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Rainfall generator for the Meuse basin: Description of simulations with and without a memory term and uncertainty analysis

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1. INTRODUCTION

The rainfall generator has been developed to generate long synthetic sequences of daily precipitation and temperature for the Meuse basin [see e.g. Leander and Buishand (2004), Leander et al. (2005), and Buishand and Leander (2011)] using the nearest-neighbour resampling (NNR) technique. These sequences have been used for discharge simulations with the semi-distributed HBV model (Aalders et al., 2004; Leander et al., 2005) to estimate the design discharge for flood protection works in the Netherlands. For the non-tidal embanked part of the river, the 1250-year return level of the discharge at Borgharen (near the city of Maastricht) is presently used as design discharge. An important source of uncertainty of this design discharge is the length of the historical records used for resampling. In order to study the sensitivity of the 1250-year return level to the choice of the historical data, several 20,000-year simulations were conducted with various 33-year subsets of the 1930-1998 period as well as a 20,000-year simulation based on the whole 1930-1998 period (Leander and Buishand, 2008). Apart from the average winter rainfall of the subset, it turned out that the presence or absence of the year 1995 in the subset strongly influenced the estimate of the 1250-year return level of the maximum 10-day winter basin-average rainfall as well as the estimated 1250-year return level of the discharge at Borgharen (Kramer et al., 2008). A difficulty with this sensitivity analysis is that it does not provide the standard deviation of the estimated return level. Therefore, for the Rhine basin a jackknife method was used to determine the uncertainty of the return level (Schmeits et al., 2014). In the present report this jackknife method is applied for the rainfall generator for the Meuse basin. Further, two alternative forms of the rainfall generator are considered to downweight the influence of the year 1995 on the estimated return levels, and the maximum 4-, 10- and 20-day basin-average rainfall amounts are studied rather than the maximum 10-day basin-average rainfall only.

This report is set up as follows. The datasets that were used in this study are summarized in section 2. The nearest neighbour resampling technique and the different forms of the rainfall generator are described in section 3. Results for the various simulations are shown in section 4 and the uncertainty analysis is presented in section 5. Finally conclusions are drawn in section 6.

2. DATASETS

As the datasets that were used are described in detail in Buishand and Leander (2011), here only a short description is given. Three datasets are used as input for the rainfall generator: a dataset consisting of daily data from 7 precipitation stations and 2 temperature stations for the period 1930-2008 (excluding the period 1940-45[1]), and 2 datasets that contain daily precipitation and temperature data for each of the 15 HBV subbasins of the Meuse basin upstream from Borgharen for the periods 1961-2007 and 1967-2008, respectively. The locations of the stations and the HBV subbasins are shown in Figures 2.1 and 2.2, respectively. The names of the HBV subbasins correspond to those used for rainfall-runoff-modelling (Hegnauer, 2013).

[1] This 5-year period was also excluded in the simulation of Buishand and Leander (2011) based on the historical data for the period 1930-2008 (sim30-08).

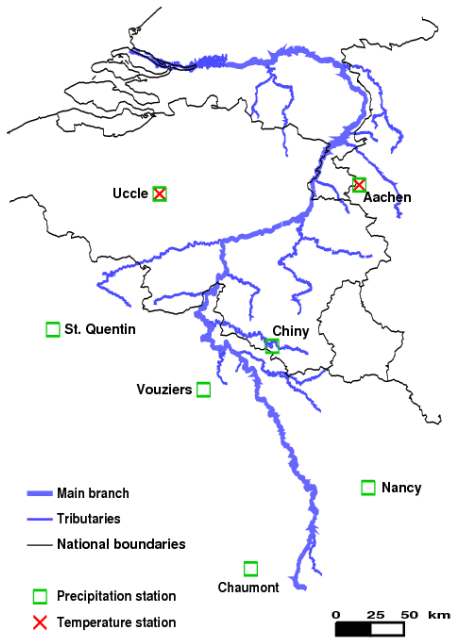

Figure 2.1: *Location of the 7 precipitation and 2 temperature stations in the Meuse basin. Chiny was closed after January 1987. A nearby station, Lacuisine, was used to continue this series.*

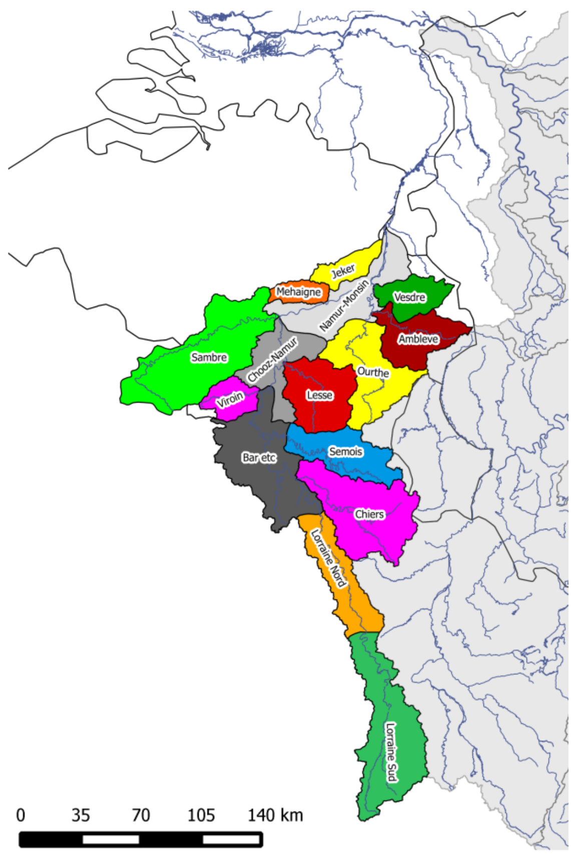

Figure 2.2: Location of the 15 HBV subbasins of the Meuse basin.

3. SIMULATION SETUP

3.1 General

With the NNR technique, weather variables are resampled simultaneously from the historical data. To incorporate autocorrelation, the resampling depends on the values of the previous resampled day. Therefore the days in the historical record that are most similar to those of the previously simulated day in terms of precipitation and temperature are pre-selected. One of these *k* nearest neighbours is randomly selected and the observed values for the day subsequent to that nearest neighbour are adopted as the simulated values for the next day. In the random selection from the *k* nearest neighbours, a decreasing kernel is used to give more weight to the closest neighbours. In line with Beersma (2002) and Leander and Buishand (2004), *k* is set to 10.

