# Cloud reflectance calculations using DAK: study on required integration points 

E.L.A. Wolters, R.A. Roebeling and P. Stammes

```
De Bilt, 2006
PO Box 201
3730 AE De Bilt
Wilhelminalaan 1o
De Bilt
The Netherlands
http://www.knmi.nl
Telephone +31(0)30-220 69 ו
Telefax +31(0) 30-2210407
```

Author: Wolters, E.L.A.
Roebeling, R.A.
Stammes, P.

# Cloud reflectance calculations using DAK: study on required integration points 

E.L.A. Wolters, R.A. Roebeling, and P. Stammes November 2006

## Contents

1 Introduction ..... 4
2 Methods ..... 5
2.1 DAK settings ..... 5
2.2 Convergence study approach ..... 5
2.3 Sensitivity study approach ..... 6
3 Results ..... 7
3.1 Fourier term convergence analysis ..... 7
3.2 Gaussian $\mu$ point convergence analysis ..... 9
3.3 Effect of varying $v_{\text {eff }}$ ..... 12
3.4 Effect of neglecting linear polarization ..... 12
4 Summary ..... 16


#### Abstract

This report presents a study on the determination of the required number of azimuth (Fourier terms) and zenith (Gaussian $\mu$ ) integration points in order to obtain a cloud reflectance as calculated by the Doubling Adding KNMI (DAK) radiative transfer model with an absolute accuracy within 0.005 . Cloud reflectance calculations at 0.632 and $1.605 \mu \mathrm{~m}$ are performed for a plane parallel water cloud having an optical thickness, $\tau$, of 10 , overlying a surface with albedo 0.10 , for effective radii of $5-24 \mu \mathrm{~m}$. Relative azimuth angles are 90,120 , 150 , and $175^{\circ}$ and (cosines of) zenith angles are 0.2-0.8. A twoparameter Gamma size distribution is applied. Fourier terms and Gaussian $\mu$ points are varied from 20-200 for each zenith and relative azimuth angle. Results indicate that accurate cloud reflectance calculations at $0.632 \mu \mathrm{~m}$ can be performed by taking 60-80 Fourier terms and 40-60 Gaussian $\mu$ points for most effective radii and viewing geometries investigated. However, for effective radii of 16 and $24 \mu \mathrm{~m}, 140-180$ Gaussian $\mu$ points are required. For the reflectance at $1.605 \mu \mathrm{~m}, 40-60$ Fourier terms and 40-60 Gaussian $\mu$ points suffice. At both wavelengths, the given values strongly increase when the backscatter viewing geometry is approached. In addition to the convergence study, the effect of a change in the variance of the size distribution, $v_{\text {eff }}$, and the error due to neglecting linear polarization in modeled cloud reflectances is examined. An absolute change in $v_{\text {eff }}$ of 0.05 results in a maximum relative difference in calculated cloud reflectance of $\sim 4 \%$. Neglecting linear polarization in the $0.632 \mu \mathrm{~m}$ cloud reflectance calculations causes an averaged relative difference of $0.1-0.3 \%$, with a maximum of $\sim 1 \%$, depending on the effective radius.


## 1. Introduction

Radiative transfer models play an important role in the retrieval of cloud physical parameters from satellites, such as cloud optical thickness and cloud particle size. The way in which the radiative transfer equation is solved, differs between the various existing models. These different approaches have, in turn, an impact on the accuracy of the simulated reflectances (e.g. Roebeling et al. 2005). The Doubling Adding KNMI radiative transfer model (DAK) (De Haan et al. 1987; Stammes 2001) simulates the short-wave reflection at the top of the atmosphere, assuming plane-parallel homogeneous clouds over a Lambertian surface. Within the Satellite Application Facility on Climate Monitoring (CM-SAF), a longterm dataset of cloud physical properties, CPP, (Roebeling et al. 2006) is constructed for the area of Europe and northern Africa using visible and near-infrared reflectances of the Spinning Enhanced Visible and Infra-Red Imager (SEVIRI) onboard the METEOSAT-8 satellite. Measured reflectances from the SEVIRI 0.6 and $1.6 \mu \mathrm{~m}$ channels are compared to Lookup Table values generated by DAK. The accuracy of a DAK simulation is among others dependent on the number of Fourier terms and Gaussian $\mu$ points, being the azimuth and zenith angle integration points, respectively. The objective of this study is to determine the required number of these points in order to obtain an absolute accuracy of the cloud reflectance within 0.005 , which is considered a typical error in the reflectance measurements of satellites.

