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Dr. C. Levert

Some problems concerning the "three-dimensional location" of a rain.

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0. INTRODUCTION

Many questions relate to rains which do not fall vertically. Such questions cannot be answered with the help of data from the conventional raingage, since this has a horizontal orifice. Sometimes data as to wind speed and wind direction during the rain are not known and even if they are it may be difficult to use such data in the correct way. Hence, a three dimensional location of the rainfall is required. It necessitates the measuring with at least two gages, of which at least one is tilted. Such a three dimensional charackterization is possible by the quantities H_o, γ and ψ, that are defined as follows:

H_o = total rainfall, measured in the ordinary gage (mm).

γ = angle between drop path and the vertical.

ψ = azimuth = angle between the vertical plane through the path of the drop and the vertical plane from N to S (see fig. 3); these angles γ and ψ represent averages over the total rainfall duration and all drops.

1. TILTED RAINGAGES IN MOUNTAINOUS REGIONS

Problems related to the measuring of rainfall in mountainous areas have been discussed in many papers. The dominating question is there whether to place the raingage orifice horizontally or parallel to the slope of the ground. We refer in particular to the studies of Serra [1; 2]. The W.M.O. prescribes the measurement of the quantity of liquid atmosferic water passing through a horizontal plane per unit of area and unit of time. If the orifice is tilted, than also a certain amount of precipitation is measured, but this may not be called <u>the</u> rainfall. Nevertheless we must bear in mind that both the meteorologist (climatologist) and the hydrologist use the rainfall figures. The second one may ask: "how much water has fallen on the whole basin?" This basin may have a certain relief. The map of the area gives the surface of the horizontal projection of the catchment area. In this way the slope of the ground is introduced. Further generally the rains do not fall vertically.

There is a striking analogy: in a flat land people may ask for the total quantity of rainwater falling on a not horizontal roof; in a mountainous area the same question may relate to a special part of the terrain.

Now let us define the intensity $I_\perp$ of a rain as the quantity of precipitation which passes through a plane perpendicular to the drop paths per unit of surface and per unit of time (m^3/m^2. sec.) See fig. 1.B. The ordinary raingage measures $I_g = I_\perp \cos\gamma$. The hydrologist is interested in - among other things - the total quantity of water H (say per sec.), which is received by the whole mountainous area (m^3/sec.).

Suppose he knows

1. the rainfall data given by a gage with horizontal circular orifice.
2. the horizontal projection $0'$ of the true basin surface 0.

Then $H = 0.I_{OB} = 0' I_g (1 + tg\,\alpha\, tg\,\gamma)$

Here I_{OB} means the intensity measured by a raingage with its orifice parallel to OB. Further $0 = 0'/\cos\alpha$ and $I_{OB} = I_\perp \cos(\alpha - \gamma)$. The factor

(1) $f_1(\alpha;\gamma) = 1 + tg\alpha .\ tg\gamma \geqslant 1$ (or $1 - tg\alpha .\ tg\gamma$, see fig. 2)

requires the knowledge of both α and γ;

N.B. $f_1 = 1$ if $\gamma = 0$ (irrespective of α) and if $\alpha = 0$ (irrespective of γ), $\gamma = 0$ denotes vertical rain, $\alpha = 0$ flat ground.

If the angle between the vertical plane through the rainfall vector and the vertical plane through the normal on the raingage orifice is μ, than

(2) $H = 0' I_g (1 + tg\alpha .\ tg\gamma .\cos\mu)$

If γ' is defined by $tg\,\gamma' = tg\,\gamma .\cos\mu$ $(\gamma' < \gamma)$, then

$H = 0' I_g (1 + tg\,\alpha\, tg\,\gamma')$

Mind the factor $f_2(\alpha;\gamma;\mu) = 1 + tg\alpha .\ tg\gamma .\cos\mu .\ (\gtreqless 1)$

Serra has constructed nomograms for f_1 and f_2.

We now refer to fig. 1 A, showing three gages a, b and c, placed on the same slope.

a) If the total daily sum measured by a is H_a, the mountain surface has received

(3) $(0'/\sigma)\ (1 + tg\alpha\ tg\gamma')\ H_a$. In this case it is necessary to know α and γ.

b) If the total sum measured by b is H_b, the mountain surface has (here for the sake of simplicity μ is taken zero; consequently $\gamma' = \gamma$) received

(4) $\left(\frac{0'/\cos\alpha}{\sigma}\right) H_b$. In this case it is necessary to know only α.

c) If the total sum measured by c is H_c, the mountain surface has received

(5) $(\frac{O'/\cos\alpha}{\sigma/\cos\alpha})\ H_c = (O'/\sigma)\ H_c$. In this case neither α nor γ have to be known.

In the three cases a, b and c it is assumed that $\mu = 0$. Of course (3) = (4) = (5) because each of these three expressions gives the total amount of water which has been received by the total true mountain surface. This results in

(6) $H_b = H_a \cos\alpha\,(1 + tg\alpha\ tg\gamma) = H_a\ f_3\,(\alpha;\gamma)$

(7) $H_c = H_a\,(1 + tg\alpha\ tg\gamma)$

Serra has constructed also a nomogram for f_3.

The same author has stressed very strongly the necessity to measure the γ -values of the rains in mountainous regions. He advises (see 5.2) to install a paired raingages. One of the gages is placed in the normal way: horizontal circular orifice (σ cm^2). The other has its orifice inclined under 45^o (irrespective of the slope of the ground), but this orifice is elliptical. It can be constructed as follows: a right cylinder (enclosing the ordinary orifice) is cut by a plane, inclined under 45^o. Then the section will be the orifice of the second gage; surface $\sigma/\cos 45 = \sigma\sqrt{2}$.

2. TILTED RAINGAGES IN A FLAT COUNTRY

2.1 A group of 4 equally tilted raingages.

Let us characterize each gage by the normal to its orifice and the angles between this normal and the three axes of a Cartesian system (origin O). See fig. 3. The x-axis is directed from S to N; the y-axis from E to W; the z-axis vertically. The four gage normals can be represented by four straight lines starting in O. Line i (representing gage i) makes angles $\alpha_i, \beta_i, \gamma_i$, with the x -, y - and z-axes; i = 1, 2, 3, 4. The rainfall vector makes angles α_o, β_o and γ_o. The raindrop speed is b; the vertical and horizontal projections may be b_r and b_h; the last one makes an angle ψ (called azimuth) with the x-axis; $\psi > 0$ in the quadrants NOW and EOS. Also:

(8) $b_h = b \sin\gamma_o$; $b_v = b\cos\gamma_o$; $tg\gamma_o = b_h : b_v$; $b_h \sin\psi = b\cos\beta_o$; $b_h\cos\psi = b\cos\alpha_o$.

We introduce:

d = quantity of liquid water (gr.) per cm^3 air; D = rain duration in sec; O = surface of orifice (cm^2).

Consequently H is expressed in grams of water (the height of rainfall is 0.1 H/O in mm). Suppose four identical gages are placed in the corners of a square, the diagonals of which are directed NS and WE; gage 1 in N; 2 in S;

3 in W and 4 in E. All normals are inclined exactly under the same angle β with the horizontal plane. Normal 1 inclines exactly to S; 2 to N; 3 to E and 4 to W. Consequently $\left[\alpha_1 = \beta_3;\ \beta_1 = 90^\circ;\ \gamma_1 = 90 - \alpha_1\right]$; $\left[\alpha_2 = 180 - \beta_3;\ \beta_2 = 90^\circ;\ \gamma_2 = \gamma_1\right]$; $\left[\alpha_3 = 90^\circ;\ \beta_3 = \alpha_1;\ \gamma_3 = \gamma_1\right]$; $\left[\alpha_4 = 90^\circ;\ \beta_4 = 180 - \beta_3;\ \gamma_4 = \gamma_1\right]$.

The general expression for the total weight of precipitation (grams) received during the total rainfall duration by a gage $\left[\alpha_i; \beta_i; \gamma_i\right]$ from a rain with vector angles $\alpha_o, \beta_o, \gamma_o$ is:

(9) $H_i = dObD\,(\cos\alpha_o \cos\alpha_i + \cos\beta_o \cos\beta_i + \cos\gamma_o \cos\gamma_i)$. Here

(10) $H_1 = ODdb_v\,(\mathrm{tg}\,\gamma \cos\beta \cos\psi + \sin\beta)$

The indices 3 in β_3 and 0 in γ_o have been omitted.

(11) $H_2 = ODdb_v\,(-\mathrm{tg}\,\gamma \cos\beta \cos\psi + \sin\beta)$

(12) $H_3 = ODdb_v\,(\mathrm{tg}\,\gamma \cos\beta \sin\psi + \sin\beta)$

(13) $H_4 = ODdb_v\,(-\mathrm{tg}\,\gamma \cos\beta \sin\psi + \sin\beta)$

These 4 equations contain only 3 unknowns db_v, γ and ψ, while we are especially interested in γ and ψ.[1)]

(14) The conventional gage gives $H_o = ODdb_v$.

So we have at our disposal 5 equations with 3 unknowns. However, the equations are interdependent, requiring that

(15) $H_1 + H_2 \equiv H_3 + H_4$

(16) $H_1 + H_2 \equiv 2\,H_o \sin\beta$

The solution, based on Δ_1, Δ_2, M_1, is:

(17) $\mathrm{tg}\,\gamma = \frac{\mathrm{tg}\beta}{2M_1}\sqrt{\Delta_1^2 + \Delta_2^2}$; $\mathrm{tg}\,\psi = \frac{\Delta_2}{\Delta_1}$; $db_v = \frac{M_1}{OD\sin\beta}$

with $2M_1 = H_1 + H_2$; $2M_2 = H_3 + H_4$; $\Delta_1 = H_1 - H_2$; $\Delta_2 = H_3 - H_4$

The solution, based on Δ_1, Δ_2, H_o, is

(18) $\mathrm{tg}\gamma = \frac{\sqrt{\Delta_1^2 + \Delta_2^2}}{2H_o\cos\beta}$; $\mathrm{tg}\psi = \frac{\Delta_2}{\Delta_1}$; $db_v = \frac{H_o}{OD}$

2.2 A group of 3 equally tilted gages.

Let the 3 raingaged be placed in the corners N, S and W of the square. In this case the 3 equations 13, 14, 15 give the following expressions for $\mathrm{tg}\,\gamma$, $\mathrm{tg}\,\psi$ and db_v,

(19) $\mathrm{tg}\,\gamma = \frac{\mathrm{tg}\beta}{2M}\sqrt{\Delta_1^2 + 4(H_3 - M)^2}$; $\mathrm{tg}\,\psi = 2\,\frac{H_3 - M}{\Delta_1}$; $db_v = M/OD\sin\beta$

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note 1)
With vertical rain ($\gamma = 0$) : $H_1 = H_2 = H_3 = H_4 = H_o \sin\beta$.