A feature vector is used to find the nearest neighbours in the historical record. In the rainfall generator for the Meuse basin the feature vector is composed of three elements:

- the average standardized daily temperature of the 2 temperature stations,
- the average standardized daily precipitation of the 7 rainfall stations, and
- the average standardized daily precipitation of the 7 rainfall stations, averaged over the 4 preceding days (4-day memory term).

Standardization is done to reduce the effect of the annual cycle on the selection of the nearest neighbours. The daily temperatures are standardized by subtracting the calendar-day mean and dividing by the calendar-day standard deviation. Daily precipitation is standardized by dividing by the calendar-day mean wet-day precipitation amount. The effect of the annual cycle is further reduced by restricting the search for nearest neighbours to days within a moving window of 121 days, centered at the last simulated day (Leander and Buishand, 2007).

In the pre-selection of the *k* nearest neighbours the feature vector elements are weighted inversely proportional to their variance. This variance was globally calculated, that is, one value was calculated for the entire record, rather than one value for each calendar day, month or season separately (hence the indication global variance and *"gvar"* in the simulation name).

At the end of the simulation procedure, the resampled standardized variables are transformed back to their original scale. This backtransformation is done with the HBV subbasin data. Because these data do not cover the same period as the historical station data, a second resampling step is used in the backtransformation (Leander and Buishand, 2004; Leander et al., 2005).

3.2 Simulation types

Buishand and Leander (2011) presented a 20,000-year reference simulation based on the period 1930-2008. This simulation was indicated as Sim30-08. Unfortunately, it turned out that the daily precipitation data for Vouziers (Figure 2.1) were not entered in the feature vector. Therefore a new reference simulation was produced in this study. To reduce the influence of the length of the simulation on the estimated return levels, this new reference simulation has a length of 50,000 years instead of 20,000 years. The new reference simulation is labelled *"mem4d"* here to distinguish it from the other simulations in this report where the 4-day memory is replaced or its influence is reduced.

Leander and Buishand (2004, 2008) observed that certain historical days often occurred in the most extreme simulated 10-day basin-average precipitation amounts, in particular a number of days in January 1995. Recent simulations for the Rhine basin show that this selection effect increases when a multi-day memory term is included in the feature vector (Schmeits et al., 2014). Therefore, two other 20,000-year simulations were produced for the Meuse basin, one in which the weighting coefficient of the 4-day memory term was halved (labelled *"halfmem4d"*), and another one in which it was replaced with the areal precipitation fraction, i.e., the fraction of precipitation stations with daily precipitation larger than 0.3 mm, in the feature vector (labelled *"nomem"*). The feature vector elements used for the latter are the same as those for the rainfall generator of the Rhine basin.

The following naming convention is used for the simulations:
<memory tag>_<variance calculation tag>_<simulation length>. For each simulation the name is explained in Table 3.1, and the feature vector elements and their weighting coefficients are given in Table 3.2.

Table 3.1: *Naming convention and explanation of the simulations that were investigated.*

Name	Meaning
mem4d_gvar_50000	4-day memory term, globally calculated weights, 50,000 yr
mem4d_gvar_20000	First 20,000 yr of mem4d_gvar_50000
halfmem4d_gvar_20000	4-day memory term (half weighting coefficient), globally calculated weights, 20,000 yr
nomem_gvar_20000	No memory term, globally calculated weights, 20,000 yr

Table 3.2: *Weighting coefficients of the feature vector elements used in the 'mem4d', the 'halfmem4d' and 'nomem' simulations. The weighting coefficients for the memory term represent the values for the 'mem4d' and 'halfmem4d' (between parentheses) simulations. Note that in simulations without a memory term ('nomem'), the weight for the memory term is set to 0, while in the simulations with a memory term the weight for the areal precipitation fraction is set to 0.*

Feature vector element	(half)mem4d	nomem
Precipitation	2.19	2.19
Temperature	1.05	1.05
Areal precipitation fraction	0	6.58
Memory	(0.15) 0.30	0

4. SIMULATION RESULTS

4.1 Comparison of the old Sim30-08 with the present mem4d simulation

Figure 4.1 compares the distributions of the maximum basin-average precipitation in the winter half-year (October –March) of the old Sim30-08 simulation (Buishand and Leander, 2011) and the present mem4d simulation for three different durations. The distribution of the precipitation maxima of the mem4d simulation is closer to that of the observations than that of the old Sim30-08 simulation. To a large extent the differences between these simulations result from Vouziers accidentally being left out in the old Sim30-08 simulation (see further next section).