## 2. Methods

### 2.1. DAK settings

In DAK, the computation of reflectances in the atmosphere requires a specification of the number of points in both the zenith angle and the azimuth angle integration. The number of points needed for convergence of the reflectance values depends on the effective radius of the cloud particles and the viewing geometries. In this study, variations in Fourier terms and Gaussian $\mu$ points were applied to a standard cloud case. This standard simulation case consisted of a plane-parallel water cloud with cloud base at 2 km and a geometrical thickness of 1 km . The optical thickness of the cloud, $\tau$, and the surface albedo, $A_{\mathrm{s}}$, were set at 10 and 0.10 , respectively. Atmospheric pressure, temperature and trace gas profiles were taken from the US Air Force Geophysics Laboratory (AFGL) mid-latitude summer profiles, thereby accounting for Rayleigh scattering by air molecules and absorption by ozone. There was no linear polarization included in the standard calculations. Reflectances were calculated for cosines of solar and viewing zenith angles, $\left(\mu_{0}, \mu\right)$, of $0.2,0.4,0.6$, and 0.8 . In order to be certain of sufficient accuracy at all viewing geometries, simulations were performed at $\mu=\mu_{\circ}$, because this geometry is more sensitive to errors. Relative azimuth angles, $\varphi-\varphi_{\circ}$, were taken at $90,120,150$, and $175^{\circ}$. Effective radii, $r_{\text {eff }}$, of the spherical water droplets were $1,3,5,8,12,16$, and $24 \mu \mathrm{~m}$, assuming a two-parameter Gamma size distribution (Deirmendjian 1969 ) with an effective variance, $v_{\text {eff }}$, of 0.15 for all $r_{\text {eff }}$ values. Mie calculations were carried out using the Meerhoff Mie Program (De Rooij and van der Stap 1984). The refraction indices for water particles were taken from Downing and Williams (1975). The resulting phase functions at $\lambda=0.632 \mu \mathrm{~m}$ and $\lambda=1.605 \mu \mathrm{~m}$ are presented in Figure 1. All DAK calculations were performed accounting for multiple scattering. Cloud reflectances were simulated at 0.632 and $1.605 \mu \mathrm{~m}$, being near the center wavelengths of the SEVIRI 0.6 and $1.6 \mu \mathrm{~m}$ channels and outside of water vapor and $\mathrm{CO}_{2}$ absorption lines.

### 2.2. Convergence study approach

The major part of this study was the determination of the optimum number of Fourier terms and Gaussian $\mu$ points. For each $r_{\text {eff }}$ and azimuth angle step the number of Fourier terms ( $n_{\text {Fourier }}$ ) was varied from 20 to 200 with a step size of 20 . In order to ascertain that observed differences were due to the variation in $n_{\text {Fourier }}$ only, the number of Gaussian $\mu$ points was set sufficiently high, namely at 20 for calculations with $r_{\text {eff }}$ of 1 and $3 \mu \mathrm{~m}$. For $r_{\text {eff }}=5-12 \mu \mathrm{~m}, 60$ Gaus-
$\operatorname{sian} \mu$ points were used, whereas for effective radii of 16 and $24 \mu \mathrm{~m}$ this number was further increased to 100 . Since at $r_{\text {eff }}=24 \mu \mathrm{~m}$ the difference in simulated reflectances between $n_{\text {Fourier }}=200$ and $n_{\text {Fourier }}=300$ was negligibly small, it was assumed that for all effective radii convergence had taken place at $n_{\text {Fourier }}=200$. Subsequently, the absolute difference between the reflectance at $n_{\text {Fourier }}=\mathrm{n}$ and $n_{\text {Fourier }}=200, R_{\mathrm{n}}-R_{200}$, was calculated. Convergence was assumed at an absolute reflectance difference $\leq 0.005$ for all calculations. This criterion was chosen in order to keep the relative error in simulated cloud reflectances smaller than the satellite measurement errors, which are typically a few percent (Nakajima and King 1990). The convergence in Gaussian $\mu$ points, $n_{\text {Gauss }}$, was examined analogously to the Fourier term convergence. Results from the Fourier term convergence study were taken in order to ascertain that $n_{\text {Gauss }}$ was the only varying factor.

