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# Diagnostic derivation of boundary layer parameters from the outputs of atmospheric models

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## DIAGNOSTIC DERIVATION OF BOUNDARY LAYER PARAMETERS FROM THE OUTPUTS OF ATMOSPHERIC MODELS

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### Abstract

In this paper a scheme is discussed which can be used to derive the boundary layer similarity parameters on the basis of information provided by atmospheric models. The quantities which are considered are the friction velocity, the Obukhov length, the boundary layer height and near surface variables such as wind, temperature and humidity. The necessary inputs are the mean values for wind, temperature and humidity on grid points of the atmospheric model. In the paper the outputs of the routines are illustrated for receptor points above land and sea, using forecasted and analyzed fields of the limited area model at KNMI. The outputs of the scheme are intended for use in air pollution dispersion studies, ocean wave and storm-surge models, and for direct use by forecasters.

### 1. Introduction

At present there is an increasing need for detailed description of the Atmospheric Boundary Layer (ABL) for a wide range of applications, such as regional air pollution simulations, ocean wave prediction modelling and for direct use by weather forecasters. In principle, weather forecast (or climate) models may provide wind, temperature and humidity with the resolution of the model's grid. Often, however, there is a need for quantities not directly provided by the atmospheric model, such as near surface wind, temperature and humidity. In addition also the height of the ABL is of importance.

The listed ABL parameters are directly related to the surface fluxes of momentum, heat and moisture. Generally, the surface fluxes will depend on the specified characteristics of the model surface, such as roughness length and moisture availability. The latter model quantities may differ from the actual values at a receptor point for which a more detailed forecast is needed. In addition, near a coast line the horizontal resolution of an atmospheric model will typically not be sufficient to represent the land/sea transition. As such there is a need to calculate the surface fluxes from the output of atmospheric models.

It is the purpose of this paper to present a comprehensive set of routines to derive the denoted boundary layer parameters from wind, temperature and humidity provided by the atmospheric model. As such we will describe the derivation of the surface fluxes and the Obukhov length (section 2), the surface layer quantities (section 3) and the boundary layer height (section 4). Subsequently, section 5 contains examples of results with the proposed routines on basis of the outputs of the limited area model at KNMI. In section 5 also a comparison with observations is made.

## 2. The surface fluxes and the Obukhov length

In atmospheric models, the surface fluxes of momentum ( $\tau$ ), sensible heat ( $H$ ), and latent heat ( $\lambda E$ ), are usually expressed as (e.g., Louis, 1979)

$$\frac{\tau}{\rho} \equiv u_*^2 = C_M U_1^2, \quad (1)$$

$$\frac{-H}{\rho c_p} \equiv u_* \theta_* = C_H U_1 (\theta_1 - \theta_s), \quad (2)$$

$$\frac{-\lambda E}{\rho \lambda} \equiv u_* q_* = C_Q U_1 (q_1 - q_s), \quad (3)$$

where  $\rho$  is the density,  $c_p$  is the specific heat and  $\lambda$  is the latent heat of vaporization.  $U_1$ ,  $\theta_1$  and  $q_1$  are wind speed, potential temperature and specific humidity at the first model level in the atmosphere;  $\theta_s$  and  $q_s$  are the corresponding surface values, and  $C_M$ ,  $C_H$ ,  $C_Q$  are exchange coefficients for momentum, heat and moisture. With Eqs (1)-(3) also the friction velocity ( $u_*$ ), the temperature scale ( $\theta_*$ ) and the moisture scale ( $q_*$ ) are defined.

The exchange coefficients for momentum, heat, and moisture can be derived by applying surface layer similarity theory, as has been discussed by e.g., Louis (1979) and Holtslag and Beljaars (1989). This leads to

$$C_M = \frac{k^2}{\left( \ln \left( \frac{z_1 + z_{0M}}{z_{0M}} \right) \right)^2} F_M (Ri_0, z_1/z_{0M}), \quad (4)$$

$$C_H = \frac{k^2}{\ln\left(\frac{z_1+z_{0M}}{z_{0M}}\right) \ln\left(\frac{z_1+z_{0H}}{z_{0H}}\right)} F_H(Ri_0, z_1/z_{0M}, z_{0M}/z_{0H}), \quad (5)$$

$$C_Q = \frac{k^2}{\ln\left(\frac{z_1+z_{0M}}{z_{0M}}\right) \ln\left(\frac{z_1+z_{0Q}}{z_{0Q}}\right)} F_Q(Ri_0, z_1/z_{0M}, z_{0M}/z_{0Q}). \quad (6)$$

Here  $k$  is Von Kármán's constant ( $k = 0.4$ ),  $z_1$  is height of first model level and  $z_{0M}$ ,  $z_{0H}$  and  $z_{0Q}$  are surface roughness lengths for momentum, heat and moisture.  $F_M$ ,  $F_H$  and  $F_Q$  denote functional dependencies of the listed quantities, where  $Ri_0$  is the bulk Richardson number between the surface and the first model level (see below). For neutral conditions  $F_M = F_H = F_Q = 1$ .

In atmospheric models the surface roughness lengths are usually taken equal to the value of  $z_{0M}$ , although important differences might be apparent (Holtslag and Beljaars, 1989). Because of the uncertainty in the specification of the roughness lengths we also tentatively assume  $z_{0Q} = z_{0H} = z_{0M}$ , where  $z_{0M}$  is related to a surface description above land. Typically  $z_{0M}$  varies between 0.01 and 1 m above land. Above sea,  $z_{0M}$  is estimated with (Charnock, 1985)

$$z_{0M} = \alpha \frac{u_*^2}{g}, \quad (7)$$

where  $\alpha = 0.0144$  (Garratt, 1977) and  $g$  is acceleration due to gravity. The minimum value used for  $z_{0M}$  is  $2 \cdot 10^{-5}$  m.

On basis of Eqs (4) and (7) the drag coefficient over sea can be calculated as a function of wind speed at a certain level. Fig. 1 shows the result for neutral conditions, in comparison with the one proposed by Smith and Banke (1975) for the 10 m wind speed. It is seen that the agreement is fairly good.

For non-neutral conditions we have to specify the functional dependencies. For unstable conditions we follow Louis et al (1982), for which

$$F_M = 1 - \frac{10 Ri_0}{1 + 75 C_N \left\{ \left( \frac{z_1+z_{0M}}{z_{0M}} \right) |Ri_0| \right\}^{1/2}}, \quad (8a)$$

