	off-nadir looking geometry (Stokes drift not taken into account).	23

7	water surface LOS velocities for combined specular and uniform diffusive reflection, for the	
	off-nadir looking geometry, taking Stokes drift into account.	24
8	water surface LOS velocities for combined specular and uniform diffusive reflection, for the	
	nadir looking geometry (Stokes drift not taken into account	24
9	water surface LOS velocities for combined specular and non-uniform diffusive reflection, for	
	the off-nadir looking geometry.	25
10	water surface LOS velocities for combined specular and non-uniform diffusive reflection, for	
	the nadir looking geometry.	25

1 Introduction

This document describes the interaction between the ADM-Aeolus zero-wind-calibration measurements and the ocean surface, and gives some details on the effect of the wave shape, and reflectivity distribution, on the expected albedo and detected net LOS water surface movement.

In [AD1] [AI4] this task is described as: "Investigate albedo above sea as function of wind speed for nadir and 37 degree; if dependence, use albedo to set threshold to select low wind speed scenes for calibration, provide a TN quantifying bias effects in GE measurements for variable wind velocities, directions and nadir/off-nadir geometries". This resulted in the current technical note. Questions to be answered are:

- does water surface movement add up to a net LOS velocity when observed at nadir and off-nadir geometry.
- does reflectivity/albedo depend from local wind speed, and if so, how.
- can observed reflectivity be used to discriminate between cases with and without a significant whitecap fraction by setting a threshold value

Upto now it has been assumed within the ADM-Aeolus project that the typical vertical movement of the ocean surface is only a modest 16 cm/s, which will average to zero when enough measurements (for example for zero-wind-calibration) are taken. Thus it was argued this vertical movement will only contribute in the noise properties of the observed quantity, and maybe neglected when taking enough observations in this average.

This 16 cm/s was based on some statistical ocean properties. It was assumed the average largest wave has a wave length λ of 220 m, a wave height h of 2.3 m, and a wave speed v of 16 m/s. The average vertical wave velocity v_z is then $v_z = \frac{h}{T} = \frac{hv}{\lambda} = 0.167$ m/s, with T the wave period. The projected velocity on the LOS will be $v_{LOS} = v_z \cos(\alpha) = 0.132$ m/s¹ for $\alpha = 37.5^{\circ}$ And the projected HLOS velocity will be $v_{HLOS} = v_{LOS}/\sin(\alpha) = 0.217$ m/s.

There are several problems with this assumption, and we suspected it may be significantly too low. Factors to be studied are:

- Waves at smaller wavelengths may contribute more to the vertical surface velocity than the largest waves present even when the amplitude of these waves is less.
- Off-nadir lidar measurements sample mostly one side of the waves, so will see either mostly upward or mostly downward movement. On a BRC level the surface conditions are generally rather uniform and a net effect may result; even on a global scale the effects will not cancel when averaging meridionally or by orbit phase, e.g. due to the trades.
- Reflectivity due to white-caps (foam) on the wave crests is not distributed in an uniform way (what source can I refer to for this statement?). This will lead to a correlation between the reflectivity and the wave-phase and thus to a net observed vertical movement, even after averaging. This is important for both nadir and off-nadir measurements.
- The horizontal component of water motion is most relevant for the off-nadir calibration. This component is significant on both BRC and regionally-averaged level, e.g. in the trade regions.
- Due to hydrodynamic modulation of small waves by larger waves and the wind interaction, the distribution of specular slopes depends on the phase of the larger waves.

The approach taken in this technical note to estimate these wave properties is as follows:

- First we take the directional reflectance model as published by Menzies et al.[LR5]. This gives reflectance (or albedo) as a function of wind speed and incidence angle. (see section 4).
- Then we model the wave shape to be able to assign a water surface velocity to the reflective facets, used by the Menzies et al. model. This gives an estimate of the water surface LOS velocity as a function of water surface slope. (see section 5)

¹not $v_{LOS} = v_z sin(\alpha) = 0.102$ as said in [RD1] (which number was also rounded down to 0.09 by skipping the 3rd digit in the 0.167 m/s).

- This is used to combine the water velocity movement and the reflectivity model to estimate the expected LOS movement and variability as will be seen my the ADM-Aeolus instrument, for different incident angles and wind speeds (see section 6).
- Finally the resulting LOS velocities are calculated for the different cases in section 7.
- The conclusions will be summarized in section 8.

2 Documents and acronyms

2.1 Applicable documents

[AD1] "Minutes of Progress Meeting #11, Aeolus Level 1B/2A Processor Refinement and Pre-Launch Validation", ESA/ESRIN, Sept. 18, 2007, AE-MN-DLR-L12-013.

2.2 Reference documents

- [RD1] "Answer to actions SYS-23 & SYS-25 (IDDR rid SYS030) Impact of altitude variation on unknown bias" AE.TN.ASF.AL.0294, version 01.00, issued 20050415.
- [RD2] "Etablishment of a backscatter coefficient and atmospheric database" by M. Vaughan et al, ESA contract 12510/97/NL/RE, June 1998.

2.3 Literature

The second

- [LR1] "Classical Electronynamics", international Student Edition, by Hans C. Ohanian, published by Allyn and Bacon, Inc., 1988, ISBN 0-205-11303-6.
- [LR2] web site: http://www.kayelaby.npl.co.uk/general_physics/2_5/2_5_7.html, visited: 20071108.
- [LR3] web site: http://en.wikipedia.org/wiki/Water_vapor, visited: 20071108. For a more extensive discussion and literature references see: http://cires.colorado.edu/voemel/vp.html.
- [LR4] "Models for the wavelength dependence of the index of refraction of water", by: Paul D. T. Huibers, Applied Optics, Vol. 36, No. 16, 1 june 1997.
- [LR5] "Lidar In-Space Technology Experiment (LITE) measurements of sea surface directional reflectance and the link to surface wind speed", by: Robert T. Menzies, David. M. Tratt and William H. Hunt, Applied Optics, Vol. 37, No. 24, 20-Aug-1998.
- [LR6] "Wind-Driven dependence of sea-surface reflectance measured with an airborne Doppler-Lidar", by: David M. Tratt and Robert T. Menzies, International Laser Radar Conference, 6-Jul-1998, Annapolis, Maryland, USA.
- [LR7] "Zeegolven", by: Dr. P. Groen and Dr. R. Dorrestein, opstellen op oceanografisch en maritiem meteorologisch gebied, No. 11, derde, herziene druk, 1976, KNMI.
- [LR8] "Effective reflectance of oceanic white caps", by: Peter Koepke, Applied Optics, Vol.23, No.11, 1 June 1984.
- [LR9] "Optimal Power-Law description of oceanic whitecap coverage dependence on wind speed", by: E.C. Monahan and I. Ó Muircheartaigh, J.Phys.Ocean. 10, 2094 (1980).
- [LR10] "Airborne Doppler lidar investigation of the wind-modulated sea-surface angular retro reflectance signature", by: David M. Tratt, Robert T. Menzies, Meng P. Chiao, Dean R. Cutten, Jeffry Rothermel, R. Michael Hardesty, James N. Howell, and Stephen L. Durden, Applied Optics, Vol.41, No.33, 20 Nov.2002.

- [LR11] "Measurement of the Roughness of the Sea Surface from Photographs of the Sun's Glitter", by: Charles Cox and Water Munk, Journal of the Optical Society of America, Vol. 44, Nr. 11, Nov. 1954, p.838-850.
- [LR12] "Stokes drift", article in wikipedia, see: http://en.wikipedia.org/wiki/Stokes_drift, visited 15-Apr-2008, which has a nice reference to an online version of the original article by Stokes, published in a book from 1880.