### 2.3. Sensitivity study approach

In addition to the Fourier and Gauss convergence analyses, the effect of a varying $v_{\text {eff }}$ on the modeled reflectance was studied. It is of importance to investigate this effect, since a larger value of $v_{\text {eff }}$ implies that the size distribution becomes broader. Especially the larger particles of the size distribution can influence the calculated reflectance significantly when varying $v_{\text {eff. }}$. More information on $v_{\text {eff }}$ is given by Hansen and Travis (1974). Since in the convergence studies $v_{\text {eff }}=0.15$ was applied, the effect of a change in $v_{\text {eff }}$ on reflectance simulations was investigated by setting $v_{\text {eff }}$ to 0.10 and 0.20 for effective radii of $5,8,12,16$, and $24 \mu \mathrm{~m}$ for all viewing geometries. The number of Fourier terms and Gaussian $\mu$ points were taken from the corresponding convergence study results.

Further, the effect of neglecting linear polarization was studied for the $0.632 \mu \mathrm{~m}$ reflectance. In principle, polarization has to be included when calculating the reflectance, since in second-order scattering (and less in higher-order scattering) the incident light is polarized and affects the scattered intensity. Therefore reflectances for effective radii of $5,8,12,16$, and $24 \mu \mathrm{~m}$ were simulated with and without including linear polarization at (cosines of) zenith angles between 0.2 and 1.0 and $\varphi-\varphi_{\circ}$ between 90 and $175^{\circ}$.

## 3. Results

### 3.1. Fourier term convergence analysis

Figures 2 and 3 show the convergence for $r_{\text {eff }}=12 \mu \mathrm{~m}$ and $\mu=\mu$ 。 values between 0.2 and 0.8 for $\lambda=0.632$ and $1.605 \mu \mathrm{~m}$, respectively. It is evident that differences at $n_{\text {Fourier }}=20$ are largest for azimuth angles of 150 and $175^{\circ}$. Further, at both 0.632 and $1.605 \mu \mathrm{~m}$ the difference $R_{\mathrm{n}}-R_{200}$ oscillates around a certain value. It is most prominent for the backscatter region at higher solar zenith angle (low $\mu_{\circ}$ ). The results for $r_{\text {eff }}=5-24 \mu \mathrm{~m}$ at $\lambda=0.632 \mu \mathrm{~m}$ and $\lambda=1.605 \mu \mathrm{~m}$ are summarized in Tables 1 and 2, respectively. Reflectance simulations for $r_{\text {eff }}=1 \mu \mathrm{~m}$ did not show any change with increasing $n_{\text {Fourier }}$. Therefore it is very likely that convergence has already taken place at $n_{\text {Fourier }}$ lower than 20. For $r_{\text {eff }}=3 \mu \mathrm{~m}$ the calculations converged around $n_{\text {Fourier }}=40$. Due to the vicinity of the backscatter peak, the largest number of Fourier terms was required at relative azimuth angles of $175^{\circ}$. It should be noted that the number of required Fourier terms at an azimuth angle of $180^{\circ}$ are significantly higher than the values presented here.

Table 1: Number of Fourier terms in azimuth at $\lambda=0.632 \mu \mathrm{~m}$ required for convergence $\left(R_{\mathrm{n}}-R_{200} \leq 0.005\right)$ at different $r_{\text {eff }}$. Values presented are maximum for the range $\mu=\mu_{\circ}=0.2-0.8, \varphi-\varphi_{\circ}=175^{\circ}$ and $150^{\circ}$.

| $r_{\text {eff }}[\mu \mathrm{m}]$ | $\varphi-\varphi_{\mathrm{o}}=175^{\circ}$ | $\varphi-\varphi_{\mathrm{o}}=150^{\circ}$ |
| :--- | :--- | :--- |
| 5 | 120 | 40 |
| 8 | 180 | 60 |
| 12 | 180 | 60 |
| 16 | 180 | 60 |
| 24 | 180 | 80 |



Figure 1: Mie phase functions for spherical water droplets at $r_{\text {eff }}=3-24 \mu \mathrm{~m}$, $v_{\text {eff }}=0.15, \lambda=0.632 \mu \mathrm{~m}$ (upper panel) and $\lambda=1.605 \mu \mathrm{~m}$ (lower panel). Refractive indices are taken from Downing and Williams (1975).