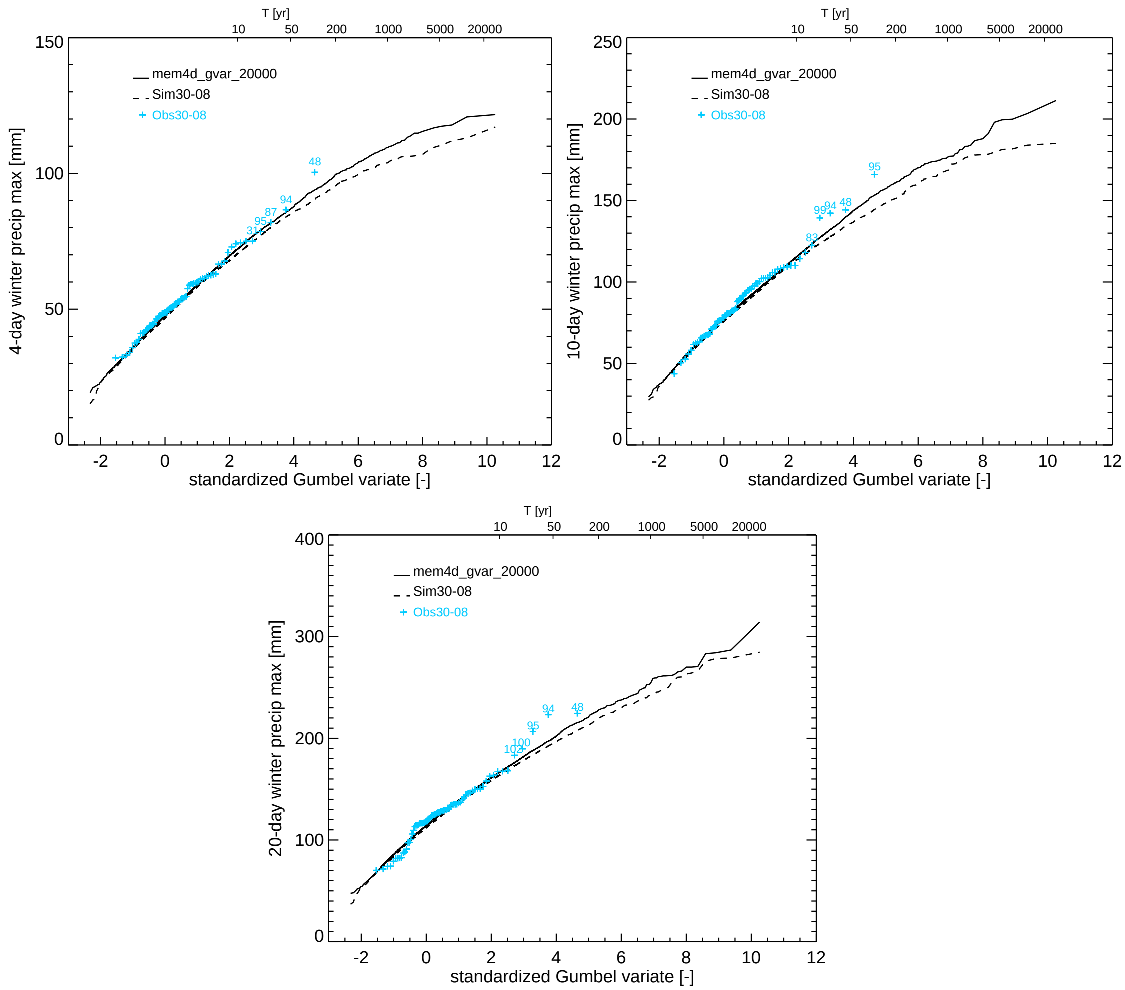

Figure 4.1: *Gumbel plots of the maximum 4-, 10-, and 20-day average precipitation over the Meuse basin in the winter half-year for the old Sim30-08 simulation (Buishand and Leander, 2011) and the first 20,000 years of the present mem4d simulation. The blue pluses indicate the ordered maxima for the historical period 1930 – 2008 (and for the top 5 the year minus 1900 is added, e.g., 95 indicates the winter half-year October 1994- March 1995). Note that for the years 1930-1960 and 2008 no daily basin averages are available and that these were replaced by the closest nearest neighbour in the period 1961-2007, cf. Leander and Buishand (2008). T denotes the return period.*

4.2 Results for simulations with and without a memory term

The 4-day memory term was included in the feature vector of the rainfall generator for the Meuse basin to improve the reproduction of the autocorrelation of the daily precipitation amounts and the standard deviation of the monthly totals (Leander and Buishand, 2004; Leander et al., 2005). Figure 4.2 compares the autocorrelation coefficients of the simulated daily rainfall sequences with those of the observations. The "nomem" simulation somewhat underestimates the autocorrelation coefficients for all lags. There is no discernable difference in autocorrelation between the "mem4d" and "halfmem4d" simulations.

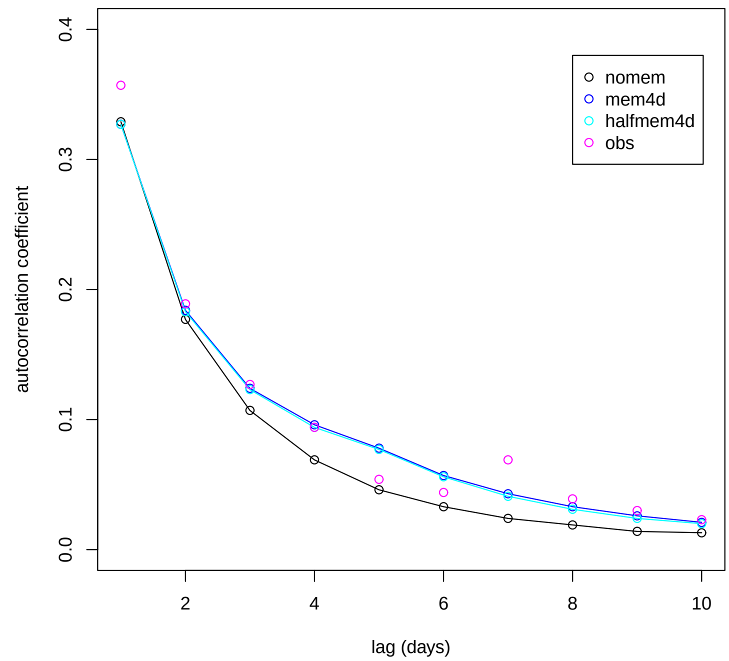

***Figure 4.2**: Basin-average autocorrelation coefficients in observed daily rainfall and in the simulations with (half) the weighting coefficient for the 4-day memory term, and without a 4-day memory term.*

Gumbel plots of the maximum 4-, 10- and 20-day basin-average precipitation amounts are shown in Figure 4.3. The differences between the two simulations with different weights for the 4-day memory term are quite small and the Gumbel plots for these simulations are closer to those for the observations than those for the simulation without the 4-day memory term. This was also found in earlier simulations for the Meuse basin (Leander et al., 2005).

As for the rainfall generator for the Rhine basin (Schmeits et al, 2014), the resampled historical days in the most extreme multi-day events in the winter half-year in the simulated series were examined. The relative frequencies of these historical days per winter are shown in Figure 4.4a, b and c for the 250 most extreme 4-day events, the 100 most extreme 10-day events, and the 50 most extreme 20-day events, respectively. For the most extreme 4-day events, days from the winter of 1948 were selected considerably more often in all simulations than historical days from other

