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A computer program for
calculating infrared fluxes



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SUMMARY

Radiation charts are widely used for computing atmospheric radiation. Computations with a radiation chart are cumbersome and laborious. A computerised version of the radiation chart is desired if many infrared fluxes have to be calculated. In the near future we may need such a computer calculation as part of an energy balance model of the earth's surface. Also for calculating the radiation flux divergence a computerised radiation chart may be useful. The calculations are based on the Elsasser chart (1942), using input data from tables of Elsasser and Culbertson's (1960). The computer program can be used for calculating infrared fluxes at every level in the atmosphere, downward as well as upward. The presence of clouds can be included in a simple way. In some unavoidable approximations (e.g. the pressure profile assumed) emphasis has been laid on the accuracy of the calculations for levels not higher than the first few kilometres. The total of the inaccuracies introduced by approximations in the calculations is not more than a few per cent. This figure should be compared with inaccuracies introduced by Elsasser's concept, which may be of the order of ten per cent. The computer time required for the calculation of one radiation flux is about one second (Burroughs B6700 computer).

SAMENVATTING

Voor de berekening van de atmosferische straling (infrarode stralingsflux) worden vaak stralingskaarten gebruikt. Het werken met deze kaarten is omslachtig en tijdrovend. We hebben daarom een computerprogramma ontwikkeld, dat de berekening met een kaart nabootst. Slechts het temperatuur- en vochtprofiel van de atmosfeer moeten in de vereiste vorm aan de computer worden toegevoerd. De berekening is gebaseerd op de stralingskaart van Elsasser (1942), waarbij gebruik is gemaakt van tabellen van Elsasser en Culbertson (1960). Het computerprogramma kan worden gebruikt voor de berekening van neerwaartse en opwaartse infrarood stralingsfluxen op elk niveau in de atmosfeer. Er is getracht fouten ten gevolge van enkele benaderingen zo klein mogelijk te houden voor de eerste paar kilometers van de atmosfeer. De fouten, geïntroduceerd door de gevolgde rekenwijze, bedragen niet meer dan enkele procenten. Benaderingen in het rekenconcept van Elsasser kunnen aanleiding geven tot fouten van zeker 10 procent.

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LIST OF SYMBOLS

B_ν	black-body spectral irradiance at frequency ν	$Wm^{-2}s$
E^\downarrow	downward longwave irradiance	Wm^{-2}
E_ν^\downarrow	idem, at frequency ν	$Wm^{-2}s$
e	water vapour pressure	mbar
g	acceleration of gravity	ms^{-2}
H	relative humidity	%
L_ν	generalized spectral absorption coefficient	-
p	atmospheric pressure	mbar
Q	radiation quantity, defined by Eq. (11)	$Wm^{-2}K^{-1}$
q	water vapour mixing ratio	$kg\ kg^{-1}$
R	radiation quantity, defined by Eq. (13)	$Wm^{-2}K^{-1}$
T	temperature	K
t	idem	$^{\circ}C$
u	reduced optical thickness	cm
z, z', z''	height	m
γ_d	dry adiabatic lapse rate	Km^{-1}
ν	frequency of the radiation	s^{-1}
ρ	gas density	kgm^{-3}
τ_F	spectral transmissivity for isotropic radiation	-

subscripts

\bar{o}	reference level
1	upper level of the atmosphere, where $T = 193\ K$
i	step in the numerical integration procedure
j	idem

A COMPUTER PROGRAM FOR CALCULATING INFRARED FLUXES

W. Kohsiek

1. THE RADIATION TRANSPORT EQUATION

Consider a layer of gas with thickness $z-z_0$ and with an arbitrary temperature distribution $T(z)$. Let us suppose an isotropic black-body irradiance at frequency $\nu, B_\nu(T(z))$, on the top of this layer.

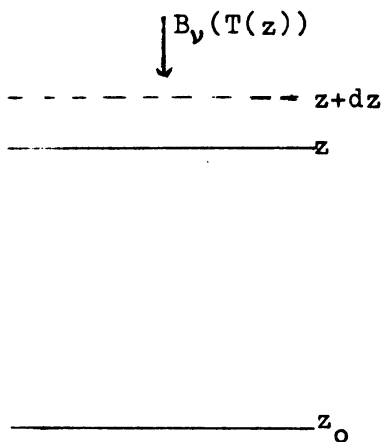


Fig. 1

The radiation reaching level z_0 due to this black-body radiation flux only is:

$$E_\nu(z_0) = B_\nu(T(z)) \tau_F(z-z_0) \quad (1)$$

where $\tau_F(z-z_0)$ is the spectral flux transmissivity of the layer $z-z_0$ for isotropic radiation. The transmissivity τ_F depends on the pressure and temperature distribution in the layer $z-z_0$, but given the distribution functions, τ_F may be considered a function of $z-z_0$ only.

Now, if we add an infinitesimal slab with thickness dz and temperature $T(z)$ on top of the gas layer, the radiation flux at z_0 will not change, because the slab dz emits the same amount of radiation as it absorbs (provided that there is no scattering of radiation, and there is a local thermodynamical equilibrium). So the radiation flux arriving at level z is still $B_\nu(T(z))$. The flux at z_0 is composed of two components: first, the transmitted part of the black-body flux:

$$B_\nu(T(z)) \tau_F(z+dz-z_0) = B_\nu(T(z)) \left[\tau_F(z-z_0) + \frac{d\tau_F(z-z_0)}{dz} dz \right] \quad (2)$$

and second, the radiation emitted by the slab dz , reaching level z_0 , which we will denote by $dE^\uparrow(z_0)$.

We still do not consider the radiation emitted by the layer $z-z_0$. So we have:

$$B_\nu(T(z)) \left[\tau_F(z-z_0) + \frac{d\tau_F(z-z_0)}{dz} dz \right] + dE_\nu^\downarrow = E_\nu(z_0) = B_\nu(T(z)) \tau_F(z-z_0) \quad (3)$$

From this follows:

$$dE_\nu^\downarrow(z_0) = - B_\nu(T(z)) \frac{d\tau_F(z-z_0)}{dz} dz \quad (4)$$

This is the radiation flux at z_0 due to the radiation emitted by the slab dz , and the subsequent absorption of the layer $z-z_0$. Then the radiation flux at z_0 due to emission and absorption of the whole layer $z-z_0$, without considering an external radiation flux at level z , is:

$$E_\nu^\downarrow(z_0) = \int_{z_0}^z dE_\nu^\downarrow(z_0) = - \int_{z_0}^z B_\nu(T(z')) \frac{d\tau_F(z'-z_0)}{dz'} dz' \quad (5)$$

and

$$E^\downarrow(z_0) = \int_\nu d\nu E_\nu^\downarrow(z_0) = - \int_\nu d\nu \int_{z_0}^z B_\nu(T(z')) \frac{d\tau_F(z'-z_0)}{dz'} dz' \quad (6)$$