Table 2: Number of Fourier terms in azimuth at $\lambda=1.605 \mu \mathrm{~m}$ required for convergence $\left(R_{\mathrm{n}}-R_{200} \leq 0.005\right)$ at different $r_{\text {eff }}$. Values are for $\mu=\mu_{\circ}=0.2-0.8, \varphi-\varphi_{\circ}=175^{\circ}$ and $150^{\circ}$.

| $r_{\text {eff }}[\mu \mathrm{m}]$ | $\varphi-\varphi_{o}=175^{\circ}$ | $\varphi-\varphi_{\mathrm{o}}=150^{\circ}$ |
| :--- | :--- | :--- |
| 5 | 80 | 60 |
| 8 | 80 | 40 |
| 12 | 140 | 40 |
| 16 | 180 | 40 |
| 24 | 180 | 60 |

### 3.2. Gaussian $\mu$ point convergence analysis

Figure 4 and 5 clearly show the rapid convergence of cloud reflectance with increasing $n_{\text {Gauss }}$. At high solar zenith angles, the difference $R_{\mathrm{n}}-R_{200}$ is large at 60 Gaussian $\mu$ points ( $0.05-0.06$ ), but an increase towards $n_{\text {Gauss }}=80$ decreases this difference by a factor of 2-3. Results for the convergence study are summarized in Table 3 and 4. From the Tables it follows that the number of $n_{\text {Gauss }}$ required for an accurate simulation strongly increases with $r_{\text {eff }}$, especially at $0.632 \mu \mathrm{~m}$. At $1.605 \mu \mathrm{~m}$, calculations converge more rapidly for $r_{\text {eff }}=5-12 \mu \mathrm{~m}$. As for the Fourier convergence, using an azimuth difference angle of $175^{\circ}$ requires a larger number of Gaussian $\mu$ points than at smaller azimuth difference angles.

Table 3: Number of Gaussian points in $\mu$ at $\lambda=0.632 \mu \mathrm{~m}$ needed for convergence ( $R_{n}-R_{200} \leq 0.005$ ) at different $r_{\text {eff. }}$. Values presented are maximum for the range $\mu=\mu_{o}=0.2-0.8, \varphi-\varphi_{\circ}=175^{\circ}$ and $150^{\circ}$.

| $r_{\text {efff }}[\mu \mathrm{m}]$ | $\varphi-\varphi_{0}=175^{\circ}$ | $\varphi-\varphi_{0}=150^{\circ}$ |
| :--- | :--- | :--- |
| 5 | 80 | 60 |
| 8 | 80 | 60 |
| 12 | 100 | 60 |
| 16 | 140 | 140 |
| 24 | 180 | 180 |



Figure 2: Absolute difference in cloud reflectance at $\lambda=0.632 \mu \mathrm{~m}$ compared to reflectance at assumed convergence point ( 200 Fourier terms), $\mu=\mu_{\circ}=0.2-0.8$, $r_{\text {eff }}=12 \mu \mathrm{~m}$, azimuth angle difference 90 (solid), 120 (dotted), 150 (dashed) and $175^{\circ}$ (dashed-dotted).


Figure 3: Absolute difference in cloud reflectance at $\lambda=1.605 \mu \mathrm{~m}$ compared to reflectance at assumed convergence point ( 200 Fourier terms), $\mu=\mu_{\circ}=0.2-0.8$, $r_{\text {eff }}=12 \mu \mathrm{~m}$, azimuth angle difference 90 (solid), 120 (dotted), 150 (dashed) and $175^{\circ}$ (dashed-dotted).