2.4 acronyms

Applicable Document (see section 2.1)
Atmospheric Dynamics Mission
Basic Repeat Cycle (covering a 200 km orbit section)
Digital Elevation Model
End-to-End Simulator
European Space Agency
Fabri-Perot (spectrometer)
Fizeau (spectrometer)
Horizontal projection of the Line-Of-Sight wind component
Koninklijk Nederlands Meteorologisch Instituut
(Royal Dutch Meteorological Institute)
Level $1B/2A/2B$ processor
Line-Of-Sight (direction)
Literature Reference (see section 2.3)
Measurement Error and Correlation Impact on ADM (ESA project)
Numerical Weather Prediction
Reference Document (see section 2.2)
period of the high frequency clock $(1./48e6 \text{ s})$

2.5 document preparation

This document was written using the IAT_EX typesetting system. All graphics have been produced by gnuplot, using an interface between the Fortran90 code used for the calculations and gnuplot.

3 Lidar equation

3.1 Atmospheric scattering

The LIDAR equation for backscatter power signal (leaving out the wavelength dependence of the back scatter, laser energy and transmission) is defined as²:

$$S_{m,r}(z) = \frac{E_L \lambda}{hc} \Delta R A_{tel} \beta_{a,m}(z) \frac{\tau(z)}{R(z)^2}$$
(1)

in which:

- $S_{m,r}(z)$ is the number of photon counts per range bin as seen by each detector due to molecular or aerosol backscatter (ignoring cross-talk effects for simplicity).
- E_L is the laser pulse energy in [J].
- $\lambda = 355 \times 10^{-9}$ [m] the wavelength emitted by the laser
- $h = 6.626 \times 10^{-34}$ [Js] Planck's constant
- c = 299792458 [m/s] the speed of light in vacuum

 $^{^2\}mathrm{Thanks}$ to Olivier Le Rille for suggesting this formulation.

- ΔR is the LOS range used to collect backscatter signal.
- A_{tel} is the telescope opening area = $\frac{\pi}{4}D^2 = 1.767 \text{ [m}^2 \text{] for D} = 1.5 \text{ [m]}.$
- $\beta_{a,m}$ is the aerosol (a) and molecular (m) backscatter in [1/(Sr.m)].
- z is the scattering altitude.
- τ is the combined two-way atmospheric transmission, and the optical transmission of the transmitter and receiver.
- R(z) is the range between the satellite and the scattering volume along the laser Line-Of-Sight. For nadir viewing geometry this equals to $z_{sat} - z$, with z the altitude of the scattering volume, and z_{sat} the satellite orbit altitude of on average 408 [km]. For off-nadir viewing geometry with earth's incidence angle $\alpha = 37.5$ degrees the range is about 1.25 times larger.

3.2 Ground backscatter

When dealing with ground return, the lidar equation 1 is rewritten as:

$$S_{m,r}(ground) = \frac{E_L \lambda}{hc} A_{tel} \frac{\rho_g}{\pi} \frac{\tau}{R^2}$$
(2)

in which:

- $S_{m,r}(ground)$ is the number of photon counts for the ground reflection as seen by each detector. (ignoring cross-talk effects for simplicity).
- ρ_g is the ground albedo (surface diffuse reflectivity)
- R is the range between the satellite and the ground along the laser Line-Of-Sight. For nadir viewing geometry this equals to the satellite orbit altitude z_{sat} of on average 408 [km]. For off-nadir viewing geometry with incidence angle $\alpha = 37.5$ degrees the range is about 1.25 times larger and becomes $R \approx 520$ [km].

The equivalent backscatter coefficient for the ground echo is needed to compare the ground albedo to the atmospheric backscatter values, without explicitly calculating the signal levels. It is defined as:

$$\beta_g = \frac{2}{cT_L} \frac{\rho_g}{\pi} = \frac{2}{\Delta R} \frac{\rho_g}{\pi} \tag{3}$$

4 Reflection

This section recapitulates some basic physics properties for reflection on surfaces, needed by the rest of this technical note.

4.1 Specular reflection

Reflection of electromagnetic radiation on a flat surface between two media is described by the so called Fresnel equations (see for example [LR1] chapter 12.5, p.390, eq. 135³).

The reflectivity for perpendicular incidence angle is given by:

$$\rho_s = \frac{(n'-n)^2}{(n'+n)^2} \tag{4}$$

with n the index of refraction of the medium in which the light ray originates (air in our case) and n' the index of refraction of the medium on which the reflection takes place (sea water in our case).

 $^{^{3}}$ Note that this reference gives the reflection in terms of the amplitude of the electric field. In order to have the reflection of the energy or intensity, the number has to be squared.

A parameterization for the refractive index for air is for example given in [LR2], and is dependent on air pressure, temperature en the partial water vapor pressure:

$$n_{tpf} = n_{tp} - f(3.7345 - 0.0401\sigma^2)10^{-10}$$
(5)

$$n_{tp} = 1 + (n_s - 1) \times \frac{p[1 + p(60.1 - 0.972T)10^{-10}]}{0.005 + 42(1 + 0.002661T)}$$
(6)

$$96095.43(1+0.003661T)$$

$$n_s - 1 = 0.0472326(173.3 - \sigma^2) - 1 \tag{7}$$

in which:

- n_{tpf} is the refractive index for air as function of temperature, pressure and humidity
- n_{tp} is the refractive index for dry air for a given temperature and pressure.
- n_s is the refractive index for standard air (at 15 °C, and 101325 Pa, containing 0.045% by volume of carbon dioxide).
- $\sigma = 1./\lambda$, with $\lambda = 0.355 \ [\mu m]$.
- *p* the air pressure in [Pa].
- T the air temperature in $[^{\circ}C]$.
- $f = RH \times p_{wv}$ the partial water vapor pressure in [Pa].

It is stated that this parameterization is only valid between wavelengths of 405 and 644 nm with a maximum discrepancy of only 1.4×10^{-8} , but I assume this may be extended to 355 nm without making a very large error.

A parameterization for the maximum partial water vapor pressure p_{wv} as function of temperature, known as the Goff-Gratch equation, is given in [LR3]:

$$\log_{10} (p_{wv}) = -7.90298 \left(\frac{373.16}{T} - 1\right) + 5.02808 \log_{10} \left(\frac{373.16}{T}\right) -1.3816.10^{-7} \left(10^{11.344(1 - \frac{T}{373.16})} - 1\right) +8.1328.10^{-3} \left(10^{-3.49149(\frac{373.16}{T} - 1)} - 1\right) + \log_{10} (1013.246)$$
(8)

With the temperature of the moist air T in [K], and its the partial water vapor pressure p_{wv} in [hPa].

After inserting reasonable minimum and maximum values for temperature T, pressure p and relative humidity RH:

- $T_{air,min} = -10 \ [^{\circ}C] = 263.16 \ [^{\circ}K]$
- $T_{air,max} = 40 \ [^{\circ}C] = 313.16 \ [^{\circ}K]$
- $p_{air,min} = 940 \ [hPa]$
- $p_{air,max} = 1040 \ [hPa]$
- $RH_{min} = 0.5$
- $RH_{max} = 1.0$

the range of refractive indices for air is found to be:

- $n_{air,min} = 1.00025037 \pm 1.4e 8$
- $n_{air,max} = 1.00032105 \pm 1.4e 8$

So for the refractive index of air I will use: $n_{air} = 1.000286 \pm 0.000004$

The refractive index of seawater is a function of wavelength and salinity and to a lesser extent of pressure and temperature. A simple parameterization for seawater, only dependent on wavelength, is given in [LR4]:

$$n_w(\lambda) = 1.31279 + \frac{15.762}{\lambda} - \frac{4382}{\lambda^2} + \frac{1.1455 \times 10^6}{\lambda^3} \tag{9}$$

With the wavelength λ given in [nm]. This model assumes no salinity, and a water temperature of 25 °C. For 355 [nm] this gives a refractive index of $n = 1.348 \pm 0.001$.

