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scientific reports WR 87-10

wetenschappelijke rapporten WR87-10

de bilt, 1987

KNMI

Nonlinear normal mode initialization of a limited area model using nonstationary Rossby modes

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July 1987

Abstract

Application of the nonlinear normal mode method to the initialization of a limited area prediction model requires the construction of the normal modes of the linearized model equations.

Linearized model equations, in differentiated form, can be derived in different ways, depending on the splitting of the terms which contain the Coriolis parameter f. Instead of setting f equal to a constant value in these equations, leading to stationary Rossby modes, we consider a simple extension, by modifying the linearized model equations so that they admit solutions with nonzero Rossby frequencies.

The results are compared with those using a nonlinear normal mode initialization with constant Coriolis parameter. They show that the two methods are virtually identical in their effect upon the initial fields and upon their development during the first hours of the forecast.

1. Introduction

The purpose of initialization of a primitive equation forecast model is to balance the initial fields of velocity and mass in order to suppress the high frequency wave oscillations in subsequent model integrations. Application of the nonlinear normal mode method to the initialization of a limited area model requires the construction of the normal modes of the linearized model equations. Linearized model equations, in differentiated form, can be derived in different ways, depending on the splitting of the terms which contain the Coriolis parameter f.

In a previous paper (Bijlsma and Hafkenscheid, 1986) we applied the nonlinear normal mode initialization method of Machenhauer (1977) to a limited area forecast model, assuming that the Coriolis parameter in the linear part of the model equations was constant. Due to the constant Coriolis parameter the frequencies of the Rossby waves are zero.

Since the inclusion of beta terms in the linearized model equations might be important in practical applications of nonlinear normal mode initialization, as was noted by Ballish (1979), we consider a simple extension of the foregoing method. We modify the linear part of the model equations so that they admit solutions with nonzero Rossby frequencies, which is the main consequence of the inclusion of beta terms in the linear model equations. As a result, the frequencies of the eastward and westward gravity waves are no longer symmetric and the corresponding modes no longer complex conjugate.

An outline of the method is given in section 2. Results of the initialization method, compared with those using a nonlinear normal mode initialization with constant Coriolis parameter, are given in section 3. Conclusions are presented in section 4.

2. Outline of the method

We consider the shallow water equations on the sphere in differentiated form

$$\frac{\partial}{\partial t} \nabla^2\chi - 2\Omega \sin\theta \nabla^2\psi + \frac{2\Omega}{r^2} \left(\frac{\partial\chi}{\partial\lambda} - \cos\theta \frac{\partial\psi}{\partial\theta}\right) + \nabla^2\phi = Q_\chi,$$

$$\frac{\partial}{\partial t} \nabla^2\psi + 2\Omega \sin\theta \nabla^2\chi + \frac{2\Omega}{r^2} \left(\frac{\partial\psi}{\partial\lambda} + \cos\theta \frac{\partial\chi}{\partial\theta}\right) = Q_\psi,$$

$$\frac{\partial}{\partial t} \phi + d \nabla^2\chi = Q_\phi,$$

where χ, ψ, ϕ and d are the velocity potential, streamfunction, geopotential and mean free geopotential height, λ and θ longitude and latitude, r and Ω the radius and angular velocity of the earth and Q_χ, Q_ψ and Q_ϕ the nonlinear terms.

The equations used in the nonlinear normal mode method under consideration, on a discrete grid $\lambda_m = \lambda_o + m\Delta\lambda$, $\theta_n = \theta_o + n\Delta\theta$, with MxN interior grid points, are

$$\frac{\partial}{\partial t} \nabla_d^2\chi - \bar{f}\nabla_d^2\psi + \frac{2\Omega}{r^2} \left(\frac{\partial}{\partial\lambda}\right)_d\chi + \nabla_d^2\phi = Q'_\chi, \tag{1}$$

$$\frac{\partial}{\partial t} \nabla_d^2\psi + \bar{f}\nabla_d^2\chi + \frac{2\Omega}{r^2} \left(\frac{\partial}{\partial\lambda}\right)_d\psi = Q'_\psi, \tag{2}$$

$$\frac{\partial}{\partial t} \phi + d\nabla_d^2\chi = Q_\phi, \tag{3}$$

where $\bar{f} = 2\Omega \sin\bar{\theta}$ is a constant Coriolis parameter, ∇_d^2 the usual five-point discrete Laplacian operator in spherical coordinates and $(\frac{\partial}{\partial\lambda})_d$ the centered difference operator in the λ-direction. Deviations of the linear terms are absorbed into the nonlinear terms on the right-hand side. In order that the linear system admits nonstationary Rossby modes, it is sufficient to include only the term $(\frac{\partial}{\partial\lambda})_d\,\psi$. However, for the sake of symmetry we also include $(\frac{\partial}{\partial\lambda})_d\,\chi$.

