Lateral mixing in shallow convection: In theory and in practice

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1 Introduction

Nowadays, practically all NWP and climate models use a mass flux concept to parameterize convective transport. In such a mass flux framework the fractional entrainment (ϵ) and detrainment (δ) coefficients are the key parameters. The fractional entrainment coefficient describes the inflow of environmental air into the cloudy area, whereas the fractional detrainment coefficient describes the outflow of cloudy air into the environment. Despite numerous efforts to parameterize ϵ and δ, there is still no concensus on a particular parameterization, satisfying in all possible conditions. Neither do we have much insight into the behavior of these lateral mixing coefficients. For example: Why is the detrainment more variable from case to case and hour to hour than the entrainment? (see e.g. [de Rooy and Siebesma(2008)]) To gain more insight into the behavior we derive analytical expressions for ϵ and δ from first principles and with a minimum of assumptions. In contrast with most other studies we are not directly targeted towards the development of a new parameterization.

2 Derivation of the expressions

For the derivation of the expressions for the lateral mixing coefficients we consider the general case of an ensemble of clouds. Therefore, and in contrast with most theoretical studies on convection (e.g. [Asai and Kasahara(1967)]), the height dependance of the cloudy area fraction can not be neglected. Starting point is the general equation for an arbitrary incloud field ϕ_c occupying a fraction a_c of a total domain of an area A [Siebesma(1998)]. We here use an equation for the total water specific humidity, q_t, the vertical velocity, w, as well as a continuity equation. Further we adopt the bulk approach, i.e. the subscript c for the cloudy area stands for the average over the cloudy area. Recent research [Heus and Jonker(2008)] described the existence of a shell surrounding the updraft. Due to this shell the air entraining the updraft does not have the properties of the environment but properties in between the environmental and the updraft values. In a simplified way, we used this insight in our derivation. With some other assumptions (already mentioned in literature) and after eliminating some unknowns with the continuity and the vertical velocity equation, we can write the equation for q_t in the following form:

$$\frac{\partial q_{t,c}}{\partial z} = -\frac{1}{2}\left(\frac{\alpha B}{w_c^2} - \frac{1}{w_c}\frac{\partial w_c}{\partial z}\right)(q_{t,c} - q_{t,e}) \tag{1}$$

where α is a buoyancy reduction factor [Simpson and Wiggert(1969)], B is the buoyancy, and subscripts c and e stand for resp. the cloudy area and the environment. In (1) we recognize the entraining plume form of [Betts(1975)]

$$\frac{\partial q_{t,c}}{\partial z} = -\epsilon_{qt}(q_{t,c} - q_{t,e}) \tag{2}$$

Note that so far we did not introduce ϵ in our derivation. This in contrast with other studies (e.g. [Gregory(2001)]) where ϵ is introduced already in a much earlier stage. Irrespectively of the validity of (2) we accept this equation here as the definition of ϵ because this equation describes how ϵ is used in parameterization schemes as well as how ϵ is usually diagnosed from LES. So this gives us the following expression for the entrainment

$$\epsilon_{w,shell} = \frac{1}{2}(\frac{\alpha B}{w_c^2} - \frac{1}{w_c}\frac{\partial w_c}{\partial z}) \quad (3)$$

where subscript $w, shell$ is used to distinguish this entrainment from ϵ diagnosed using (2), noted as ϵ_{qt}. The latter will be considered as pseudo observation in the validation section. Note that the properties involved in (2) and (3) are different. Therefore these equations can be considered independent which is desirable for validation purposes. Furter, it is important to mention that the $\frac{\partial a_c}{\partial z}$ term, still present in the starting equations, is disappeared in the expression for $\epsilon_{w,shell}$.