This leads to a specular reflection of: $\rho_s = 0.0219 \pm 0.0001$ on a flat seawater surface for perpendicular irradiation.

4.2 Diffusive reflection

For a Lambertian surface, which appears equally bright in al directions when illuminated, the situation is completely different, even if the reflectivity is similar.

In [LR8] the reflection of foam in white caps and wind streaks is discussed in detail. Here it is stated that fresh foam has a reflectance close to 55 %, based on theoretical models assuming several layers of stacked bubbles, and confirmed by laboratory measurements. However, this paper makes clear that as foam grows older the reflectance decreases. Besides white-caps, also wind streaks exist, which have much smaller reflectance, in the order of 10 %, and which can be modeled by a single layer of bubbles. Combining the two types of foam, and the aging effect, an average reflectivity of 22 % is found, which has almost no wind dependency, and for wavelengths below 700 nm also no wavelength dependency.

This reflectivity R_f of 22 % is close, and in the right order of magnitude to the albedo of 0.14 used within this project [RD2]. (assuming reflectivity and albedo are the same thing)

When the geometry is off-nadir the situation changes slightly. The reflectance can now be written as (for $\alpha = 37.5^{\circ}$) (see [LR10]):

$$R_{wc} = \frac{R_f \cos(\alpha)}{\pi} = 0.253 R_f \tag{10}$$

This is due to the definition of albedo as based on Lambertian scattering.

4.3 Sub-surface scattering

Scattering from below the surface (underlight) can be assumed to be isotropic [LR8], and cannot be neglected for short wavelengths (below 700 nm). This would add an additional wind speed independant term in equation 21 (see section 4.6).

It gives a constant background level which becomes noticable for larger incidence angles (above 15 degrees, see [LR5]). The model used by Menzies and Trat, described below (see section 4.6) does not take this term into account, since it is not possible to distinghuish it from the instrument noise level.

In the simulations described in section 7.3 below a constant het been added in equation 33 to simulate this effect. However, we have currently no example data available for this effect so the magnitude assumed here is just a guess.

4.4 Albedo

Surface albedo ρ_g is defined as the ratio of reflected to incident radiation. It is the combined effect of specular, diffusive and subsurface reflections. We use the definition given by equation 2, so rewritten this is:

$$\rho_g = \frac{P_g \pi R^2}{P_L A_{tel} \tau} = \frac{P_g}{P_L} \frac{1}{4\sigma_{tel} \tau}$$
(11)

[I am not sure about this factor of 4, is this correct?] [should there not be a $\cos(\theta)$ term in this equation to convert albedo to backscatter, as is done in for example eq. 10?]

so it is the fraction of the laser power reaching the detector after reflecting on the surface, normalized by the solid angle of the telescope $\sigma_{tel} = \frac{A_{tel}}{4\pi B^2}$ and corrected for all transmission losses.

4.5 White-cap occurrence

The model by menzies et al. [LR5], discussed in more detail in section 4.6, uses a parameterization for white-cap coverage mentioned by Koepke [LR8], who used an expression published by Monahan and O'Muircheartaigh [LR9]. The whitecap coverage W as a function of wind speed U_{10} at 10 meter above sea level, is given as follows:

$$W = 2.95 \times 10^{-6} (U_{10})^{3.52} \tag{12}$$

and has been validated for wind speeds between 4 and 25 [m/s]. This expression is plotted in figure 1.



Figure 1: White-cap coverage as a function of wind speed at 10 m above the sea surface, according to the expression by Monahan and O'Muircheartaigh.

From this it is clear that:

- White-caps are absent below wind speeds of 5 [m/s].
- White-cap coverage grows from 0.1 % to 1 % in the wind speed range between 5 and 10 [m/s].
- It grows further to 10 % for a wind speed of around 20 [m/s], and to 25 % for a wind speed of around 25 [m/s].
- above 38 [m/s] this parametrisation grows to more than 100 %, so then it clearly is not realistic any more. For simulation purposes the coverage is set to a fixed value of 100 % when this happens.

4.6 The Menzies and Tratt model

The model by menzies et al. [LR5] combines specular and white-cap reflection. It is based on the idea that specular reflection occurs on facets for which the orientation (slope) can be parameterized as a function of wind speed.

It is assumed that the slope distribution is a Gaussian function, with a width depending on the local wind speed. This results in the following parametrisation:

$$\langle s^2 \rangle = 3.16 \times 10^{-3} U_{10} \tag{13}$$

4 REFLECTION

This formulation was originally proposed by Cox and Munk[LR11], and was experimentally tested on photographs of the sun's glitter on the ocean surface, taken from from a plane, for wind speeds between 1 and 14 m/s.

Note that the original formulation by Cox and Munk was for a wind speed at 41 feet (12.5 m) above sea level. The difference between the wind at both levels is very small though. Assuming neutral conditions and zero surface wind, the wind at 10 m is only about 2% smaller than the wind at 12.5 m. Therefore this effect is neglected for now, and U_{10} is used in stead.

Note here that in a later article by Tratt and Menzies [LR10] a modified relation is used to describe different slope distributions for upwind/downwind and crosswind looking geometry.

$$\sigma_u^2 = 3.16 \times 10^{-3} U_{12.5} \qquad \text{for upwind/downwind} \qquad (14)$$

$$\sigma_c^2 = 0.003 + 1.92 \times 10^{-3} U_{12.5} \qquad \text{for crosswind} \qquad (15)$$

The wind invariant contribution for crosswind is attributed to ocean swells.

For the current study a constant addition of 0.001 (similar to the 0.003 attributed to ocean swells above) is adopted, because it prevents the numerical problem of the asymptotic behaveour for the backscatter when the wind speed approaches zero. This introduces a difference of only 4 % at 5 m/s and less for all larger wind speeds. So the equation actually used for the slope variance is:

$$\langle s^2 \rangle = 0.001 + 3.16 \times 10^{-3} U_{10} \tag{16}$$

For later use, some numbers of the slope variance are collected in table 1

wind speed at 10 m $[m/s]$	σ_{slope}	$\sigma_{slope} \times \sqrt{2}$
2	0.0856	0.121
4	0.117	0.165
6	0.141	0.200
8	0.162	0.229
10	0.181	0.255
15	0.220	0.311
20	0.253	0.358
25	0.283	0.400

Table 1: slope variance for different wind speeds

Then the lidar backscatter R_s [1/Sr] is calculated from this using:

$$R_s = \frac{\rho \sec^4(\theta)}{2\pi \langle s^2 \rangle} \exp\left(-\frac{\tan^2(\theta)}{\langle s^2 \rangle}\right) \tag{17}$$

In which ρ is the Fresnel reflectance.

For the directional parametrisation this changes to:

$$R(\theta, \phi) = \frac{\rho \sec^4(\theta)}{4\pi\sigma_u \sigma_c} \exp\left(-\frac{\tan^2(\theta)}{2\sigma^2(\phi)}\right)$$
(18)

with

$$\sigma^2(\phi) = \frac{\sigma_u^2 \sigma_c^2}{\sigma_c^2 \cos^2(\phi) + \sigma_u^2 \sin^2(\phi)}$$
(19)

in which ϕ is the azimuth direction of the local wind, relative to the viewing direction.

For white-cap coverage the parametrisation by Monahan and O'Muircheartaigh as defined in section 4.5 is used. For whitecap reflectivity ρ_{wc} , the value of 22% suggested by Koepke [LR8] is used. The white-cap reflectivity R_{wc} is then obtained from:

$$R_{wc} = \rho_{wc} \frac{\cos(\theta)}{\pi} \tag{20}$$

The whitecap reflection R_{wc} , specular reflection R_s and coverage W are combined into a single reflectivity as follows:

1

$$R_{combined} = R_{wc}W + R_s(1 - W) \tag{21}$$

Figure 2 gives the lidar backscatter resulting from the model by Menzies et al. for different wind-speeds and different incidence angles.