Because of the presence of the terms $(\frac{\partial}{\partial\lambda})_d\psi$ and $(\frac{\partial}{\partial\lambda})_d\chi$ in Eqs. (1) and (2), the eigenfunctions of the linear spatial operator on the left-hand side of Eqs. (1) to (3) are complex exponential functions in the E-W direction. Therefore we should like to write the solution in the form

$$\chi = \chi_o + \hat{\chi}, \quad \psi = \psi_o + \hat{\psi}, \quad \phi = \phi_o + \hat{\phi},$$

where the fields $\hat{\chi}$, $\hat{\psi}$ and $\hat{\phi}$ have periodic boundary conditions in the E-W direction and zero boundary conditions in the N-S direction, and where the fields χ_o, ψ_o and ϕ_o satisfy the discrete Laplace equations

$$\nabla_d^2\chi_o = 0, \; \nabla_d^2\psi_o = 0, \; \nabla_d^2\phi_o = 0$$

and are equal to the initial values of $\chi-\hat{\chi}$, $\psi-\hat{\psi}$ and $\phi-\hat{\phi}$ on the boundary.

Including the terms $(\frac{\partial}{\partial\lambda})_d\chi_o$ and $(\frac{\partial}{\partial\lambda})_d\psi_o$ in the nonlinear terms on the right-hand side, we could solve the remaining system of equations for the fields $\hat{\chi}$, $\hat{\psi}$ and $\hat{\phi}$, just like in the case of the f-plane approximation, where these fields have zero boundary conditions.

However, because the periodic boundary values of $\hat{\chi}$, $\hat{\psi}$ and $\hat{\phi}$ are not known beforehand but follow from an iterative procedure, as we see at the end of this section, we proceed as follows. As a first approximation, we introduce the (time-independent) functions χ_o, ψ_o and ϕ_o, which satisfy the discrete Laplace equations

$$\nabla_d^2\chi_o = 0, \quad \nabla_d^2\psi_o = 0, \quad \nabla_d^2\phi_o = 0$$

and are equal to the initial values of χ, ψ and ϕ on the boundary. Then, setting $\chi = \chi_o + \hat{\chi}$, $\psi = \psi_o + \hat{\psi}$ and $\phi = \phi_o + \hat{\phi}$ and removing the primes of

Q'_χ and Q'_ψ, Eqs. (1) to (3) become

$$\frac{\partial}{\partial t} \nabla^2_d \chi - \bar{f} \nabla^2_d \hat{\psi} + \frac{2\Omega}{r^2} (\frac{\partial}{\partial \lambda})_d \hat{\chi} + \nabla^2_d \hat{\phi} = Q_\chi - \frac{2\Omega}{r^2} (\frac{\partial}{\partial \lambda})_d \chi_o, \quad (4)$$

$$\frac{\partial}{\partial t} \nabla^2_d \psi + \bar{f} \nabla^2_d \hat{\chi} + \frac{2\Omega}{r^2} (\frac{\partial}{\partial \lambda})_d \hat{\psi} = Q_\psi - \frac{2\Omega}{r^2} (\frac{\partial}{\partial \lambda})_d \psi_o, \quad (5)$$

$$\frac{\partial}{\partial t} \phi + d \nabla^2_d \hat{\chi} = Q_\phi. \quad (6)$$

Let the normal modes of the linear operator on the left-hand side of Eqs. (4) to (6) have the following spatial behaviour

$$S_{k\ell}(m,n) = f_{k\ell}(n) \exp(2\pi i \, km/(M+1)),$$

satisfying $\nabla^2_d \, S_{k\ell}(m,n) = -\alpha^2_{k\ell} \, S_{k\ell}(m,n)$, with $f_{k\ell}(0) = f_{k\ell}(N+1) = 0$.

Then, subsitution shows that the Rossby and gravity modes are determined by the eigenvectors and eigenvalues of the matrix

$$\underset{\approx}{M}_{k\ell} = \begin{pmatrix} -i\varepsilon_{k\ell} & -\bar{f} & 1 \\ \bar{f} & -i\varepsilon_{k\ell} & 0 \\ -\alpha^2{}_{k\ell} d & 0 & 0 \end{pmatrix}$$

with

$$\varepsilon_{k\ell} = \frac{2\Omega}{r^2 \Delta\lambda \alpha^2{}_{k\ell}} \sin \frac{2k\pi}{M+1}.$$

The eigenvectors of $\underset{\approx}{M}_{k\ell}$ are proportional to (if T denotes the transpose)

$$(i(\sigma + \varepsilon_{k\ell}), \bar{f}, \bar{f}^2 - (\sigma + \varepsilon_{k\ell})^2)^T$$

with eigenvalues $i\sigma$, where σ satisfies the equation

$$\sigma(\sigma + \varepsilon_{k\ell})^2 - \bar{f}^2\sigma - (\sigma + \varepsilon_{k\ell})\ \alpha_{k\ell}^2 d = 0.$$