After the derivation of $\epsilon_{w,shell}$ we now derive an expression for δ. For that we use the well-known mass flux budget equation

$$\frac{\partial M}{\partial z} = (\epsilon - \delta)M \quad (4)$$

where $M = \rho w_c a_c$ denotes the upward mass flux with the density ρ (here approximated by 1). Similarly to (2), we will consider (4) (together with (2)) as a definition of δ because it is used in this way in mass flux schemes as well as in LES diagnoses for δ (and noted as δ_{qt}). Substituting (3) in (4) gives the following expression for δ

$$\delta_{w,shell} = \frac{\alpha B}{2w_c^2} - \frac{3}{2w_c}\frac{\partial w_c}{\partial z} - \frac{1}{a_c}\frac{\partial a_c}{\partial z} \quad (5)$$

The first two terms on the RHS of (5) are quite similar to the ones in (3). However, now the $\frac{\partial a_c}{\partial z}$ term appears and, as we will show, this term can be very dominant.

3 Validation

The expressions are validated using LES results with the Dutch Atmospheric LES model (DALES; [Cuijpers and Holtslag(1998)]) for the 1997 ARM case [Brown et al.(2002)], the BOMEX case [Siebesma and Cuijpers(1995)], and the RICO case [Rauber et al.(2007)]. Because the ARM case describes a non-steady state diurnal cycle of shallow convection above land, it is pre-eminently suited for a thorough test of our expressions. The BOMEX and RICO case are a more or less steady state shallow convection cases above sea.

Figure 1 show on the x-axis the ϵ and δ coefficients as usually diagnosed from LES, here considered as best possible estimates, whereas on the y-axis the coefficients applying expressions (3) and (5) are plotted. Besides the generally good correspondence, the outliers are almost all related to too small ensembles of clouds for which it is known that the bulk approach breaks down (also for the best possible estimates). Although the variation in ϵ is much smaller than in δ, as we will see, the ϵ values for the ARM case are significantly smaller than for the BOMEX and RICO case. With the use of the expression (3) we can now investigate the contribution of the different terms to the value of ϵ. It turned out that the smaller ϵ values can be explained by the smaller first term on the RHS of (3) in the ARM case. More specific it is the higher vertical velocity in the ARM case which is responsible for the smaller ϵ values compared to the BOMEX and RICO case.

In Fig. 2 profiles of ϵ and δ are plotted for the ARM case only. Note that the x-axis scale of the detrainment plots is ten times larger than the scale for the entrainment. So the variation from hour to hour is much larger