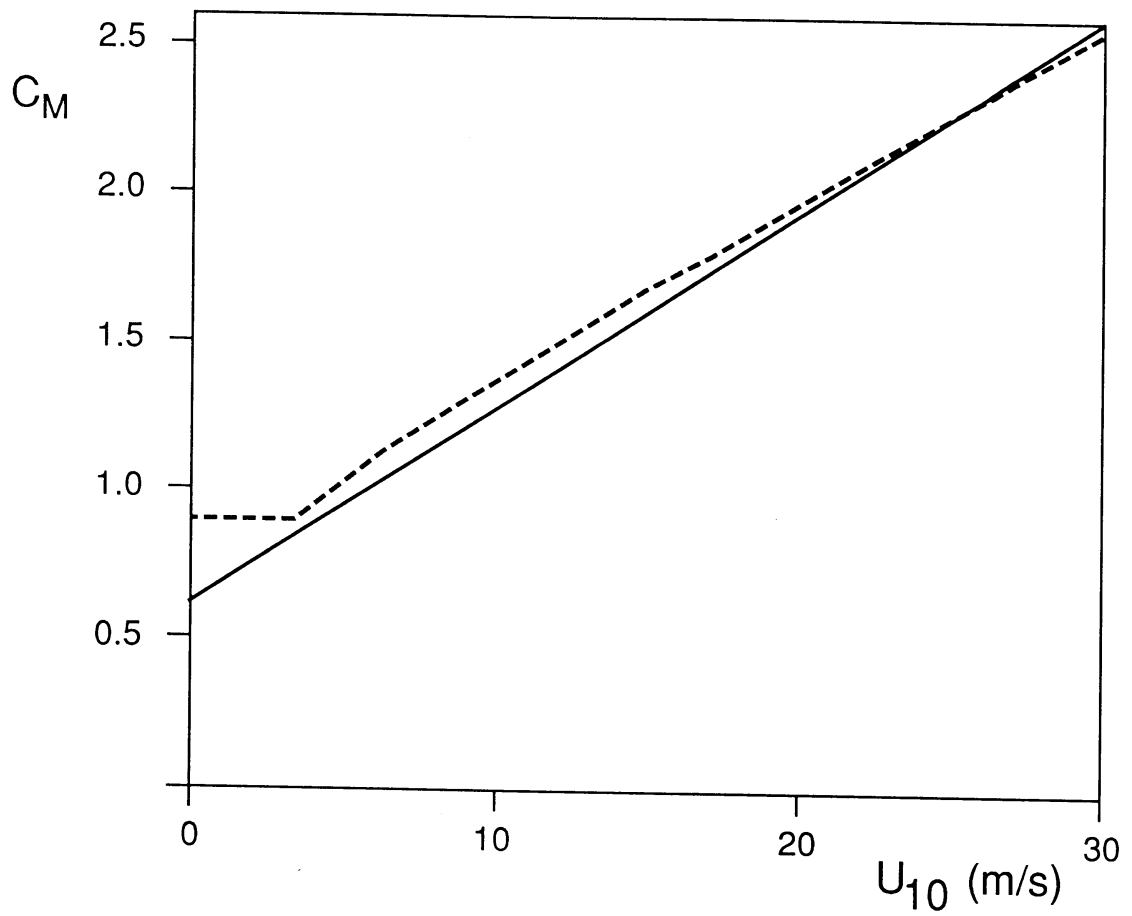


Fig 1 The drag coefficient for the 10 m wind speed above sea according to Smith and Banke (1975) (full line) and the present results (dashed line).

$$F_H = 1 - \frac{15 Ri_0}{1 + 75 C_N \left\{ \left( \frac{z_1 + z_{0M}}{z_{0M}} \right) | Ri_0 | \right\}^{1/2}}, \quad (8b)$$

$$F_Q = F_H, \quad (8c)$$

where

$$C_N = \left( \frac{k}{\ln \left( \frac{z_1 + z_{0M}}{z_{0M}} \right)} \right)^2, \quad (9)$$

and the Richardson number  $Ri_0$  is defined by

$$Ri_0 = \frac{g (S_1 - S_0) z_1}{S_1 U_1^2}. \quad (10)$$

Here  $S_i$  ( $i = 1, 0$ ) is the dry static energy given by

$$S_i = c_p T_i (1 + 0.61 q_i) + g z_i, \quad (11)$$

where  $T_i$  is temperature (K) and  $g$  is the acceleration due to gravity.

The exchange of momentum, heat and moisture in stable conditions, has been the subject of many publications (e.g. Carson and Richards, 1978; Louis, 1979; Louis et al, 1982; Delage, 1988). In Figs. 2a and 2b we have summarised some of the findings for  $F_M$  and  $F_H$  (Holtslag and Beljaars, 1989). The latter authors propose

$$F_Q = F_H = F_M = \frac{1}{1 + 10 Ri_0 (1 + 8 Ri_0)}, \quad (12)$$

which is consistent with wind and temperature observations at the Cabauw tower up to ~100 m (Holtslag, 1984; Holtslag and De Bruin, 1988) and which is also consistent with the observations of Hicks (1976). It can be shown that Eq. (12) also describes the data presented by



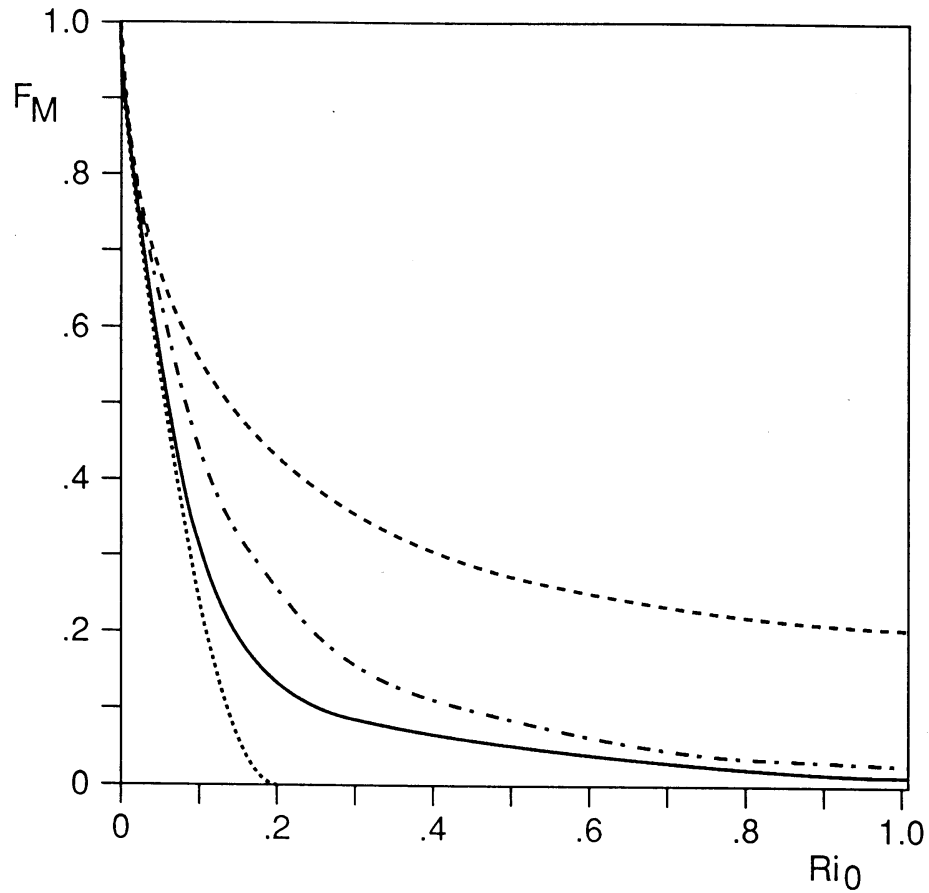


Fig 2a The normalised drag coefficient for momentum  $F_M$  as a function of Richardson number  $Ri_0$ , for stable conditions according to

Louis (1979) : dashed-dotted line,  
Louis et al (1982) : dashed line,  
Dyer (1974) : dotted line, and  
Holtslag and Beljaars (1989): full line.

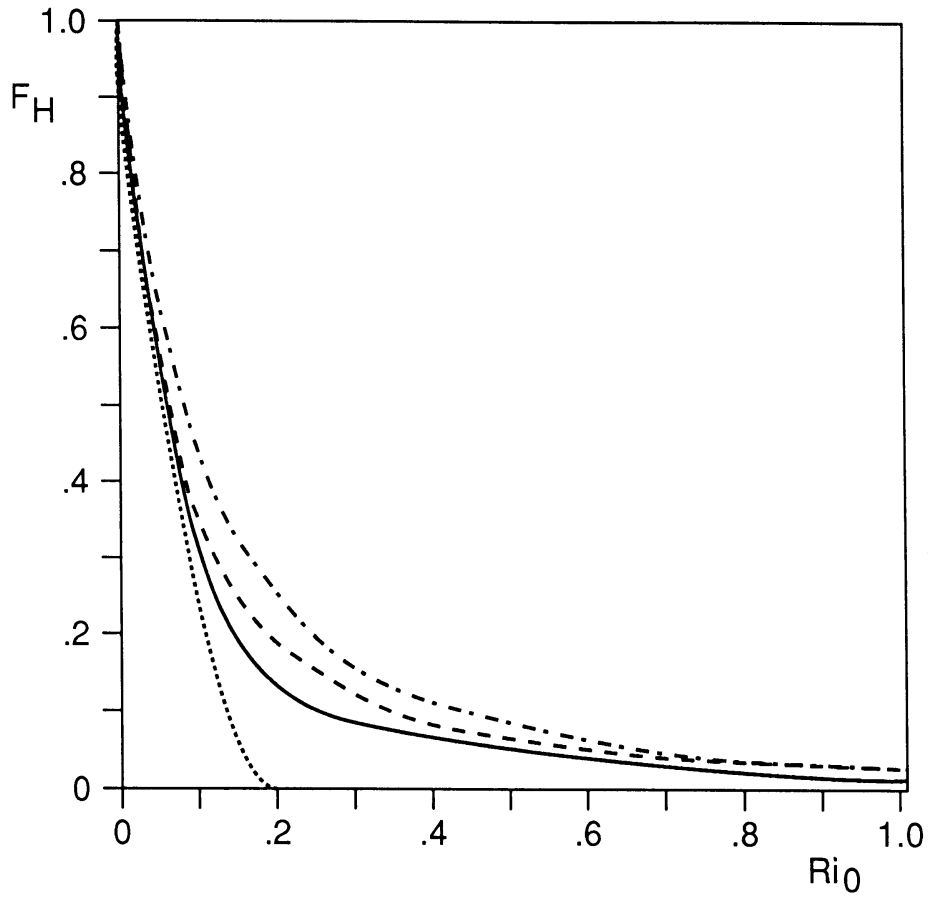


Fig 2b As Fig 2a but for the exchange for coefficient heat.