This is the integrated version of the radiation transport equation. The transmissivity τ_F shows a pressure- and temperature dependency by line-broadening effects, which may be approximated by $\propto p \sqrt{T}$. It is therefore convenient to express the path in the atmosphere in terms of the so-called reduced optical thickness u , defined by:

$$u(z-z_0) = \int_{z'=z_0}^z \left[\frac{T_0}{T(z')} \right]^{\frac{1}{2}} \frac{p(z')}{p_0} dh(z'-z_0), \quad (7)$$

$$h(z'-z_0) = \int_{z_0}^{z'} \rho(z'') dz'' \quad (8)$$

T_0 and p_0 is the reference state (usually 293 K and 1013 mbar). $\rho(z'')$ is the gas density at level z'' . In practice, the vertical thickness is not expressed in units kg m^{-2} , but in cm precipitable water, or, for other gases, in the equivalent thickness of the gas layer if it were at standard pressure (1013 mbar) and temperature (293 K). The dependency of τ_F on the frequency is modelled by the generalized spectral absorption coefficient L_ν , which is a smoothed quantity over the many spectral absorption lines, so that L_ν varies only slowly with frequency ν . τ_F may now be expressed as a function of the product of L_ν and u :

$$\tau_F = \tau_F(L_\nu \cdot u) \quad (9)$$

and the radiation transport equation is written as:

$$E^\dagger(u_0) = - \int_{\nu} d\nu \int_{u_0}^{u_1} B_\nu(T(u)) \frac{d\tau_F(L_\nu(u-u_0))}{du} du \quad (10)$$

The upper integration level u_1 is the upper boundary of the atmosphere (above which there is no optically active material). The dependency of τ_F on $(L_\nu \cdot u)$ follows from laboratory measurements, so if the profiles of temperature, density and pressure are known, it is possible to calculate L^\dagger with Eqs (10), (9), (8) and (7), in case of a pure gas (in practice only H_2O , CO_2 and O_3 are of importance. Other gases, like chlorofluoromethanes, are not considered). In case of a mixture of gases, it is not correct to add the results for the constituents because of overlap of spectral lines; special correction procedures have to be followed. We shall return to this matter in section 2.2.

2. ELSASSER'S TREATMENT OF THE RADIATION TRANSPORT EQUATION

2.1 A pure gas

Elsasser (1942) reduces the double integration in (6) to a single integration by introducing the quantity $Q(u,T)$, defined as follows:

$$Q(u, T) = \int_0^{\infty} \frac{dB_{\nu}(T)}{dT} \tau_F(L_{\nu}, u) d\nu \quad (11)$$

After some mathematical manipulations, the radiation transport equation is transformed to:

$$E^+(z_0) = \int_0^{T_0} Q(0, T) dT + \int_{T_0}^{T_1} Q(u(T), T) dT + \int_{T_1}^0 Q(u_1, T) dT \quad (12)$$

Here, the integration variable is T instead of u in (6).

$T_0 = T(z=z_0)$, $T_1 = T(z_1)$ and $u(T_0) = 0$. Expression (12) is the basis of Elsasser's radiation chart. The integrations in (12) constitute an integration along a closed path in the Q, T plane. The first term on the right hand side of Eq. (12) is the black-body radiation at temperature T_0 (this follows from the definition of $Q(u, T)$, and from $\tau_F(0) = 1$). The second term can be determined if the temperature- and humidity profile of the atmosphere are known.

Eq. (12) may be simplified by introduction of the quantity $R(u, T)$:

$$R(u, T) = \frac{dB}{dT} - Q(u, T) \quad (13)$$

$$E^+ = - \left[\int_{T_0}^{T_1} R(u(T), T) dT + \int_{T_1}^0 R(u_1, T) dT \right] \quad (14)$$

where $B(T)$ is the total black-body emittance at temperature T . Elsasser and Culbertson (1960) present tables for R as a function of u and T , and for $\int_{T_1}^0 R(u_1, T) dT$ as a function of u_1 , where T_1 is chosen at 193 K, for three spectral regions of the water vapour spectrum and the total H_2O spectrum, CO_2 and O_3 .

2.2 Mixture of gases

The atmospheric radiation at the earth's surface is principally emitted by water vapour, but also CO_2 and O_3 contribute. In calculating the radiation, mostly only H_2O and CO_2 are considered.

The contribution of O_3 , which is 1-2 % at ground level, is left out. Elsasser and Culbertson (1960) account for the CO_2 contribution by replacing $\tau_F(H_2O)$ by $\tau_F(H_2O) \cdot \tau_F(CO_2)$. (See Appendix I). Next, the quantities Q and R are corrected by ΔQ and ΔR respectively, which are defined by

$$\Delta Q = - \Delta R = \int_0^{\infty} \frac{dB_{\nu}(T)}{dT} \tau_F(H_2O) [1 - \tau_F(CO_2)] d\nu \quad (15)$$

Since the effect of the CO_2 correction is in practice most important in the lowest layer of the atmosphere, Elsasser and Culbertson only present tables of ΔR as a function of u_{H_2O} and u_{CO_2} at a temperature of 293 K. At very low temperatures, however, the correction must go to zero. For instance, Yamamoto (1952) curves his CO_2 correction lines down below $T = 160$ K. It can be shown that the CO_2 corrections of Elsasser and Yamamoto are formally the same. (See Appendix I). Therefore, we recommend to apply the correction ΔR not to the whole integration path from $T = T_1$ to $T = 0$ in Eq. (14), but to part of it. From Yamamoto's CO_2 curve we estimated the equivalent block-shaped CO_2 correction extending from 193 to 133 K.

2.3 Upward terrestrial radiation

The upward terrestrial radiation is defined as the sum of the components (1) surface emission; (2) long-wave radiation reflected by the surface and (3) upward atmospheric radiation as received at the level of observation. If the emission coefficient of the earth is assumed to be equal to 1, only the first and the third component contribute, and the upward radiation can be computed with the Elsasser scheme in a simple way. If T_1 (Eq. (14)) is supposed to be the surface temperature, $u_1(T_1) = \infty$ (black body), so:

$$\int_{T_1}^0 R(u_1, T) dT = \int_{T_1}^0 R(\infty, T) dT = \int_{T_1}^0 \frac{dB(T)}{dT} dT = - B(T_1) \quad (16)$$

since $Q(\infty, T) = 0$ as a consequence of $\tau_F(\infty) = 0$ (Eq. (11)).

The other integral of Eq. (14) has to be determined with the temperature- and humidity profile of the atmosphere between z_0 and the surface.

2.4 Clouds

If there is a cloud cover present with base at level z_1 , the downward terrestrial radiation may be computed along the same lines as the upward terrestrial radiation. It is assumed that the cloud cover is a black body at level z_1 , with temperature T_1 , so again $u_1(T_1) = \infty$, and Eq. (16) can be applied. In case there is a fractional cloud cover with different base heights, the separate contributions may be computed.

3. COMMENTS ON ELSASSER'S FORMULATION

Zdunkovski et al. (1966) pointed out that the transmission function τ_F shows a double dependency on temperature: first, τ_F is a function of the generalized absorption coefficient L_ν , which depends slightly on temperature (by the Boltzmann population factor), and second, it depends on $u(T)$. Elsasser and Culbertson omitted these implicit dependencies in the construction of their radiation tables. Zdunkovski et al. corrected Elsasser's formulation, and present new tables for the quantity $R(u, T)$ for the total water vapour spectrum CO_2 . The tables for $\int_{T_1}^0 R(u_1, T) dt$ (see Eq. (14)) need no correction, because this quantity is empirically determined.