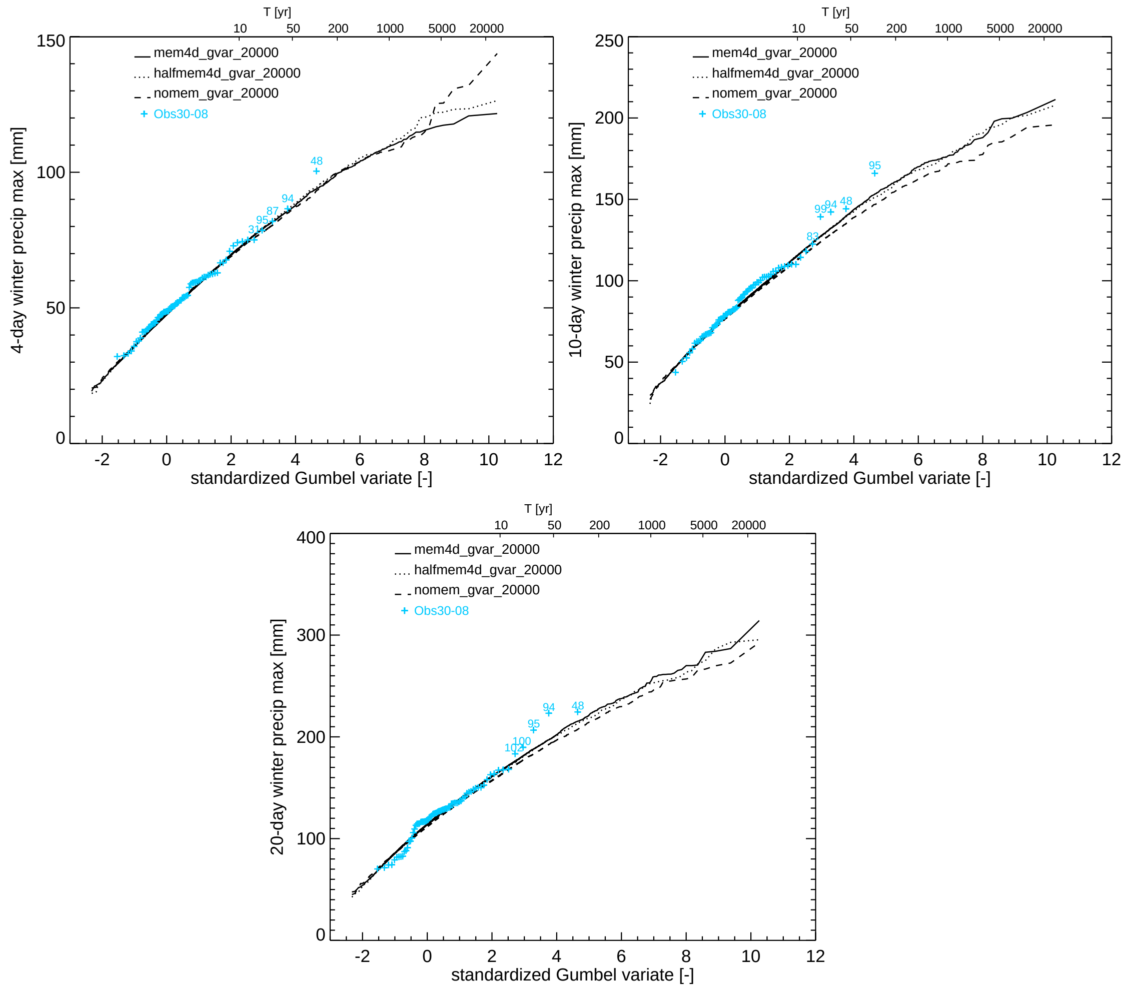

Figure 4.3: *As in Figure 4.1, but now for simulations with half the weighting coefficient for the 4-day memory term (dotted) and without a memory term (dashed). The plot for the first 20,000 years of the simulation with a 4-day memory term (mem4d) is included for reference and is indicated by the solid line.*

winters, with a relative frequency of more than 30%. Note that this large relative frequency involves only a few days from December 1947. A secondary maximum frequency is visible in 1995. By contrast, for the 100 most extreme 10-day events the simulations with a 4-day memory term show a maximum frequency of about 20% in 1995, but the frequency of selected historical days from 1995 is much lower (about 7%) for the simulation without a memory term. Days from the winter of 1948 are also frequently found in the most extreme simulated 10-day events. For the most extreme 20-day events all frequencies are below 11%.

As was also noted by Schmeits et al. (2014) for the Rhine basin, the results of Figure 4.4 give rise to suspect the resampling technique having a strong preference to specific periods with extremely high multi-day precipitation amounts. This strong preference to certain historical days in the maximum multi-day precipitation amounts, especially in simulations with a memory term, probably leads to an increase of the standard error of the extreme quantiles of these multi-day precipitation amounts compared to simulations without a memory term. This is further investigated in section 5.

Despite the fact that days from the winter of 1995 are more often selected if a 4-day memory term is included in the feature vector (Fig. 4.4), we can conclude from the other results in this section (Figs. 4.2 and 4.3) that the rainfall generator with a 4-day memory term is preferable, which is in line with earlier studies (Leander and Buishand, 2004; Leander et al., 2005).

The frequent occurrence of days from December 1947 in the simulated extreme 4-day and 10-day events is also the key for understanding the influence of omitting the Vouziers data on the simulated multi-day precipitation maxima. In December 1947 flooding occurred in the French part of the basin (de Wit, 2008). For that month also the largest historical 4-day basin-average precipitation is found and precipitation in Vouziers, in particular, was extreme in that period. Consequently, the resampled days from the December 1947 event are generally coupled with more extreme precipitation over the sub-basins in the back transformation if the Vouziers data are included in the feature vector. The extreme multi-day events in the present "mem4d" simulation are therefore larger than in the old Sim30-08 simulation.