Figure 2: Lidar backscatter against incidence angle calculated using the model by Menzies et al.

Figure 3 gives the lidar backscatter resulting from the model by Menzies et al. for 2 different incidence angles as a function of wind speed. The green crossing lines mark the 5 [m/s] wind speed result, which may be used as threshold to exclude cases with whitecap reflection. This threshold value would then be 0.215 [1/Sr]. [todo: change to reflect the 2 m/s and 8 m/s threshold discussed later] It is important to note here that the off-nadir curve does not really show asymptotic behaviour when the wind speed drops to zero. Due to the underwater reflection a minimum level does exists, even for perfectly windstill conditions. The location of this level is not known.

5 Wave-shape

The most simple wave shape imaginable is a (co)sine or a superposition of (co)sines. However, for our purpose this is too simple, because such a wave will only have vertical movement. What we are after is also the horizontal movement of the water surface, which is important when using off-nadir measurement geometry.

The obvious candidate to use then is the trochoid wave-shape [LR7] (p.22), which approaches the (co)sine shape for small amplitudes, but has much steeper crests for higher amplitudes.

5.1 Trochoid definition

A 1D trochoid wave shape is defined by a parametrisation of both x and y to a third parameter p. It describes the movement of a water surface element by imagining that this element is attached to a wheel with radius R which rolls along a line above the surface. If the element is attached at radius r in this wheel, this will result in a wave amplitude A of r. The radius R of the wheel determines the wavelength $L = 2\pi R$



Figure 3: Lidar backscatter against wind speed calculated using the model by Menzies et al.

and the ratio between r and R determines how much the wave shape deviates from a cosine shape. The mathematical definition of the x and y coordinates of the wave surface, x(p) and y(p), is:

$$\begin{array}{ll} x(p) = pL - & A\sin(\omega t + 2\pi p) \\ y(p) = & A\cos(\omega t + 2\pi p) \end{array} \right\} \text{ trochoid parametrisation}$$
(22)

In which ω is the wave angular frequency, defined by $\omega = \frac{2\pi}{T}$. The relation between wave-length L and wave-period T depends on the wave speed C which is (for deep water) a function of wave-length itself:

$$L = CT$$

$$C = \sqrt{\frac{g}{2\pi}L}$$
(23)

in which g the gravitational acceleration, for which I will use the constant value of $g = 9.81 \text{ [m/s^2]}$. A nice property of eq. 22 is that it immediately gives the x and y velocity of the water surface elements, v_x and v_y , and the slope s of the surface:

$$v_x = \frac{dx(p)}{dt} = -\omega A \cos(\omega t + 2\pi p)$$

$$v_y = \frac{dy(p)}{dt} = -\omega A \sin(\omega t + 2\pi p)$$

$$s = \frac{dy(p)}{dx(p)} = \frac{\frac{dy(p)}{dp}}{\frac{dx(p)}{dp}} = \frac{-2\pi A \sin(\omega t + 2\pi p)}{L - 2\pi A \cos(\omega t + 2\pi p)}$$
(24)

From this equation it is clear that singularities will occur in the equation for the slope, if the amplitude becomes too large, i.e. if $2\pi A \ge L = 2\pi R$ so when r > R. In this case the parametrisation does no longer define a function y = f(x), but a spiral shape is formed with at some locations multiple y values for a given x. This case is no longer realistic, and is not used in the current study.

5.2Stokes drift

Another significant effect that must be taken into account is the net water movement caused by the forcing of the wind. This effect is known as "Stokes drift" and is usually parametrised as follows (see for example [LR7], p. 17, or [LR12]):

$$u_s = \frac{(2\pi A)^2}{LT} \tag{25}$$

Note that this effect has no direct relation to the surface wind, but depends on the wave length, period and amplitude, and will differ for the different waves used in the calculations. More general it will depend on the local wave spectrum, so on the choices we make on the wavelengths and amplitudes to take into account.

When 22 and 25 are combined, this results in:

$$\begin{aligned} x(p) &= pL - A\sin(\omega t + 2\pi p) + u_s t \\ y(p) &= A\cos(\omega t + 2\pi p) \end{aligned}$$
 trochoid parametrisation including stokes drift (26)

Also the derivative $\frac{dx(p)}{dt}$ changes now as follows:

$$v_x = \frac{dx(p)}{dt} = -\omega A \cos(\omega t + 2\pi p) + u_s$$
(27)

However, the slope distribution itself is not changed.

5.3Hydrodynamic modulation

_

Hydrodynamic modulation is parametrized in a very simple way. We modulating the amplitude of the smaller waves, by multiplying it with the phase of the largest wave in the simulation normalised to the range [0,1]).

Calculation of LOS surface velocity 5.4

Using the results obtained in section 5.2 above for v_x and v_y the water surface velocity in the LOS direction is given by:

$$v_{LOS} = -v_x \sin(\alpha) + v_y \cos(\alpha) \tag{28}$$

with α the incidence angle of the laser.

The desribed waves move towards the LOS, so correspond to a wind blowing towards the laser LOS. The opposite case of a wind blowing away from the laser LOS can easily be obtained from the given formulation by multiplying the time parameter t in equation 26 by a factor of -1, and modifying the derivatives accordingly. Note that a number of surface properties are not present in this model:

• ocean currents may add another net movement to the surface velocity, which is not taken into account.

- Rhe upwind/downwind asymmetry mentioned above (see eqs. 14 and 15) cause a bias as well. This is not yet taken into account.
- wind statistics are latitude-dependent. This is ignored for now.
- 3D effects are not simulated.

Numerical issues 5.5

The parametrisation for the trochoid wave shape as defined in the previous section gives a complete description of the wave-shape for a given wave-length and amplitude, but since it is not a function in the form y = f(x) it is not very convenient to use in an application. This has been solved numerically by constructing a look-up-table (LUT). To do this $x(p), y(p), v_x(p), v_y(p)$ and s(p) have been calculated for a large number of p values, typically in the order of 100.000. This number is adjustable to achieve higher precision if needed. Then an interpolation between these points is performed to a obtain values for a regularly spaced x-array. This is needed to allow later summation of several waves with different wave-lengths and amplitudes. Finally the LUT's for all defined waves are summed into one LUT, which is made accessible by an interpolation function to achieve the functionality of a function y = f(x) for each of the defined wave properties.

5.6 Examples

Figure 4 gives an example of the wave-shape and slope calculated for a single trochoid wave without stokes drift (based on eqs. 22 and 24).



Figure 4: Example of a wave-shape composed a single trochoid wave with wave-length L=6 [m], and amplitude A=0.335 [m] (without stokes drift). The blue line gives the wave surface. The green line gives the wave slope s. The black arrows indicate the surface movement of the water.

Figure 5 gives an example of the wave-shape and slope calculated for two trochoid waves of different wavelength and amplitude superimposed. The amplitude of the second wave is modulated by the the phase of the largest wave, so by the height above the trough of the first wave. On the wave crest of the first large wavelength wave the second wave has maximum amplitude.

Figure 6 gives an example of the wave-shape and slope calculated for three trochoid waves of different wavelength and amplitude superimposed. Again the amplitude of the two smaller waves is modulated by the phase of the largest wave.

6 Connecting wave shape and reflectivity

The wave model described in section 5 has to be connected to the reflectivity model. The idea is for a given wind speed, to use the slope distribution of the surface as defined by the Menzies and Tratt model. Then we try to find a combination of trochoid waves that have almost the same slope distribution. This gives a relation between surface LOS velocity and slope. Using the same wind speed the reflectivity model can be used to estimate the reflectivity for a given surface slope. Combined this gives for each possible slope, the frequency of occurrence for this slope, its LOS velocity and its reflectivity. This is sufficient to calculate the observed LOS velocity, which is a weighted average over all possible slopes of the LOS velocity, using the occurrence of the slope and the reflectivity as weights.