Delage (1988). The advantage of Eq. (12) over the ones proposed by Delage is that only one equation is needed to cover the variation with  $Ri_0$ .

On the basis of Eqs. (1)-(12) we finally compute the Obukhov length as (Busch, 1973)

$$L = \frac{u_*^2}{k \frac{g}{\bar{T}} (\theta_* + 0.61 \bar{T} q_*)}, \quad (13)$$

where  $\bar{T}$  is the mean temperature across the lowest model layer.

The surface fluxes and the Obukhov length as calculated with the current routines, may differ from the ones in the atmospheric model for several reasons. First, we may use a more detailed horizontal specification of surface characteristics than is possible and useful within the atmospheric model. In that case the mean model output may be interpolated to the specific locations of interest before applying the present scheme.

Secondly, the formulation of the exchange coefficients may be different to correct for errors made in the original model due to low vertical resolution (Delage, 1988). In all cases, however, it is assumed that the mean variables predicted by the atmospheric model that are used as inputs to the present scheme are correct.

### 3. The surface layer variables

In atmospheric models, the height of the lowest model layer is typically in the range of 20 to 100 m. As such Eqs. (1)-(3) will describe a significant part of the atmospheric surface and boundary layer. Routine observations, however, are made at heights of 2 m for temperature and humidity, and a height of 10 m for wind. To verify numerical weather predictions against such observed values and also for operational applications, we need to interpolate between the lowest model level and the surface.

Recently, Geleyn (1988) provided a precise scheme for diagnostic interpolation of the vertical profiles of wind, temperature and humidity. With the scheme an expensive iteration procedure is avoided, and the outputs are consistent with the previous used exchange coefficients (e.g., Eqs. (1)-(3)). The inputs of the scheme are the same as for the ones in the preceding section. In addition the values for the exchange coefficients are needed.

The surface layer wind speed  $U(z)$  at height  $z$ , is calculated with

$$U(z) = \frac{U(z_1)}{b_M} \left[ \ln \left( 1 + \frac{z}{z_1} \left( e^{b_N} - 1 \right) \right) - \frac{z}{z_1} (b_N - b_M) \right] \quad (14a)$$

for stable conditions and with

$$U(z) = \frac{U(z_1)}{b_M} \left[ \ln \left( 1 + \frac{z}{z_1} \left( e^{b_N} - 1 \right) \right) - \ln \left( 1 + \frac{z}{z_1} \left( e^{b_N - b_M} - 1 \right) \right) \right] \quad (14b)$$

for unstable conditions. Here  $U(z_1)$  is wind speed of first model level at height  $z_1$ ,  $b_M = k/C_M^{1/2}$  and  $b_N = k/C_N^{1/2}$  (see Eqs. 4 and 9).

Similarly, potential temperature, specific humidity and dry static energy are calculated with

$$S(z) = S_0 + \frac{S(z_1) - S_0}{b_H} \left[ \ln \left( 1 + \frac{z}{z_1} \left( e^{b_N} - 1 \right) \right) - \frac{z}{z_1} (b_N - b_H) \right] \quad (15a)$$

for stable conditions, and with

$$S(z) = S_0 + \frac{S(z_1) - S_0}{b_H} \left[ \ln \left( 1 + \frac{z}{z_1} \left( e^{b_N} - 1 \right) \right) - \ln \left( 1 + \frac{z}{z_1} \left( e^{b_N - b_H} - 1 \right) \right) \right] \quad (15b)$$

for unstable conditions. Here  $S(z)$ ,  $S_0$  and  $S(z_1)$  denote one of the listed surface layer quantities for the actual height  $z$ , the surface and at the height  $z_1$  of the first model level. Moreover,  $b_H$  is defined by  $b_H = k C_M^{1/2} / C_H$  (see Eqs. 4 and 5).

In section 5 some results will be presented for the calculation of the 10 m wind speed, and the temperature and specific humidity at a height of 2 m on basis of the model outputs.

#### 4. The boundary layer height

The height of the atmospheric boundary layer defines the thickness of the turbulent layer and is therefore a crucial parameter for air pollution studies (e.g., Verver and Scheele, 1988). The boundary layer height is also important to characterize the structure of the boundary layer (e.g. Holtslag and Nieuwstadt, 1986).

Atmospheric models usually do not provide the ABL height directly. Therefore, a diagnostic routine is developed which provides the height on basis of the outputs of atmospheric models.

Since, normally the vertical resolution of atmospheric models is not very large, the routine may be relatively simple.

In neutral conditions, the boundary layer height  $h_N$  can be expressed as (Deardorff, 1972)

$$h_N = c_1 \frac{u_*}{f}, \quad (16)$$

where  $f$  is Coriolis parameter and  $c_1$  is a constant. Duynkerke (1988) shows that the value of  $c_1$  is sensitive to temperature gradients aloft, which inhibit the growth of the ABL. As such he obtains  $c_1$  in the range of 0.12 to 0.20 for small temperature gradients of 2 K/km and 1 K/km, respectively. In the light of these results we take  $c_1 = 0.15$ .

In stationary stable conditions ( $L > 0$ ) the ABL-height  $h_s$  is often described with (Zilitinkevich, 1972; Nieuwstadt, 1981)

$$h_s = c_2 \left( \frac{u_*}{f} L \right)^{1/2}, \quad (17)$$

where  $c_2$  is a constant. An interpolation formula between Eqs. (16) and (17) is given by Nieuwstadt (1981), which reads in dimensionless form as

$$\frac{h}{L} = \frac{c_1 u_*/(fL)}{1 + c_3 h/L}, \quad (18)$$

where  $c_3 = c_1/c_2^2$ .

Fig. 3 shows a comparison of Eq. (18) with the data of Nieuwstadt (1981). The solid line represents (18) with  $c_1 = 0.3$  and  $c_2 = 0.4$  as used by Nieuwstadt, and the dashed line represents  $c_1 = 0.15$  and  $c_2 = 0.7$ . The latter value has been proposed by Businger and Arya (1974). From Fig. 3 it is seen that the dashed line is in better agreement with the data, although the differences with the actual data are not small for both parameter sets. Note that a linear relationship between  $h/L$  and  $u_* / (fL)$  is also in reasonable agreement (see Koracin and Berkowicz, 1988). However, here we prefer Eq. (18) because this equation combines two well established formulas (notably Eqs. (16) and (17)).