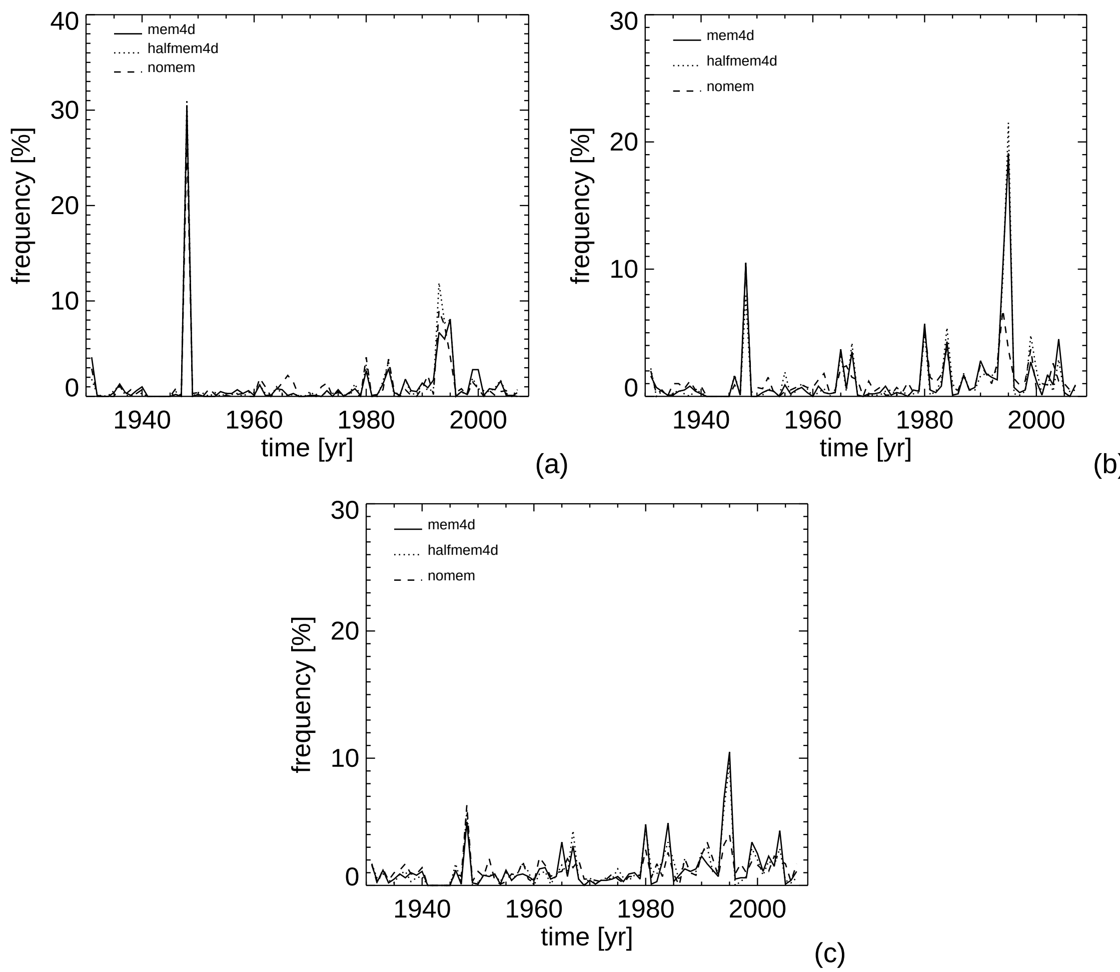

Figure 4.4: *Frequency of simulated days from historical years for (a) the 250 most extreme 4-day precipitation winter maxima, (b) the 100 most extreme 10-day precipitation winter maxima, and (c) the 50 most extreme 20-day precipitation winter maxima in the simulations without a memory term, and with (half) the weighting coefficient for the 4-day memory term. The frequencies plotted at each year y were computed using data from the last months of year y-1 through the first months of year y.*

5. UNCERTAINTY ANALYSIS

In this section the uncertainty analysis for the rainfall generator for the Meuse basin is presented. The jackknife method used by Schmeits et al. (2014) for the Rhine basin was also applied to determine the standard deviation of the estimated return levels for the simulated data for the Meuse basin. First, the uncertainty analysis is discussed in detail for the rainfall generator with a 4-day memory term in the feature vector. Then the results for the rainfall generator without this memory term are given. Finally, sampling variability resulting from the finite length of the simulation run is explored.

Twenty-four jackknife series of 69 years were formed by leaving out subsequent non-overlapping 3-year blocks from the original series of 72 years (with the first block being 1931-33[2]). For each jackknife series a 20,000-year simulation with a 4-day memory term was conducted similar to the 50,000-year reference simulation that includes all 72 years (the names of the NetCDF files of these simulations and the reference simulation are given in Appendix A1). Let $\hat{x}_{T(i)}$ be the estimated T-year return level from the i-th jackknife series. Then the jackknife standard deviation of the estimated T-year return level is given by:

$$s_{jack} = \left\{ \frac{n-1}{n} \sum_{i=1}^{n} \left[\hat{x}_{T(i)} - \hat{x}_{T(\bullet)} \right]^2 \right\}^{1/2} \qquad (1)$$

where

$$\hat{x}_{T(\bullet)} = \sum_{i=1}^{n} \hat{x}_{T(i)} / n$$

and n (= 24) is the number of jackknife series. Because of less computational restraints on the hydrological part for the Meuse basin than for the Rhine basin (Schmeits et al. 2014), smaller blocks could be used in the jackknife method (3 years instead of 5 years). This yields a more accurate estimate of the standard deviation since the variance of s_{jack} decreases with decreasing blocksize. Figure 5.1 shows the Gumbel plots of the 20,000-year simulations based on these 24 jackknife series together with the Gumbel plot of the 50,000-year mem4d reference simulation. The plots for two jackknife series are highlighted in this figure: one without the 1946-48 block (black dotted) and one without the 1994-96 block (red dotted). These show the lowest precipitation maxima at long return periods, which is not surprising because a winter from which days were often selected in situations of extreme multi-day rainfall (1948 or 1995, see Figure 4.4) was left out in the respective jackknife series.

Table 5.1 compares two estimates of the 1250- and 4000-year return levels of the 4-, 10- and 20-day precipitation maxima in the winter half-year. The empirical estimate is computed as the 16th and 5th largest value in each of the 20,000-year simulations for T = 1250 and 4000 years, respectively, while the Weissman estimate is based on the joint distribution of the r largest values $x_{[1]} \geq x_{[2]} \geq \ldots \geq x_{[r]}$ (Weissman, 1978). In the Weissman method the T-year return level x_T is estimated as

$$\hat{x}_T = x_{[r]} + \hat{\sigma} \ln(rT / N) \qquad (2)$$

[2] Or, more precisely, 1 October 1930 -30 September 1933. The start of a block was set at 1 October to avoid splitting the winter half-years.

with N (= 20,000) the length of the simulation, and

$$\hat{\sigma} = \bar{x}_r - x_{[r]} \tag{3}$$

where $\bar{x}_r$ is the average of the *r* largest values. In line with the uncertainty analysis for the Rhine basin (Schmeits et al., 2014), *r* was set to 100. The estimated return levels in the table are the averages $\hat{x}_{T(\bullet)}$ from the 24 jackknife series and their standard deviations are based on Eq. (1).

The estimated 1250- and 4000-year return levels of the 4-, 10- and 20-day precipitation maxima in the winter half-year for the Weissman method agree quite well with the empirical estimates, but the standard deviation of the Weissman estimate is somewhat smaller for the 4000-year return level. The relative standard deviation, $s_{jack} / \hat{x}_{T(\bullet)}$, ranges from 11 to 17% for the empirical estimate and from 11 to 15% for the Weissman estimate.

The same analysis was done for the rainfall generator in which the 4-day memory term was replaced by the areal precipitation fraction. The results are given in Table 5.2. The standard deviations are lower than those in Table 5.1 for the rainfall generator with the 4-day memory term, except for the empirical estimates of the 4000-year return level. For the Weissman estimate the relative standard deviation ranges from 8 to 12%. The reduction in standard deviation may be attributed to the fact that the rainfall generator without the 4-day memory term does not suffer from generating extreme multi-day rainfall events by repeatedly selecting certain days in January 1995. Nevertheless, the relative standard deviations in Table 5.2 are nearly twice as large as those given by Schmeits et al. (2014) for a rainfall generator for the Rhine basin without a 4-day memory term. Though the relative variability of area-average rainfall over the Meuse basin is somewhat larger than that for the Rhine basin because of the smaller size of the Meuse basin, this cannot explain the large differences between the relative standard deviations of the estimated return levels. A more important factor is the large uncertainty of these standard deviations, in particular those for the Rhine basin which were based on only 11 jackknife series.