2.3 A group of 2 equally tilted gages.

Let the two gages be installed in the corners N and S of the square in question. In this case we have 2 equations, see (10) and (11), with 3 unknowns. If it is permitted to equal ψ to the azimuth of the wind (and probably it is) than we can solve γ and db_v:

$$tg\,\gamma = \Delta\, tg\beta / 2M\cos\psi \quad \text{and } db_v = M/OD\sin\beta .$$

If the gages are placed on a turning table[2], which is directed always by the wind in such a way that the vertical plane through the gage normals contains the wind direction, then the rain ψ-value equals the wind ψ-value (averaged over the rainfall) and

$$tg\,\gamma = \Delta\, tg\beta/2M \quad \text{and } db_v = M/OD\sin\beta .$$

3. THE QUANTITIES WHICH CAN BE DETERMINED: γ, ψ, d AND b_v.

In all three cases (2.1), (2.2), (2.3) it is possible to compute γ, ψ (and db_v). Of course γ and ψ are values averaged over the whole rainfall duration and all drops. If b_h (b_v) is known, than b_v (b_h) can be computed via γ.

1. suppose it is allowed to replace b_h by the windspeed W; then $tg\,\gamma$ gives b_v = mean vertical fall speed of the drops.
2. suppose it is permitted to derive the median drop diameter $\hat{\phi}$ from the mean intensity I of the rainfall (found by means of the recording raingage), with the aid of a relation $\hat{\phi} = a\, I^b$, such as found by some investigators, where a and b are constants. In this case $\hat{\phi}$ gives the average fall speed b_v (by means of the relation between dropdiameter and vertical fall speed); then b_h is known via $tg\,\gamma$. It is important to compare this b_h with W. (See Appendix B)

Finally d, the water density over the gage, during the rainfall, can be computed by means of db_v, substituting the value of b_v, found by 1) or 2). (See Appendix B)

4. CONSIDERATIONS OF ACCURACY.

4.1 With regard to γ.

4.1.1 For two equally tilted, identical raingages (see 2.3).

As β is the angle between the raingage normal and the horizontal plane $\alpha = 90 - \beta$ is the angle between the plane of the orifice and the horizontal plane. (Inclination of the orifice)

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note 2)
In "Rainfall sampling on rugged terrain" 1954 [pg.38] a paired raingage is described, consisting of a horizontal and a vertical funnel mounted on a rotating head which was kept pointed to the wind by a vane.

(20) $\operatorname{tg} \gamma = \frac{\Delta}{2M \operatorname{tg}\alpha}$, with $\Delta_1 = H_1 - H_2$; $2M_1 = H_1 + H_2$ or (see 16) also.

(20a) Now while $\operatorname{tg} \gamma = \Delta \cos\alpha / 2 H_o$, what will the accuracy (the error) be of γ, called σ_γ?

Answer: starting from (20) we must bear in mind that each of the quantities H_1, H_2 and α (and hence Δ, M, α) are measured with certain errors. This may be explained as follows: suppose we want to incline both gages under the same angle α; this is impossible. The actual α will be situated between $\alpha^* - \frac{1}{2} R_\alpha$ and $\alpha^* + \frac{1}{2} R_\alpha$ and all values within the range R_α will be equally probable. This rectangular distribution has a standarddeviation $\sigma_\alpha = R_\alpha / \sqrt{12}$.

In the same way there exists a range of values for R_H. Suppose the true total quantity of precipitation (passing through an imaginary orifice) is H^*; this H^* is measured as some value H, between $H^* - \frac{1}{2} R_H$ and $H^* + \frac{1}{2} R_H$ 3); all values within R_H being equally probable. Now $\sigma_H = R_H/\sqrt{12}$. Suppose $\sigma(H_1) = \sigma(H_2) = \sigma_H = R_H/\sqrt{12}$, called σ_d; suppose the measuring errors of H_1, H_2 and α are independent; suppose σ_d does not depend on H and σ_α does not depend on α. Then $\sigma(\Delta) = \sigma_d\sqrt{2}$; $\sigma(M) = \frac{1}{2}\sigma_d\sqrt{2}$.

Then σ_γ can be expressed in Δ, M, α, σ_o and σ_α as follows (by means of the law of propagation of errors):

(21) $$\sigma_\gamma = \left[\frac{2(\Delta^2 + 4M^2)\operatorname{tg}^2\alpha}{(\Delta^2 + 4M^2\operatorname{tg}^2\alpha)^2}\,\sigma_d^2 + \frac{4\,\Delta^2 M^2}{(\Delta^2 + 4M^2\operatorname{tg}^2\alpha)^2\cos^4\alpha}\,\sigma_\alpha^2\right]^{\frac{1}{2}}$$

(σ_γ in radials)

If σ_γ is expressed in γ, M, α, σ_d and σ_α then

(22) $$\sigma_\gamma = \left[\frac{\cos^4\gamma}{2M^2}\left(\operatorname{tg}^2\gamma + \frac{1}{\operatorname{tg}^2\alpha}\right)\sigma_d^2 + \left(\frac{\sin 2\gamma}{\sin 2\alpha}\right)^2 \cdot \sigma_\alpha^2\right]^{\frac{1}{2}}$$

N.B. Losses caused by wind effect have been neglected.

The next question is: will there be a "best" α? Definition: α^* is called the best α if, for given M, γ, σ_d and σ_α, the value of σ_γ is as small as possible (here the rain is given by M and γ). The answer is

if $\alpha = 0 \rightarrow 45^o$, then σ_γ decreases
if $\alpha = 45 \rightarrow 90^o$, then σ_γ decreases and passes through a minimum.
Now, for given α, only γ's, with $\gamma \leq 90 - \alpha$ can be measured.

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note 3)
Better: between $H^* - R_H$ and H^*.

The present author prefers some α between o and 45°, because

i. also rains with $\gamma > 45^\circ$ are interesting

i.i. it is not possible to incline the conventional gages steeper than 20°

i.i.i. the larger the inclination, the more important may be the losses of water by wind effect

Fig. 4 and 5 illustrate nomograms for tg γ (and γ) as a function of Δ, with M as parameter and for σ_γ as a function of tg γ, with M as parameter. In these figures $\sigma_\alpha = 0.6^\circ$ and $\sigma_d = 0{,}06$ mm ($R_\alpha = 2^\circ$; $R_H = 0{,}2$ mm). The nomograms have been drawn for $\alpha = 45^\circ$ and $\alpha = 15^\circ$.

Some numerical examples may illustrate these nomograms:

a) $H_o = 10$ mm; $\alpha = 45^\circ$; $\gamma = 3^\circ$. Consequently $M = H_o \cos 45 = 7.0$ mm; $\Delta = 2M\mathrm{tg}\,\gamma\,.\mathrm{tg}\,\alpha = 0.7$ mm (this difference can be measured); $H_1 = H_o \cos\alpha + \frac{1}{2}\Delta = 7.4$ and $H_2 = H_o \cos\alpha - \frac{1}{2}\Delta = 6.7$ mm; $\sigma_\gamma = 0.4^\circ$. This means that a true value $\gamma = 3^\circ$ is measured somewhere between $3 - 2 \times 0.4 = 2.2^\circ$ and $3 + 2 \times 0.4 = 3.8^\circ$ [4] (with a certainty of 95%) and σ_γ/γ = percentual accuracy = 11%.

b) $H_o = 2$ mm; $\alpha = 45^\circ$; $\gamma = 3^\circ$. Consequently $M = 1.4$ mm; $\Delta = 0.18$ mm (this difference can be measured hardly); $H_1 = 1.5$; $H_2 = 1.3$ mm; $\sigma_\gamma = 1.5^\circ$; $\sigma/\gamma = 51\%$. Such large an inaccuracy may be called impermissible. If it is desirable to determine γ sufficiently accurately (e.g. $\sigma/\gamma \leqslant 5\%$), than H_o should be sufficiently large. This necessity is stronger, the smaller the α (the less inclined the orifices).

c) $\Delta = H_1 - H_2 = 0{,}2$ mm and $\alpha = 45^\circ$ (this is almost the smallest difference that can be measured),
Question: how does σ_γ/γ vary with M?
Answer: $\sigma_\gamma/\gamma \cong 35\%$, irrespective of M;
Further, if $\Delta = 1.0$ mm, than $\sigma_\gamma/\gamma = 9\%$, irrespective of M.

Rule of thumb:

If γ is small (nearly vertical rain) and α is small (nearly horizontal orifices) then

$$\sigma_\gamma \cong \frac{\frac{1}{2}\sigma_d\sqrt{2}}{H_o \sin\alpha} \cong \frac{\frac{1}{2}\sigma_d\sqrt{2}}{H_o\,.\,\alpha} \qquad (23)$$

Next (22) is simplified by supposing $\sigma_\alpha = o$ (i.e. the raingage inclination can be measured exactly). Then, starting from 20a,

$$\sigma_\gamma = \sigma_d \frac{\cos^2\gamma}{H_o\sqrt{2}} \left[2\,\mathrm{tg}^2\gamma + \frac{1}{\sin^2\alpha} \right]^{\frac{1}{2}} \qquad (24)$$

See fig. 6.

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note 4)
Only if the γ-error satisfies a normal law. We suppose that this holds approximately.

The rain is now characterized by H_o and γ. Again, of all values of α from 0 up to and including 45^o, the value 45^o is the "best" one.

(25) For $\alpha = 45^o$ $$\sigma_\gamma = \frac{\cos\gamma}{H_o}\sigma_d$$

Fig. 7 relates to the following question. Suppose it is required that $\sigma_\gamma \leqslant k.\ \sigma_d$ (again $\sigma_\alpha = o$), what is the level that should be exceeded by H_o?

The answer is $$H_o \geqslant \frac{\cos\gamma}{k}\sqrt{\sin^2\gamma + \frac{\cos^2\gamma}{2\sin^2\alpha}}$$

The curves have been drawn for k = 1 and σ_d = 0,06 mm = 33^o

Two examples:

a) let be given $\gamma = 45^o$; suppose we want $\sigma_\gamma \leqslant \sigma_d = 3.3^o$.
Then for $\alpha = 10^o$ H_o must equal or exceed 2.1 mm; for $\alpha = 30^o$ $H_o \geqslant 0.8$; for $\alpha = 45^o$ $H_o \geqslant 0,7$ mm.

b) if $k \neq 1$, then still use the figure for k = 1. Read off the H_o-value and divide this by k.