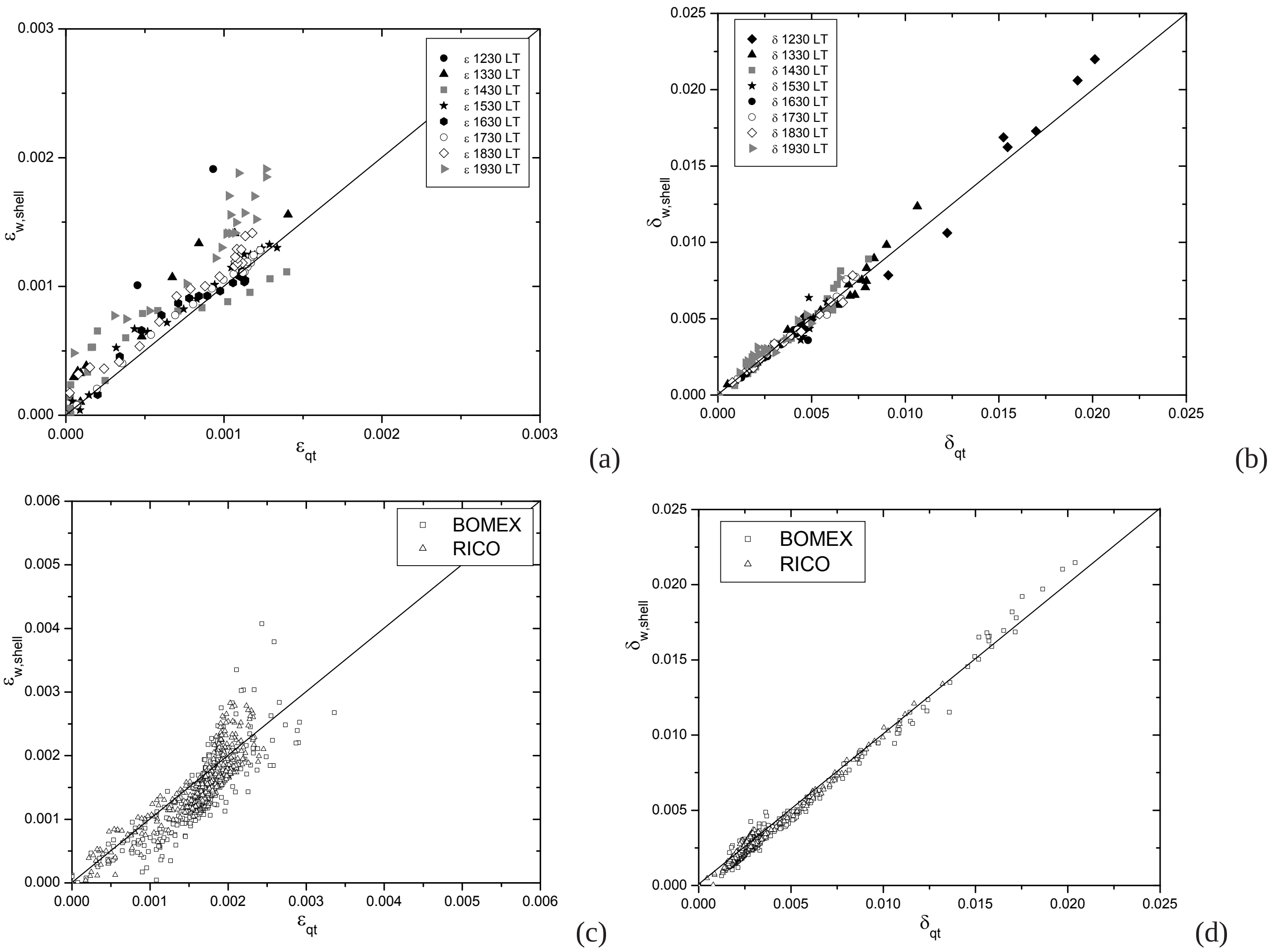

Figure 1: Comparison of estimated ϵ and δ, noted with subscript $w, shell$, with directly LES diagnosed ones, noted as ϵ_{qt} and δ_{qt}, for all hours and heights. Panels (a) and (b) show the results for the ARM case whereas panels (c)-(d) are the results for the BOMEX and RICO case.

for δ. If we look at the contribution of the different terms in the expression (5) we see that these variations as well as the value for δ itself are dominated by the $\frac{\partial a_c}{\partial z}$ term.

4 Discussion and Conclusions

Analytical expressions, in principle also valid for an ensemble of clouds, are derived for ϵ and δ from first principles and with a minimum of assumptions. A good correspondence is found when these expressions are validated against ϵ and δ as usually diagnosed from LES (here considered as best possible estimates). Further, the expressions give us insight into the behavior of the mixing coefficients. For example, analysis of LES for the non-steady ARM case revealed that the $\frac{\partial a_c}{\partial z}$ term dominates the detrainment coefficient and is mainly responsible for the larger variation in δ than in ϵ. As this term is strongly linked to the mass flux, via $M = a_c w_c$, it therefore seems plausible to let the variability of the mass flux profile be controlled by the detrainment only (as argued by [de Rooy and Siebesma(2008)]). Besides giving insight, the expressions can help us to judge existing parameterizations as well as be a source of inspiration for future parameterization developments.