Table 4: Number of Gaussian points in $\mu$ at $\lambda=1.605$ needed for convergence $\left(R_{n}-\right.$ $R_{200} \leq 0.005$ ) at different $r_{\text {eff }}$. Values presented are maximum for the range $\mu=\mu_{\circ}$, $\varphi-\varphi_{\circ}=175^{\circ}$ and $150^{\circ}$.

| $r_{\text {eff }}[\mu \mathrm{m}]$ | $\varphi-\varphi_{\mathrm{o}}=175^{\circ}$ | $\varphi-\varphi_{\mathrm{o}}=150^{\circ}$ |
| :--- | :--- | :--- |
| 5 | 20 | 20 |
| 8 | 20 | 40 |
| 12 | 40 | 40 |
| 16 | 40 | 40 |
| 24 | 60 | 40 |

### 3.3. Effect of varying $v_{\text {eff }}$

In general, for the $0.632 \mu \mathrm{~m}$ reflectance the effect of varying $v_{\text {eff }}$ is small. Differences were calculated as absolute $\left(R_{\mathrm{X}}-R_{0.15}\right)$ as well as relative differences, $\frac{R_{0.10}-R_{0.15}}{R_{0.15}}$, for all geometries ( $\mu=\mu_{0}, \varphi-\varphi_{0}$ ), with suffix X referring to the value of $v_{\text {eff }}$, being 0.10 or 0.20 , and suffix 0.15 referring to the standard value of $v_{\text {effr }}$. The results are summarized in Table 5.

Table 5: Maximum absolute and relative differences between modeled $0.632 \mu \mathrm{~m}$ cloud reflectances using $v_{\text {eff }}=0.10,0.15$, and 0.20 , standard water cloud case applied.

| $r_{\text {eff }}[\mu \mathrm{m}]$ | $R_{0.10}-R_{0.15}$ | $\frac{R_{0.10}-R_{0.15}}{R_{015}}[\%]$ | $\left(R_{0.20}-R_{0.15}\right)$ | $\frac{R_{0.20}-R_{0.15}}{R_{015}}[\%]$ |
| :--- | :--- | :--- | :--- | :--- |
| 5 | -0.0041 | -0.58 | 0.0045 | 0.63 |
| 8 | -0.0095 | -1.06 | 0.0073 | 0.82 |
| 12 | -0.0053 | -0.91 | 0.0059 | 1.02 |
| 16 | -0.0052 | -1.07 | 0.0047 | 0.96 |
| 24 | -0.0157 | -3.82 | 0.0146 | 3.56 |

It follows from the Table that absolute differences are generally in the order of 0.005-0.01. When a look is taken at the relative differences, it follows that from $5-16 \mu \mathrm{~m}$ differences are typically in the order of $1 \%$. However, at $24 \mu \mathrm{~m}$ the relative difference has increased to $3.56 \%$ for $v_{\text {eff }}=0.20$ and $-3.82 \%$ for $v_{\text {eff }}=0.10$.

### 3.4. Effect of neglecting linear polarization

Table 6 presents the differences of cloud reflectance calculations at $\lambda=0.632 \mu \mathrm{~m}$ performed with and without linear polarization included. Obviously, the effect of
neglecting linear polarization is small at all effective radii. The maximum relative difference is $1.05 \%$ at $r_{\text {eff }}=8 \mu \mathrm{~m}$. To obtain a general indication of the effect of neglecting linear polarization, absolute differences were averaged over all zenith and azimuth angles. The results are presented in the last column of Table 6. It is evident that this averaged relative difference is almost negligible, with values of 0.1-0.3\%.

Table 6: Maximum absolute, maximum and mean relative difference in $0.632 \mu \mathrm{~m}$ cloud reflectance modeled with and without linear polarization included.

| $r_{\text {eff }}[\mu \mathrm{m}]$ | max. abs. diff. | max. rel. diff. [\%] | mean rel. diff. [\%] |
| :--- | :--- | :--- | :--- |
| 5 | 0.0032 | 0.42 | 0.12 |
| 8 | 0.0063 | 1.05 | 0.32 |
| 12 | 0.0035 | 0.45 | 0.13 |
| 16 | 0.0037 | 0.48 | 0.15 |



Figure 4: Absolute difference in cloud reflectance at $\lambda=0.632 \mu \mathrm{~m}$ compared to reflectance at assumed convergence point ( 200 Gauss points in $\mu$ ), $\mu=\mu_{\circ}, r_{\text {eff }}=12 \mu \mathrm{~m}$, azimuth angle difference 90 (solid), 120 (dotted), 150 (dashed) and $175^{\circ}$ (dasheddotted).