In well developed unstable conditions ( $H > 0$ ;  $L < 0$ ), a simple determination of ABL height ( $h_c$ ) is given by the dry parcel intersection method (Holzworth, 1964). This method is illustrated in Fig. 4. For a given virtual temperature profile within the atmospheric model, a parcel with virtual temperature  $\theta_p$  will be lifted up to  $h_c$ . Troen and Mahrt (1986) propose to derive  $\theta_p$  from

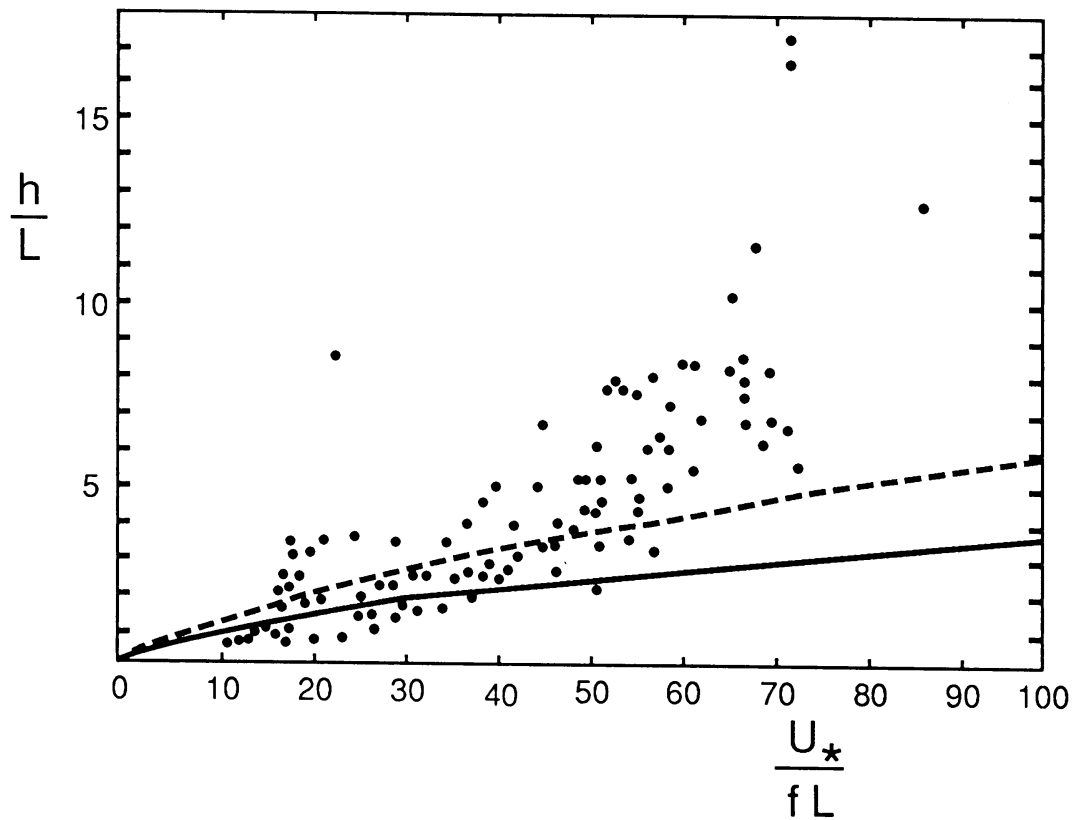


Fig 3 A comparison of the normalised boundary layer height  $h/L$  in stable conditions with  $u_*/(fL)$  for Cabauw data discussed by Nieuwstadt (1980). Full line is Eq (18) with  $c_1 = 0.3$  and  $c_2 = 0.4$ . Dashed line is Eq (18) with  $c_1 = 0.15$  and  $c_2 = 0.7$ .

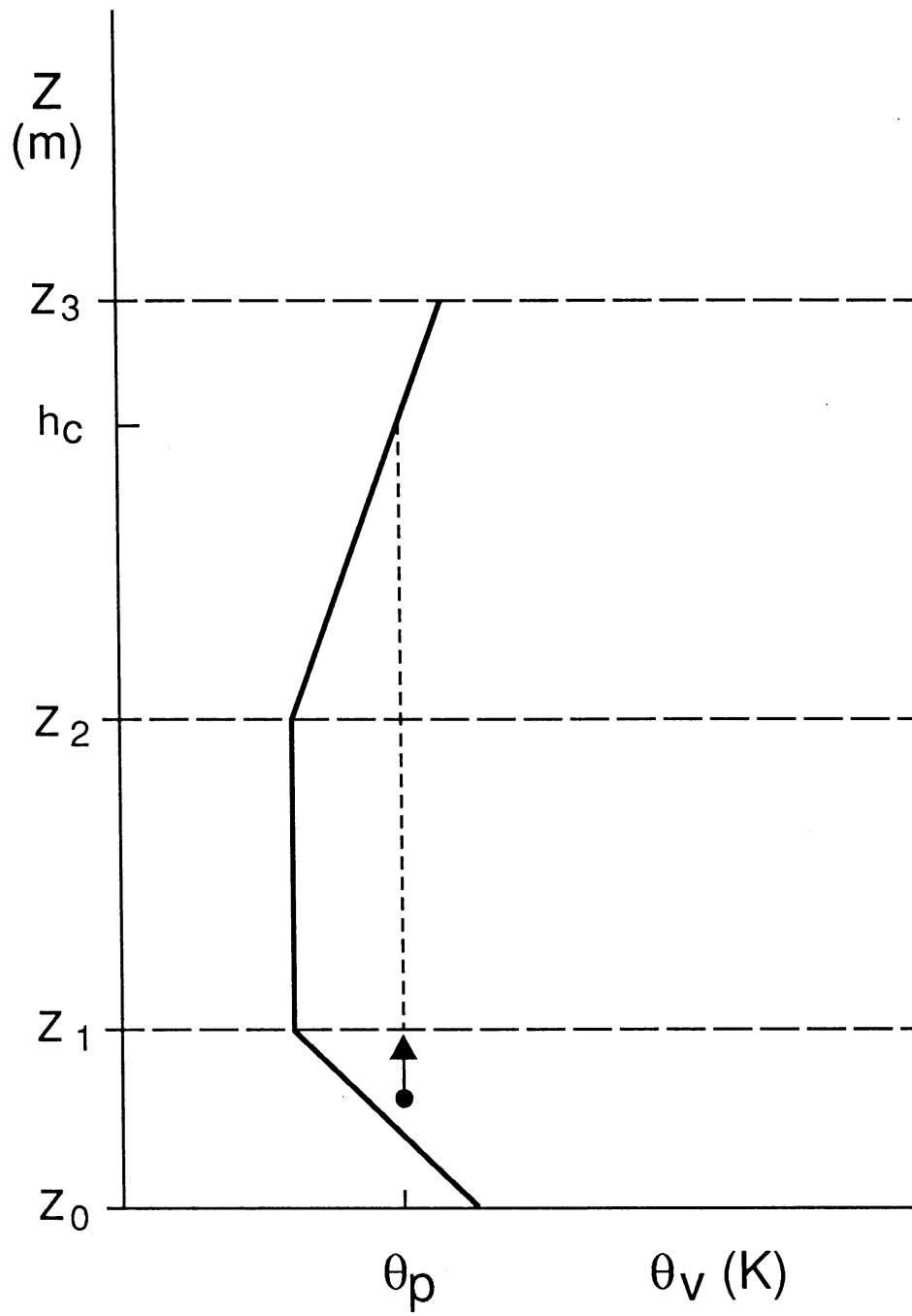


Fig 4 Determination of the convective boundary layer height  $h_c$  by lifting a parcel with virtual temperature  $\theta_p$ .

$$\theta_p = \theta_s + c_4 \overline{w\theta_0} / w_* , \quad (19)$$

where  $\theta_s$  is a surface layer virtual temperature,  $c_4$  is a constant taken as 6.5,  $\overline{w\theta_0}$  is the kinematic surface heat flux related to  $H$  of Eq. (2) by  $H = \rho c_p \overline{w\theta_0}$ , and  $w_*$  is the convective velocity scale defined by

$$w_* = \left( \frac{g}{T} \overline{w\theta_0} h_c \right)^{1/3} . \quad (20)$$

With (19) and (20),  $h_c$  can be determined by iteration. A first guess is obtained by using  $\theta_p = \theta_s$ , which then defines  $w_*$ . This procedure normally converges within 3 steps. For  $\theta_s$  we use the virtual temperature at the height of 10 m as derived with the routines of section 3. When during unstable conditions  $h_c < h_N$ , we use  $h_N$  as the best estimate for the boundary layer height  $h$ . Here  $h_N$  is estimated with Eq. (16). In all cases a lower limit of 50 m is used for  $h$ .

## 5. Some results

In this section we will illustrate the outputs of the present scheme for a grid point above land and above sea. The KNMI limited area model (LAM) has been used to produce the inputs of the present scheme. The LAM is in many aspects similar to the formulation of the ECMWF global model, as has been summarized by Cats et al (1987). The horizontal resolution, however, is about 60 km. Table 1 contains a list of the parameters obtained from the LAM with their vertical resolution.