The discrepancies between the new tables for $R(u, T)$ and the uncorrected ones are of the order of 10 per cent. As a consequence, the calculated infrared fluxes also differ by about 10 per cent. It is also argued that the correction for the $\text{H}_2\text{O}-\text{CO}_2$ overlap is not independent on temperature, as Elsasser assumed. The differences in the ΔR values may be considerable, up to a factor 2 or 3. The corrections of Zdunkovski were questioned by Charlock and Herman (1976). These authors show that the Elsasser scheme contains two internal mathematical errors, which are self-cancelling. As a result, the procedure of Elsasser and Culbertson for the evaluation

of downward fluxes for a "real" atmosphere gives correct numerical results, whereas Zdunkovski's correction for isothermal slab fluxes should be proper. In a joint comment, Charlock, Herman and Zdunkovski (1976) reduce the differences in point of view to different approximations in the simplification of the radiation transport equation. They conclude that both solutions are approximate, and the last word should be to the measurements. We may add to the discussion that, first, the conclusions of Charlock and Herman regarding isothermal and non-isothermal slab fluxes lead to unrealistic physical discontinuities. This is explained in more detail in Appendix II. Second, Staley and Jurica (1970), who calculated isothermal slab emissivities from Elsasser and Culbertson's original generalized absorption coefficients and flux transmissivities, without using Zdunkovski's corrected R function, show that their results are in good agreement with Zdunkovski's.

Concluding, it seems that Elsasser and Culbertson's calculations for isothermal slab fluxes are about 10 per cent in error. The results of Zdunkovski et al., Charlock and Herman and of Staley and Jurica agree on this part. Charlock and Herman's expressions for isothermal and non-isothermal slab fluxes show a non-physical discontinuity, whereas Zdunkovski's do not. So one is inclined to prefer the latter calculations to the former, but it should be kept in mind that for a non-isothermal atmosphere all the calculations discussed here are only approximate solutions of the radiation transport equation.

4. THE CALCULATION OF THE ATMOSPHERIC RADIATION IN PRACTICE

4.1 The H₂O contribution

The quantities $\int_{T_0}^{T_1} R(u(T), T) dT$ and $\int_{T_1}^0 R(u_1, T) dT$ for water vapour have to be calculated. The relation $u(T)$ is given by the temperature-, humidity- and pressure profile of the atmosphere. (See next paragraph). The values of the quantity $R(u, T)$ are determined from tables of Elsasser and Culbertson's (1960) (or equivalent tables of Zdunkovski's et al. (1966), see section 3) by a quadratic interpolation procedure.

The integration is carried out numerically. It should be noted that in practice the highest observed level generally has a temperature greater than $T_1 = 193$ K; the way we provided for the lacking temperature interval is discussed in section 4.3. The quantity $\int_{T_1}^0 R(u_1, T) dT$ is obtained from tables of Elsasser and Culbertson's, again by quadratic interpolation.

The determination of u is as follows:

- (i) the atmosphere is divided in layers $j = 1, 2, \dots, i$, corresponding to the integration steps in the numerical determination of $\int_{T_0}^{T_1} R(u, T) dT$;
- (ii) it is assumed that the pressure profile of the atmosphere is that of an atmosphere with a dry-adiabatic temperature profile, with temperature $T_0 + \gamma_d z_0$ and pressure $p_0 = 1013$ mbar at $z = 0$:

$$p(z) = p_0 \left(1 - \frac{\gamma_d}{T_0 + \gamma_d z_0} z \right)^{7/2} \quad (17)$$

where γ_d is the dry-adiabatic lapse rate. The exponent in Eq. (17) stands for c_p/R , which is, to a very good approximation, equal to $7/2$;

- (iii) the reduced optical path u_i between level $j = 1$ and $j = i$ is calculated with

$$u_i = \sum_{j=1}^i \Delta u_j, \quad \Delta u_j = \bar{q}_j \frac{\Delta p_j}{g} \frac{\bar{p}_j}{p_0} \left(\frac{T_0}{T_j} \right)^{1/2} \quad (18)$$

$\bar{q}_j = [q(z_j) + q(z_{j-1})]/2$ is the average mixing ratio of layer j (in units $\text{kg H}_2\text{O}$ per kg dry air); $g = 9.81 \text{ ms}^{-2}$, $\Delta p_j = p(z_j) - p(z_{j-1})$ and $\bar{p}_j = [p(z_j) + p(z_{j-1})]/2$.

As $\Delta p_j/g \approx \bar{\rho}_j(z_j - z_{j-1})$, where $\bar{\rho}_j$ is the average density of the air of the layer j , $q_j \cdot \frac{\Delta p_j}{g}$ is the amount of water in layer j in kg m^{-2} . (It is convenient to express this quantity in units cm ; $1 \text{ kg m}^{-2} \hat{=} 10^{-1} \text{ cm}$).

The humidity measurements of the radiosonde are often not in terms of kg H₂O per kg dry air, but in a relative measure. The specific humidity is calculated from the relative humidity H following Robitzch (1941):

$$q = 0.622 \frac{e}{p-e} ,$$

$$e = H \cdot 10^{\left(\frac{7.5t}{237.3+t} + 0.78571 \right)} \quad (19)$$

where t is the temperature in °C. The constants in (19) hold for the case that the relative humidity is expressed as the actual vapour pressure divided by the saturated pressure above water, also for temperatures below 0 °C.

4.2 The CO₂ contribution

We start from the following assumptions:

- (i) the CO₂ concentration is 0.033 vol. %;
- (ii) the pressure profile of the atmosphere is that of an atmosphere with a dry-adiabatic temperature profile (as for the case of H₂O; see equation (17)).

The reduced optical path between z and z₀ is calculated with:

$$u_{\text{CO}_2}(z-z_0) = 33 \cdot 10^{-5} \int_{z_0}^z \frac{p(z')}{p_0} \cdot \frac{T_0}{T(z')} \cdot \frac{p(z')}{p_0} \cdot \left(\frac{T_0}{T(z')} \right)^{\frac{1}{2}} dz' \quad (20)$$

The first two factors of the integrand represent the variation of the CO₂ density with height, the two following factors are the pressure- and temperature corrections. Assuming c_p/R = 7/2, and neglecting terms with z₀ in the integrand, the integral can be determined; the result is:

$$u_{\text{CO}_2}(z-z_0) = -33 \cdot 10^{-5} \cdot \frac{2}{13} \frac{T_0}{\gamma} \left[\left(1 - \frac{\gamma_d}{T_0} z \right)^{13/2} - \left(1 - \frac{\gamma_d}{T_0} z_0 \right)^{13/2} \right] \quad (21)$$

with u_{CO₂} expressed in m. The omission of z₀ in the integrand of (20) is not a serious restriction if z₀ ≪ 1 km.

Given a pair of u_{CO_2} , $u_{\text{H}_2\text{O}}$ values, the quantity ΔR is determined by interpolation from a table of Elsasser and Culbertson's. The contribution $\int_{T_0}^{T_1} \Delta R dT$ is calculated with the same numerical integration procedure as for H_2O . The determination of the interval T_0-0 is discussed in the next section.

4.3 Additional procedures for H_2O and CO_2 at low temperatures

As mentioned in section 3.1, the lowest temperature observed in the atmosphere (T_n) will in practice be often higher than $T_1 = 193$ K. The contribution of the temperature interval (T_n, T_1) is calculated with:

$$\begin{aligned} \int_{T_n}^{T_1} R(u, T) dT &\approx \int_{T_n}^{T_1} R(u(T_n), T) dT \approx \int_{T_n}^{T_1} R(u(T_n), T_n) \frac{T^3}{T_n^3} dT = \\ &= \frac{1}{4} \frac{1}{T_n^3} (T_1^4 - T_n^4) R(u(T_n), T_n) \end{aligned} \quad (22)$$

The first step in (22) assumes that the atmosphere is completely dry above level n. This is a reasonable assumption in case level n is at the tropopause. For the second step, it is assumed that the quantity $R(u = \text{constant}, T) \propto T^3$. This is only approximately true.