We solve this equation for $\varepsilon_{k\ell} \geqq 0$; for $\varepsilon_{k\ell} < 0$ we find the complex conjugate eigenvalues (see, for instance, van der Waerden, 1955, p. 187 for the solution of a cubic equation). The values of σ corresponding to Rossby waves and westward and eastward gravity waves are (if $3\alpha_{k\ell}^2 d - 6\bar{f}^2 + \frac{2}{3}\varepsilon_{k\ell}^2 > 0$)

$$\sigma_{k\ell 1} = -\frac{2}{3}\varepsilon_{k\ell} - \frac{2}{3}(3p)^{\frac{1}{2}}\cos(\pi+\mu)/3,$$

$$\sigma_{k\ell 2} = -\frac{2}{3}\varepsilon_{k\ell} - \frac{2}{3}(3p)^{\frac{1}{2}}\cos(\pi-\mu)/3,$$

$$\sigma_{k\ell 3} = -\frac{2}{3}\varepsilon_{k\ell} + \frac{2}{3}(3p)^{\frac{1}{2}}\cos\mu/3,$$

where

$$\mu = \arctan\left((12p^3 - q^2)^{\frac{1}{2}}/q\right), \quad 0 < \mu \leq \pi/2,$$

$$p = \bar{f}^2 + \alpha_{k\ell}^2 d + \frac{1}{3}\varepsilon_{k\ell}^2,$$

$$q = 3\varepsilon_{k\ell}\ \alpha_{k\ell}^2 d - 6\varepsilon_{k\ell}\ \bar{f}^2 + \frac{2}{3}\varepsilon_{k\ell}^3.$$

Note that $\varepsilon_{k\ell} = 0$ for $k = 0$ and $k = \frac{M+1}{2}$ (if M is odd), so that $\sigma_{k\ell 1} = 0$, $\sigma_{k\ell 2} = -(\alpha_{k\ell}^2 d + \bar{f}^2)^{\frac{1}{2}}$ and $\sigma_{k\ell 3} = (\alpha_{k\ell}^2 d + \bar{f}^2)^{\frac{1}{2}}$. Let us write the solution in the form $\hat{\underset{\sim}{\eta}}(m,n) = (\hat{\chi}(m,n), \hat{\psi}(m,n), \hat{\phi}(m,n))^T$ and define the scalar product

$$\langle\hat{\underset{\sim}{\eta}}_1, \hat{\underset{\sim}{\eta}}_2\rangle = \frac{1}{M+1}\sum_{m=0}^{M}\sum_{n=1}^{N}[\hat{\phi}_1\hat{\phi}_2^* - d(\hat{\chi}_1\ \nabla_d^2\hat{\chi}_2^* + \hat{\psi}_1\ \nabla_d^2\hat{\psi}_2^*)]\cos\theta_n,$$

where the asterisk denotes complex conjugation.

Then the normalized Rossby mode and westward and eastward gravity modes are given by

$$\underset{\sim}{P}_{k\ell r} = N_{k\ell r}^{-\frac{1}{2}} \underset{\sim}{A}_{k\ell r} S_{k\ell}(m,n), \qquad r = 1,2,3$$

where

$$\underset{\sim}{A}_{k\ell r} = \left(i(\sigma_{k\ell r} + \varepsilon_{k\ell}),\ \bar{f},\ \bar{f}^2 - (\sigma_{k\ell r} + \varepsilon_{k\ell})^2\right)^T,$$

$$N_{k\ell r} = \left(\bar{f}^2 - (\sigma_{k\ell r} + \varepsilon_{k\ell})^2\right)^2 + \alpha_{k\ell}^2 d \left((\sigma_{k\ell r} + \varepsilon_{k\ell})^2 + \bar{f}^2\right).$$

We may expand $\hat{\eta}$ in the normal modes

$$\underset{\sim}{\hat{\eta}}(m,n) = \sum_{k=0}^{M} \sum_{\ell=1}^{N} \sum_{r=1}^{3} \hat{\gamma}_{k\ell r} \underset{\sim}{P}_{k\ell r},$$

where

$$\hat{\gamma}_{k\ell r} = \langle \underset{\sim}{\hat{\eta}}, \underset{\sim}{P}_{k\ell r} \rangle.$$

Substituting this into Eqs. (4) to (6) and projection on the gravity modes yields,

$$\dot{\gamma}_{k\ell r} = -\nu_{k\ell r} \hat{\gamma}_{k\ell r} + F_{k\ell r}, \qquad r = 2,3$$

where $\quad \nu_{k\ell r} = i\sigma_{k\ell r}$ and

$$F_{k\ell r} = \frac{1}{M+1} N_{k\ell r}^{-\frac{1}{2}} \sum_{m=0}^{M} \sum_{n=1}^{N} \left[\left(\bar{f}^2 - (\sigma_{k\ell r} + \varepsilon_{k\ell})^2\right) Q_{\phi}\right.$$

$$-d[\bar{f}(Q_\psi - \frac{2\Omega}{r^2}(\frac{\partial}{\partial\lambda})_d\ \psi_o) - i(\sigma_{k\ell r} + \varepsilon_{k\ell})(Q_\chi - \frac{2\Omega}{r^2}(\frac{\partial}{\partial\lambda})_d\chi_o)]]S^*_{k\ell}\cos\theta_n,$$

$$\dot{\Upsilon}_{k\ell r} = \frac{1}{M+1}\ N^{-\frac{1}{2}}_{k\ell r} \sum_{m=0}^{M} \sum_{n=1}^{N} [(\bar{f}^2 - (\sigma_{k\ell r} + \varepsilon_{k\ell})^2)\ \frac{\partial\phi}{\partial t}$$

$$-d[\bar{f}\ \frac{\partial}{\partial t}\ \nabla^2_d\psi - i(\sigma_{k\ell r} + \varepsilon_{k\ell})\ \frac{\partial}{\partial t}\ \nabla^2_d\chi]]S^*_{k\ell}\cos\theta_n.$$