General conclusion: with two equally tilted raingages, if the inclination is very small, only the γ of heavy falls (and consequently rare rains) can be measured with sufficient accuracy.

4.1.2 For four equally tilted, identical raingages.

Start from (18) (γ expressed in H_o, $\beta = 90 - \alpha$, Δ_1, Δ_2). It is possible to compute σ_γ as a function of γ, H_o, α, σ_d (neglecting σ_α), resulting again in the expression (24).

4.2 With regard to ψ and db_v.

Of course also the orifice O (in the Netherlands 400 cm^2) and rainfall duration D are measured with errors, but generally these errors may be neglected. Consequently:

(26) $$\sigma(db_v) = \sigma_d/O.D \quad \text{and} \quad \sigma_\psi = \sigma_d\sqrt{2}/\sqrt{\Delta_1^2 + \Delta_2^2}$$

5 SOME OTHER SPECIAL COMBINATION OF GAGES.

5.1 The vecto pluviometer of Pers [7].

This combination consists of four identical gages with vertical orifices directed towards N, E, S and W. Then in (13), (14), (15), (16) the following values should be substituted: $\alpha_1 = o$; $\beta_1 = \gamma_1 = 90$; $\alpha_2 = 180$; $\beta_2 = \gamma_2 = 90^o$; $\alpha_3 = 90$; $\beta_3 = 180$; $\gamma_3 = 90$; $\alpha_4 = o$; $\beta_4 = \gamma_4 = 90$.
Unless the horizontal velocity of the raindrop is exactly in one of the chief directions (N, E, S, W), it is certain that always two of the four gages measure zero mm.

Example: the horizontal component is situated between W and N; then $H_1 = dOD\, b_h \cos\psi = H_o tg\gamma .\cos\psi$; $H_2 = o$; $H_3 = dOD\, b_h \sin\psi = H_o tg\gamma .\sin\psi$; $H_4 = o$; $H_o = dODb_v$.

(27) Then $tg\gamma = b_h/b_v = \sqrt{H_1^2 + H_3^2}/H_o$; $tg\psi = H_3/H_1$; $db_v = H_o/OD$

$\sigma_\gamma = \sigma_d . \cos\gamma / H_o$, called $\sigma_\gamma(P)$ (if α is exactly 90^o)

Expression (24) gives σ_γ (2)

Comparing $\sigma_\gamma(P)$ with $\sigma_\gamma(4)$, we conclude:

i) if $\alpha < 45^o$, then $\sigma_\gamma(P) < \sigma_\gamma(4)$; the Pers-combination is "better"

ii) if $\alpha > 45^o$, then $\sigma_\gamma(P) > \sigma_\gamma(4)$; then the configuration of 4 equally inclined gages is "better".

However, Pers does not consider the wind effect (causing water losses) and almost certainly this effect is larger, the steeper the orifices. For this reason the present author prefers a small value of α (say between o and 30^o).

5.2 The paired horizontal and tilted raingages of Serra [2]

There are two identical gages; the one has a horizontal orifice ($\alpha_1 = o$). The other gage is tilted under some angle α_2 (here called α). In such a combination we have

(28) $tg\gamma = (H_1 - H_o \cos\alpha)/H_o \sin\alpha$. Stress is laid on the fact that this expression holds good only if $\mu = o$,

with μ = angle between vertical plane through rainvector and vertical plane through normal on the second orifice. In case $\mu \neq o$, then $\sin\alpha$ must be replaced by $\sin\alpha .\cos\mu$. 5)

With (28)

(29) $$\sigma_\gamma = \frac{\cos^2\gamma}{H_o \sin\alpha}\, \sigma_d \sqrt{(\cos\alpha + \sin\alpha . tg\gamma)^2 + 1}$$, neglecting σ_α.

(30) For $\alpha = 45^o$ $$\sigma_\gamma(S) = \frac{\cos\gamma}{H_o}\, \sigma_d \sqrt{1 + 2(1 + tg\gamma)\cos^2\gamma}$$ (the S refers to Serra)

note 5)

1. In case $\mu \neq o$ the same expression (28) holds good, but γ should be replaced by γ^1; γ^1 is the projection of γ on the vertical plane through the orifice normal, so that $tg\gamma^1 = tg\gamma . \cos\mu$. Mind: $\gamma^1 < \gamma$ if $\mu \neq o$.

2. Serra prefered to choose $\alpha = 45^o$ and to give raingage 2 (quantity H_S) an elliptical orifice with surface O/cos 45, if O = surface of circular orifice of the conventional gage. In this way he finds $tg\gamma = (H_S - H_o) : H_o$. This expression is somewhat simpler than (28). Consequently also the expression for σ_γ is changed somewhat, but the value of σ_γ remains the same for given H_o, α and γ.

5.3 The raingages of Lacy [4].

There are two gages with equal orifices, one is horizontal, the other vertical. Then $tg\gamma = H_1 : H_o$; $\sigma_\gamma = (\cos\gamma)\ \sigma_d/H_o$, say $\sigma_\gamma(L)$.
Again $\mu = o$ (see under 5.2).

It is important to compare $\sigma(2)$, $\sigma(L)$, $\sigma(S)$ (see fig. 8). We find:

i) if $\alpha < 45^o$, then $\sigma(S) > \sigma(2) > \sigma(L)$; that means the Lacy-configuration is better than the configuration of 2 equally tilted gages and the second one is better than that of Serra.

ii) if $\alpha > 45^o$, then $\sigma(S) > \sigma(L) > \sigma(2)$; that means the 2-configuration is better than the L-configuration and this is better than the S-configuration.

iii) For $\alpha = 45^o$ $\sigma(2) = \sigma(L)$ en $\sigma(S) > \sigma(2)$;
In all three cases the S-configuration is the worse one (that is: with regard to σ_γ); mind:
here a configuration is called better if γ can be measured more accurately.

5.4 The stereo pluviometer of Pers [5; 6; 7]

The orifice of the raingage is constructed in such a way that the rim (c) is exactly congruent with the boundary (C) of the catchment area. If H mm is measured with this gage, then the whole area has received q.H mm (supposing that the "structure" of the rain was homogeneous over the whole catchment). The multiplication factor (which is independent on the slope of the ground and on the rainfall inclination) equals the ratio of the surfaces of the areas enclosed by the horizontal projections of the curves c and C.

For further details see the literature.

6. THE CONFIGURATION OF TILTED RAINGAGES IN OUR INVESTIGATION.

The author prefered to measure with four equally tilted identical gages, placed in the corners of a square, with 4 meter-diagonals, see 2.1. In the first experimental configuration the gages are inclined under 20°. It was not necessary to construct a new type of raingage. The conventional gages can be tilted easily, although not more than 20°. Results:pg. 18 etc.

7. IS THE HORIZONTAL COMPONENT OF THE RAINDROP VELOCITY EQUAL TO THE WIND SPEED?

7.0 The problem can be formulated as follows:

A spherical raindrop, radius r, diameter ϕ , moves horizontally with the speed of the wind W_H cm/sec at a height of Hm.This drops begins to fall. Suppose the wind speed decreases with decreasing height z according to some function $W = f(z)$("wind profile"). Because of its inertia the drop cannot follow immediately the gradual change in W and its horizontal speed V (while decreasing during the fall) will exceed W in such a way that the difference V - W will increase with diminishing height.

The following questions may be asked:

(1) Wat is the function $V = \varphi(z)$?

(2) How does $y = V - W$ depend on z?

(3) What horizontal speed V_o has the drop when reaching the ground (or reaching the raingage level)?

(4) What is the difference y_o between V_o and W_o?

(5) How do these answers depend on the drop diameter ϕ ?

7.1 It is not easy to give a mathematical exact solution of this problem, not only because of mathematical difficulties, but also because of the indefiniteness of the problem. It is necessary to simplify. Of course this can be done in several ways; for instance as follows:

1. during the whole fall the vertical velocity V_v of the drop is constant and equal to the so called terminal fall speed, which is a function of ϕ only (see 9)
2. the wind profile is stationary, at least during the fall
3. the drop does not evaporate or grow larger during the fall
4. the temperature is constant during the fall (for instance 20°C) and consequently the values of η and R are constant (see 32a)
5. $y = V - W \ll V_v$ during the fall.

We start with the basic differential equation

(31) $m\dot{V} = -(\frac{1}{2}\pi r^2 \rho^1)\, C_D\, (V - W)^2$; the righthand side represents the "drag force"

C_D = drag coefficient

ρ^1 = specific density of the air = 0.0013 g.cm^3

η = viscosity of the air = 0.00018 g.cm^{-1} (20°C)

$m = \frac{4}{3}\pi r^3 \rho$; ρ = specific density of water $\cong$ 1.

(32) An empirical relation is $C_D = 24(R^{-1} + 0.2\,R^{-0.37} + 2.6.10^{-4}R^{0.38})$ (see for instance "Survey in mechanics" by C. Taylor, 1956)

(32a) R = Reynolds coefficient $= \frac{2r\rho^1}{\eta}\sqrt{V_v^2 + (V - W)^2}$

If the origin of the time ($t = o$) is taken at height H, then $z = H - V_v t$. Consequently W, R and C_D are functions of t (or z). The differential equation (31) is solvable only approximately.

There are two extreme cases

i) during the whole fall $V - W \gg V_v$ or $R = \frac{2r\rho^1}{\eta}(V - W)$

ii) during the whole fall $V - W \ll V_v$ and $R = \frac{2r\rho^1}{\eta} V_v$ = constant for given r.

In this report only case ii) is treated, although we know it represents an approximation because there are situations in which the inequality $V - W \ll V_v$ does not hold for the whole path of the drop. In fact it occurs that the path of drops (at rain gage level) is inclined more than 45^o with the vertical.

Equation (31) leads to:

$$\dot{V} = -r^{-2}V_v^{-1}\left[0.008 + 0.0013\,(rV_v)^{0.63} + 0.0000124\,(rV_v)^{1.37}\right] y^2 = -r^{-2}V_v^{-1} \cdot By^2 = -Ay^2 \tag{33}$$

If Stokes' law holds true, then $\dot{V} = -0.0008\, r^{-2}V_v y^2$. However, the value of B increases with increasing r, and soon it is not correct to equal B to 0.0008, not even approximately. Since indeed most rainfall have drops with radii larger than 0.01 cm, it is necessary to solve (33) in its complete form.

7.2 Several not too difficult functions $W = f(z)$ may be substituted. The simplest case is $W = az = a(H - V_v t)$, a very rough approximation of real conditions.