The CO_2 contribution for the interval (T_n, T_1) is treated in the same way. However, the quantity $\Delta R(u(\text{CO}_2) = \text{constant}, u(\text{H}_2\text{O}) = \text{constant}, T)$ is not proportional to T^3 , but almost independent of the temperature in this interval. The error made is small. The CO_2 contribution for the interval ($T_1, 0$) should not be equalled to $193 \times \Delta R$, because, as mentioned in section 2.2, ΔR decreases sharply below $T \approx 160$ K. We made the following estimation from Yamamoto's CO_2 -correction curve:

$$\int_{T_1}^0 \Delta Q(u_1(\text{H}_2\text{O}), u_1(\text{CO}_2)) dt \approx 60 \Delta Q(u_1(\text{H}_2\text{O}), u_1(\text{CO}_2)) \quad (23)$$

5. SOME REMARKS ON THE CALCULATION PROCEDURE FOLLOWED

5.1 Inaccuracies introduced by the calculation procedure

- (1) The procedure for the H_2O and CO_2 contributions at temperatures between T_n and T_1 . (See section 4.3).

The contribution of the interval (T_n, T_1) may be calculated by introducing artificial temperature levels in this interval, instead of using the approximative expression (Eq. 22). It is still assumed that there is neither H_2O nor CO_2 at levels lower in temperature than T_n . The difference between the two procedures is not more than one per cent.

- (2) The CO_2 contribution for T_1-0 .

In Eq. (23) the effective temperature interval is estimated at 60 K. If this had been 50 K, which is not impossible, the total radiation would have been lowered by 1-2 per cent.

- (3) The pressure profile of the atmosphere.

We assume that the pressure profile is that of an atmosphere with an adiabatic temperature profile. (See section 4.1). If we suppose the pressure profile to be that of an isothermal atmosphere (so $p/p_0 = \exp\{-\frac{gM}{RT} z\}$), the atmospheric radiation is changed by only about 0.1 per cent. $p_0 = 1000$ mbar in place of $p_0 = 1013$ mbar changes the result by about 0.06 per cent. So the calculation is insensitive to inaccuracies in the pressure profile assumed. The reason is that the calculated optical path u is rather insensitive to the pressure profile assumed, and that the radiation function $R(u, T)$ is not very sensitive to u .

- (4) The CO_2 concentration.

We assumed $[CO_2] = 0.033$ vol. %. The calculated atmospheric radiation is rather insensitive to variations of this figure; e.g. in the extreme case of $[CO_2] = 0.030$ vol. %, the variation in the atmospheric radiation only is 0.3 %.

(5) O_3

Inclusion of O_3 in the calculations has a small effect (1-2 %) on the calculated atmospheric radiation at ground level.

5.2 Sensitivity of the calculated atmospheric radiation to the input parameters (temperature, humidity, height)

In almost all cases the data from the routine radiosonde are sufficient to calculate the atmospheric radiation with an error not greater than a few per cent. Only in exceptional cases, if e.g. there is a sharp, low-level moisture jump, a better resolution of the first few hundred metres than most radiosondes can provide for may be necessary. The reason is that $R(u,T)$, with which L^\downarrow is calculated (see Eq. (14)), is only a slowly varying function with u , and L^\downarrow is therefore rather insensitive to errors made in the reduced optical thickness u . For instance, if all the observed values for the specific humidity are lowered by 20 %, the calculated atmospheric radiation will be about 1 % less in a normal case. An error of 1 % is also produced by a systematic temperature deviation of 1 K.

If one is interested in the calculation of radiation divergence, a detailed knowledge of the temperature- and humidity profile in the region of interest is truly essential. In those cases differences of atmospheric radiation have to be calculated, which are often not more than 1 % of the radiation values proper.

6. CONCLUSIONS

A computer program has been developed for computing longwave atmospheric radiation fluxes. The calculation is based on the Elsasser chart. The input quantities for the program are: the temperature- and humidity distribution with height (these data may be provided by a routine radiosonde), and values for radiation quantities of H_2O and CO_2 (from tables of Elsasser and

Culbertson's for instance). The presence of clouds can be included in a simple way. The error in the calculations, due to approximations in the calculation procedure at lower temperatures, the omission of the contribution of ozone, and other approximations, is limited to a few per cent. The error introduced by Elsasser's treatment of the radiation transport equation may be 10 per cent or more. The calculation of one flux requires about one second on a Burroughs B6700 computer. The computer program may be of value if many fluxes have to be calculated, e.g. as part of a boundary layer model including the energy balance of the earth.

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APPENDIX I : THE CO₂ CORRECTION

In this appendix we will show that the CO₂ correction procedures of Elsasser and Culbertson (1960) and of Yamamoto (1952) are formally the same. We needed this conclusion in order to arrive at the procedure for the CO₂ correction at low temperatures, Eq. (23).

Yamamoto's calculation of the atmospheric radiation at level z is:

$$E^\downarrow(T_z) = B(T_z) - \int_0^{B(T_\infty)} \tau_F\{u_\infty(T_\infty)\} dB - \int_{B(T_\infty)}^{B(T_z)} \overline{\tau_F\{u(T)\}} dB \quad (\text{AI.1})$$

where

$$\overline{\tau_F\{u(T)\}} \frac{dB(T)}{dT} = \int_0^\infty \frac{dB_\nu(T)}{dT} \tau_F(L_\nu, u) d\nu \quad (\text{AI.2})$$

Here, Yamamoto's notation is followed in part; T_∞ is the temperature at the boundary of the atmosphere, corresponding to Elsasser's T₁.

The effect of CO₂ is taken into account by reducing the transmissivity $\overline{\tau_F}$ with $\Delta\tau_F(u, u_{CO_2})$:

$$\Delta\tau_F(u, u_{CO_2}) \frac{dB(T)}{dT} = \int_0^\infty \frac{dB_\nu(T)}{dT} \{ \tau_F(L_\nu, u) - \tau_F(L_\nu, u) \tau_F(u_{CO_2}) \} d\nu \quad (\text{AI.3})$$

The right hand side terms of Eq. (AI.1) can be converted to the corresponding terms in Elsasser's formulation (Eq. 12, section 2.1):

$$E^\downarrow(z) = \int_0^z Q(0, T) dT - \int_0^{T_1} Q(u_1, T_1) dT - \int_{T_1}^z Q(u(T), T) dT \quad (\text{AI.4})$$

We put T₁ = T_∞ and u₁ = u_∞. For instance, the third term is:

$$\int_{T_1}^Z Q(u(T), T) dT = \int_{T_1}^Z \int_0^{\infty} \frac{dB_{\nu}(T)}{dT} \tau_F(L_{\nu} u) dT d\nu =$$

$$= \int_{T_1}^Z \frac{1}{\tau_F\{u(T)\}} \frac{dB(T)}{dT} dT = \int_{B(T_{\infty})}^{B(T_Z)} \frac{1}{\tau_F\{u(T)\}} dB$$

The CO₂ correction, in Elsasser's terms, is:

$$\Delta Q = \int_0^{\infty} \frac{dB_{\nu}(T)}{dT} \tau_F(H_2O) [1 - \tau_F(CO_2)] d\nu \quad (AI.5)$$

Analogous to L[↓](z) itself, the correction may be expressed as the sum of three terms. The third term is e.g.:

$$\int_{T_1}^Z \Delta Q dT = \int_{T_1}^Z \int_0^{\infty} \frac{dB_{\nu}(T)}{dT} \tau_F(H_2O) [1 - \tau_F(CO_2)] dT d\nu$$

Analogous to the procedure for L[↓](z), the correction may be converted to the corresponding correction of Yamamoto:

$$\int_{T_1}^Z \Delta Q dT = \int_{T_1}^Z \frac{1}{\Delta\tau_F(u, u_{CO_2})} \frac{dB(T)}{dT} dT = \int_{B(T_{\infty})}^{B(T_Z)} \frac{1}{\Delta\tau_F(u, u_{CO_2})} dB \quad (AI.6)$$

So, the CO₂ corrections of Elsasser and of Yamamoto are mathematically identical. The numerical values of $\Delta\tau_F \frac{dB}{dT}$ and ΔQ are given in table AI.1 for some values of u_{H_2O} and u_{CO_2} . The differences are due to a different choice of the transmission functions.