Since Eqs. (4) to (6) apply at interior grid points, the boundary values of the forcing and time tendency terms are assumed to be zero. Following Machenhauer (1977), the initial tendencies of gravity wave components are set to zero, yielding

$$-\nu_{k\ell r}\ \hat{\Upsilon}_{k\ell r} + F_{k\ell r} = 0, \quad r = 2,3.$$

This nonlinear equation can be solved iteratively, as follows

$$\hat{\Upsilon}^{(q+1)}_{k\ell r} = F^{(q)}_{k\ell r}/\nu_{k\ell r}, \quad r = 2,3.$$

The new fields are constructed from $\chi^{(q+1)} = \chi_o^{(q+1)} + \hat{\chi}^{(q+1)}$, $\psi^{(q+1)} = \psi_o^{(q+1)} + \hat{\psi}^{(q+1)}$ and $\phi^{(q+1)} = \phi_o^{(q+1)} + \hat{\phi}^{(q+1)}$, where the fields $\hat{\underset{\sim}{n}}^{(q+1)}$ are given by the equations

$$\hat{\underset{\sim}{n}}^{(q+1)} = \hat{\underset{\sim}{n}}^{(q)} + \sum_{k=0}^{M} \sum_{\ell=1}^{N} \sum_{r=2}^{3} (F^{(q)}_{k\ell r}/\nu_{k\ell r} - \hat{\Upsilon}^{(q)}_{k\ell r})\ \underset{\sim}{P}_{k\ell r}$$

and where the fields $\chi_o^{(q+1)}$, $\psi_o^{(q+1)}$ and $\phi_o^{(q+1)}$ satisfy the discrete Laplace equations

$$\nabla_d^2 \chi_o^{(q+1)} = 0, \quad \nabla_d^2 \psi_o^{(q+1)} = 0, \quad \nabla_d^2 \phi_o^{(q+1)} = 0$$

and are equal to the initial values of $\chi - \hat{\chi}^{(q+1)}$, $\psi - \hat{\psi}^{(q+1)}$ and $\phi - \hat{\phi}^{(q+1)}$ on the boundary.

These new fields may be used to evaluate $F_{k\ell r}^{(q+1)}$. Since $\hat{\gamma}_{M+1-k\ell r} = \hat{\gamma}^*_{k\ell r}$, for $k = 1(1)\,\frac{M-1}{2}$ if M is odd or $k = 1(1)\,\frac{M}{2}$ if M is even, it is sufficient to consider the wave numbers $k = 0(1)\,\frac{M+1}{2}$ if M is odd or $k = 0(1)\,\frac{M}{2}$ if M is even.

3. Results

In order to test the initialization method described above, a test run was made with the five-level limited area version of the ECMWF grid point model employed by Temperton and Williamson (1979), on a grid having a mesh spacing ($\Delta\lambda$, $\Delta\theta$) = (2°, 1°) and covering approximately the area between 45° and 65°N and 20°W and 20°E.

By a vertical decomposition of the model equations, initialization of a baroclinic model becomes equivalent to the initialization of the resulting system of shallow water equations. For details the reader is referred again to the paper of Temperton and Williamson.

Initial data are obtained from analyses of the geopotential at pressure levels. Temperatures are derived hydrostatically. Wind components u and v at sigma levels are calculated with the linear balance equation. Orography is included.

Initialization starts with the computation of divergence and vorticity. From the divergence and vorticity the stream function ψ and velocity potential χ are derived. After initialization the velocity components are obtained from ψ and χ.

The retrieval of the temperature and pressure changes at each iteration step of the initialization procedure is accomplished by a variational method (Daley, 1979).

The effect of initialization is made visible by means of a longitude-time diagram of the surface pressure tendencies along the latitude line 55°N, from 22°W to 20°E.

An experiment with the initialization method of section 2 was performed on initial data valid at 1200 GMT, 5 January 1982. The same data were used for a comparison between the nonlinear normal mode and bounded derivative methods in Bijlsma and Hafkenscheid (1986). For details the reader is referred to that paper.

In Table 1 the gravitational frequencies of the method of section 2 are compared with those using the f-plane approximation $(\sigma_{k\ell 2} = -(\alpha_{k\ell}^2 d + \bar{f}^2)^{1/2},\ \sigma_{k\ell 3} = (\alpha_{k\ell}^2 d + \bar{f}^2)^{1/2})$ for the first two vertical modes (satisfying $3\alpha_{k\ell}^2 d - 6\bar{f}^2 + \frac{2}{3}\varepsilon_{k\ell}^2 > 0$) and zonal wave numbers $k = 0(1)\frac{M+1}{2}$.