A small complication is the fact that our wave model is only 1 dimensional, while the model for reflection uses 2 dimensional variance. Therefore the slope variance by the wave model has to be a factor $\sqrt{2} = 1.414$ larger than the slope variance of the reflectivity model.

Table 1 shows the waves that have been constructed to show the same slope variance as the variance calculated by the Menzies model, (using eq. 16). For each modeled wave the slope-to- v_{LOS} has been fitted to a line by using a least-squares-fitting method, with slope m and zero crossing c. This fitting has been



Figure 5: Example of a wave-shape composed of two superimposed trochoid waves. The first one has a wavelength L=6 m, and amplitude A=0.30 [m] and the second one has a wave-length L=1 [m], and amplitude A=0.050 [m]. The blue line gives the wave surface. The green line gives the wave slope s. The black arrows indicate the surface movement of the water.



Figure 6: Example of a wave-shape composed of 3 superimposed trochoid waves. The first one has wave-length L=6 m, and amplitude A=0.269 [m], the second one has wave-length L=1 [m], and amplitude A=0.045 [m], and the third one has wave-length 0.05 [m] and amplitude 0.0045 [m]. The blue wave gives the wave surface. The green line gives the wave slope s. The black arrows indicate the surface movement of the water.

$U_{10} [{\rm m/s}]$	$\sigma_{slope,model} \times \sqrt{2}$	L_1	A_1	L_2	A_2	L_3	A_3	$m_{37.5}$	$C_{37.5}$	m_0
	$=\sigma_{slope,wave}$									
0	0.0232	220	1.150	-	-	-	-	23.356	0.0061	18.533
2	0.121	6	0.163	-	-	-	-	3.837	-0.0272	3.061
2	0.121	6	0.141	1.01	0.0235	-	-	3.564	-0.0205	2.841
2	0.121	6	0.128	1.01	0.0213	0.102	0.00213	3.465	-0.0174	2.765
4	0.165	6	0.221	-	-	-	-	3.820	-0.0498	3.061
4	0.165	6	0.192	1.01	0.0321	-	-	3.548	-0.0399	2.849
4	0.165	6	0.175	1.01	0.0292	0.102	0.00292	3.451	-0.0342	2.774
6	0.200	6	0.268	-	-	-	-	3.802	-0.0732	3.061
6	0.200	6	0.234	1.01	0.0390	-	-	3.533	-0.0599	2.855
6	0.200	6	0.214	1.01	0.0357	0.102	0.00357	3.438	-0.0521	2.781
8	0.229	6	0.305	-	-	-	-	3.785	-0.0949	3.061
8	0.229	6	0.268	1.01	0.0446	-	-	3.519	-0.0791	2.860
8	0.229	6	0.245	1.01	0.0409	0.102	0.00409	3.427	-0.0691	2.788
10	0.255	6	0.339	-	-	-	-	3.768	-0.117	3.061
10	0.255	6	0.299	1.01	0.0498	-	-	3.506	-0.0985	2.865
10	0.255	6	0.274	1.01	0.0457	0.102	0.00457	3.415	-0.0869	2.794
15	0.311	6	0.409	-	-	-	-	3.727	-0.171	3.061
15	0.311	6	0.364	1.01	0.0607	-	-	3.475	-0.149	2.877
15	0.311	6	0.336	1.01	0.0560	0.102	0.00560	3.388	-0.132	2.807
20	0.358	6	0.468	-	-	-	-	3.686	-0.224	3.061
20	0.358	6	0.420	1.01	0.0700	-	-	3.446	-0.199	2.888
20	0.358	6	0.414	1.01	0.0690	0.102	0.00690	3.351	-0.201	2.827
25	0.400	6	0.518	-	-	-	-	3.647	-0.275	3.061
25	0.400	6	0.469	1.01	0.0782	-	-	3.419	-0.248	2.900
25	0.400	6	0.435	1.01	0.0725	0.102	0.00725	3.400	-0.222	2.832

done both for nadir and off-nadir looking geometry. Note that due to the symmetry for the nadir looking case, c_0 is zero for all cases, and is not summarized in the table.

Table 2: Summary of the simulated waves used to obtain the given list of slope variances. For up to 3 waves the length and amplitude are given. The last 3 columns give the regression results from comparing the LOS surface velocity results to the slope values, for nadir and off-nadir geometry (case without Stokes drift).

Using the a single wave with the trochoid wave-shape, it is impossible to find a slope-distribution anywhere near a Gaussian shape. An example for a wave with wavelength of 6 m and amplitude of 0.339 m, giving a slope variance of 0.255 (identical to $\sqrt{2}$ times the slope variance found by the Menzies model for $u_{10} = 10 \text{ [m/s]}$) is given in figure 7. Clearly the slope distribution is very different from a Gaussian one, and is more a bi-modal one. Different wave-lengths and amplitudes only change the width of the distribution, but not the shape. Clearly a single wave is a too simple model for our purpose.

A more complicated wave-form, build from 3 superimposed trochoids with different wavelength, and for which the amplitudes of the 2 smaller ones are modulated on the phase of the first trochoid (with the largest wave-length) gives a result as given in figure 8.

For this example the following 3 waves where used:

- wavelength 6 m, amplitude 0.271 m
- wavelength 1.01 m, amplitude 0.0452 m
- wavelength 0.102 m, amplitude 0.00452 m

Note that the wave length was choosen this way on purpose, to prevent the different wavelengths to be a multiple of each other. This would lead to a fast return of the curve in the slope-to-LOS plots shown below, and would not sample the problem properly.

Combined, they again give a slope variance of 0.255 (identical to the slope variance found by the Menzies model for $u_{10} = 10 \text{ [m/s]}$ times $\sqrt{2}$). Clearly the slope distribution is now much closer to a Gaussian shape.

$U_{10} [{\rm m/s}]$	$\sigma_{slope,model} \times \sqrt{2}$	L_1	A_1	L_2	A_2	L_3	A_3	$m_{37.5}$	$c_{37.5}$	m_0
	$=\sigma_{slope,wave}$									
2	0.121	6	0.128	1.01	0.0213	0.102	0.00213	3.552	0.0240	2.765
4	0.165	6	0.175	1.01	0.0292	0.102	0.00292	3.570	0.0432	2.774
6	0.200	6	0.214	1.01	0.0357	0.102	0.00357	3.582	0.0631	2.781
8	0.229	6	0.245	1.01	0.0409	0.102	0.00409	3.590	0.0815	2.788
10	0.255	6	0.274	1.01	0.0457	0.102	0.00457	3.597	0.101	2.794
15	0.311	6	0.336	1.01	0.0560	0.102	0.00560	3.607	0.148	2.807
20	0.358	6	0.414	1.01	0.0690	0.102	0.00690	3.613	0.220	2.827
25	0.400	6	0.435	1.01	0.0725	0.102	0.00725	3.613	0.241	2.832

Table 3: Summary of the simulated waves used to obtain the given list of slope variances. For up to 3 waves the length and amplitude are given. The last 3 columns give the regression results from comparing the LOS surface velocity results to the slope values, for nadir and off-nadir geometry, when Stokes drift is taken into account.



Figure 7: Histogram of slopes for a single trochoid wave with wavelength of 6 m and amplitude of 0.339 m.

For comparison, when calculating the slope distribution of the wave with wavelength of 220 [m] and amplitude of 1.15 [m] (= $\frac{1}{2}$ wave height of 2.3 [m]) a slope variance of only 0.023 is found. This clearly is far from any realistic wave distribution. In fact this is even below the slope distribution for zero wind speed.