In Figs 5a-5f we have given the outputs of the present scheme for a grid point near Cabauw, the Netherlands (51°58'N, 4°56'E). The roughness length  $z_0$  is taken as  $z_0 = 0.2$  m. The scheme has been applied both to forecasts and analyses of the LAM. The initial time of the forecasts is April 1, 1989, 00 h. The model has been integrated for 30 h. Every 3 hours output is generated, which can be compared with the actual analyses. The analyses are obtained from meteorological data observed around the analysis time and a first-guess field. The latter field results form a 3 h forecast with the model from the previous analysis. This means that the analyses are influenced by the quality of the numerical model (Cats et al, 1987).



Table 1 Vertical resolution of the limited area model (LAM) for the input parameters of the present scheme. Here  $\sigma = P/P_0$  ( $P$  is pressure,  $P_0$  is surface pressure),  $z$  is height in standard atmosphere,  $T_i$  is air temperature at level  $i$ ,  $q_i$  is specific humidity at level  $i$ ;  $U_1$  and  $V_1$  are the wind components at the first model level in the atmosphere.

$\sigma$ -level	Height $z$ (m)	Quantity
1.0	0	$T_0, q_0$
0.9926	60	$T_1, q_1, U_1, V_1$
0.9405	490	$T_2, q_2$
0.8522	1280	$T_3, q_3$
0.7419	2390	$T_4, q_4$
0.6214	3800	$T_5, q_5$

In Figs 5a-5c, also the point observations (10 minute averages) at Cabauw are given. Despite of the fact that the forecasts and analyses are representative of a grid square of about 60 x 60 km, the agreement with the point measurements is relatively good. This is particularly true for temperature and specific humidity at 2 m. The observed wind speed at Cabauw is somewhat larger than the results of the LAM and the scheme for this case.

In Figs 5d and 5e also estimates of friction velocity and reciprocal Obukhov length are given, which are directly obtained from observations at Cabauw utilizing the profile method (see Van Ulden and Holtslag, 1985). The agreement with the model predictions and analyses is good. Fig 5f shows the results of the scheme for the boundary layer height ( $h$ ). Unfortunately, no direct observations of  $h$  were available to compare with.

In Figs 6a-6f we have given the outputs of the present scheme for a gridpoint near the platform Ekofisk at the North Sea (56°33' N, 3°13' E) for the same period as in Figs 5a-5f. Generally the agreement between forecasts and analyses is reasonable. In Figs 6a-6e we have also included estimates of the quantities, derived with local observations of wind speed at 85 m, temperature and relative humidity at 28.8 m, and the sea water temperature. On basis of these data the output variables were estimated using the profile method (Van Ulden and Holtslag, 1985), and assuming that the roughness length of the sea is given by Eq (7). The calculation of specific humidity of 2 m was not always possible, because the relative humidity at 28.8 m was not continuously available. The derived quantities are generally in good agreement with the model results. For instance, Fig 6c shows that the variation of 10 m wind speed with time is well predicted.

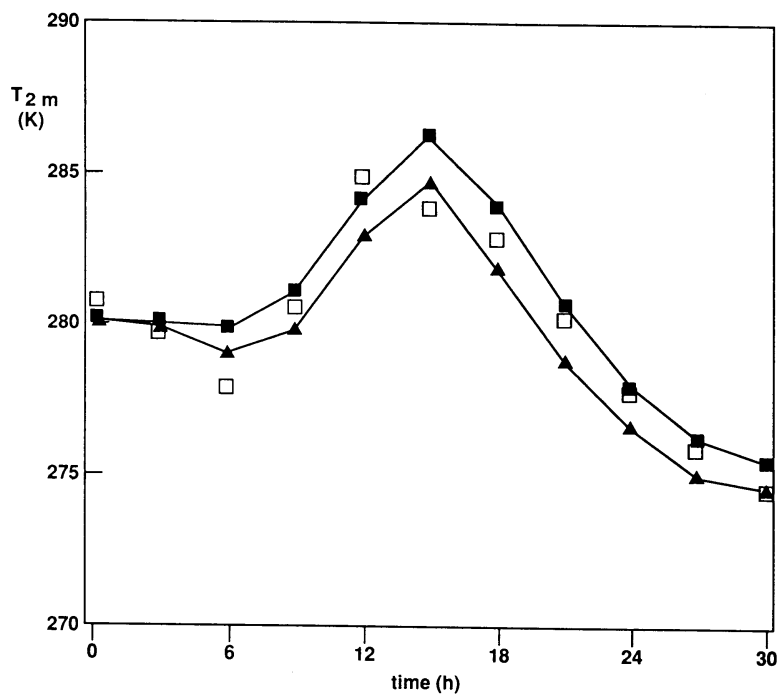


Fig 5a Forecasts of the air temperature at 2 m ( $T_{2m}$ ) (solid squares) up to 30 h ahead, in comparison with analyses (solid triangles) and observations at Cabauw (open squares) for the period of 1 April 00 h - 2 April 06 h, 1989.

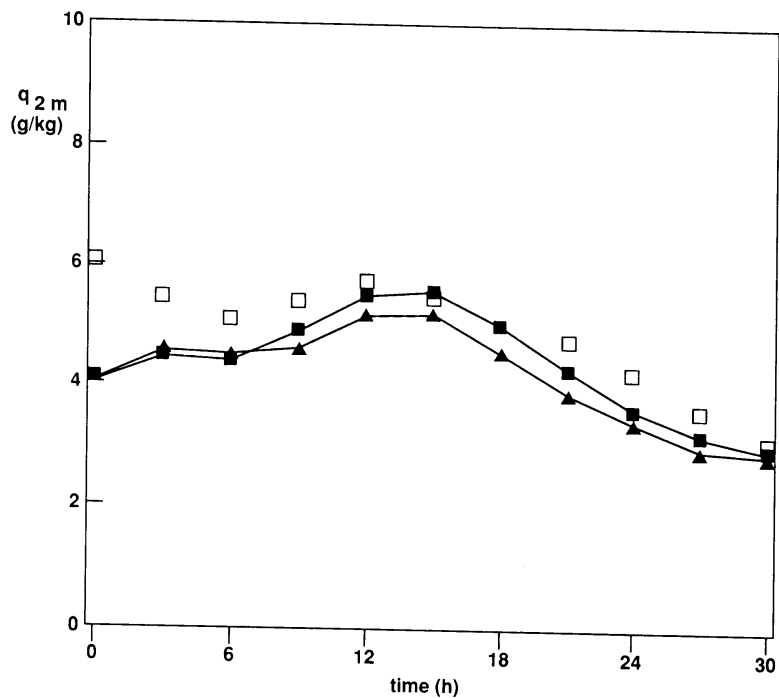


Fig 5b As Fig 5a for the specific humidity at 2 m ( $q_{2m}$ ).

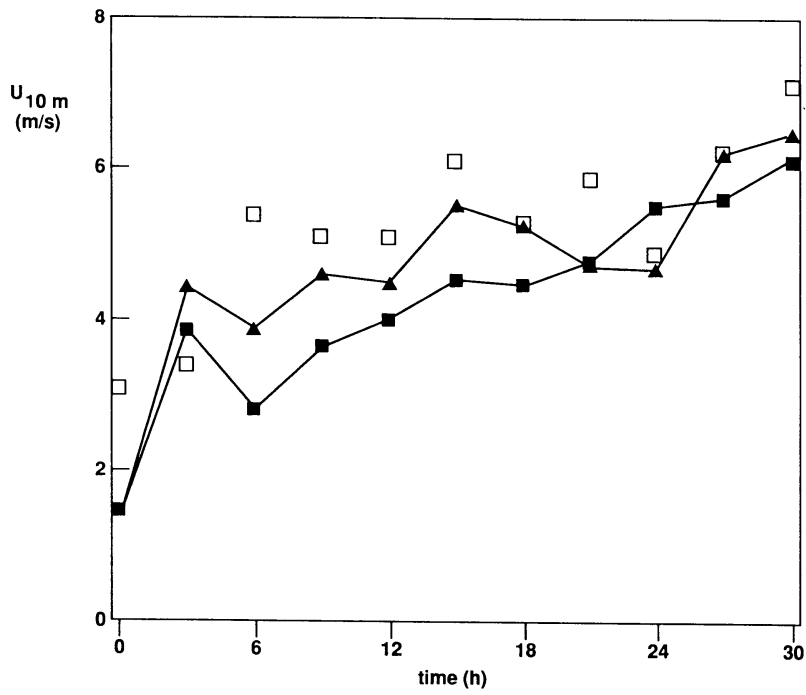


Fig 5c As Fig 5a for the wind speed at 10 m ( $U_{10m}$ ).