For a particular k, the wave number ℓ has been chosen so that $\varepsilon_{k\ell}$ has a maximum value. The parameters used have the following values: the geopotential heights are d = 91932.53, 12478.39 m^2s^{-2} respectively, the angular velocity $\Omega = 7.29x10^{-5}s^{-1}$, the radius of the earth $r = 6367x10^3$ m, M = 21, N = 20 and $\bar{\theta} = 55\frac{1}{2}°N$.

Figure 1 gives the longitude-time diagram for the first 6 hours of the limited area model run, showing the gravity wave pattern before initialization.

In Fig. 2 the longitude-time diagram is shown after initialization of the first vertical mode with two iterations using the initialization method of section 2. Fig. 3 shows results of this method after initialization of the first two vertical modes with two iterations. The diagram for normal mode initialization with constant Coriolis parameter, after initialization of the first two vertical modes with two iterations, is shown in Fig. 4.

From these experiments we may conclude that the method described in section 2 is almost identical to the nonlinear normal mode method with constant Coriolis parameter in its effect on the initial field and in its success in suppressing noise in the early forecast. This results is consistent with results of Lynch (1985, section 4).

4. Conclusions

A nonlinear normal mode initialization method, employing nonstationary Rossby modes, is applied to a baroclinic limited area forecast model.

Results of the initialization method have been compared with those using a nonlinear normal mode initialization with stationary Rossby modes.

These results show that the two methods are virtually identical in their effect upon the development of the fields during the initial forecast hours.

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Table 1.

Frequencies of the gravity waves of the f-plane approximation and the method of section 2, for the external and first internal mode respectively. For further explanation, see section 3.

k	$(\alpha^2_{k\ell}d + \bar{f}^2)^{1/2}$		$-\frac{2}{3}\varepsilon_{k\ell} - \frac{2}{3}(3p)^{1/2}\cos(\pi-\mu)/3$		$-\frac{2}{3}\varepsilon_{k\ell} + \frac{2}{3}(3p)^{1/2}\cos\mu/3$	
0	4.2233×10^{-4}	1.9154×10^{-4}	-4.2233×10^{-4}	-1.9154×10^{-4}	4.2233×10^{-4}	1.9154×10^{-4}
1	8.0448×10^{-4}	3.1674×10^{-4}	-8.0664×10^{-4}	-3.1916×10^{-4}	8.0233×10^{-4}	3.1434×10^{-4}
2	1.3736×10^{-3}	5.1824×10^{-4}	-1.3750×10^{-3}	-5.1968×10^{-4}	1.3722×10^{-3}	5.1680×10^{-4}
3	1.9186×10^{-3}	7.1563×10^{-4}	-1.9196×10^{-3}	-7.1663×10^{-4}	1.9176×10^{-3}	7.1463×10^{-4}
4	2.4247×10^{-3}	9.0025×10^{-4}	-2.4254×10^{-3}	-9.0100×10^{-4}	2.4239×10^{-3}	8.9951×10^{-4}
5	2.8836×10^{-3}	1.0682×10^{-3}	-2.8841×10^{-3}	-1.0688×10^{-3}	2.8830×10^{-3}	1.0677×10^{-3}
6	3.2873×10^{-3}	1.2163×10^{-3}	-3.2878×10^{-3}	-1.2167×10^{-3}	3.2869×10^{-3}	1.2158×10^{-3}
7	3.6286×10^{-3}	1.3415×10^{-3}	-3.6289×10^{-3}	-1.3418×10^{-3}	3.6283×10^{-3}	1.3412×10^{-3}
8	3.9010×10^{-3}	1.4415×10^{-3}	-3.9012×10^{-3}	-1.4418×10^{-3}	3.9008×10^{-3}	1.4413×10^{-3}
9	4.0994×10^{-3}	1.5144×10^{-3}	-4.0995×10^{-3}	-1.5146×10^{-3}	4.0992×10^{-3}	1.5143×10^{-3}
10	4.2199×10^{-3}	1.5587×10^{-3}	-4.2200×10^{-3}	-1.5588×10^{-3}	4.2198×10^{-3}	1.5586×10^{-3}
11	4.2603×10^{-3}	1.5736×10^{-3}	-4.2603×10^{-3}	-1.5736×10^{-3}	4.2603×10^{-3}	1.5736×10^{-3}

Figure Captions

Fig. 1.

Longitude-time diagram of surface pressure tendencies along the latitude line of 55°N, starting at 1200 GMT 5 January 1982, before initialization. Units are millibars per hour.

Fig. 2.

As in Fig. 1, but after initialization with the method of section 2 of the external mode, with two iterations.

Fig. 3.

As in Fig. 2, but after initialization with the method of section 2 of the external and first internal mode, with two iterations.

Fig. 4.

As in Fig. 3, but after normal mode initialization with constant Coriolis parameter of the external and first internal mode, with two iterations.