Table AI.1 Elsasser's and Yamamoto's CO₂ corrections

$u_{\text{H}_2\text{O}}$ (cm)	u_{CO_2} (cm)	$\overline{\Delta\tau}_F \frac{dB}{dT}$	Q (cal cm ⁻² day ⁻¹)
0.01	0.1	0.20	0.307
0.01	1	0.68	0.804
0.1	1	0.58	0.713
0.1	10	1.08	1.218
1	10	0.62	0.784
1	100	0.94	1.112

-o-o-o-

APPENDIX II : THE FLUX DISCONTINUITY OF CHARLOCK AND HERMAN (1976)

Charlock and Herman's expression for the downward flux is:

$$\begin{aligned}
 L^{\downarrow} = & \int_0^{T_1} \int_0^{\infty} \frac{dB_{\nu}}{dT} d\nu dT + \int_{T_0}^{T_1} \int_0^{\infty} \frac{dB_{\nu}}{dT} \tau_F(u(T), T) dT d\nu \\
 & - \int_0^{T_1} \int_0^{\infty} \frac{dB_{\nu}}{dT} \tau_F(u_1, T) dT d\nu - \int_0^{T_1} \int_0^{\infty} B_{\nu} \frac{\partial \tau_F(u_1, T)}{\partial T} dT d\nu \quad (\text{AII.1})
 \end{aligned}$$

$T_1 = 193$ K. Charlock and Herman show that this expression leads to the same results as the expression of Elsasser and Culbertson (1960), which is identical to (AII.1) except for the last (fourth) term. This at first sight mysterious result is explained by pointing out that the sum of the last two terms on the right hand side of (AII.1) is the flux of an isothermal layer with temperature $T_1 = 193$ K and thickness u_1 , minus $B(T_1)$, which quantity is equal to the third term alone of Elsasser's expression, due to the fact that both quantities are determined in an empirical way. Now, if we let a "real" atmosphere with a temperature interval (T_0, T_1) approach to an isothermal layer with temperature T_0 and thickness u_1 , by deforming the original temperature profile to a block-shaped one (see figure AII.1), expression (AII.1) will approach to its limit:

$$\begin{aligned}
 L^{\downarrow}(T_0) = & \int_0^{T_0} \int_0^{\infty} \frac{dB_{\nu}}{dT} dT d\nu + \int_{T_0}^{T_1} \int_0^{\infty} \frac{dB_{\nu}}{dT} \tau_F(u_1, T) dT d\nu \\
 & - \int_0^{T_1} \int_0^{\infty} \frac{dB_{\nu}}{dT} \tau_F(u_1, T) dT d\nu - \int_0^{T_1} \int_0^{\infty} B_{\nu} \frac{\partial \tau_F}{\partial T}(u_1, T) dT d\nu =
 \end{aligned}$$

$$\begin{aligned}
 &= \int_0^{T_0} \int_0^\infty \frac{dB_\nu}{dT} dT d\nu - \int_0^{T_0} \int_0^\infty \frac{dB_\nu}{dT} \tau_F(u_1, T) dT d\nu \\
 &- \int_0^{T_1} \int_0^\infty B_\nu \frac{\partial \tau_F}{\partial T} (u_1, T) dT d\nu \tag{AII.2}
 \end{aligned}$$

The interpretation of an isothermal layer following Charlock and Herman is different from this procedure; they consider level T_0 the top of the "atmosphere" (in place of T_1 , as we did above) and give the following expression for the isothermal layer:

$$\begin{aligned}
 L^\downarrow(T_0) &= \int_0^{T_0} \int_0^\infty \frac{dB_\nu}{dT} dT d\nu - \int_0^{T_0} \int_0^\infty \frac{dB_\nu}{dT} \tau_F(u_1, T) dT d\nu \\
 &- \int_0^{T_0} \int_0^\infty B_\nu \frac{\partial \tau_F}{\partial T} (u_1, T) dT d\nu \tag{AII.3}
 \end{aligned}$$

The difference with (AII.2) is:

$$\begin{aligned}
 L^\downarrow(\text{isotherm}) - \lim_{\text{isotherm}} L^\downarrow(\text{atmosphere}) &= \\
 &= \int_{T_0}^{T_1} \int_0^\infty B_\nu \frac{\partial \tau_F}{\partial T} (u_1, T) dT d\nu \tag{AII.4}
 \end{aligned}$$

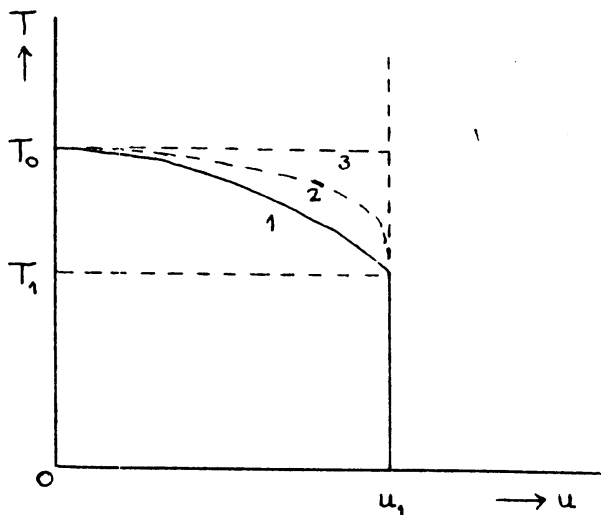


Fig. AII.1

Transition from a non-isothermal layer (1) to an isothermal layer with temperature T_0 and thickness u_1 (3).

The inconsistency of Charlock and Herman's treatment may also be demonstrated by showing that the downward flux L^\downarrow depends on the choice of the temperature T_1 : a different choice of T_1 is completely compensated in the second and third term of (AII.2) (first part), but the fourth term is not compensated.

It is interesting to note that Zdunkovski's (1966) treatment of the radiation transport equation leads to an expression for L^\downarrow identical to (AII.2), except for an extra term:

$$\int_{T_0}^{T_1} \int_{\nu_1}^{\nu_2} B_\nu \frac{\partial \tau_F}{\partial T} (u, T) dT d\nu$$

The sum of this term and the fourth term is independent of the choice of T_1 , so there is no inconsistency in Zdunkovski's equation. Also the treatment of Elsasser and Culbertson, which hides mathematical errors, leads to a consistent expression for the radiation flux.

It may be noted, finally, that the origin of the inconsistency mentioned above may be traced back to the approximative solution of the radiation transport equation. (See also the discussion by Charlock, Herman and Zdunkovski (1976)). Zdunkovski's approximation is different from that of Charlock and Herman, and also different from that of Elsasser and Culbertson. Elsasser and Culbertson did avoid discontinuities of the kind discussed above at the cost of mathematical inconsistencies.