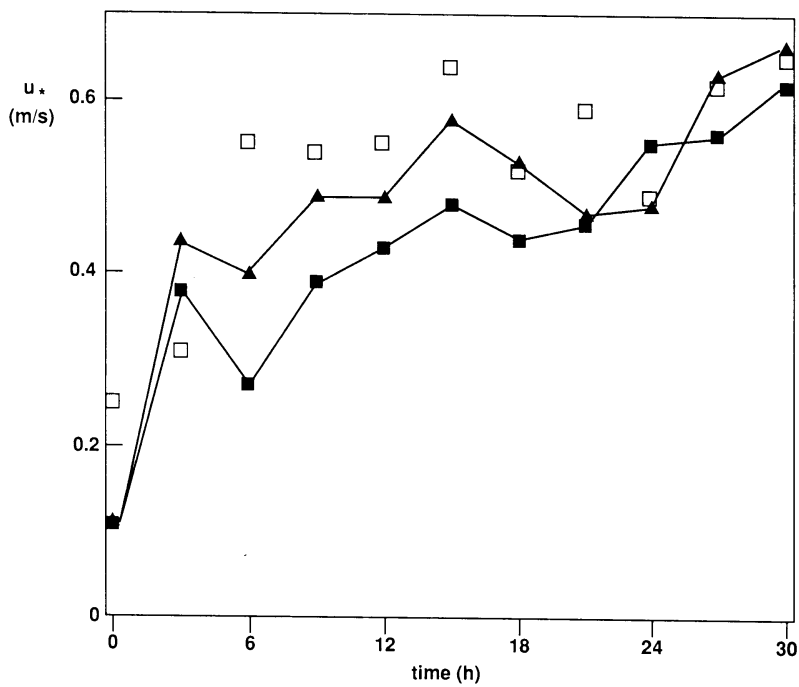


Fig 5d As Fig 5a for the friction velocity ( $u_*$ ).

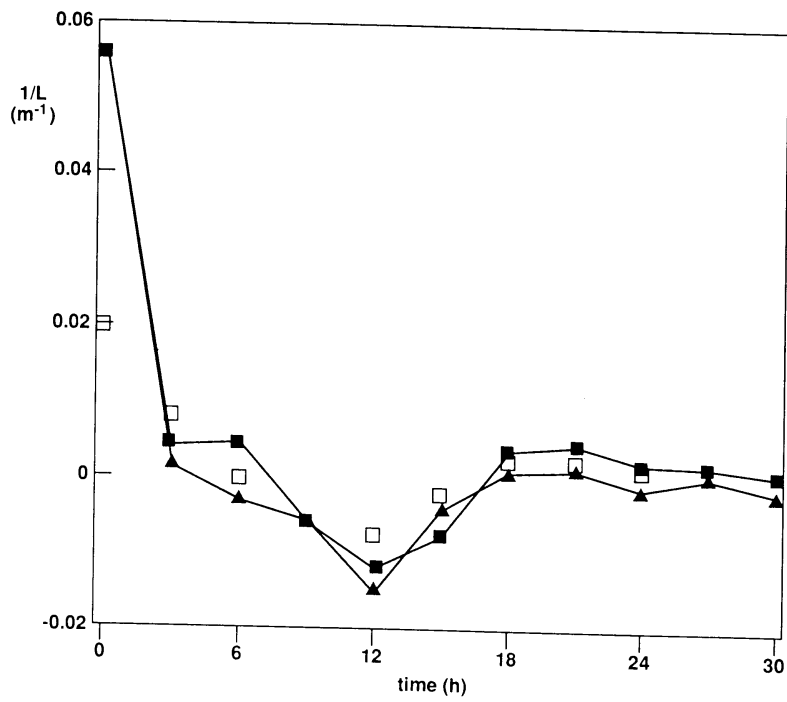


Fig 5e As Fig 5a for the reciprocal Obukhov length ( $1/L$ ).

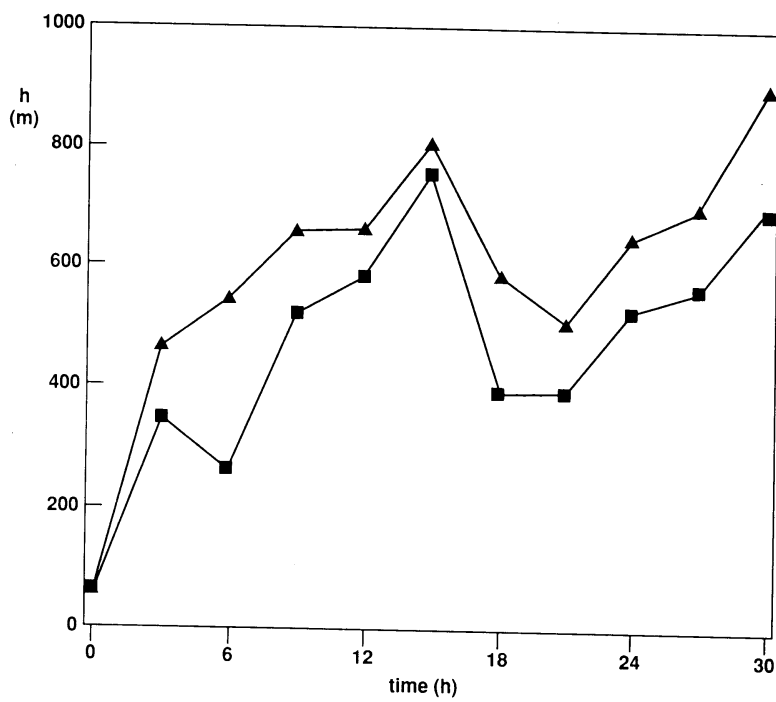


Fig 5f As Fig 5a for the boundary layer height ( $h$ ).

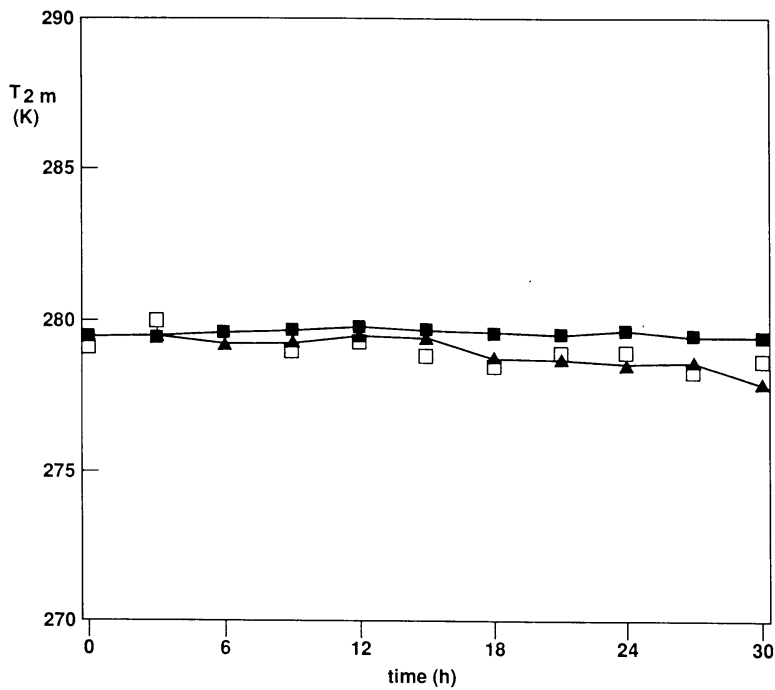


Fig 6a Forecasts of the air temperature at 2 m ( $T_{2m}$ ) (solid squares) up to 30 h ahead, in comparison with analyses (solid triangles) at the platform Ekofisk for the period of 1 April 00 h - 2 April 06 h, 1989. Open squares represent the derived temperatures from observations.