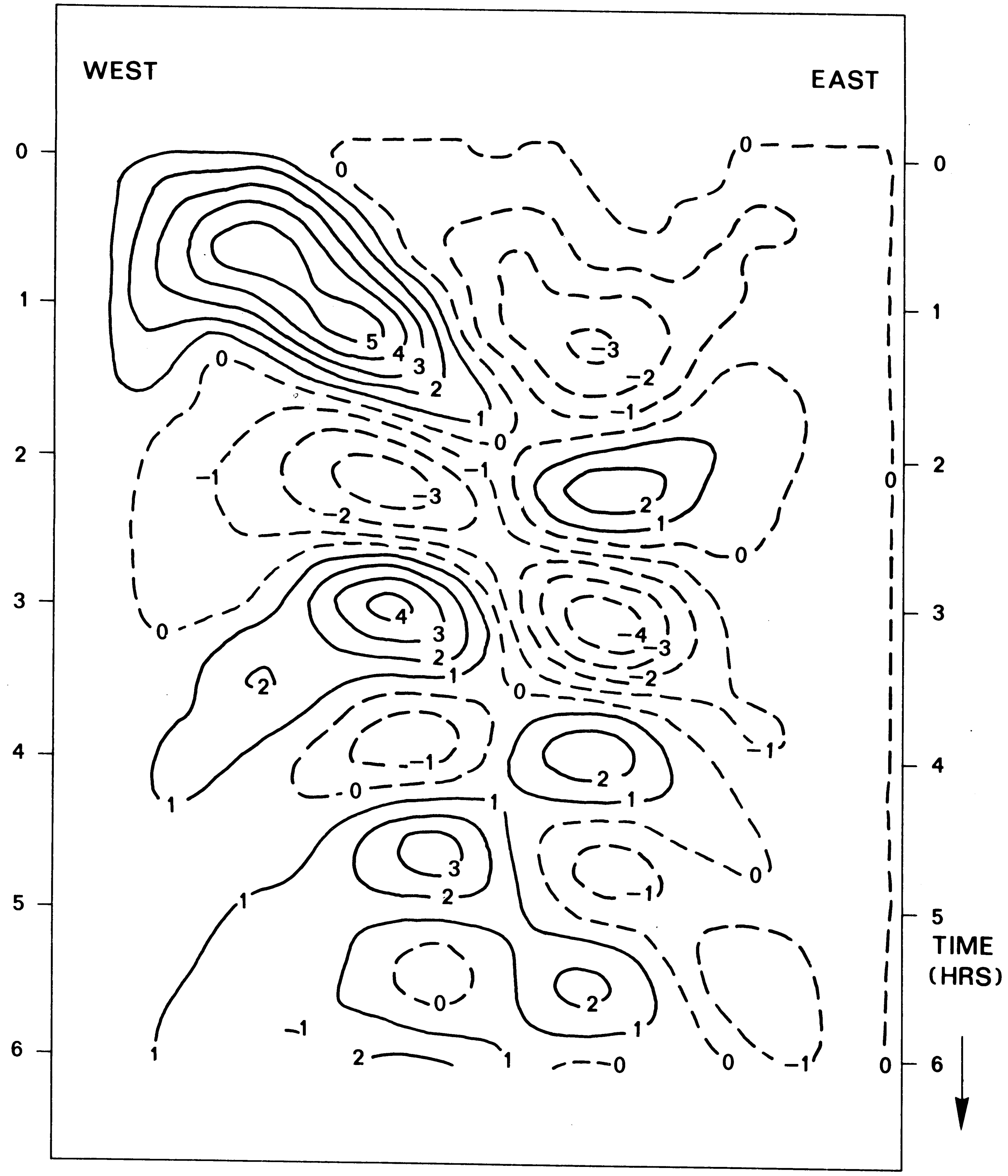

Figure.1

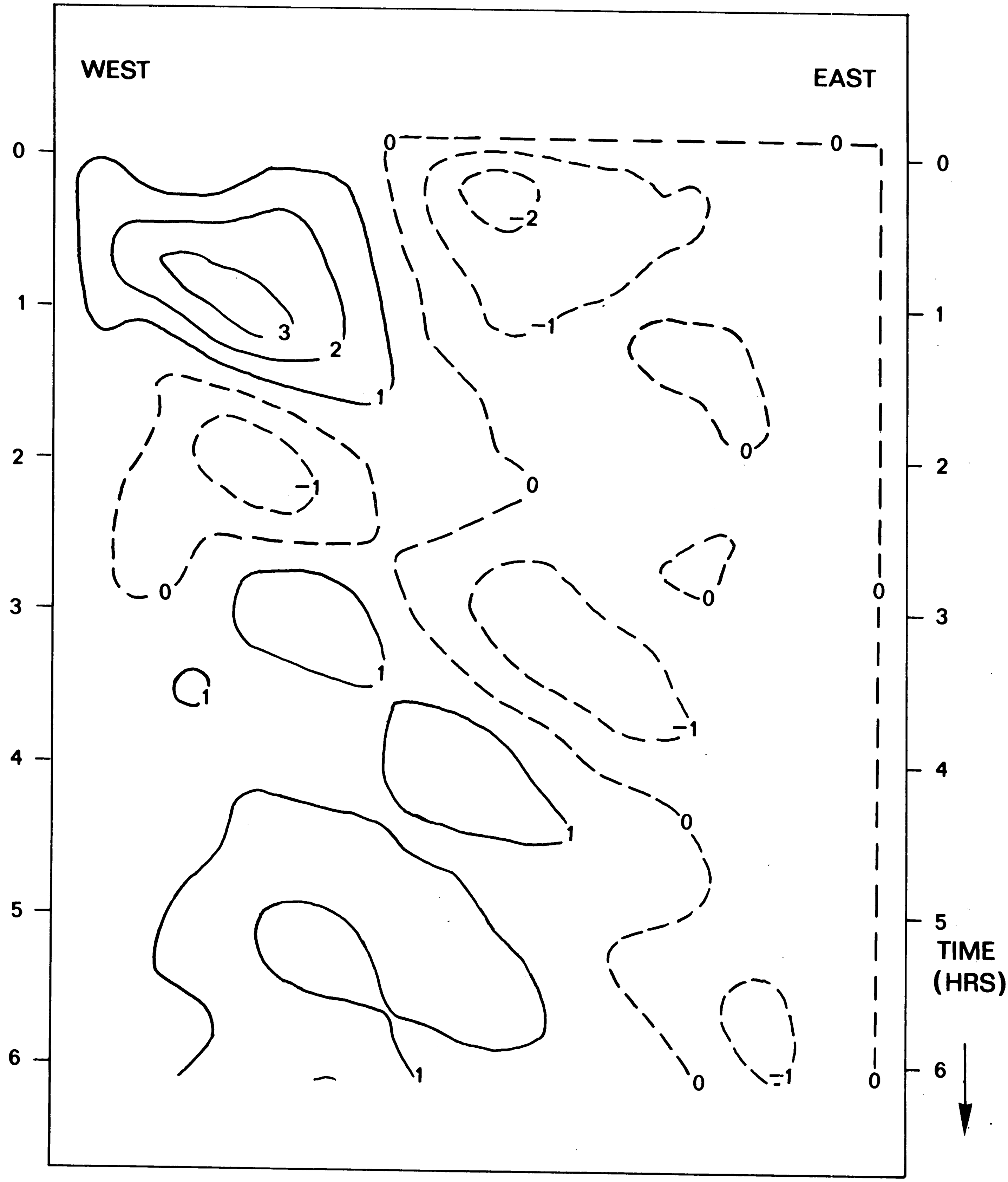