In this case, the solution of (33) is

$$y = V - W = \sqrt{\frac{a\,V_v}{A}}\left[\frac{e^{-2d(z-H)} - 1}{e^{-2d(z-H)} + 1}\right]; \text{ with } d = \sqrt{\frac{aA}{V_v}};\ z = H - V_v t \tag{34}$$

At $t = 0$, $z = H$ and $V = W_H$ (initial condition). Computation shows that the factor between brackets is nearly one (always smaller than 1) for all drops $r = 0.01$ to 0.5 cm for nearly the whole path (i.e. from the ground to near the starting level).

Consequently at the ground level ($z = 0$; $W = o$)

$$V_o = \sqrt{aV_v/A} \text{ cm/sec.} \tag{35}$$

This expression only contains the slope of the linear wind profile and not this wind profile itself. Further, V_v is fixed as soon as the drop is given and A depends on both r and V (see 33). The result (34) means that for nearly the whole path (from $z = o$ to nearly $z = H$) the horizontal speed V of the drop exceeds the windspeed W by a constant amount $\sqrt{aV_v/A}$ cm/sec.

The difference V - W is somewhat smaller, the higher the drop, since in (34) the factor [ ] increases to one during the fall of the drop. This fact is taken into account when solving the two and three differential equations in the section 7.3 and 7.4.

A numerical example will illustrate these theoretical computations. We met in practice a wind profile $W^* \doteq 100 \lg z^*$ with W^* in cm/sec. and z^* in cm. Consequently, but very approximatively, W = 0.0115 z, with W in m/sec. and z in m. At a height H = 1000 m the wind speed is $W_H = 11\frac{1}{2}$ m/sec. Some results are collected in table 1.

TABLE I

r cm.	V_v cm/sec.	V_o cm/sec.	$\hat{z}$ m.
0.01	76	1.9	1.6
0.05	390	21	18
0.1	690	48	42
0.5	980	152	132

The table gives values of $\hat{z}$ = "effective" height = height at which the wind velocity equals the horizontal speed V_o of the drop at ground level.

N.B. The values of V_o are very large! Was the approximation too rough?

7.3 Next the same wind profile will be approximated by two linear functions; W = 0.0040 z + 7.6 between the heights H_1 = 1000 m and H_2 = 20 m, and W = 0,38 z between 20 m and the ground. Then at heights H_1 and H_2 the wind speeds are 11.5 m/sec. and 7.6 m/sec, in agreement with actual conditions. Now two differential equations must be solved.
Table 2 gives some results.

TABLE 2

r cm	V_v cm/sec.	at height H_2=20 m.		on the ground	$\hat{z}$ m.
		V_v cm/sec.	W cm/sec.	V_o cm/sec.	
0.01	76	760 + 1	760	11	0.3
0.05	390	760 + 12	760	121	3.0
0.1	690	760 + 28	760	258	6.5
0.5	980	760 + 90	760	444	11.1

It appears that especially the second linear part (ending at the ground) of the wind profile influences the value of V_o.

7.4 Now, let the same logarithmic wind profile be approximated by three functions:
$W = 0.004\,z + 7.6$ between $H_1 = 1000$ and $H_2 = 20$ m; $W = 0.07\,z + 6.9$ between $H_2 = 20$ and $H_3 = 10$ m, and $W = 0.69\,z$ between H_3 and the ground, with wind speeds 11.5, 7.6 and 6.9 m/sec at the heights H_1, H_2 and H_3. Then three differential equations must be solved. Table 3 contains some results.

TABLE 3

r cm	V_v cm/sec.	height $H_2 = 20$ m.		height $H_3 = 10$ m.		ground	$\hat{z}$ m
		V cm/sec.	W	V	W	V_o	
0.01	76	760 + 1	760	690 + 4.6	690	14	0.2
0.05	390	760 + 12	760	690 + 48	690	160	2.3
0.1	690	760 + 28	760	690 + 81	690	370	5.4
0.5	980	760 + 90	760	690 + 119	690	580	8.5

Computation shows that for almost all drops (except for drops with, say, $r \geqslant 0.5$ cm.) the difference $y = V - W$ has reached its limiting value at gage or ground level. Moreover, as was said already, it proves very important to use within, say, the 10 m-region immediately above the ground the actual exact wind profile, but this leads to mathematical difficulties. The author dit not succeed in solving the differential equation when substituting the general expression $W = m \lg [(z + z_o)/z_o]$, with z_o and m constants in (33), however, with $W = Az^B$ (A and B constants) the equation could be solved.

Evidently V_o depends both on the drop diameter ϕ and the wind profile $W = f(z)$. At every height $V > W$, in other words it is as if the drop has taken downwards some wind speed from higher levels. Since, however, our mathematical model is too simple (this was unavoidable), the estimated differences $V - W$ near to the ground or near to the gage may be too large. When asking "how much" we may remark that probably the stronger the wind and the quicker its speed fluctuates (a wind profile is never stationary) the less is the difference $V - W$ (at least on an average) and especially for small drops.

In addition, when looking at the actual rains in nature and following the rotations of the anemometer cups we easily see how the inclinations of the drop paths change almost immediately with changing wind speeds and how at one and the same moment all paths are not equally inclined. Consequently, probably the effective height $\hat{z}$ will be situated beneath 3 m (in Appendix B an "effective height", averaged over a range of drop diameters, of about 2 m. is computed, although in quite a different way).

8. THE RELATION BETWEEN MEAN RAINFALL INTENSITY AND MEAN DROP DIAMETER.

8.0 It seems possible to derive the median drop diameter in the rain from the mean rainfall intensity.

8.1 A.C. Best [8].

(36) Best has found empirically the relation $F = 1 - \exp. \left[-\left(\frac{x}{a}\right)^n \right]$ with $a = \alpha . p^{\beta}$

α, β, n are constants.

F = fraction of all liquid water in the air (during the rainfall and over the gage), due to drops with a diameter $\phi < x$ mm

p = mean intensity in mm/h.

Best has computed n, α and β for several countries. The overall mean values are $\bar{\alpha} = 1.30$; $\bar{\beta} = 0.232$ and $\bar{n} = 2.25$. However, England : 1,61; 0.227; 2.26; Germany : 1.42; 1.272; 2.59. The median value $\hat{\phi}$ is defined by $0.50 = \exp. \left[-\left(\frac{\hat{\phi}}{a}\right)^n \right]$, leading to

(37) $\hat{\phi} = 0.69^{\frac{1}{n}} . \alpha . 60^{\beta} \; I^{\beta}$ mm, with mean intensity I in mm/min.

(38) England : $\hat{\phi} = 3.5\ I^{0.227}$ and Germany : $\hat{\phi} = 3.7\ I^{0.272}$; overall mean $\hat{\phi} = 2.8\ I^{0.232}$.

All these relations hold only for continuous rains (rains which do not show diameter frequency distributions with more than one mode). It is therefore questionable whether it is allowed to apply these exponential expressions to rainfalls in the Netherlands.

8.2 J.O. Laws [9].

This investigator found $\hat{\phi} = 2.6\ I^{0.182}$; $\hat{\phi}$ in mm; I in mm/min. (Washington; 1938, 1939). He stresses the conclusion: the larger the area and the larger the rainfall duration over which is averaged, the better the exponential relation is satisfied.

The range is $\phi = 1.1 - 7.0$ mm; $I = 0.01 - 2.5$ mm/min.

8.3 J.S. Marshall and W. Palmer [10].

(39) These authors found $\hat{\phi} = 2.2\ I^{0.210}$ (Ottawa, 1946).

8.4 What relation is to be used in the Netherlands?

The three examples show unequal constants in an exponential relation $\hat{\phi} = a\ I^{b}$, representing one and the same "phenomenon". This probably is due to the fact that the investigators have analyzed different data (different "types" of rain). Of course it should also be borne in mind that all such exponential relations are only highly approximate. Studying for instance the figure in the paper of Laws and Parsons it even seems questionable whether there is any relation between $\hat{\phi}$ and I, because the so called 90%

region (situated between the 5 and 95% bounderies) is, for each specified value of $\hat{\phi}$, nearly as broad as 140% of this $\hat{\phi}$ value itself!

It seems preferable to proceed in our investigation along the two following lines:

i) The four gages-configuration gives a value of γ. The pluviogram gives I. One of the four relations $\hat{\phi} = a\, I^b$ (see fig. 9) is adopted. It gives $\hat{\phi}$ and consequently also V_v is known. Then V_h is known by means of tg γ. Next, we will have to compare V_h with W and try to find out whether this W is the wind speed at ground level, at rain-gage-level or some higher level (see appendix B)

ii) We are not going to use the exponential relation mentioned above, since this relation has not yet been verified in the Netherlands, but we want to develop an empirical relation $\hat{\phi} = \phi$ (I). To this end we may take W from gage level and suppose $V_h = W$. Then γ (found by means of H_1, H_2, H_3, H_4) gives V_v and consequently also $\hat{\phi}$ (see fig. 9). The registration gives I and hence a pair $I_2, \hat{\phi}$ is found. Perhaps some analytical function $\hat{\phi} = \phi$ (I) can be found.
A much better way would be to construct a special instrument for counting the drops and measuring their diameters.

9. THE RELATION BETWEEN DROP DIAMETER AND VERTICAL FALL SPEED.

In 7.1 it was supposed, for the sake of simplicity, that for each drop during its whole course the vertical component of its speed is constant. Indeed the vertical speed has become "constant" very quickly. Laws says: 95% of the final fall speed (with vertically falling drops) were reached after 2.2 m for $\phi = 1$ mm; after 5.0 m for $\phi = 2$ mm and after 7.2 m for $\phi = 6$ mm.

Many investigators have studied the relation between the vertical fall speed (limiting value) V_v and the drop diameter ϕ (in air at rest) Table 4 contains some results of measurements published by Gunn and Kinzer [11].
See also fig. 9.

TABLE 4

$r = \frac{1}{2}\phi$ mm	V_v cm/sec.
0.2	162
0.5	403
1.0	649
2.0	883
2.8	916

A very rough approximation is $V_v = 620 \sqrt{r}$ cm/sec. (r in mm), which expression holds only for not too small droplets, $r < 2$ mm.

Although different authors have published different date as to these fall velocities, this fact does not affect our computations, as they are only approximate.