APPENDIX III : COMMENT ON THE COMPUTER PROGRAM

1. A copy of the program, and an example of the output may be found hereafter. The program is written in ALGOL for a Burroughs B6700 computer.
2. The input format is: $\langle 3I2, I4, X1, F6.1, F.5.1, F6.2 \rangle$, day, month, year, time, height, temperature, humidity.
3. For every height one card is required.
4. The last card with input data has to be followed by a blank card.
5. Then, the procedure may be started again with a new set of data cards.
6. The humidity may be expressed in per cent as well as in gram H_2O/kg air. In the latter case, the number should be raised by 100.
The temperature is in $^{\circ}C$, the height in m.
7. In case the first height interval is very small, the water vapour path for this interval may be less than 10^{-5} cm, which is the lowest tabulated value. The program gives the error message "array limit exceeded". For example, this happens if the height interval is less than 0.16 m at a specific humidity of 10 g/kg.
8. Data cards with an input temperature lower than $-74.9^{\circ}C$ are skipped to prevent that the limit of the radiation array $R_{H_2O}(u,T)$ is exceeded.
9. The tables for $R_{H_2O}(u,T)$ (from Elsasser or Zdunkovski), for $\int_{193}^0 RdT$ and for ΔR (both from Elsasser) are to be read from card every time the program is executed.
10. The first output line gives the number of the day, month, year and time.
11. The next columns correspond to the steps in the numerical integration procedure.

12. The quantity $\int_{193}^0 RdT$ is denoted by "aanvulling 193-OK".
13. The quantity $\int_{T_i}^0 RdT$ (T_i is the temperature on the last data card) is denoted by "aanvulling tot 193 K en 193-0 K".
14. The radiation flux, in units $\text{cal cm}^{-2}\text{day}^{-1}$, is denoted by "totale straling". The next line gives the same quantity, in units Wm^{-2} , and the corresponding emission coefficient, i.e. the radiation flux divided by σT_0^4 , where T_0 is the temperature on the first input card. Also the black-body temperature, equivalent to the calculated flux, is given ("stralingstemperatuur").

In the following some comments on the computer program.

00009000 - 00025000. Procedure INTERPOL ($Y_1, Y_3, \text{VERSX}, S$).

This procedure computes by a quadratic interpolation procedure the value of $Y(X)$, where $X_2(X(Y_1), X(Y_3))$, and $\text{VERSX} = \min(|X-X(Y_1)|, |X-X(Y_2)|, |X-X(Y_3)|)$; Y_1, Y_2 and Y_3 are successively tabulated values. S indicates whether the interpolation is in $U(1 < S < 22)$, or in $T(S=100)$.

00026000 - 00057000. Procedure BEPAAL (I, VERSU).

The procedure determines the integer number I , which is the number of the u -row of the matrix (2-dimensional array) $R(u, T)$, with a value for u nearest to a given u -value. The difference of the given u -value and the u -value of the I th row is VERSU .

00058000 - 00079000. Real procedure RADIAT(U, T).

This procedure calculates the value for $R(u, T)$ by interpolation in u and T . In this procedure the two foregoing ones are used.

00080000 - 00081000. Read statement for $R(u, T)$ ($R[I, J]$).

00082000. Read statement for $\int_{193}^0 R(u, T)dT$ ($\text{RLAAG}[I]$).

00083000 - 00084000. Read statement for $\Delta R(\text{CO}_2 \text{ correction})$ ($\text{RCp2}[J, I]$).

00086000. Read statement for the first input card of the temperature- and humidity profile of the atmosphere.

00087000 - 00088000. Pressure calculation for the first level (PB).

00090000 - 00094000. Calculation of the mixing ratio (QB),
first level.

00106000 - 00114000. Like 00086000 - 00094000, for the next level.

00115000 - 00117000. Calculation of $u(\text{H}_2\text{O})$.

00118000. Calculation of $R(u, T)$ (FHOOG).

00119000 - 00128000. Calculation of $u(\text{CO}_2)$ and of $\Delta R(\text{RADIATC}\phi_2)$.

00130000. Step of the numerical integration procedure.

00139000 - 00140000. Calculation of $\int_{193}^0 \text{RdF}$.

00141000 - 00142000. Calculation of $\int_{T_1}^0 \text{RdT}$ and of the (total)
radiation (F).

REFERENCES

- Charlock, T., and B.M. Herman, 1976: Discussion of the Elsasser formulation for infrared fluxes. *J. Appl. Meteor.*, 15, 657-661.
- Charlock, T., B.M. Herman and W. Zdunkovski, 1976: Comments on "Discussion of the Elsasser formulation for infrared fluxes". *J. Appl. Meteor.*, 15, 1317-1319.
- Elsasser, W.M., 1942: Heat transfer by infrared radiation in the atmosphere. *Harvard Meteor. Studies*, No. 6, Cambridge, Harvard Univ. Press.
- Elsasser, W.M., and M.F. Culbertson, 1960: Atmospheric radiation tables. *Meteor. Monogr.*, 4, No. 23.
- Robitzch, M., 1941: Ausführliche Tafeln zur Berechnung der Luftfeuchtigkeit. Keller, Leipzig.
- Staley, D.O., and G.M. Jurica, 1970: Flux emissivity tables for water vapour, carbon dioxide and ozone. *J. Appl. Meteor.*, 9, 365-372.
- Yamamoto, G., 1952: On a radiation chart. *Sci. Rep. Tohoku Univ.*, Ser. 5, 4, 9-23.
- Zdunkovski, W.G., R.E. Barth and F.A. Lombardo, 1966: Discussion on the atmospheric radiation tables by Elsasser and Culbertson. *Pure Appl. Geophys.*, 63, 211-219.