Figure 5: Absolute difference in cloud reflectance at $\lambda=1.605 \mu \mathrm{~m}$ compared to reflectance at assumed convergence point ( 200 Gauss points in $\mu$ ), $\mu=\mu_{\circ}, r_{\text {eff }}=12 \mu \mathrm{~m}$, azimuth angle difference 90 (solid), 120 (dotted), 150 (dashed) and $175^{\circ}$ (dasheddotted).

## 4. Summary

The analysis presented focused on finding the required number of Fourier terms and Gaussian $\mu$ points to accurately model cloud reflectances at 0.632 and $1.605 \mu \mathrm{~m}$ by the Doubling Adding KNMI (DAK) radiative transfer model. The convergence study was restricted to a standard water cloud case with an optical thickness of 10 and a surface albedo of 0.10 .

At $0.632 \mu \mathrm{~m}$, for most effective radii and viewing geometries 60-80 Fourier terms suffice, whereas at $1.605 \mu \mathrm{~m}$ the required number of Fourier terms decreases to $40-60$. The required number of Gaussian $\mu$ points is $40-60$ for most effective radii investigated at both 0.632 and $1.605 \mu \mathrm{~m}$. It should be stressed that due to the vicinity of the backscatter peak, at both wavelengths the number of Fourier terms and Gaussian $\mu$ points strongly increases for $\varphi-\varphi_{\circ}$ approaching $180^{\circ}$.

The effective variance of the two-parameter Gamma size distribution was varied between 0.10 and 0.20 and the absolute and relative difference in cloud reflectance to values with $v_{\text {eff }}=0.15$ was calculated. The largest relative difference was found for $r_{\text {eff }}=24 \mu \mathrm{~m}$, having a value of $\sim 4 \%$.

Lastly, the effect of neglecting linear polarization was investigated. The relative difference between cloud reflectance calculations at $0.632 \mu \mathrm{~m}$ performed with and without linear polarization appeared to be small with a maximum relative difference of $\sim 1 \%$ and mean relative differences of $0.1-0.3 \%$, which indicates that linear polarization can be neglected in cloud reflectance calculations.

## References

De Haan, J. F., P. B. Bosma, and J. W. Hovenier, 1987: The adding method for multiple scattering calculations of polarized light. Astron. \& Astrophys., 183, 371-391.

De Rooij, W. A. and C. C. A. H. van der Stap, 1984: Expansion of Mie scattering matrices in generalized spherical functions. Astron. \& Astrophys., 131, 237248.

Deirmendjian, D., 1969: Electromagnetic scattering on spherical polydispersions. American Elsevier, 312 pp.

Downing, H. D. and D. Williams, 1975: Optical constants of water in the infrared. J. Geophys. Res., 80, 1656-1661.

Hansen, J. E. and L. D. Travis, 1974: Light scattering in planetary atmospheres. Space Sci. Rev., 16, 527-610.

Nakajima, T. and M. D. King, 1990: Determination of the optical thickness and effective particle radius of clouds from reflected solar radiation measurements, part 1: Theory. J. Atmos. Sci., 47, 1878-1893.

Roebeling, R. A., A. Berk, A. J. Feijt, W. Frerichs, D. Jolivet, A. Macke, and P. Stammes, 2005: Sensitivity of cloud property retrievals to differences in radiative transfer simulations. Scientific report, KNMI WR 2005-02.

Roebeling, R. A., A. J. Feijt, and P. Stammes, 2006: Cloud property retrievals for climate monitoring: Implications of differences between Spinning Enhanced Visible and Infrared Imager (SEVIRI) on METEOSAT-8 and Advanced Very High Resolution Radiometer (AVHRR) on NOAA-17. J. Geophys. Res., 111, doi:10.1029/2005JD006990.

Stammes, P., 2001: Spectral radiance modelling in the UV-Visible range. IRS 2000: Current problems in Atmospheric Radiation, W. L. Smith and Y. M. Timofeyev, eds., A. Deepak, Hampton, VA, 385-388.