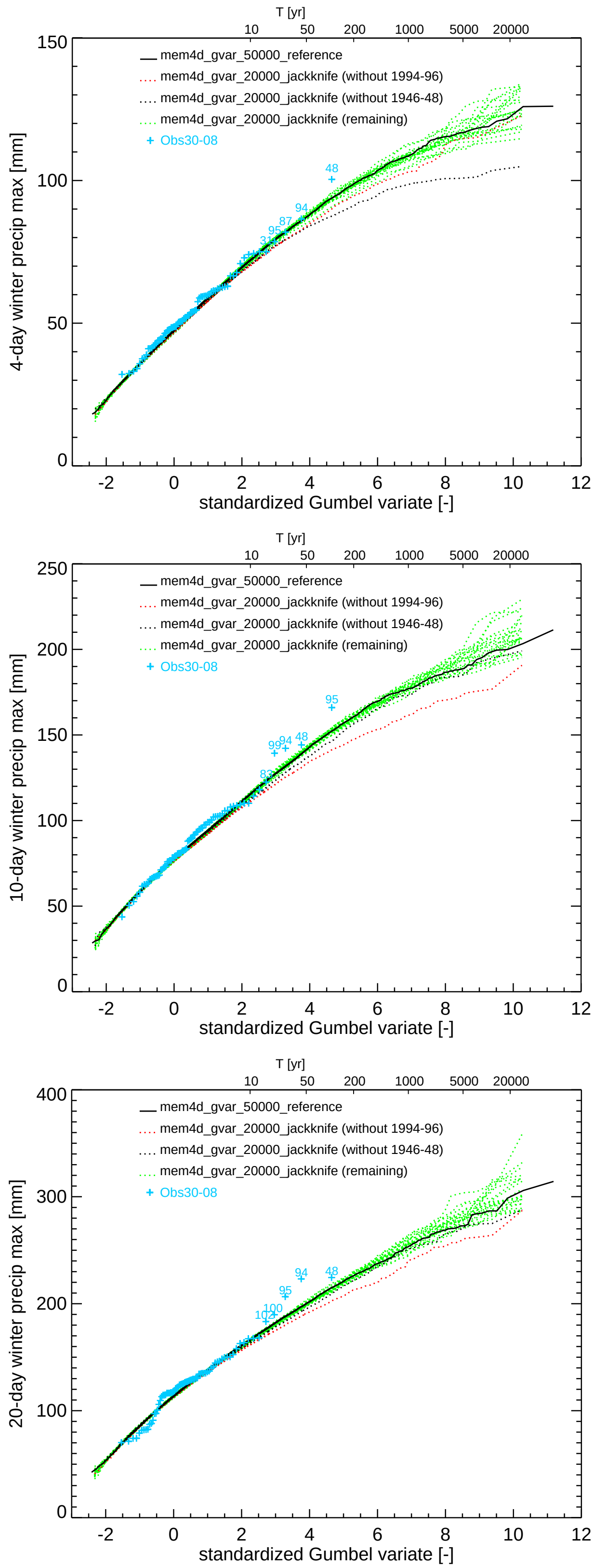

Figure 5.1: *As in Figure 4.1, but now for 20,000-year simulations with a 4-day memory term based on 24 jackknife series of 69 years (black, red and green dotted; see legend), and the complete 50,000-year "mem4d" simulation as a reference (black solid).*

Table 5.1: *Estimated return levels of 4-, 10- and 20-day winter half-year precipitation maxima and their standard deviations for the 20,000-year simulations with a 4-day memory term, based on 24 jackknife series of 69 historical years and for return periods of 1250 and 4000 years. Both empirically determined and using the Weissman method (r = 100), see text.*

T (yr)	Return level (mm)		Standard deviation	
	Empirical	Weissman	Empirical	Weissman
		4-day precipitation		
1250	109	109	14 mm (13%)	14 mm (13%)
4000	115	116	19 mm (17%)	17 mm (15%)
		10-day precipitation		
1250	179	179	19 mm (11%)	19 mm (11%)
4000	190	191	27 mm (14%)	23 mm (12%)
		20-day precipitation		
1250	259	257	29 mm (11%)	27 mm (11%)
4000	278	277	40 mm (15%)	35 mm (13%)

Table 5.2: *As Table 5.1 but now for the 20000-year simulations without a memory term.*

T (yr)	Return level (mm)		Standard deviation	
	Empirical	Weissman	Empirical	Weissman
		4-day precipitation		
1250	109	109	8 mm (7%)	11 mm (10%)
4000	116	116	20 mm (17%)	12 mm (11%)
		10-day precipitation		
1250	176	176	17 mm (10%)	17 mm (10%)
4000	189	189	28 mm (15%)	22 mm (12%)
		20-day precipitation		
1250	248	247	24 mm (10%)	19 mm (8%)
4000	267	265	40 mm (15%)	28 mm (11%)

Part of the uncertainty of the return levels is due to the limited length of the baseline series used for resampling. Another source of uncertainty is the finite length of the simulation run. The magnitude of this uncertainty can be estimated using asymptotic expressions for the variances of order statistics and the Weissman estimate. The empirical estimates in Tables 5.1 and 5.2 were based on the s-largest value $x_{[s]}$, where $s = N/T$. The variance of this order statistic can be approximated as (see Appendix A2):

$$\mathrm{var}\left(x_{[s]}\right) \approx \frac{T}{N}\sigma^2 \qquad (4)$$

where σ is the scale parameter of an underlying Gumbel distribution. This scale parameter is estimated from Eq. (3). The variance of the Weissman estimate can be approximated as (see Appendix A3):

$$\mathrm{var}(\hat{x}_T) \approx \frac{\sigma^2}{r}\left\{1+\left[\ln(rT/N)\right]^2\right\} \tag{5}$$

This approximation relies on properties of order statistics from an exponential distribution and may therefore not apply in case of departures from an exponential tail. Departures from exponentiality may lead to systematic errors in the estimated variances. The bootstrap is an alternative that provides an estimate of the variance of order statistics without making assumptions about the underlying distribution. It also provides an estimate of the variance of the Weissman estimate, which is still valid if exponentiality is not precisely met. In this method samples of size N are generated by sampling with replacement from the simulated maxima. The standard deviation is then based on the estimated return levels for these bootstrap samples.