LANGSTRAAL / KOHLSIEK
 = = = = =

SOURCE TAPE: (KOHV)LANGSTRAAL ON TEST.

```

BEGIN
    COMMENT BEREKENING LANGGOLVIGE STRALING;
    FILE CARJ (KIND=READER),LP (KIND=PRINTER);

    INTEGER J,K,N,INTERVAL,L,M,LH,A,DAG,MND,JAAR,TYD;
    REAL ARRAY RC(1:21,1:13),PLAAG(1:21),RCO2(1:16,1:19),RADC2(1:13);
    REAL U,T,F,FLAAG,VERSU,VERSUCO2,RADIATCO2,Z,TB,TE,FHODGB,FHODGE,UGEN,
    TOA,RV,P,PE,GAMMA,PHATER,QB,QE,UA,FREST,ZO,FHODG,ZB;
    LABEL OP,NOG,AF,NIEUW,AANV,REST;
    REAL PROCEDURE INTERPOL(Y1,Y3,VERSX,S);
    INTEGER J;
    REAL Y1,Y3,VERSX;
    BEGIN
        REAL X1,X3;

        X1:=-.10; X3:=.10;
        IF S.LEQ 100 THEN
            BEGIN
                X1:=-0.4; X3:=0.3;
                IF ENTIER((S-1)/3)*3=S-1 THEN
                    BEGIN X1:=-0.3; X3:=0.3 END;
                IF ENTIER((S-1)/3)*3=S-2 THEN
                    BEGIN X1:=-0.3; X3:=0.4 END;
            END;
        INTERPOL:=((Y1*X3-Y3*X1)*VERSX - Y1*X3**2+Y3*X1**2)*VERSX/
        (X1*X3*(X1-X3));
    END INTERPOL;

    PROCEDURE BEPAAL(I,VERSU);
    INTEGER J;
    REAL VERSU;
    BEGIN
        LABEL LA;

        INTEGER L;
        REAL DEC;
        I:=(ENTIER(U)+5)*3+1;
        DEC:=U-ENTIER(U);
        IF DEC>0.3 THEN
            BEGIN
                IF DEC<0.7 THEN I:=I+1 ELSE I:=I+2
            END;
        ;
    LA:
        L:=((I-1)/3-ENTIER((I-1)/3))*9;
        IF L=0 THEN L:=7;
        VERSU:=L/10;
        DEC:=DEC-VERSU;
        IF VERSU>0.15 AND ENTIER(I/3) NEQ I/3 THEN
            BEGIN
                I:=I+1;
                GJ TO LA
            END
    END BEPAAL;
    INTERPOL IS SEGMENT 00003
    1 0002000 003:0000:1
    0003000 003:0000:1
    DATA IS 0005 LONG
    DATA IS 0004 LONG
    0004000 003:0000:1
    0005000 003:0000:1
    0006000 003:0006:1
    0007000 003:0006:1
    0008000 003:0006:1
    0009000 003:0006:1
    0010000 003:0006:1
    0011000 003:0006:1
    0012000 003:0006:1
    0013000 003:0006:1
    INTERPOL IS SEGMENT 00004
    2 0001400 004:0000:1
    0001500 004:0002:0
    0001600 004:0002:5
    3 0001700 004:0003:2
    0001800 004:0007:3
    0001900 004:0008:1
    4 0002000 004:000F:3
    0002100 004:0013:2
    4 0002200 004:0017:3
    3 0002300 004:0017:3
    0002400 004:001D:0
    0002500 004:001F:3
    INTERPOL(004) IS 0021 LONG
    2 0002600 003:0006:1
    0002700 003:0006:1
    0002800 003:0006:1
    0002900 003:0006:1
    0003000 003:0006:1
    BEPAAL IS SEGMENT 00005
    2 0003100 005:0000:1
    0003200 005:0000:1
    0003300 005:0000:1
    0003400 005:0003:5
    0003500 005:0006:1
    0003600 005:0008:1
    3 0003700 005:0008:4
    0003800 005:000E:3
    3 0003900 005:000F:2
    0004000 005:000F:2
    0004100 005:000F:2
    0004200 005:0014:1
    0004300 005:0016:0
    0004400 005:0017:3
    0004500 005:0019:0
    0004600 005:001E:2
    3 0004700 005:001E:5
    0004800 005:0020:2
    
```

```

END
  ELSE IF VERSU>0.15 THEN
  BEGIN
    I:=I+1;
    VERSU:=VERSU-0.3
  END
  ;
END
BEP AAL;

REAL PROCEDURE RADIAT(U,T);
VALUE U,T;
REAL U,T;
BEGIN
  INTEGER J,IH;

  REAL ARRAY RAD[1:21];
  REAL SEC,VERST;
  J:=ENTIER(T/10)+9;
  VERST:=T+90-J*10;
  IF VERST>5 THEN
  BEGIN
    J:=J+1;
    VERST:=VERST-10
  END
  ;
  BEP AAL(I,VERSU);
  FOR IH:=I-1 STEP 1 UNTIL I+1 DO
  RAD[IH]:=R[IH,J]+INTERPOL(R[IH,J-1]-R[IH,J],R[IH,J+1]-R[IH,J],,
  VERST,100);
  RADIAT:=RAD[I]+INTERPOL(RAD[I-1]-RAD[I],RAD[I+1]-RAD[I],VERSU,,I);
END
RADIAT;

READ(CARJ,/,FOR I:=1,I+1 WHILE I<22 DO FOR J:=1,J+1 WHILE J<14
DJ R(I,J));
READ(CARJ,/,FOR I:=1,I+1 WHILE I<22 DO RLAAG(I));
READ(CARJ,/,FOR I:=1,I+1 WHILE I<20 DO FOR J:=1,J+1 WHILE J<17
DO RCO2(I,J));
NI EUW:
READ(CARJ,<3I2,I4,X1,F6.1,F5.1,F6.2>,DAG,MND,JAAR,TYD,Z,TUA,RV)(LUP);
GAMMA:=9.85*10**(-3)/(273+TUA+0.00985*Z);
PB:=1013*(1-GAMMA*Z)**3.5;
TB:=TUA;
IF RV > 100 THEN QB:=RV-100 ELSE BEGIN
RV:=RV/100;
IF TB NE. 0 THEN PWATER:=RV*10** (7.5+TB/(237.3+TB)+0.78571) ELSE
PWATER:=RV*10** (9.5+TB/(265.5+TB)+0.78571);
QB:=PWATER/(PB-PWATER)*622;END;
WRITE(LP,</,/,I2,2I3,I6>,DAG,MND,JAAR,TYD);
WRITE(LP,</,/, "HOOGTE",X3,"TEMPERATUUR",X3,"SPEC.VOCHT",X3,"WAT",
"EMWEG",X3,"WATERSTRALING",X3,"CO2WEG",X3,"CO2STRALING",X3,"TOTALE",
X1,"STRALING",/,X2,"M",X10,"C",X10,"GKG*-1",X7,"CM",X24,"CM",/);
WRITE(LP,</,/,F7.1,X3,F5.1,X9,F5.2>,Z,TUA,QB);
UA:=0;
F:=0;
FHOOQB:=0;
ZO:=Z;
NOG:
READ(CARJ,<3I2,I4,X1,F6.1,F5.1,F6.2>,DAG,MND,JAAR,TYD,Z,TE,RV);
IF JAAR = 0 THEN GO TO AF;
IF TE<-74.9 THEN GO TO REST;
PE:=1013*(1-GAMMA*Z)**3.5;
IF RV > 100 THEN QE:=RV-100 ELSE BEGIN

```

```

00049000 005:0020:5
3 00050000 005:0020:5
00051000 005:0023:1
3 00052000 005:0023:4
00053000 005:0025:1
00054000 005:0026:0
3 00055000 005:0028:2
00056000 005:0028:2
2 00057000 005:0028:2
BEP AAL(005) IS 002A LONG
00058000 003:0006:1
00059000 003:0006:1
00060000 003:0006:1
00061000 003:0006:1
00062000 003:0006:1
RADIAT IS SEGMENT 00006
2 00063000 006:0000:1
00064000 006:0000:1
00065000 006:0000:1
00066000 006:0003:2
00067000 006:0005:4
00068000 006:0006:1
3 00069000 006:0006:4
00070000 006:0008:0
00071000 006:0008:2
3 00072000 006:0009:2
00073000 006:0009:2
00074000 006:0008:5
00075000 006:000F:5
00076000 006:0010:1
00077000 006:0020:1
00078000 006:002A:5
2 00079000 006:002A:5
RADIAT(006) IS 003B LONG
00080000 003:0006:1
00081000 003:0015:4
00082000 003:0028:2
00083000 003:0044:2
00084000 003:0053:4
00085000 003:0069:2
P 00086000 003:0080:1
P 00088000 003:0086:2
00089000 003:008A:5
00090000 003:008B:4
2 00091000 003:008E:5
00092000 003:0090:1
00093000 003:0099:1
00094000 003:00A1:1
2 00095000 003:00A3:4
00096000 003:00AF:2
00097000 003:00B1:1
00098000 003:00B1:1
00099000 003:00B1:1
DATA IS 000E LONG
00100000 003:00B4:2
00101000 003:00BE:2
00102000 003:00BF:0
00103000 003:00BF:4
00104000 003:00C0:2
00105000 003:00C1:1
00106000 003:00C1:1
00107000 003:00D4:2
00108000 003:00D5:3
P 00109000 003:00D7:4
00110000 003:00DB:5