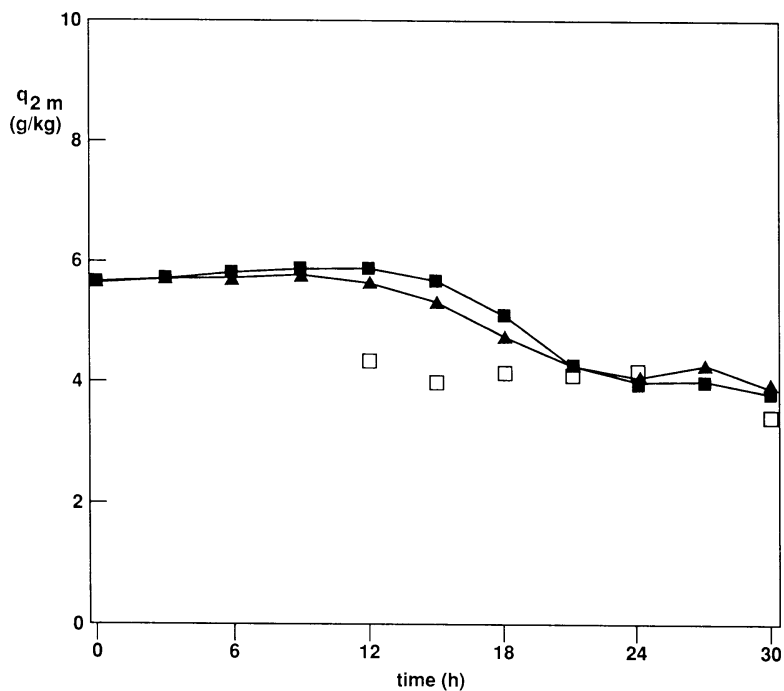


Fig 6b As Fig 6a for the specific humidity at 2 m ( $q_{2m}$ ).

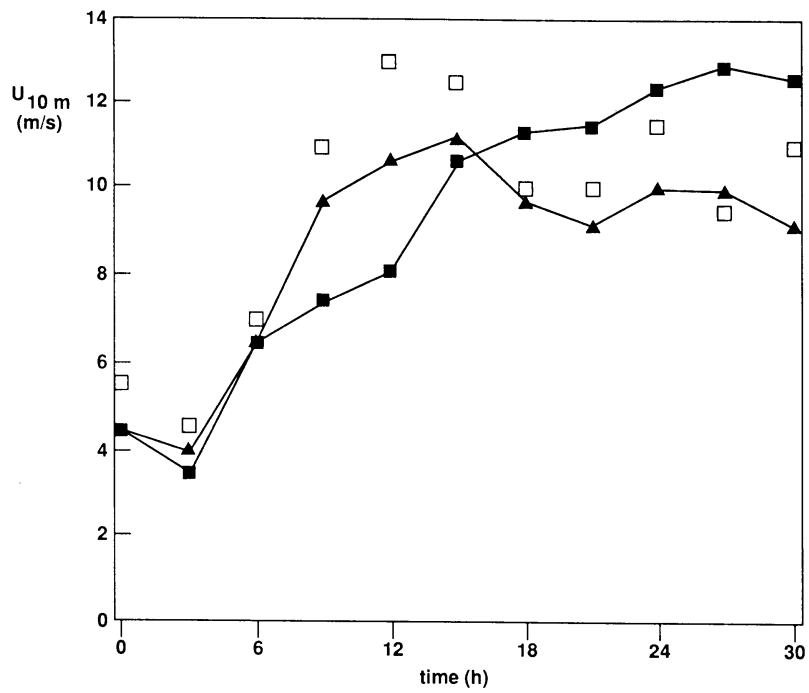


Fig 6c As Fig 6a for the wind speed at 10 m ( $U_{10m}$ ).

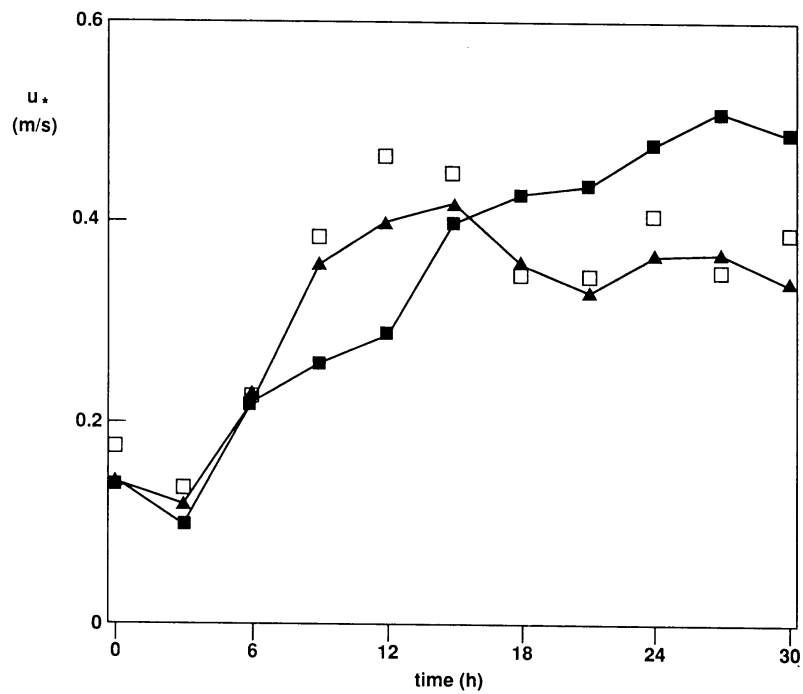


Fig 6d As Fig 6a for the friction velocity ( $u_*$ ).

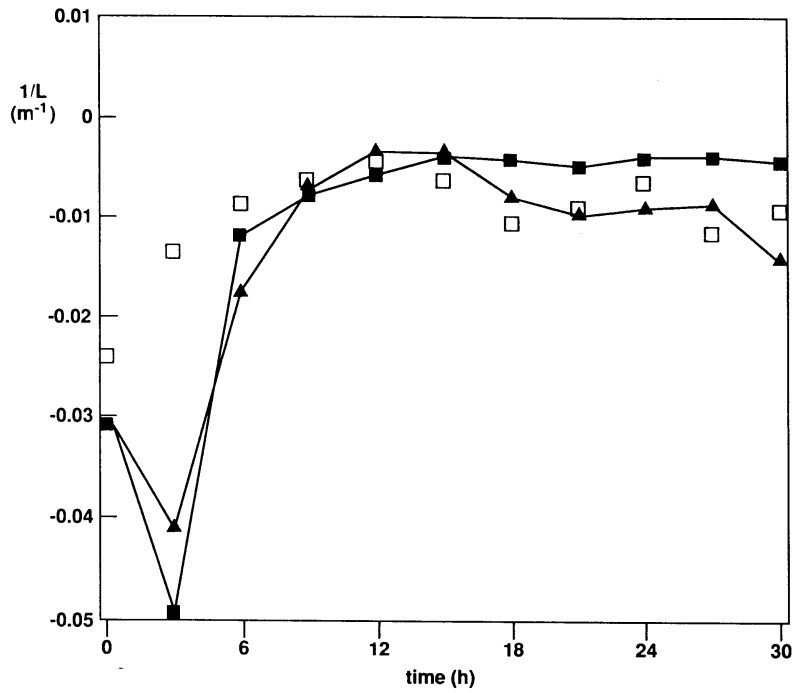


Fig 6e As Fig 6d for the reciprocal Obukhov length ( $1/L$ ).