Table 5.3: *Relative standard deviation (%) of the estimated 1250- and 4000-year return levels of 4-, 10- and 20-day winter half-year precipitation maxima owing to the finite length of the simulation, Results are given for the empirical estimate of the return levels as well as for the Weissman estimate (r = 100) and apply for a 20,000-year simulation with a 4-day memory term in the feature vector.*

T (yr)	Empirical		Weissman	
	Asymptotic, Eq. (4)	Bootstrap	Asymptotic, Eq. (5)	Bootstrap
		4-day precipitation		
1250	1.3	1.3	0.9	1.1
4000	2.2	1.4	1.4	1.5
		10-day precipitation		
1250	1.5	1.6	1.1	1.3
4000	2.5	3.1	1.6	1.8
		20-day precipitation		
1250	1.5	1.4	1.2	1.2
4000	2.5	3.0	1.8	1.8

Table 5.3 presents the relative standard deviation of the estimated 1250- and 4000-year return levels of the 4-, 10-, and 20-day precipitation maxima for a mem4d simulation of 20,000 years. The bootstrap estimates are based on 500 bootstrap samples of 20,000 years. For the empirical estimate of the 1250-year return level, the standard deviation from Eq. (4) corresponds well with the bootstrap estimate, but this does not hold for the empirical estimate of the 4000-year return level. For that return level the bootstrap estimate is unreliable, because the number of different values that the estimated 4000-year return level in the bootstrap samples take is limited, namely it coincides with $x_{[5]}$ and order statistics close to $x_{[5]}$, making it vulnerable to unusual data points (Davison and Hinkley,1997). For the Weissman estimate, the bootstrap results in slightly larger values of the standard deviation than Eq. (5). In that case it is saver to use the bootstrap results. The relative standard deviations in Table 5.3 are much smaller than those in Table 5.1, which means that only a small part of the

uncertainty is due to the finite length of the simulation. Nevertheless, the length of the simulation may have some influence on the jackknife estimate of the standard deviation, which is sensitive to random fluctuations in the simulation for a subseries wherein an influential year is deleted. In particular, for these subseries longer simulations may be useful. For a 50,000-year simulation the relative standard deviation is reduced by a factor of $\sqrt{5/2} \approx 1.6$.

6. PET SIMULATION

All simulations with the rainfall generator for the Meuse basin presented in this report are supplemented with corresponding time series for potential evapotranspiration (PET). For this a regression between daily temperature and daily PET for each HBV subbasin is used. Using this regression and the simulated temperature for each subbasin the corresponding PET is simulated. Note that PET is not an integral part of the rainfall generator for the Meuse basin, as precipitation and temperature, but that PET is obtained from a post-processing procedure. The (regression) relation for daily PET and daily temperature T used reads:

$$\mathrm{PET} = \left[1 + \alpha_m (T - \overline{T}_m)\right] \overline{\mathrm{PET}}_m \qquad (6)$$

with $\overline{T}_m$ the mean observed temperature (˚C) and $\overline{\mathrm{PET}}_m$ the mean observed PET (mm/day) for calendar month m in the period 1968-1989 for which both daily temperature and daily PET was available. α_m varies from 0.18 ˚C^{-1} in February to 0.07 ˚C^{-1} in September and October. This regression equation is introduced and first used in Leander and Buishand (2007).

7. CONCLUSIONS

A series of simulations with and without a 4-day memory term was performed. A new simulation with a 4-day memory term based on the historical data for the period 1930-2008 was done, because in the old Sim30-08 simulation (Buishand and Leander, 2011) the data from Vouziers were accidentally left out. Gumbel plots of the 4-, 10-, and 20-day precipitation maxima of this new simulation are closer to those of the observed precipitation maxima than those of the maxima of the old Sim30-08 simulation. The nature of the differences between simulations with and without the 4-day memory term is similar to findings from analogous earlier simulations (Leander et al., 2005). For the Meuse basin, the use of a 4-day memory term improves the reproduction of the autocorrelation of the daily precipitation amounts and the distributions of the multi-day precipitation maxima. However, the rainfall generator with a 4-day memory term frequently selects days from the winter of 1995 in situations of extreme multi-day precipitation. In all simulations, certain days of December 1947 were often found in these extreme situations, no matter whether a 4-day memory term was included in the feature vector or not. It is, however, not fully clear what causes such selection effects. Halving the weight of the 4-day memory term does not reduce the selection of days from the winter of 1995.

As in Schmeits et al. (2014), a jackknife approach was followed in the uncertainty analysis in this study. Using 24 (delete 3-year) jackknife series of 69 years as input for the rainfall generator, the relative standard deviation of the estimated 1250-year return level of the 4-, 10-, and 20-day precipitation maxima varied between 11 and 13% when a 4-day memory term was included in the feature vector, and between 7 and 10% if such a memory term was not included. These values are (almost) a factor of two larger than those for the Rhine basin (Schmeits et al, 2014), which should mainly be attributed to the large uncertainty of the jackknife estimates of the standard deviation.

The main conclusion from the results in this report is that for the Meuse basin the rainfall generator with a 4-day memory term serves best as a reference for GRADE, which is in line with earlier results (Leander and Buishand, 2004; Leander et al., 2005).

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APPENDICES

A1. Names of the NetCDF files

The names of the NetCDF files start with the string Meuse_2013_v01.1_mem4d_reference_50K for the reference simulation (with mem4d_gvar_50000[_reference] as the corresponding name in this report). The 25 NetCDF files each contain 2000 years of data and are called:
Meuse_2013_v01.1_mem4d_reference_50K_part01.nc, …,
Meuse_2013_v01.1_mem4d_reference_50K_part25.nc.

For the 24 jackknife simulations the names of the NetCDF files start with the string
Meuse_2013_v01.1_mem4d_jackknife_20K_subseries01,…,
Meuse_2013_v01.1_mem4d_jackknife_20K_subseries24
(with mem4d_gvar_20000_jackknife as the corresponding name in this report).
In each subseries a different 3-year block is deleted; details of which 3-year block is deleted are given in the metadata of the NetCDF files. The 10 NetCDF files for each of the 24 jackknife simulations each contain 2000 years of data and are called:
Meuse_2013_v01.1_mem4d_jackknife_20K_subseries01_part01.nc,…..,
Meuse_2013_v01.1_mem4d_jackknife_20K_subseries01_part10.nc
to
Meuse_2013_v01.1_mem4d_jackknife_20K_subseries24_part01.nc,…..,
Meuse_2013_v01.1_mem4d_jackknife_20K_subseries24_part10.nc.

The NetCDF files contain the precipitation, temperature and evaporation data for each of the 15 HBV_Meuse subbasins and use the Gregorian calendar. Each simulation starts in the year 2001 to avoid the Gregorian correction in the year 1582.