10. THE DISTURBANCES OF THE WIND FIELD AND THE DROP PATHS NEAR HORIZONTAL AND TILTED RAINGAGES.

10.1 The wind effect may be defined as the percentage difference between the measured quantity of precipitation H_m, which passed through the orifice of the actual gage per unit of time and unit of surface and the unknown quantity H_t, which would have passed through an identical imaginary orifice. The meteorologist wants to know H_t, but he measures H_m. Except for rare and special conditions the airflow of the atmosphere is turbulent. When the scale of turbulence is of the order of centimeters, the eddies near the gage orifice are usually a result of the geometry of the gage itself and vary in size with the mean wind speed (gage eddies). The final result is $H_m < H_t$. The (percentual) difference $H_t - H_m$ depends on several factors: (1) the surface of the orifice, (2) the wind speed, (3) the height of establishment above the ground, (4) the inclination of the gage, (5) the orientation of the wind in case the gage is tilted and last but not least (6) the character of the precipitation (with appreciable wind the deficiency of the gage catch may increase greatly from the case of large drops to that of fine droplets or even dry snowflakes).

10.2 Since it was not possible to study this effect in a windtunnel the author applied the following statistical method of estimating its influence (better: to verify its reality). The expressions (10), (11), (12), (13) have been derived under the assumption of absence of any windeffect. These expressions led to (15) and (16). It turns out necessary to compute the errors in the differences $\Delta_M = M_1 - M_2 = \frac{1}{2}(H_1 + H_2) - \frac{1}{2}(H_3 + H_4)$ and in the ratios $q = \frac{1}{2}(H_1 + H_2) : H_o \sin\beta$. The differences Δ_M and $q - 1$ should vary only by chance if there is no wind effect. Their deviations from zero are caused by

(1) the measuring errors in H_o, H_1, H_2, H_3, H_4, characterized by the quantity σ_d.

(2) the fact that it is impossible to install all five gages at exactly the same spot in the field. Although their mutual distances are only 4 or $2\sqrt{2}$ m (gages 1, 2, 3, 4 are placed in the corners of a square with diagonals of 4 meter), the true (unknown) amounts, produced by one and the same rain, in these five places may differ slightly (see further Appendix A).

If there were no wind effect and no field differences, the standard deviations of the deviations of Δ from zero and of q from 1, caused only by inaccuracies concerning the measurements, would be

$$\sigma(M_1 - M_2) = \sigma_d \quad \text{and} \quad \sigma(q - 1) = \sigma(q) = \frac{\sigma_d}{2 H_o \sin\beta} \sqrt{4q^2 \sin^2\beta + 2} \tag{40}$$

So it is necessary to verify (in a statistical way) whether the differences Δ_M and q - 1 are caused only by chance.

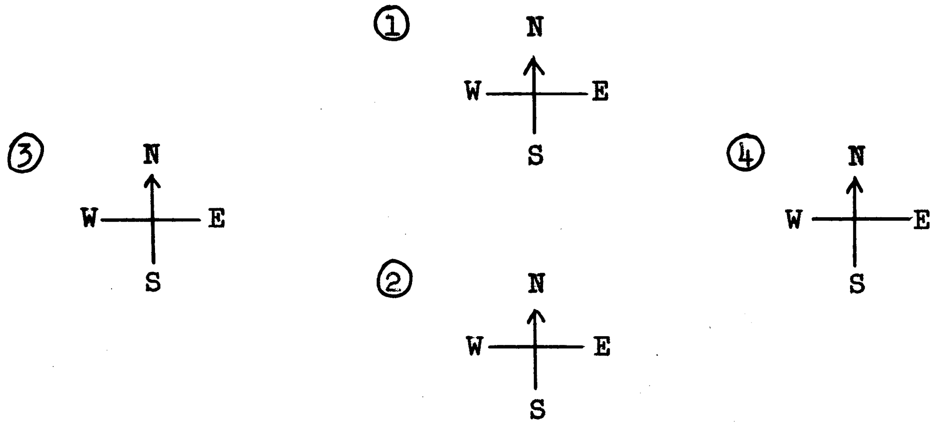
In Appendix A an experiment with 12 identical, horizontal gages in the field is reported, the results of which necessitate to enlarge the above mentioned $\sigma(M_1 - M_2)$ and $\sigma(q)$ on the basis of real field differences, occurring in one and the same rainfall.

In 12. more is said about the assumption that the wind effect might be neglected or be equally large for the four gages.

11. SOME NUMERICAL RESULTS OBTAINED WITH AN EXPERIMENTAL CONFIGURATION OF 4 EQUALLY TILTED IDENTICAL GAGES.

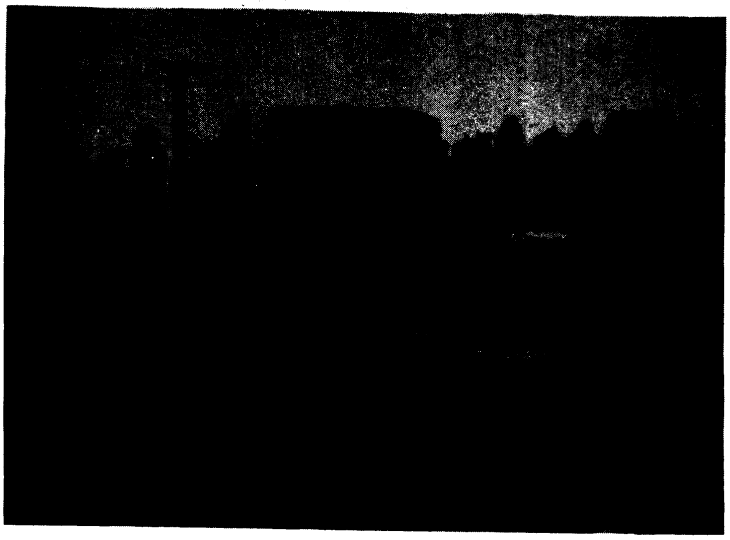
11.1 First results.

Four identical raingages (with circular orifices of 400 cm^2) have been installed at heights of 40 cm above the ground at the corners of a square in the meteorological field of the Institute. The diagonals are directed NS and EW; their lengths were 4 m. The situation is presented in the sketch below

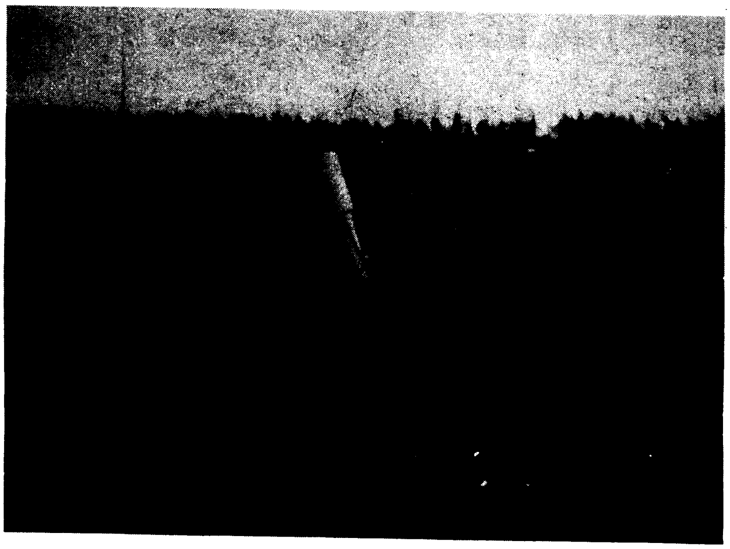


The W-E diameters of the orifices of the gages 1 and 2 and the N-S diameters of the orifices of the gages 3 and 4 are horizontal. The N-S diameters of 1 and 2 and the E-W diameters of 3 and 4 are inclined towards the centre under 20^o with the horizontal plane.

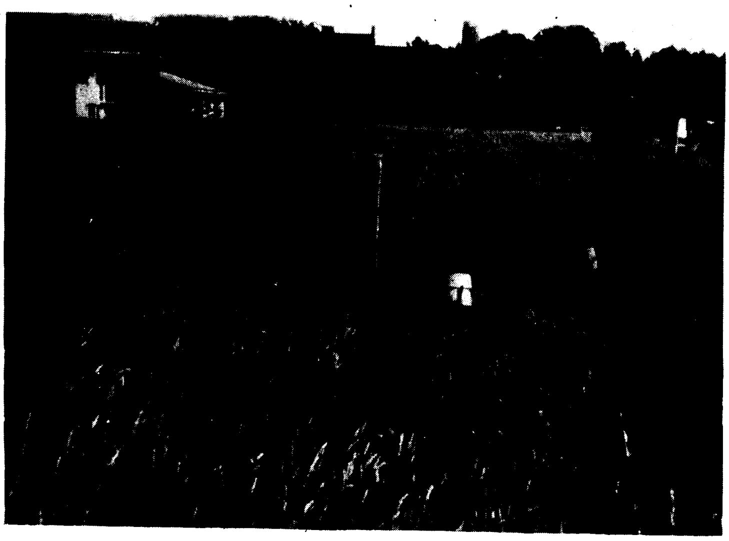
If γ = angle between raindrop path and vertical and
ψ = azimuth of raindrop path = angle between horizontal projection of the drop speed and the N-S-axis
($\psi > o$ in quadrants NW and ES) then



Netherlands' standard raingauge. Orifice 400 cm^2.
Upperrim 40 cm above the ground.



The same gauge in a tilted position.
Angle between plane of orifice and horizontal plane is 20^o.



The group of four equally tilted identical raingauges.
(text section 11)

$$\text{tg}\,\gamma = \frac{1.375}{\bar{M}}\sqrt{\Delta_1^2 + \Delta_2^2} \quad ; \quad \sigma_\gamma = \frac{2.44}{\bar{M}}\cos^2\gamma\sqrt{\text{tg}^2\gamma + 7.58} \; \circ$$

$$\text{tg}\,\psi = \Delta_2/\Delta_1 \qquad \sigma_\psi = 4.85/\sqrt{\Delta_1^2 + \Delta_2^2} \; \circ$$

with $\bar{M} = M_1 = M_2 = \frac{1}{4}(H_1 + H_2 + H_3 + H_4)$; $M_1 = \frac{1}{2}(H_1 + H_2)$; $M_2 = \frac{1}{2}(H_3 + H_4)$

$\Delta_1 = H_1 - H_2$; $\Delta_2 = H_3 - H_4$;

$\bar{M} = H_o \sin(90^\circ - 20^\circ) = 0.94\ H_o$

H_o = amount (weight in grams) measured with the conventional gage (horizontal circular orifice of 400 cm^2)

σ_d = 0.06 mm = standard deviation of measuring errors.