Now using the modeled waves, the LOS velocity is calculated as a function of the surface slope, for the most realistic case mentioned above using 3 wavelengths. The geometry used to do this is depicted in figure ... [make a figure to illustrate this!]

The result for the combination of 3 super imposed trochoid waves, for nadir looking geometry is shown in figure 9. The result for the combination of 3 super imposed trochoid waves, for off-nadir looking geometry is shown in figure 10. The dependence is rather wild, but on a coarse scale a clear correlation between slope and LOS velocity is present. The over plotted red line is a linear least-squares fit, minimizing the difference in x-direction. This fit result will be used to couple the reflectivity model to the LOS velocity.

The resulting parameters m and c of the regression line $v_{LOS} = ms + c$ are given in table 2 for the case without Stokes drift, and in table 3 for the case with Stokes drift. For off-nadir geometry $m_{37.5}$ and $c_{37.5}$ are given. For nadir geometry only m_0 is given, since c_0 is always zero in this case.

EOLUS



Figure 8: Histogram of slopes for three trochoid waves superimposed. Wavelengths are 6 m, 1.01m and 0.102 m.



Figure 9: LOS surface velocity against surface slopes for three trochoid waves superimposed. Wavelengths are 6 [m], 1.01 [m] and 0.102 [m], for nadir geometry.

Since the waves are still perfectly symmetric, and also the stokes drift reverses sign when the wind direction reverses, the resulting m and c values are symmetric as well, so for negative wind speeds the sign of m and c simply reverses.

Finally, it is interesting to note that the majority of these simulated waves do not show the slopes



Figure 10: LOS surface velocity against surface slopes for three trochoid waves superimposed. Wavelengths are 6 [m], 1.01 [m] and 0.102 [m], for off-nadir geometry.

needed to cause specular reflection for the off-nadir measurement geometry. Only for the 15, 20 and 25 [m/s] wind speeds, and 3 superimposed waves some specular reflection is found. All other cases have no specular reflection. At the real ocean there are of course much more wavelengths present, and interfering which each other, so a small amount of specular reflection may be present anyway. This is automatically taken into account in the next section, by the tail of the Gaussian slope distribution used by the Menzies model.

7 Results

7.1 Results specular reflection

Now from geometry of the problem it is clear that a wave-slope of $\frac{dy}{dx} = \tan(37.5) = 0.767$ (0) is needed to obtain a specular reflection at 37.5° (0°) incidence angle. From the m and c coefficients in table 2 the LOS water surface movement can be estimated directly. The results for the case without Stokes drift are given in table 4. Disregarding Stokes drift means that these results are an indication for what is to be expected in cross-wind situations.

The same calculation, now taking the Stokes drift into account leads to the results given in table 5. Note that the decreasing LOS surface velocity is caused by the fact that the wave motion counteracts the Stokes drift for this geometry. This is the case when the wind direction is towards the laser LOS.

From this it is clear that a typical surface velocity observed by specular reflection in the off-nadir looking geometry will be in the order of 2.5 to 3 m/s, depending on local windspeed and wind direction. For the nadir looking geometry this will always be zero. The variance around this number may be considerable. From figures 9 and 10 it is clear that for a given slope the variance maybe in the order of 0.5 m or more.

Wind direction distributions accumulated over the world's oceans vary by a factor of two. Over specific latitudes, e.g. the trades, the wind direction pdf modulation will be even larger. Therefore, a straightforward averaging will not be effective in reducing these HLOS effects. Also the variance of the surface velocity, when taking such a large dataset, will remain in the order of 3 m/s if only specular reflection is considered.



$U_{10} [{\rm m/s}]$	$m_{37.5}$	$C_{37.5}$	m_0	$v_{LOS,37.5} \mathrm{[m/s]}$	$v_{LOS,0} \mathrm{[m/s]}$
25	-3.400	0.222	2.832	-2.39	0
20	-3.351	0.201	2.827	-2.37	0
15	-3.388	0.132	2.807	-2.47	0
10	-3.415	0.0869	2.794	-2.53	0
8	-3.427	0.0691	2.788	-2.56	0
6	-3.438	0.0521	2.781	-2.59	0
4	-3.451	0.0342	2.774	-2.61	0
2	-3.465	0.0174	2.765	-2.64	0
2	3.465	-0.0174	2.765	2.64	0
4	3.451	-0.0342	2.774	2.61	0
6	3.438	-0.0521	2.781	2.59	0
8	3.427	-0.0691	2.788	2.56	0
10	3.415	-0.0869	2.794	2.53	0
15	3.388	-0.132	2.807	2.47	0
20	3.351	-0.201	2.827	2.37	0
25	3.400	-0.222	2.832	2.39	0

Table 4: water surface LOS velocities for specular reflection only.

$U_{10} [{\rm m/s}]$	$m_{37.5}$	$c_{37.5}$	m_0	$v_{LOS,37.5} \mathrm{[m/s]}$	$v_{LOS,0} \mathrm{[m/s]}$
-25	-3.613	-0.241	2.832	-3.01	0
-20	-3.613	-0.220	2.827	-2.99	0
-15	-3.607	-0.148	2.807	-2.92	0
-10	-3.597	-0.101	2.794	-2.86	0
-8	-3.590	-0.0815	2.788	-2.84	0
-6	-3.582	-0.0631	2.781	-2.81	0
-4	-3.570	-0.0432	2.774	-2.78	0
-2	-3.552	-0.0240	2.765	-2.75	0
2	3.552	0.0240	2.765	2.75	0
4	3.570	0.0432	2.774	2.78	0
6	3.582	0.0631	2.781	2.81	0
8	3.590	0.0815	2.788	2.84	0
10	3.597	0.101	2.794	2.86	0
15	3.607	0.148	2.807	2.92	0
20	3.613	0.220	2.827	2.99	0
25	3.613	0.241	2.832	3.01	0

Table 5: water surface LOS velocities for specular reflection only, when Stokes drift is taken into account.

7.2 Results combined specular and uniform white-cap reflection

Now this has to be combined with the diffusive reflection on white-caps. Equation 12 gives the occurance of white caps as a function of wind speed. First, assuming the white caps are distributed uniformly over the waves, and cover a fraction W of the surface. Then the Gaussian slope distribution must be multiplied with the linear slope-to- v_{LOS} relation, and integrated, to obtain the average v_{LOS} value from the white-cap reflectivity. From the very small zero crossing values c in table 2 it can already be seen that this result will be very close to zero for all cases. This integration has been done numerically for a number of wind speeds U_{10} and both nadir and off-nadir geometry.

The results for off-nadir geometry are given in table 6. To obtain these results first the specular reflection R_s and diffusive reflection R_wc were calculated using equations 20 and 17. The LOS water velocity as seen by the specular reflected light is obtained by using the LOS-to-slope parametrisation determined in section 6, using a slope $s = tan(\theta)$. The LOS water velocity as seen by the diffusive reflected light on the white-caps (assuming they are uniformly distributed on the waves) is obtained by using the LOS-to-slope



parametrisation determined in section 6, for a range of slopes between -1 and +1, and integrating over the Gaussian slope distribution as discussed above. During this integration a weighting was performed on the local incidence angle. Local incidence angle is defined here as the difference between the normal to the water segment surface β and the incidence angle α , and for the weighting the factor $\cos(\alpha - \beta)$ was used. Finally, using the white-cap coverage W obtained from equation 12, the combined LOS water velocity is obtained by using the reflectivity, and the white-cap coverage as a weighting factor. This gives:

$$v_{LOS,sum} = \frac{(v_{LOS,diff} R_{wc} W + v_{LOS,spec} R_s (1 - W))}{R_{wc} W + R_s (1 - W)}$$
(29)