Note that a version “Meuse_2013_v01_”
(Meuse_2013_v01_mem4d_reference_50K_... and
Meuse_2013_v01_mem4d_jackknife_20K_...) was provided earlier to Deltares that used the Julian calendar[3] and which turned out to be incompatible with the Gregorian calendar used in FEWS[4]. Version “Meuse_2013_v01_” should therefore not be used anymore.

[3] In this earlier version each NetCDF file contains 1000 years of simulated data rather than 2000 years.
[4] FEWS: Flood Early Warning System (by Deltares).

A2. The variance of empirical return levels

For a simulation of length N, the T-year return level can be estimated as the s largest winter maximum $x_{[s]}$, where $s = N/T$. For the variance of $x_{[s]}$, the following asymptotic result holds (David, 1981):

$$\mathrm{var}\left(x_{[s]}\right) \approx \frac{1/T\left(1-1/T\right)}{Nf^{2}\left(x_T\right)} \tag{A1}$$

where $f(.)$ is the probability density of the maxima. For the estimation of $\mathrm{var}(x_{[s]})$ it is assumed that the maxima follow a Gumbel distribution.

The distribution function of the Gumbel variable is given by:

$$F\left(x\right) = \exp\left[-\exp\left(-\frac{x-\mu}{\sigma}\right)\right] \tag{A2}$$

and the density

$$f\left(x\right) = \frac{1}{\sigma}\exp\left(-\frac{x-\mu}{\sigma}\right) \times \exp\left[-\exp\left(-\frac{x-\mu}{\sigma}\right)\right]. \tag{A3}$$

The T-year return level follows from:
:

$$F\left(x_T\right) = 1 - 1/T\ . \tag{A4}$$

giving

$$x_T = \mu - \sigma \ln\left[-\ln\left(1-1/T\right)\right]. \tag{A5}$$

Substituting this into Eq. (A3) gives for the density at $x = x_T$:

$$f\left(x_T\right) = -\frac{1}{\sigma}\left(1-1/T\right)\ln(1-1/T)\,. \tag{A6}$$

The variance of $x_{[s]}$ can then be approximated as

$$\mathrm{var}\left(x_{[s]}\right) \approx \frac{1/T\left(1-1/T\right)\sigma^{2}}{N\left(1-1/T\right)^{2}\left[\ln\left(1-1/T\right)\right]^{2}}\ . \tag{A7}$$

For large T, $1 - 1/T \approx 1$ and $\ln(1 - 1/T) \approx -1/T$, and Eq. (A7) reduces to:

$$\mathrm{var}\left(x_{[s]}\right) \approx \frac{T}{N}\sigma^{2}\,. \tag{A8}$$

A3. The variance of the Weissman estimate

The Weissman estimate of the T-year return level is given by Eq. (2). It can be shown that the statistics $\bar{x}_r$ and $\hat{\sigma}$ in this equation are independent with variances (Weissman, 1978; Buishand, 1989):

$$\mathrm{var}(\bar{x}_r) = \sigma^2 \psi'(r) \tag{A9}$$

$$\mathrm{var}(\hat{\sigma}) = \frac{r-1}{r^2}\sigma^2 \tag{A10}$$

where $\psi'(.)$ is the trigamma function. This leads to the following expression for the variance of $\hat{x}_T$:

$$\mathrm{var}(\hat{x}_T) = \sigma^2 \left\{ \psi'(k) + \frac{r-1}{r^2}\left[\ln(rT/N)\right]^2 \right\}. \tag{A11}$$

For large r, $\psi'(r) \approx 1/r$ and Eq. (A11) can be approximated as:

$$\mathrm{var}(\hat{x}_T) = \frac{\sigma^2}{r}\left\{1 + \left[\ln(rT/N)\right]^2\right\}. \tag{A12}$$

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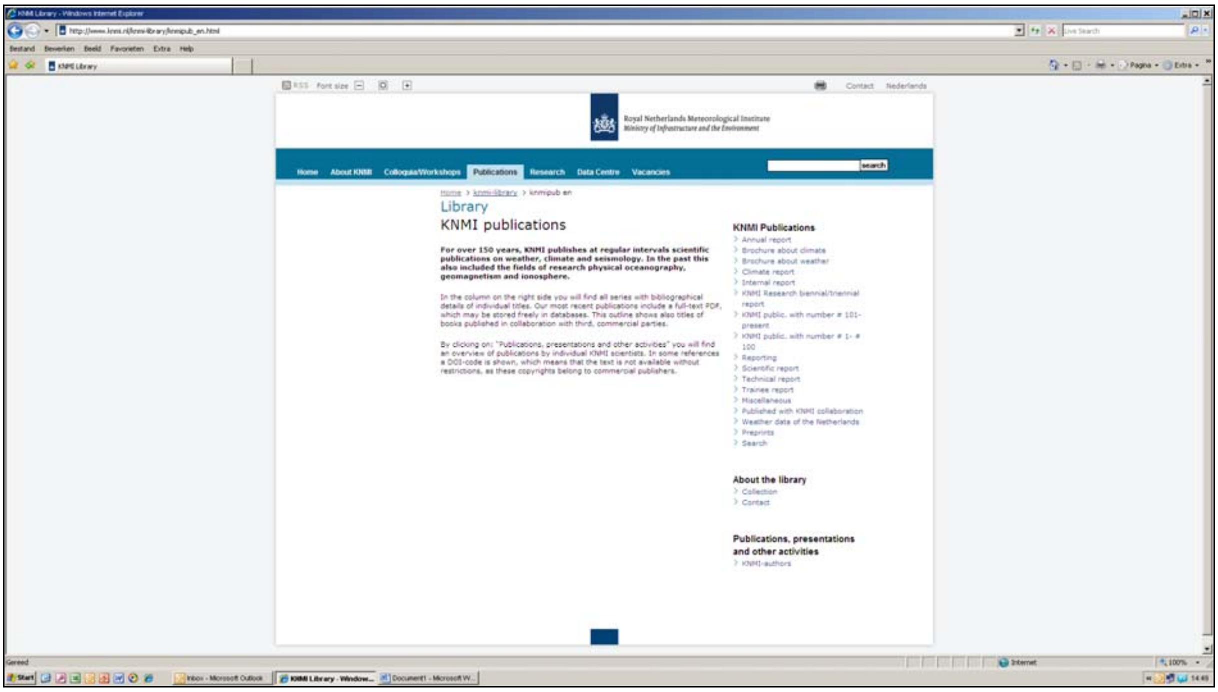

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