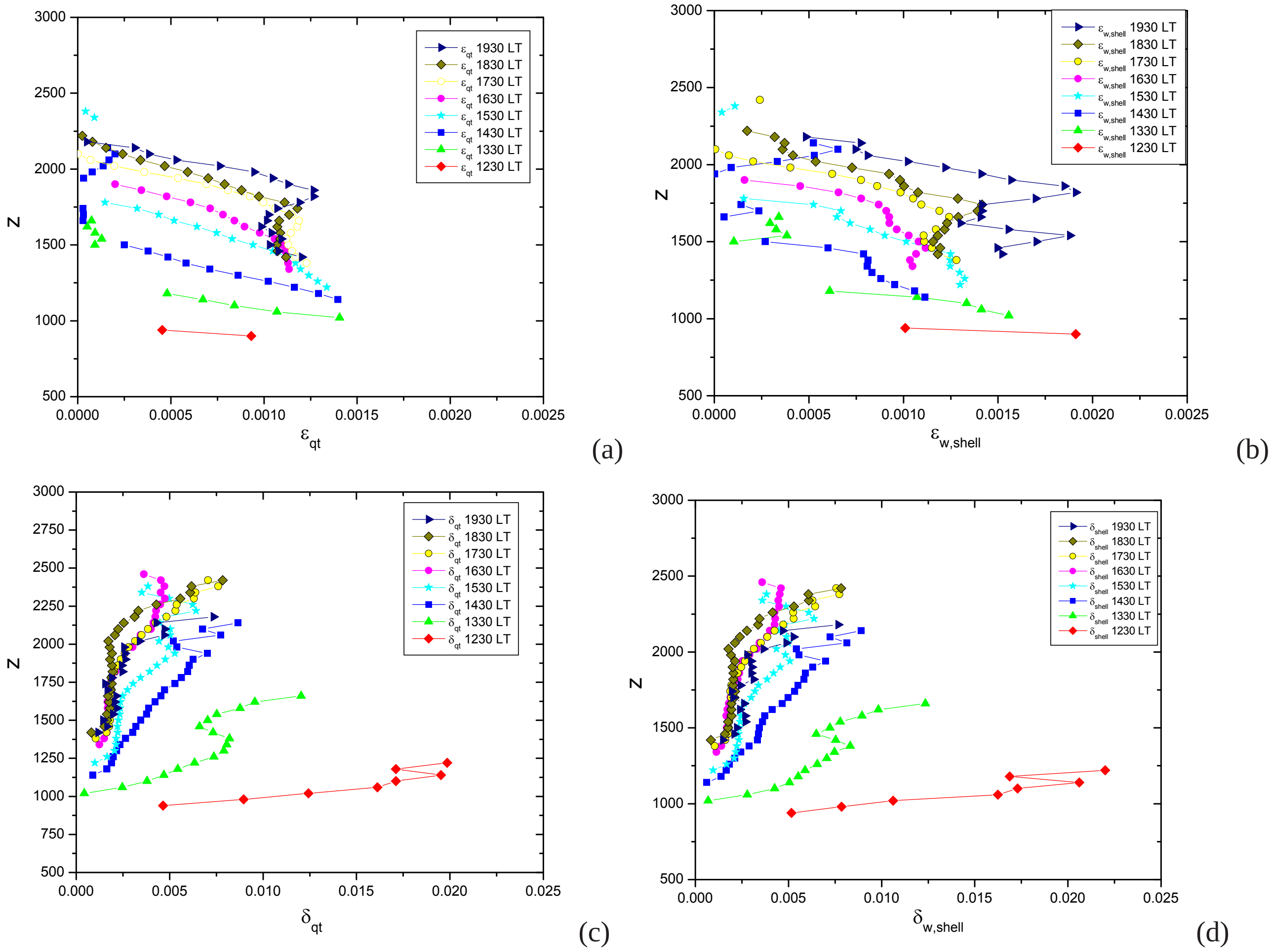

Figure 2: Profiles of ϵ_{qt} (a), $\epsilon_{w,shell}$ (b), δ_{qt} (c), and $\delta_{w,shell}$ (d) (see text) for the ARM case. Different colors correspond with different hours during the simulation. Note the different x-axis scale for the entrainment and detrainment plots

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