$U_{10} [{\rm m/s}]$	R_s	R_{wc}	$v_{LOS,spec}$	$v_{LOS,diff}$	W	$v_{LOS,sum}$	σ_{LOS}
-25	7.25×10^{-5}	0.0556	-2.39	0.0271	0.246	0.0175	0.962
-20	1.48×10^{-5}	0.0556	-2.37	0.0447	0.112	0.0396	0.849
-15	9.82×10^{-7}	0.0556	-2.47	0.0114	0.0407	0.0114	0.745
-10	4.01×10^{-9}	0.0556	-2.53	0.00395	9.77×10^{-3}	0.00395	0.617
-8	6.46×10^{-11}	0.0556	-2.56	0.00164	4.45×10^{-3}	0.00164	0.556
-6	7.06×10^{-14}	0.0556	-2.59	0.00042	1.62×10^{-3}	0.00042	0.486
-4	1.20×10^{-19}	0.0556	-2.61	-0.00145	$3.88 imes 10^{-4}$	-0.00145	0.403
-2	1.45×10^{-35}	0.0556	-2.64	-0.00192	$3.38 imes 10^{-5}$	-0.00192	0.296
2	1.45×10^{-35}	0.0556	2.64	0.00192	3.38×10^{-5}	0.00192	0.296
4	1.20×10^{-19}	0.0556	2.61	0.00145	$3.88 imes 10^{-4}$	0.00145	0.403
6	7.06×10^{-14}	0.0556	2.59	-0.00042	$1.62 imes 10^{-3}$	-0.00042	0.486
8	6.46×10^{-11}	0.0556	2.56	-0.00164	$4.45 imes 10^{-3}$	-0.00164	0.556
10	4.01×10^{-9}	0.0556	2.53	-0.00395	9.77×10^{-3}	-0.00395	0.617
15	9.82×10^{-7}	0.0556	2.47	-0.0114	0.0407	-0.0103	0.745
20	1.48×10^{-5}	0.0556	2.37	-0.0447	0.112	-0.0396	0.849
25	7.25×10^{-5}	0.0556	2.39	-0.0271	0.246	-0.0175	0.962

Table 6: water surface LOS velocities for combined specular and uniform diffusive reflection, for the off-nadir looking geometry (Stokes drift not taken into account).

From table 6 it is clear that if uniform distribution of white-caps is assumed on the waves (and Stokes drift is disregarded), only minimal LOS water velocity is seen, always below 0.1 m/s for wind speeds upto 25 m/s for off-nadir geometry.

When this precedure is repeated for Wave model that includes the Stokes drift From table 7 it is clear that if uniform distribution of white-caps is assumed on the waves and Stokes drift is taken into account, a clear net LOS water velocity is seen, upto 0.5 m/s for wind speeds upto 25 m/s for off-nadir geometry.

The results for nadir geometry are given in table 8. From table 8 it is clear that if uniform distribution of white-caps is assumed on the waves, no LOS water velocity is seen at all for nadir looking geometry.

For both geometries it is clear that the white-cap reflection is the dominant term. Therefore the variance in the LOS water velocity σ_{LOS} follows directly from the slope variance using the LOS-to-slope parametrisation. This is indicated in the last column of both tables. However, note that the actual variance may even be larger because of the simplification introduced by this LOS-to-slope parametrisation (especially for the lower wind speeds).

7.3 Results combined specular and non-uniform white-cap reflection

Finally we have to consider the case in which the white-caps are not distributed uniformly on the waves. This is the case at the real ocean since white-caps are formed at the wave crests by breaking waves, after which the largest waves move ahead of the foam. [what source can I refer to for this statement?] Unfortunately I do not have any published examples for this effect, so I have to make some assumptions here.

First it is assumed that a fraction of the diffusive reflection is caused by sub-surface scattering, and uniformly distributed foam or bubbles on the water surface (as was discussed in section 4.3). For the moment a constant weight of 0.2 is assumed for this, independent of the actual surface slope. On top of



$U_{10} [{\rm m/s}]$	R_s	R_{wc}	$v_{LOS,spec}$	$v_{LOS,diff}$	W	$v_{LOS,sum}$	σ_{LOS}
-25	7.25×10^{-5}	0.0556	-3.01	-0.448	0.246	-0.448	0.962
-20	1.48×10^{-5}	0.0556	-2.99	-0.389	0.112	-0.394	0.849
-15	9.82×10^{-7}	0.0556	-2.92	-0.276	0.0407	-0.278	0.745
-10	4.01×10^{-9}	0.0556	-2.86	-0.188	$9.77 imes 10^{-3}$	-0.188	0.617
-8	6.46×10^{-11}	0.0556	-2.84	-0.152	4.45×10^{-3}	-0.152	0.556
-6	7.06×10^{-14}	0.0556	-2.81	-0.117	1.62×10^{-3}	-0.117	0.486
-4	1.20×10^{-19}	0.0556	-2.78	-0.0801	3.88×10^{-4}	-0.0801	0.403
-2	1.45×10^{-35}	0.0556	-2.75	-0.0428	3.38×10^{-5}	-0.0438	0.296
2	1.45×10^{-35}	0.0556	2.75	0.0428	3.38×10^{-5}	0.0438	0.304
4	1.20×10^{-19}	0.0556	2.78	0.0801	3.88×10^{-4}	0.0801	0.417
6	7.06×10^{-14}	0.0556	2.81	0.117	1.62×10^{-3}	0.117	0.506
8	6.46×10^{-11}	0.0556	2.84	0.152	4.45×10^{-3}	0.152	0.582
10	4.01×10^{-9}	0.0556	2.86	0.188	$9.77 imes 10^{-3}$	0.188	0.649
15	$9.82 imes 10^{-7}$	0.0556	2.92	0.276	0.0407	0.278	0.794
20	1.48×10^{-5}	0.0556	2.99	0.389	0.112	0.394	0.915
25	7.25×10^{-5}	0.0556	3.01	0.448	0.246	0.448	1.022

Table 7: water surface LOS velocities for combined specular and uniform diffusive reflection, for the off-nadir looking geometry, taking Stokes drift into account.

$U_{10} [{\rm m/s}]$	R_s	R_{wc}	$v_{LOS,spec}$	$v_{LOS,diff}$	W	$v_{LOS,sum}$	σ_{LOS}
-25	0.0452	0.0700	0	0	0.246	0	0.801
-20	0.0563	0.0700	0	0	0.112	0	0.716
-15	0.0746	0.0700	0	0	0.0407	0	0.618
-10	0.111	0.0700	0	0	$9.77 imes 10^{-3}$	0	0.504
-8	0.137	0.0700	0	0	$4.45 imes 10^{-3}$	0	0.452
-6	0.181	0.0700	0	0	$1.62 imes 10^{-3}$	0	0.393
-4	0.265	0.0700	0	0	3.88×10^{-4}	0	0.324
-2	0.494	0.0700	0	0	3.38×10^{-5}	0	0.237
2	0.494	0.0700	0	0	3.38×10^{-5}	0	0.237
4	0.265	0.0700	0	0	3.88×10^{-4}	0	0.324
6	0.181	0.0700	0	0	1.62×10^{-3}	0	0.393
8	0.137	0.0700	0	0	4.45×10^{-3}	0	0.452
10	0.111	0.0700	0	0	$9.77 imes 10^{-3}$	0	0.504
15	0.0746	0.0700	0	0	0.0407	0	0.618
20	0.0563	0.0700	0	0	0.112	0	0.716
25	0.0452	0.0700	0	0	0.246	0	0.801

Table 8: water surface LOS velocities for combined specular and uniform diffusive reflection, for the nadir looking geometry (Stokes drift not taken into account.