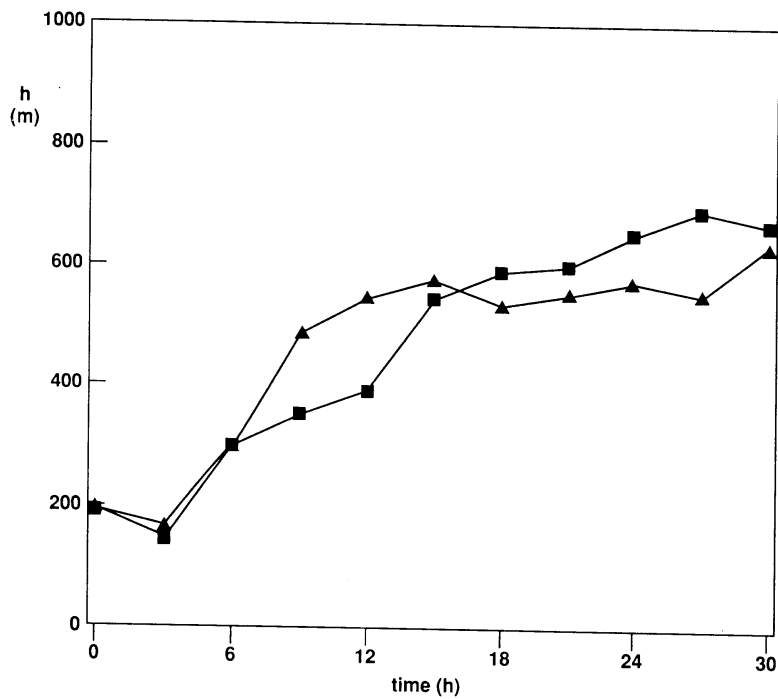


Fig 6f As Fig 6d for the boundary layer height ( $h$ ).

## 6. Summary

In this paper we have presented a scheme for the derivation of boundary layer parameters from the outputs of atmospheric models. With the scheme the surface fluxes, the boundary layer height and related mean variables in the atmospheric surface layer can be calculated. For a grid point above land and sea, a comparison is given for the outputs of the scheme by using forecasts and analyses of a limited area model. Also a comparison with point measurements has been presented for all quantities except boundary layer height. The agreement is generally good. Before definitive conclusions can be drawn, however, it will be necessary to run the scheme for a longer period on the outputs of an atmospheric model. With the outcome of such a study the present scheme may be adjusted. Also the atmospheric model itself may benefit from such a study, because more insight will be given on the quality of its outputs.

## 7. Acknowledgements

We would like to thank L. Hafkenscheid and G. Cats for running the limited area model for the cases considered in this paper. A. Beljaars and G. Verver are thanked for discussions and comments on a draft of this paper. Miss M. Kaltofen is acknowledged for typing the manuscript.



Appendix: The turning of the wind with height

In sections 2 and 3 we have discussed how the magnitudes of the friction velocity and the surface layer wind speed are obtained. To derive the wind direction and its variation with height we use the following procedure.

Fig 7 shows the mean variation of wind direction with height according to the data of Holtslag (1984) and Van Ulden and Holtslag (1985) for stable conditions in different classes of Obukhov length  $L$ . The height is normalised with the calculated ABL-height of Eq (18), using  $c_1 = 0.15$  and  $c_2 = 0.70$ . Both the normalised direction at height  $z$  relative to the direction at the top of the ABL, and the difference between the direction at height  $z$  and the 20 m wind are shown.

The indicated line of Fig. 7 is described with

$$\frac{D_z}{D_h} = d_1 \left( 1 - e^{-d_2 z/h} \right), \quad (A1)$$

where  $D_h = 45^\circ$ ,  $d_1 = 1.23$  and  $d_2 = 1.75$ . A similar procedure has been discussed by Van Ulden and Holtslag (1985), but they used Eq (17) with  $c_2 = 0.4$  to normalise the actual height.

With Eq (A1) it follows that for  $z = 0$ ,  $D_z = 0$  and for  $z = h$ ,  $D_z = D_h = 45^\circ$ . This means that the total variation of wind direction across the boundary layer is  $45^\circ$  in stable conditions, in agreement with the solution of the classical Ekman spiral (e.g. Holton, 1979) and in good agreement with the model calculations of Nieuwstadt (1983).

The latter author obtains  $D_h \approx 20^\circ$  for very unstable conditions. To obtain a smooth variation of  $D_h$  with stability we use  $D_h = 20^\circ$  for  $h/L \leq -10$ , and

$$D_h = 20 + 25 \left( 1 + \frac{h/L}{10} \right) \quad (A2)$$

for  $-10 < h/L < 0$ . For  $h/L \geq 0$  we use Eq (A1).

With (A1) and (A2) we can calculate the wind direction at the height of the first model level ( $z_1$ ) relative to the direction of the friction velocity. Using the actual value of the wind direction within the atmospheric model we arrive at the absolute value for the direction of the friction velocity. Subsequently, the direction of the wind at any height within the ABL follows from (A1) and (A2).

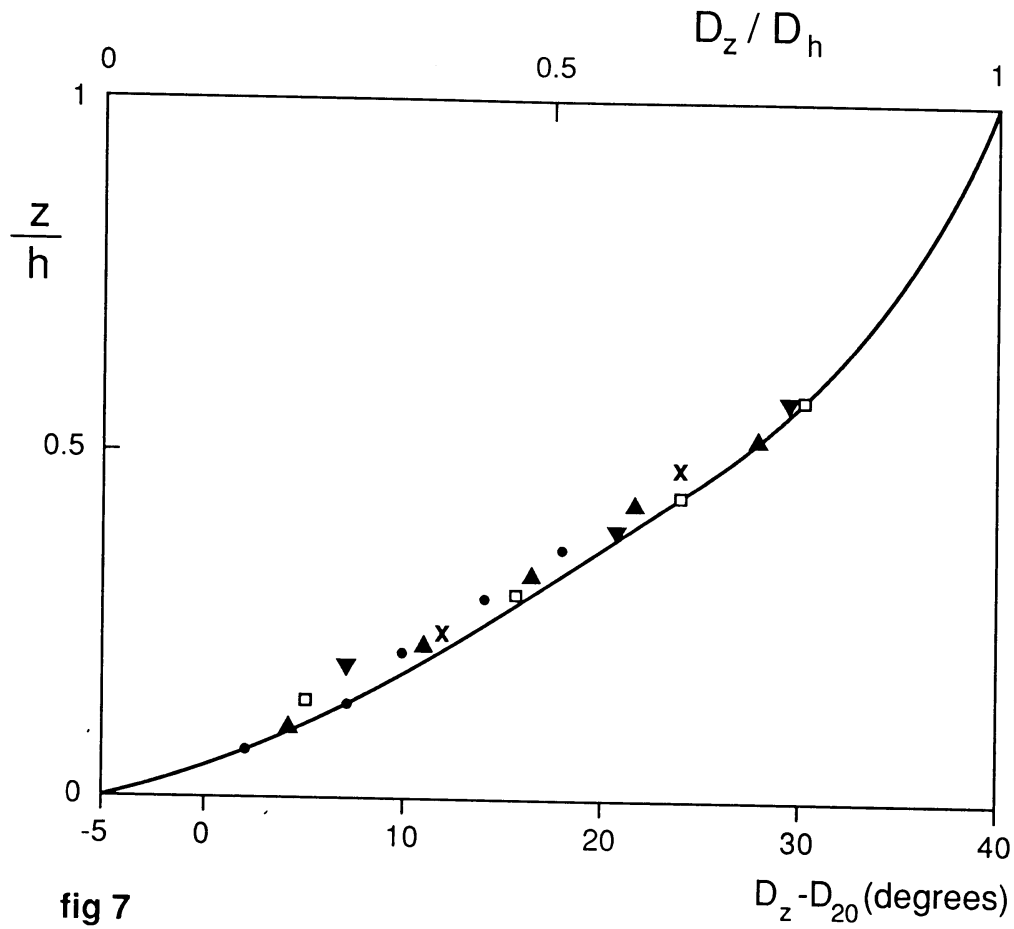


fig 7 The mean turning of wind direction  $D_z$  at height  $z$  for the Cabauw data discussed by Holtslag (1984) and Van Ulden and Holtslag (1985). The full line represents Eq (A1). Dots represent cases with  $200 < L \leq 1000$  m, triangles refer to  $100 < L \leq 200$  m, triangles upside down represent  $10 < L < 40$  m, and crosses  $0 \leq L \leq 10$  m.

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