These expressions hold only if there is no effect of the wind and if there are no differences in the true values of the precipitation falls at distances of 4 m or thereabout in other words: if the areal distribution of the precipitation is not associated with radical discontinuities. Since there are measuring errors (with σ_d = 0.06 mm) and since the above mentioned effects do occur, both $M_1 \neq M_2$ and $\bar{M} \neq 0.94\ H_o$ in general. For these reasons $\Delta_M = M_1 - M_2$; $q = \bar{M}/0.94\ H_o$; σ_Δ and σ_q are computed. By means of a nomogram (see fig. 10) the values of γ, σ_γ, ψ, σ_ψ can be determined (only approximately) as soon as the values of Δ_1, Δ_2 and $\bar{M}$ are given by the experiment. These values are sufficient in first approximation. The mean wind direction during the rainfall (s) which produced the amounts H_o, H_1, H_2, H_3, H_4 in question, is read from the small daily wheathermap in the chief directions N, NNE, NE, etc. In this way we could verify whether the rainfall-ψ agreed with the wind-ψ, taking into account the value of σ_ψ (there is a probability 0.95 that the true value of ψ is situated between $\psi - 2\sigma_\psi$ and $\psi + 2\sigma_\psi$). In table 5 only 10 rainfalls out of 38 are summarized. The falls refer to periods between to successive measurements, these being made at 8.40, 14.40 and 19.40h M.E.T. They generally were produced by only one rain.

Table 5 shows:

1. The larger the $|\Delta_1|$ and $|\Delta_2|$, the steeper the rain (larger γ) and the more accurate ψ, (smaller σ_ψ).
2. The differences $M_1 - M_2$ are both negative and positive, as if they were caused by chance. See, however, 11.2. The value $|M_1 - M_2|$ generally is so small, that in many cases $|M_1 - M_2| \leq 2\ \sigma_d$. Here 0.06 mm was substituted for σ_d, but it seems better to enlarge this value on the ground of the field differences in one and the same rainfall, which are treated in Appendix A. pg. 23.

TABLE 5

	mm														rainfall duration					$2.1 I^{0.84}$	$1.73 I^{0.23}$	$97 I^{0.16}$
	H_o	H_1	H_2	H_3	H_4	γ^o	σ_γ^o	ψ^o	ψ^o	wind	M_1	M_2	M_1-M_2	q	D min	$I=H_o/D$ mm/min	W_2 m/sec	V_v m/sec	$V_v tg\gamma$ m/sec	d gr/m^3	$\hat{r}$ mm	N dr/m^3
1	0.8	0.8	0.9	0.8	0.9	14.5	8.4	45	34	NNW	0.75	0.75	0.00	0.995	245	0.003	1.4	3.8	1.0	0.0096	0.47	38
2	1.2	1.7	1.0	1.5	1.2	37.7	3.2	23.2	6.4	SSE	1.35	1.35	0.00	1.195	195	0.007	1.9	4.4	3.4	0.043	0.56	46
3	2.7	3.0	2.2	2.3	2.8	26.9	2.1	-32.5	5.1	SSW	2.60	2.54	0.06	1.005	128	0.024	2.5	5.4	2.7	0.080	0.74	53
4	3.7	3.7	3.3	3.2	3.8	15.5	1.8	-56.3	6.7	WSW	3.50	3.50	0.00	1.005	35	0.106	1.8	6.7	1.8	0.279	1.05	68
5	4.1	4.2	3.6	4.0	3.7	13.4	1.6	26.6	7.2	SSE	3.90	3.85	0.05	1.005	127	0.032	3.4?	5.6	1.3	0.119	0.79	56
6	8.6	8.8	7.4	7.7	8.6	15.2	0.8	-3.7	2.6	WSW	8.10	8.15	-0.05	1.001	325	0.026	?	5.5	1.5	0.098	0.76	78
7	9.2	9.1	8.2	9.4	8.2	13.0	0.8	52	3	SE	8.65	8.78	-0.13	1.001	225	0.041	1.5	5.8	1.3	0.105	0.84	60
8	9.3	9.5	8.5	7.9	10.4	22.2	0.7	-68.2	2.6	SW	9.00	9.15	-0.15	1.030	122	0.076	2.5	6.4	2.6	0.208	0.97	59
9	17.8	16.0	16.6	16.3	16.4	2.9	0.4	9.6	8.1	NNW	16.30	16.35	-0.05	0.976	246	0.072	0.6	6.4	0.3	0.102	0.96	66
10	18.0	18.2	14.8	15.6	18.5	20.2	0.4	-40.5	1.1	SSW	16.50	17.05	-0.55	0.991	80	0.212	2.3	5.3	2.0	0.561	1.20	76

Explanation: the differences $/M_1-M_2/$ and $/q-1/$ should be not larger than resp. $2\sigma_M$ and $2\sigma_q$, computed in the correct manner, described in Appendix A. Only the case $H_o = 18.0$ mm gives some troubles.

If only measuring errors are present, then $\sigma_M = \sigma_d = 0.06$ mm and $/\Delta/ = 0.55 \gg 0.06$. Considering also field differences, σ_M becomes about 0.15, but still $/\Delta/ > 2\sigma_M$. In all other cases $/\Delta/ \leq 2\sigma_M$. However, only by chance, cases in which $/\Delta/ > 2\sigma_M$ may be expected with a percentual frequency 5%. Now 1 in 38 cases does not disagree with this statistical requirement.

3) The ratio q varies around 1; in many cases $/q - 1/ \leqslant 2\sigma_q$, with $\sigma_q = \frac{0.032}{H}\sqrt{3.53\, q^2 + 2}$ which expression is based again on $\sigma_d = 0.08$ mm. If there are real field differences this value should be increased and then all q's appear to vary around 1 as if by chance. (See Appendix A.)

11.2 On the required equality of the inclinations of the four gages.

As was said already in the beginning of this chapter, the inclinations of the north-south and of the west-east diameters of the equally sized circular orifices of the four gages should be inclined exactly under 20° and 0°. We will never succeed in doing so. Therefore it is important to calculate the consequences of small differences. After some weeks the angles 20° and 0° were remeasured with the following results

gage no.	NS under	WE under	
1	20°	0°	
2	20°	2°	(E higher)
3	19°	$1\frac{1}{2}^\circ$	(S ")
4	$19\frac{1}{2}$	1°	(S ")

We will illustrate the consequences of such "small" deviations by substituting for instance $\gamma = 35^\circ$ and $\psi = 22\frac{1}{2}$. Then the expressions 10 - 13 yield in the given conditions:

$H_1 = 1.12\, H_o$; $H_2 = 0.73\, H_o$; $H_3 = 0.87\, H_o$; $H_4 = 1.08\, H_o$. Hence

$M_1 = \frac{1}{2}(H_1 + H_2) = 0.925\, H_o$ and $M_2 = \frac{1}{2}(H_3 + H_4) = 0.975\, H_o$;

we see $M_1 \neq M_2$; $M_1 - M_2 = -0.05\, H_o$ (and not zero). This example illustrates that even if there are no measuring errors, no losses caused by the wind effect, no true differences between the point measurements at 4 m distances within one and the same rain, then the small errors made in the angles 20° and 0° may cause a difference $/M_1 - M_2/$ of 0.1 mm if $H_o = 2$ mm or of 0.5 mm if $H_o = 10$ mm or of 1.0 mm if $H_o = 20$ mm. This example involves the warning to fix these angles 20° and 0° as exactly as possible (say: deviations of $\frac{1}{2}^\circ$ or more are not permitted). If this is not possible, then it is not so easy to verify whether a difference $M_1 - M_2$ deviates from zero in consequence of a combination of the three above mentioned "effects" or only because of inaccuracies in the desired angles of inclination.

11.3 Some other results.

The overall average of all inclinations of the 38 studies rainfalls was 19.8°.

The smallest γ was $2^\circ 56'$ with a standard deviation $\sigma_\gamma = 0.4^\circ$; $H_o = 17.8$; $H_1 = 160$; $H_2 = 16.6$; $H_3 = 16.3$; $H_4 = 16.4$ mm; D = 246 minutes.

The largest γ was $48^\circ 30'$, with $\sigma_\gamma = 7{,}1^\circ$; $H_o = 0.8$; $H_1 = 0.78$; $H_3 = 0.70$; $H_4 = 0.90$.

The most frequent γ was situated between 10 and 20°.

11.4 Future points.

Table 5 contains only 10 out of only 38 measurements, which have been made up to the moment of writing this report.

These measurements are far too few to draw a definite conclusion. Stress is laid on the following points

1. much larger quantities fallen within short durations (consequently: large mean intensities) should be preferred
2. each of these falls should be produced by only one rain
3. the rains should fall with large wind gradients
4. rains with almost equally sized drops should be preferred.

12. ON THE WIND EFFECT.

In 4.1.1 it was assumed that losses of rainfall caused by the so called wind effect(defined in 10.1) could be reglected. Let us now consider this assumption in some detail. If all measurements H_o, H_1, H_2, H_3, H_4 must be enlarged for this wind effect by equal percentages, it is obvious that the difference $\Delta = M_1 - M_2 = \frac{1}{2}(H_1 + H_2) - \frac{1}{2}(H_3 + H_4)$ and the ratio $q = M_1 : H_o \sin\beta$ are not affected by this correction. We do not know whether this special suppossition holds, but "all" differences and all ratio's q may be understood as produced only by measuring errors and field differences. This means: only in 5 percent of the cases $/\Delta/ > 2\sigma_\Delta$ and $/q - 1/ > 2\sigma_q$, provided that these standard deviations σ_Δ and σ_q are calculated in the right manner (see Appendix A). However, it remains questionable whether the assumption mentioned above is fulfilled; an inclined raingage may suffer a wind effect quite different from that for the conventional horizontal one, and this wind effect may depend on the wind direction. However, we do not dispose of figures. Then in general $p_o \neq p_1 \neq p_2 \neq p_3 \neq p_4$, if the corrected measurement may be written as $H_i^* = (1 + p_i) H_i$. The way in which p_o depends on the windspeed is rather well known from several investigations[*pg.38]. Almost certainly p_i exceeds p_o ($i = 1, 2, 3, 4$). However, the way in which p_i depends on the inclination of the gage i, on the wind speed and on the wind direction, is unknown. Consequently also the final correction on both Δ and q, if ever it must be made, is unknown. It proves extremely difficult to foresay how much the ratios $/\Delta/ : \sigma_\Delta$ and $/q - 1/ : \sigma_q$ must be changed (decreased or enlarged?) only on the ground of these wind corrections.

THE GENERAL, HOWEVER PROVISIONAL, CONSLUSION IS:

The differences $\Delta = \frac{1}{2}(H_1 + H_2) - \frac{1}{2}(H_3 + H_4)$ and the differences $q - 1$, with $q = \frac{1}{2}(H_1 + H_2) : H_o \sin\beta$ can be explained sufficiently well by measuring errors and field differences. Although the reality of losses by wind effects cannot be denied, the results suggest that, if such wind corrections should be applied, they are for horizontal and inclined raingages almost numerically equal.

APPENDIX A

A 1. On areal differences of the total amounts of precipitation produced by one and the same rainfall.