```

```

RV:=RV/100;
IF TE NEQ 0 THEN PWATER:=RV*10**(7.5*TE/(237.3+TE)+0.78571); ELSE
PWATER:=RV*10**(9.5*TE/(265.5+TE)+0.78571);
QE:=PWATER/(PE-PWATER)*622 END;
UGEM:=ABS((QE+QB)/2*(PB-PE)*(PB+PE)/(2*1013)/981)*(273/(273+TE))**
0.5;
UA:=UA+UGEM;
U:=LOG(UA);
FHOOG:=RADIAT(U,TE);
U:=ABS(5.08*10**(-3)/GAMMA*((1-GAMMA*Z)**6.5-(1-GAMMA*Z)**5.5))*
(273/(273+TOA))**1.5;
U:=LOG(U);
BEPAL(M,VERSUCO2);
L:=I-0; IF L LEQ 1 THEN L:=2;
M:=M-1; IF M LEQ 1 THEN M:=2;
FOR LH:=L-1 STEP 1 UNTIL L+1 DO
RADC02[LH]:=RADC02[LH,M]+INTERPOL(RADC02[LH,M-1]-RADC02[LH,M],
RADC02[LH,M+1]-RADC02[LH,M],VERSUCO2,M);
RADIAC02:=RADC02[L]+INTERPOL(RADC02[L-1]-RADC02[L],RADC02[L+1]-
RADC02[L],VERSUCO2);
FHOOGE:=FHOOG+RADIAC02;
F:=F+(FHOOGE+FHOOGB)/2*(TB-TE);
WRITE(LP,</>,"F7.1,X3,F5.1,X9,F5.2,X8,F6.4,X5,F6.3,X6,F7.2,X4,F6.3,
X8,F8.3">Z,TE,QE,UA,FHOOG,10**U,PADIAC02,F);

TB:=TE;
FHOOGB:=FHOOGE;
PB:=PE;
QB:=QE;
GO TO NOG; AF;

REST:
FLAAG:=RLAAG[I]+INTERPOL(RLAAG[I-1]-RLAAG[I],RLAAG[I+1]-RLAAG[I],
VERSUCO2)+60*RADIAC02;
FREST:=FLAAG+0.25*(TB+273)**(-3)*((TB+273)**4-193**4)*FHOOGE;
F:=F+FREST;
WRITE(LP,</>,"AANVULLING 193=0 K",X18,F8.3,</>,"AANVULLING TOT 193"
," K EN 193=0 K",X3,F8.3,</>,"TOTALE STRALING",X21,F8.3,X2,
"CALC 4**2*DAG**1">,"FLAAG,FREST,F);

WRITE(LP,SKIP 11,<X36,F8.3,X2,"WM**2 STRALINGSTEMPERA TUUR ",
F5.1," K EMISSIE ",F4.2,>F*0.4845,(F*0.4845/5.67*10**8)**0.25,
F*0.4845/(5.67*10**(-8)*(TLA+273)**4));
WRITE(LP,SKIP 11);
GO TO NIEUW;
UP:
END .

```

```

2 00111000 003:00DF:0
00112000 003:00E0:2
00113000 003:00E9:1
00114000 003:00F1:1
2 P 00115000 003:00F3:4
C 00115100 003:00F9:4
00116000 003:00FC:4
00117000 003:00FE:0
00118000 003:00FF:3
P 00119000 003:0101:2
C 00119100 003:0108:2
00120000 003:010F:5
00121000 003:0111:2
00122000 003:0113:0
00123000 003:0116:1
00124000 003:0119:2
00125000 003:011D:5
00126000 003:0126:1
00127000 003:0120:1
00128000 003:0134:1
00129000 003:0137:5
00130000 003:0139:3
00131000 003:013C:1
00132000 003:013E:2
DATA IS 0047 LO
00133000 003:014E:2
00134000 003:014F:1
00135000 003:0150:0
00136000 003:0150:5
00137000 003:0151:4
00138000 003:0152:1
00139000 003:0152:1
00140000 003:015B:0
00141000 003:0150:5
00142000 003:0166:3
00143000 003:0167:5
00144000 003:0169:4
00145000 003:0169:4
DATA IS 0012 LO
00146000 003:0171:2
00147000 003:0173:4
DATA IS 0020 LO
00148000 003:0180:5
00149000 003:018A:2
00150000 003:018E:2
00151000 003:018E:5
00152000 003:018E:5
B.0000(0.3) IS 0190 LO
STACKCODE IS SEGMENT 000
STACKCODE(011) IS 003A LO
DATA IS 0012 LO

```

```

=====
NUMBER OF LPRGRS DETECTED = 0.
NUMBER OF SEGMENTS = 15. TOTAL SEGMENT SIZE = 754 WORDS. CORE ESTIMATE = 2223 WORDS. STACK ESTIMATE = 63
PROGRAM SIZE = 154 CARDS, 1418 SYNTACTIC ITEMS, 53 DISK SEGMENTS.
PROGRAM FILE NAME: (KOH)LANGSTRAAL/KOHSIEK ON TEST.
COMPILATION TIME = 9.976 SECONDS ELAPSED; 2.528 SECONDS PROCESSING; 0.982 SECONDS I/O.
=====

```


9 5 78 1200

HOOGTE M	TEMPERATUUR C	SPEC.VOCHT GKG**-1	WATERWEG CM	WATERSTRALING	CO2WEG CM	CO2STRALING	TOTALE STRALING
2.0	16.0	4.25					
5.0	15.5	4.26	0.0017	1.540	0.09	0.300	0.460
10.0	15.0	4.22	0.0042	2.010	0.24	0.480	1.543
20.0	14.5	4.09	0.0092	2.432	0.54	0.658	2.937
40.0	14.0	3.97	0.0188	2.835	1.15	0.822	4.624
80.0	13.7	3.91	0.0375	3.247	2.35	0.966	5.905
120.0	13.5	3.88	0.0559	3.494	3.54	1.037	6.679
200.0	13.3	3.86	0.0919	3.815	5.89	1.111	7.625
333.0	13.1	3.68	0.1491	4.143	9.72	1.164	8.648
639.0	9.5	2.85	0.2585	4.353	18.19	1.202	28.200
1291.0	5.1	1.45	0.3986	4.425	34.64	1.232	52.868
2198.0	1.3	0.54	0.4757	4.348	54.29	1.265	74.279
2797.0	-2.2	0.59	0.5003	4.188	65.43	1.282	93.673
3684.0	-7.7	1.58	0.5545	3.965	79.58	1.284	123.151
4549.0	-12.9	0.28	0.5910	3.739	91.01	1.286	149.865
5498.0	-17.7	0.35	0.6032	3.514	101.27	1.297	173.472
6744.0	-28.4	0.10	0.6121	3.037	111.73	1.308	222.456
7837.0	-34.8	0.06	0.6142	2.779	118.63	1.316	249.463
8877.0	-43.4	0.03	0.6150	2.463	123.63	1.322	283.348
10437.0	-55.0	0.01	0.6153	2.092	128.93	1.329	325.147
11478.0	-60.3	0.01	0.6154	1.942	131.35	1.332	342.889

AANVULLING 193-0 K
 AANVULLING TOT 193 K EN 193-0 K
 TOTALE STRALING

177.736
 233.803
 576.692
 279.407

CAL*CM**-2*DAG**-1
 WM**-2

STRALINGSTEMPERATUUR 264.9 K EMISSIE 0.71