Figure.2

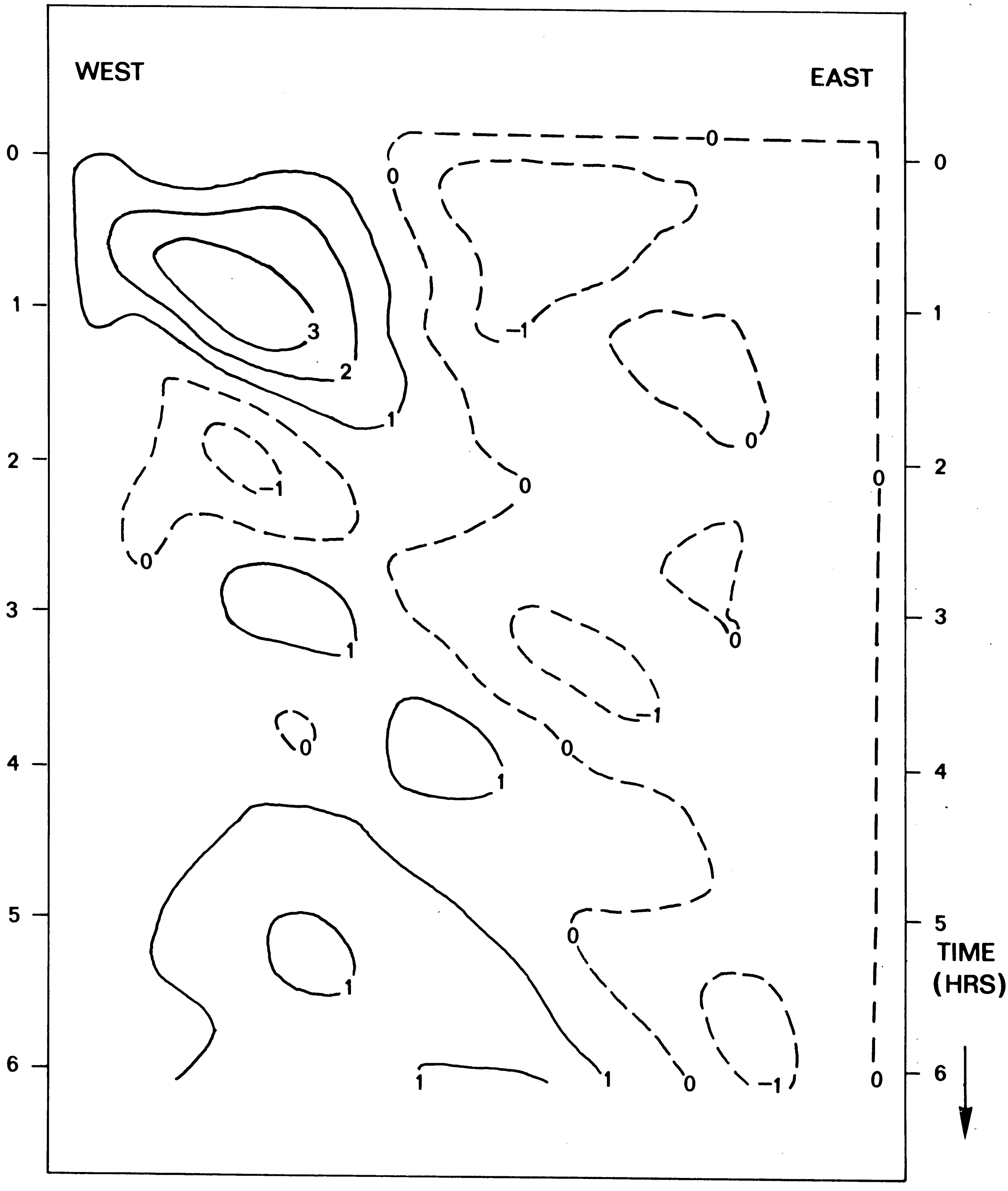

Figure.3