A 2. On the influence of these differences on the accuracy of the difference $\Delta = M_1 - M_2$ and on the ratio $q = M_1 : H_o \sin\beta$.

A 1.

Even if the four identical raingages of our experimental site were installed (in the corners of a square with diagonals of a length of 4 m) in an exactly horizontal position, they would have yielded different readings, since there may exist real local differences between the true amounts of precipitation produced by one and the same rainfall, although the distances between neighbouring gages are only $2\sqrt{2}$ m. (Supposing that no measuring errors are made or that the readings have been corrected for inaccuracies of measurement.)

If the four gages were placed in the near vicinity of each other, it is highly probable that they would affect each other aerodynamically. Consequently the losses by the "wind effect" (treated in 10.1) would have been unequal, which generally would result in mutual differences of the readings, unless one could combine in some way the four horizontal orifices into some sort of "quatro pluviometer". If the gages were set far from each other, they do not influence each other aerodynamically, but then their differences are uncertain to an extent depending on the spatial variation of the precipitation.

A precipitation measurement is a "sample" of a precipitation "pattern" obtained from one or more "pointmeasurements", and it is of some importance to estimate this "sampling effect". For this purpose the following experiment was made during August and September 1956. Twelve identical raingages of the ordinary type were set out in a triangular grid (see fig.11). The triangle formed by the points 10, 11, 12 has sides of 60, 60 and 100 m. The distances between each pair of neighbouring gages within the triangle formed by the points 1, 7, 10 were 10 m. Though these gages were made of plastic material they were of the same size and form as the conventional climatological gage. The orifices (of 400 cm^2) were at 60 cm above the ground. The rainwater was collected in a bottle underneath, which was replaced at each observation. It is assumed that by this kind of measurement the true amounts of rainfall could be obtained with a greater accuracy than when measured by a standard gage. The standard deviation of the measuring errors probably amounted to 0.02 mm, whereas that of the standard gage reached about 0.06 mm.

Only those cases (a number of 27) were considered in each of which all point falls equalled or exceeded 0.1 mm. This number of measurements has turned out to be far too small to draw detailed conclusions. These would have required a far greater number of gages in the field (for instance 60 or 100) and a far longer period of measurement. Still the few results reveal some interesting features, which are mentioned now.

The field value "g" is defined as the arithmetical average of all gages. Such a field value was computed for each set of observations. They have been grouped in table 6.

TABLE 6

field values	
class in mm	number
0 - 1	12
1 - 2	5
2 - 4	3
4 - 6	4
6 - 8	1
8 - 10	1
10 - 12	-
12 - 14	-
14 - 16	1
all	27

In the figures 12, 13, 14, 15, 16 typical examples are shown, with g = field values of 4.2 6.0 7.2 9.6 and 14.2 mm respectively.

As was already formulated above, the reality of field differences can be proved as follows: the amounts of precipitation referring to the same period, which can be measured with the help of N identical horizontal rain-gages installed on a field of say 100 x 100 m^2, usually show differences which cannot be caused only by measuring errors. When changing the mutual distances or (and) the total number N of the gages, and even if the measuring errors could be reduced to a minimum, these mutual differences remain. Obviously the cause of these field differences is due to the "sampling effect". The question arises how, among others, the difference $\Delta = M_1 - M_2$ and the ratio $q = M_1 : H_o \sin\beta$ may be influenced by this particular effect. In order to answer this question it is necessary first to formulate some numerical expression of these areal differences. For instance:

a) We may investigate the variation of the difference Δ_d between the readings made in two "points" at a given mutual distance of d metres. What is the probability distribution $Q(\Delta_d)\ d\Delta_d$ and how does this distribution depend on the relative distance d and the field value g? In particular we are interested in the standard deviation $\sigma_d = [\sum_1^N (\Delta_d - \bar{\Delta}_d)^2/(N-1)]^{\frac{1}{2}}$ and the mean deviation $\varepsilon_d = \sum_1^N / \Delta_d - \bar{\Delta}_d)/N$, with $\bar{\Delta}_d = \sum_1^N \Delta_d/N$, for $N \to \infty$.

b) We can solve the above-mentioned problem for different parts of the field and investigate whether the various probability distributions as formulated above, and in particular σ_d and ε_d, depend on the position in the field. If they do, $\varepsilon_d \neq 0$ between special points of the field; then there are "systematic" differences.

c) We may choose a "point" in the field and consider the difference of the reading in this point and the true field value. As the true field value is unknown, it is replaced by the value of g, the average of all gages.

For a given area of N gages the "true" field value can be defined as the limit of the averages of the readings, when N is increased indefinitely, provided this limit does exist. Many investigators have studied the problem referring to the question whether a "point measurement" of rainfall is representative for the whole area (areas of 100 x 100 m^2 as well as of 100 x 100 km^2). In our experiments N = 12. Assuming that the average of these 12 readings did approximate the true field value sufficiently well, we found an answer to the questions a and c as follows:

<u>Question a.</u>

For each of the 27 observations 12 independent differences Δ_{10} between two gages at a mutual distance of 10 m could be calculated. These produced a mean deviation e_{10} and a standard deviation s_{10}. In the same way we obtained for each observation an e_{20} and a s_{20} from 8 independent differences Δ_{20}; and an e_{30} and a s_{30} from 2 independent differences Δ_{30}; and a e_{60} and a s_{60} from also 2 independent differences Δ_{60}. Finally each observation gave only one difference Δ_{100}; the 27 observations gave an e_{100} and a s_{100}. The results have been assembled in table 7. Also the largest differences Δ_d (max) are mentioned. The 27 g values were grouped in some intervalls; the mean in each class is denoted as $\bar{g}$.

<u>Question c.</u>

For each of the 27 observations the standard deviation s of the 12 readings was computed. This procedure is analogous to drawing a sample from a given universe, with unknown mean value μ and standard deviation σ.

In our case μ and σ are estimated by $\bar{x} = \frac{1}{12} \sum_1^{12} x_i$ and $s = \left[\frac{1}{11} \sum_1^{12} (x_i - \bar{x})^2\right]^{\frac{1}{2}}$ resp. The analogy can be made clear as follows: for given true field value T it is possible to make measurements x in a very great number of "points" in the field. Then $\lim_{N \to \infty} \bar{x} = \mu \equiv T$ and $\lim_{N \to \infty} \left[\frac{1}{N-1} \sum_1^N (x_i - \bar{x})^2\right]^{\frac{1}{2}} = \sigma$.

A sample should be "random", that is the elements should be mutually independent. This requires that the 12 gages should be installed far enough apart. The results are also given in table 7.

SEE TABLE 7.

SEE TABLE 8.

The ultimate range of deviations, caused by inaccuracies made at the measurement itself, may be put equal to almost 0.05 mm for these plastic gages. This means: if the true point fall amounts to T mm, then the reading should be situated between T and T - 0.05 mm and in this region everywhere equally probable. Such a rectangular distribution has a standard deviation $\sigma = 0.05/\sqrt{12} = 0.014$ mm.

To frequency distribution of the differences $/\Delta_d/$ is given in table 8. Since the mutual differences between the readings surpassed the value 2σ in a much larger number of cases than can be expected on the basis of pure chance, we may well conclude that these differences are caused by other effects than by the inaccuracies of the measurements only.

Probably the so called field correlation ρ between simultaneous measurements in two "points" in the field must decrease with increasing distance d and increasing true field value T; however, the number of observations was much too small to construct any relation $\rho = \rho(d; T)$. For the same reason one should not attach much significance to the differences observed in the computed values for e_d and s_d (d = 10, 20, 30, 60, 100 m). It seems reasonable, however, that the universum values $\mathcal{E}_d$ and σ_d increase with increasing d and increasing T.

A 2.1. THE MORE DETAILED COMPUTATION OF $\sigma(M_1 - M_2)$.

Let us suppose that the true, but unknown, areal average of rainfall amount in some given period (e.g. a day or half a day) is T mm. This value has to be estimated by averaging linearly over a sufficiently large number of gages, set out at sufficiently large distances from each other. We again consider the four points 1, 2, 3 and 4 at the corners of the square with diagonals of 4 m, and suppose that their mutual distances are sufficiently large. Neglecting measuring errors the four readings would

TABLE 7 legenda: see pg. 28 mean and standard deviations of "field differences"

	g	d = 10 m		20 m		30 m		60 m		100 m							
		e = $\overline{\|\Delta\|}$;	$\|\Delta\|_M$	e = $\overline{\|\Delta\|}$;	$\|\Delta\|_M$	e = $\overline{\|\Delta\|}$;	$\|\Delta\|_M$	e = $\overline{\|\Delta\|}$;	$\|\Delta\|_M$	e = $\overline{\|\Delta\|}$;	$\|\Delta\|_M$	s_{10}	s_{20}	s_{30}	s_{60}	s_{100}	
A	0.08	0.010	0.05	0.012	0.05	0.025	0.05	0.060	0.09								
	0.13	0.080	0.20	0.033	0.08	0.050	0.05	0.055	0.09								
	0.10	0.045		0.022		0.038	0.05	0.057				0.079	0.050	0.057	0.063		0.03
B	0.24	0.043	0.10	0.033	0.08	0.010	0.01	-	-								
	0.32	0.027	0.05	0.007	0.02	0.060	0.02	0.170	0.27								
	0.32	0.036	0.12	0.042	0.12	0.010	0.01	0.020	0.02								
	0.34	0.024	0.07	0.035	0.09	0.035	0.06	0.070	0.14								
	0.30	0.032		0.028		0.029		0.087				0.039	0.041	0.047	0.101		0.05
C	0.60	0.016	0.07	0.015	0.05	0.035	0.04	0.095	0.19								
	0.68	0.040	0.10	0.060	0.08	0.100	0.17	0.140	0.16								
	0.69	0.019	0.08	0.068	0.11	0.055	0.10	0.675	0.13								
	0.71	0.047	0.09	0.050	0.08	0.005	0.01	0.065	0.13								
	0.77	0.050	0.15	0.035	0.07	0.095	0.14	0.110	0.17								
	0.79	0.020	0.04	0.022	0.06	0.040	0.04	0.050	0.07								
	0.71	0.032		0.042		0.055		0.089				0.045	0.045	0.057	0.086		0.04
D	1.05	0.036	0.10	0.040	0.07	0.035	0.04	0.095	0.15								
	1.38	0.086	0.19	0.065	0.10	-	-	-	-								
	1.47	0.040	0.07	0.078	0.15	0.090	0.10	-	-								
	1.51	0.040	0.20	0.038	0.11	0.095	0.10	0.145	0.16								
	1.94	0.043	0.08	0.047	0.07	0.015	0.02	0.005	0.01								
	2.36	0.074	0.30	0.147	0.36	0.060	0.10	0.060	0.10								
	2.63	0.137	0.31	0.155	0.37	0.050	0.06	0.055	0.06								
	1.76	0.065		0.081		0.058		0.072				0.129	0.119	0.104	0.091		0.07
E	3.97	0.167	0.30	0.130	0.30	0.100	0.20	-	-								
	4.21	0.167	0.40	0.200	0.40	0.050	0.10	0.150	0.20								
	4.33	0.100	0.20	0.183	0.30	0.100	0.20	-	-								
	4.36	0.192	0.50	0.230	0.50	0.150	0.20	-	-								
	4.22	0.156		0.183		0.100		0.150				0.196	0.229	0.110	0.210		0.11
F	5.97	0.133	0.40	0.167	0.30	0.100	0.10	0.050	0.10								
	7.23	0.300	0.60	0.250	0.40	0.050	0.00	0.200	0.20								
	9.55	0.222	0.40	0.117	0.30	0.150	0.30	-	-								
	7.58	0.218		0.178		0.083		0.125				0.268	0.229	0.146	0.204		0.12
G	14.2	0.413	0.80	0.420	0.80	0.200	0.30	0.250	0.40								
	14.2	0.413		0.420		0.200		0.250				0.430	0.380	0.280	0.210	0.120	0.24
irresp. of g		0.095	0.80	0.099	0.80	0.080	0.30	0.096	0.40	0.151	0.40						
number of differences		258		165		52		40		25							