this a Gaussian weighting function is placed, using a width of $\sigma_w = 0.3\sigma$, an offset from zero of $\Delta s = 0.3\sigma$, and an amplitude of 0.8. This results in the following LOS calculation:

$$v_{LOS} = \frac{\sum_{i=1}^{nsteps} \left[\{ m_{37}(u_{10})s + c_{37}(u_{10}) \} P_s(s_i) P_w(s_i) \right]}{\sum_{i=1}^{nsteps} \left[P_s(s_i) P_w(s_i) \right]}$$
(30)

with

$$s_i = -1 + \frac{2(i-1)}{(nsteps - 1)} \tag{31}$$

and

$$P_s(s) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-s^2}{2\sigma^2}\right) \tag{32}$$

and

$$P_w = 0.2 + 0.8 \frac{1}{\sigma_w \sqrt{2\pi}} \exp\left(\frac{-(s - \Delta s)^2}{2\sigma_w^2}\right)$$
(33)

These equations have been evaluated using nsteps=1001, for a set of different wind speeds. The results are given in table 9. From table 9 it is clear that if non-uniform distribution of white-caps is assumed on

$U_{10} [{\rm m/s}]$	R_s	R_{wc}	$v_{LOS,spec}$	$v_{LOS,diff}$	W	$v_{LOS,sum}$	σ_{LOS}
2	1.45×10^{-35}	0.0556	2.66	0.0954	3.38×10^{-5}	0.0954	0.383
4	1.20×10^{-19}	0.0556	2.67	0.138	3.88×10^{-4}	0.138	0.385
6	7.06×10^{-14}	0.0556	2.68	0.175	1.62×10^{-3}	0.175	0.387
8	6.46×10^{-11}	0.0556	2.68	0.208	4.45×10^{-3}	0.208	0.389
10	4.01×10^{-9}	0.0556	2.69	0.238	9.77×10^{-3}	0.238	0.390
15	9.82×10^{-7}	0.0556	2.70	0.308	0.0407	0.309	0.393
20	1.48×10^{-5}	0.0556	2.71	0.370	0.112	0.375	0.394
25	7.25×10^{-5}	0.0556	2.74	0.467	0.246	0.476	0.390

Table 9: water surface LOS velocities for combined specular and non-uniform diffusive reflection, for the off-nadir looking geometry.

the waves, a significant LOS water velocity is seen for off-nadir looking geometry. For these arbitrarily chosen parameters LOS velocities up to 0.5 m/s are expected. Note that the parameters of this model still need to be tuned. Preferably experimental data should be used to do this. P_s and P_w are to some extend independent, since P_w determines mostly the observed variance in the LOS velocity, and P_s determines the average observed value of the LOS velocity. Therefore tuning the model to experimental data should in principle be possible.

In a similar way results can be obtained for the nadir looking geometry. These results are given in table 10. At first sight the results presented in table 10, especially for $v_{LOS,sum}$, seem a bit counter-intuitive,

$U_{10} [{\rm m/s}]$	R_s	R_{wc}	$v_{LOS,spec}$	$v_{LOS,diff}$	W	$v_{LOS,sum}$	σ_{LOS}
2	0.494	0.0700	0	0.0614	3.38×10^{-5}	0	0.307
4	0.265	0.0700	0	0.0824	3.88×10^{-4}	0	0.311
6	0.181	0.0700	0	0.0984	1.62×10^{-3}	0	0.315
8	0.137	0.0700	0	0.112	4.45×10^{-3}	2.54×10^{-4}	0.318
10	0.111	0.0700	0	0.123	9.77×10^{-3}	7.63×10^{-4}	0.320
15	0.0746	0.0700	0	0.147	0.0407	5.62×10^{-3}	0.327
20	0.0563	0.0700	0	0.166	0.112	0.0225	0.332
25	0.0452	0.0700	0	0.181	0.246	0.0609	0.335

Table 10: water surface LOS velocities for combined specular and non-uniform diffusive reflection, for the nadir looking geometry.

because the numbers are far below the off-nadir geometry results. This can be understood by realizing that for nadir geometry the specular reflection is the dominant factor, even for high wind speeds up to 25 m/s. Therefore the LOS speed present in the light originating from the diffusive reflection on the white-caps is diluted significantly by the light originating from specular reflection. And since this specularly reflected light originates almost only from the wave tops and crests, no vertical water movement is seen in this light. Therefore in the combined signal the observed vertical water motion will be significantly below the number observed in the off-nadir geometry case (for which the diffusive reflection is the dominant factor).

8 Conclusion

Assuming the presented model describes the water surface slopes and the reflection on the surface well enough, the following conclusions may be drawn for the off-nadir looking geometry:

- Off-nadir reflection is dominated by diffusive reflection on white-caps.
- A clear net water velocity will be observed, either positive or negative depending on the propagation direction of the waves compared to the looking direction.
- This effect will already be above the acceptable threshold of 0.1 [m/s] for very low wind speeds just above 2 [m/s]. If this threshold is released to 0.2 [m/s] measurements at wind speeds up to 8 [m/s] might be acceptable.
- For large wind speeds of 25 [m/s] a net surface velocity in the order of at least 0.5 [m/s] will be seen.
- From figure 3 it is clear that the total observed backscatter increases with wind speed for off-nadir looking geometry. Therefore a threshold on this property could in principle be used to select low wind speed cases. However, the signal levels for the usable region are very low, Following the model by Menzies and Tratt, for 2 [m/s] wind speed an equivalent ground backscatter of 1.88×10^{-6} [1/(Sr] is expected (using eq. 3 on page 8 and assuming a lowest rangebin size of 250 m). This is in the same order of magnitude as the typical expected ground backscatter levels. Molecular backscatter is expected to be around 1×10^{-5} [1/m.Sr] and aerosol backscatter is expected to be in the range between 1×10^{-6} and 1×10^{-5} [1/m.Sr] close to the surface (see [RD2], table 3.11 at page 41). This makes it very unlikely that these weak ground reflections can be discriminated from the atmosphere signal from the lowest range bin. Therefore selection using a backscatter threshold to obtain wind speeds below 2 [m/s] will probably not be possible.
- For 8 [m/s] the ground backscatter is expected to be 2.47×10^{-4} [1/(Sr]. It has to be seen whether using this value setting a threshold is feasable.
- Regarding these thresholds is must also be note that the underwater scattering will provide a floor in this backscatter curve for very low wind speeds at a currently unknown level. This may pose an additional limit to the windspeed selection process using a threshold.
- From for example scatterometry measurements it is known that on a global scale the u and v wind components can be described by a Gaussian distribution with a width of 5.5 m/s⁴ [add a source for this] Using this distribution it can be calculated that wind speeds below 2 m/s only occur in 6.4 % of all cases. Wind speeds below 8 m/s occur in 65 % of all cases. Therefore performing zero-wind calibration with the condition of having a bias below 0.1 m/s seems extremely difficult. Relaxing the threshold to 0.2 m/s would make this type of measurement more realistic.

For nadir geometry the following conclusions may be drawn:

- The net LOS water velocity will always be very small, even for very large wind speeds of 25 m/s this velocity is only 0.06 m/s. So the simple estimate of 0.16 m/s mentioned in the introduction seems even over-estimated for this case.
- This nadir result is largely independent off the white-cap properties so not susceptible to the large uncertainty in the knowledge of the white-cap location, mentioned for the off-nadir case.
- the main uncertainty in this result lies in the model by Menzies and Tratt. This model was not tested for incidence angles below about 4 degrees. The reason was that the dynamic range of the detector was tuned to give good results for the off-nadir measurements up to 30 degrees, which lead to saturation for angles close to nadir.

Note however that there are large uncertainties in the model used. Especially the distribution of the white-caps on the waves, and their relation to the wave-phase is not well known at all. Also the effect of underlight is not well known.

Clearly we need a good experimental validation of the estimates presented in this report. The results should only be taken as an estimate of the order of magnitude of the expected surface movement, and not as a precise and detailed simulation result.

- end of document -

⁴ignoring the trade-winds for the moment, which distort the u component distribution depending on the season.