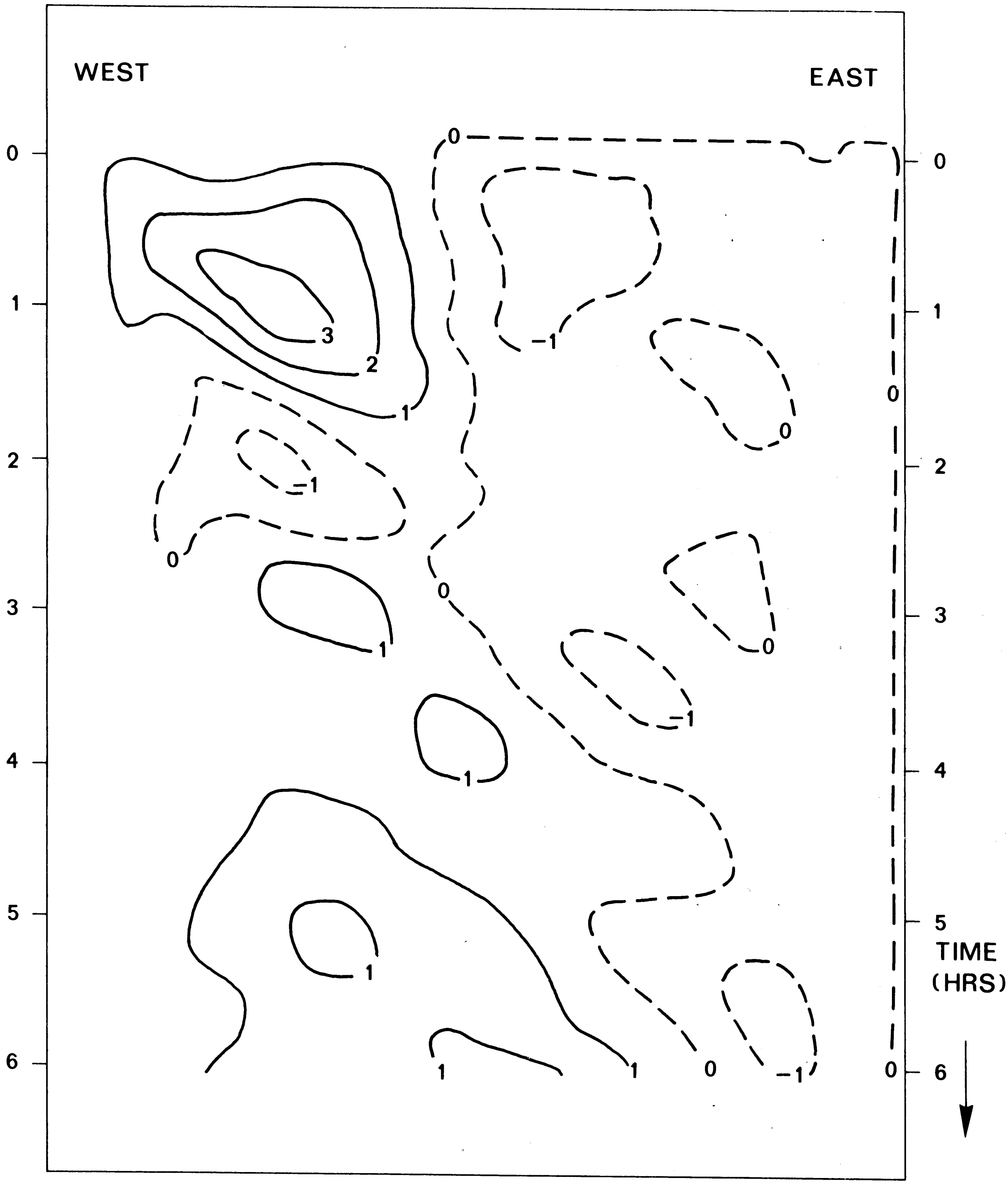
WEST
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Figure. 4