LEGENDA: with respect to table 7.

g = average of the measurements with the 12 gages in the field, on each of 27 observations.

$\underline{g}$ = average of the g-values in the groups of observations.

$e = \overline{/\Delta/}$ = mean of all independent absolute differences between gages at a distance of d m.

$/\Delta/_M$ = largest of these differences.

$\bar{s}_d$ = quadratic mean of the values of s_d on the observations in the group.

s_d = standard deviation of all independent absolute differences between gages at a distance of d m; each observation gives one s_d.

$\bar{s}$ = quadratic mean of the values of s on the observations in the group.

s = standard deviation of all measurements at one observation; each observations gives one s.

σ_d = standard deviation of measuring errors.

TABLE 8

Frequency distribution of $/\Delta/$-values.

$/\Delta/$ mm	for field average $\leqslant$ 1.50 mm					$>$ 1.50 mm					all field averages				
	distance between points					distance between points					distance between points				
	10	20	30	60	100 m	10	20	30	60	100 m	10	20	30	60	100 m
0 - 0.10	127	83	26	16	5	74	56	23	12	3	201	139	49	28	8
0.11 - 0.20	7	4	2	9	8	29	19	4	8	4	36	23	6	17	12
0.21 - 0.30				1	1	23	12	3	1	3	23	12	3	2	4
0.31 - 0.40						9	10		1	1	9	10		1	1
0.41 - 0.50						2	2				2	2			
0.51 - 0.60						1					1				
0.61 - 0.70						2	1				2	1			
0.71 - 0.80						1	1				1	1			
sum	134	87	28	26	14	141	101	30	22	11	275	188	58	48	25

be $T + a_1$, $T + a_2$, $T + a_3$ and $T + a_4$, if the gages are in exactly horizontal position. The additive term a_i represents some unknown deviation which will vary in general around zero according to some error distribution. For the sake of simplicity this distribution is assumed to be a normal one. The standard deviation $\sigma(a_i)$ is assumed to be independent on i (that is the spot in the field), but dependent on the true field value T. It is estimated by $\sqrt{s^2 - 0.014^2}$ with s taken from table 6. Since in general $s \gg 0.014$ the value of $\sigma(a)$ will approximately equal s. Here s^2 is the computed variance of the observed differences. Since $\sigma(a)$ by definition does not contain deviations due to measuring errors, the computed variance s^2 should be diminished by the roughly estimated variance of the latter, i.e. by 0.014^2.

In our experiments the raingages at the "points" 1, 2, 3 and 4 are inclined as described in the foregoing sections, and moreover measuring errors do exist (yielding new additive terms d_i). Then the measured amounts of rainfall can be written as.
$H_1 = (T + a_1)(A + B) + d_1$; $H_2 = (T + a_2)(-A + B) + d_2$; $H_3 = (T + a_3)(C + B) + d_3$; $H_4 = (T + a_4)(-C + B) + d_4$ and at "point" 0 (convertional, horizontal gage)
$H_o = (T + a_o) + d_o$. With $A = \text{tg}\,\gamma \cos\beta \cos\psi$; $C = \text{tg}\,\gamma \cos\beta \sin\psi$; $B = \sin\beta$; γ = inclination of rainfall (angle between vertical and rainfall vector); ψ = azimuth of rainfall; $90 - \beta$ = inclination of a normal to the orifice. In general $T + a_o \neq T + a_1 \neq T + a_2 \neq T + a_3 \neq T + a_4$. In our former expressions the factors $T + a_i$ had been put equal to H_o, but since field differences are now taken into account, this is no more true. Further, by definition,

$$\Delta = M_1 - M_2 = \tfrac{1}{2}(H_1 + H_2) - \tfrac{1}{2}(H_3 + H_4) =$$
$$\tfrac{1}{2}\left[(A + B)\,a_1 + (-A + B)\,a_2 - (C + B)\,a_3 - (-C + B)\,a_4 + d_1 + d_2 - d_3 - d_4\right]$$

[N.B. In the absence of any field difference of the rainfall itself (all $a_i = 0$) then $\Delta = \tfrac{1}{2}(d_1 + d_2 - d_3 - d_4)$]

Next the variance $\sigma^2(M_1 - M_2)$ will be derived. This means: for a given T, a population of varying Δ values ($\Delta = M_1 - M_2$) can be expected caused both by field differences and by measuring errors (the terms a_i and d_i). For reasons of simplicity we suppose $\sigma(a_i) = \sigma_a$ (depending on T) for each i and $\sigma(d_i) = \sigma_d$ (not depending on T), for each i. For this σ_d the value 0.06 mm, relating to the conventional gage (all gages in our γ, ψ-experiments were of the ordinary type), must be substituted. Then we find (assuming independence [6]) between the a_i's and d_i's)

note 6)
This assumption includes that the raingages are situated far enough from each other, but how many meters are equivalent to "far enough" could not be concluded from table 6 sufficiently sharp.

$\sigma^2 (M_1 - M_2) = \frac{1}{4} [4 \sigma_d^2 + 2 (A^2 + 2B^2 + C^2) \sigma_a^2] = \sigma_d^2 +$
$(\frac{1}{2} \text{tg}^2\gamma \cos^2\beta + \sin^2\beta) \sigma_a^2 > \sigma_d^2.$

Without field differences this variance was σ_d^2 (only being the result of measuring inaccuracies), but now, because of field differences, this σ_d^2 is increased with $(\frac{1}{2} \text{tg}^2\gamma \cos^2\beta + \sin^2\beta)\sigma_a^2$.

Let us investigate this extra term in some detail.

Of course, again $\overline{\Delta} = 0$, because $\bar{a}_i = \bar{d}_i = 0$, since the distributions of a_i and d_i were supposed to be symmetrical around zero.

We note:

1) $\sigma(M_1 - M_2)$ depends on γ, β, σ_d and σ_a, and by means of σ_a also on T.

2) If rainfall is vertical ($\gamma = 0$) then $\sigma(M_1 - M_2) = \sqrt{\sigma_d^2 + \sigma_a^2 \sin^2\beta}$
The larger the angle between the orifice and the horizontal plane ($\beta = 90 \rightarrow 0^\circ$) the smaller $\sigma(M_1 - M_2)$.

In our case $\beta = 70^\circ$; $\sigma_d \cong 0.06$ mm. Then according to table 6

for T = 1 mm $\sigma_a \cong 0.04$ and consequently $\sigma(M_1 - M_2) = 0.07$ mm and

for T = 8 mm $\sigma_a \cong 0.11$ and $\sigma(M_1 - M_2) \cong 0.12$ mm.

3) When using four horizontal gages $\sigma(M_1 - M_2) = \sqrt{\sigma_d^2 + \sigma_a^2}$, irrespective of the inclination of the rainfall

4) When using four vertical gages $\sigma(_1 - M_2) = \sqrt{\sigma_d^2 + \frac{1}{2} \sigma_a^2 \text{tg}^2\gamma}$ (with $\gamma > 0$). Then the orifices are directed towards N, S, W, E. Now $\sigma(M_1 - M_2)$ will increase with increasing inclination of the rainfall.

In our case $\beta = 70^\circ$ and $\sigma(M_1 - M_2) = \sqrt{0.06^2 + (0.06 \text{ tg}^2\gamma + 0.88) \sigma_a^2}$.

In general $\gamma < 45^\circ$ and consequently approximately

$\sigma(M_1 - M_2) = \sqrt{0.06^2 + 0.90 \sigma_a^2}$, with σ_a taken from table 6.

As was said already, σ_a depends on T.

It was found that $\sigma_a > \sigma_d$; in other words values $a_i > d_i$ are more frequent than values $a_i < d_i$. Then for field values increasing from say 0.1 to 10 mm the value of $\sigma(M_1 - M_2)$ will increase from say 0.07 to 0.14 mm.

If we take these new values of $\sigma(M_1 - M_2)$ into account, we are able to show that "none" of the measured differences $\Delta = M_1 - M_2$ differed from zero more than twice the value of this $\sigma(M_1 - M_2)$. Consequently these differences can be caused by measuring errors and field differences only. In other words: We may assume that $\Delta = M_1 - M_1$ varied only by chance.

A 2.2. THE MORE DETAILED COMPUTATION OF σ_q.

Definition: $q = M_1 : H_o \sin\beta = \frac{1}{2}(H_1 + H_2) : H_o \sin\beta$.

Now, for given true field value T,

$H_1 = (T + a_1)(A + B) + d_1$; $H_2 = (T + a_2)(-A + B) + d_2$; $H_o = (T + a_o) + d_o$.

Thus

$$q = \frac{1}{2\sin\beta} \; \frac{2\,T.B + (A + B)\,a_1 + (-A + B)\,a_2 + d_1 + d_2}{T + a_o + d_o}$$

For a given value of T

$$\sigma_q^2 = \frac{(4\,q^2\sin^2\beta + 2)\,\sigma_d^2 + \left\{4\,q^2\sin^2\beta + 2\,(tg^2\gamma\cos^2\beta\;\cos^2\psi + \sin^2\beta\;)\right\}\sigma_a^2}{4\,(T + a_o + d_o)^2\sin^2\beta}$$

If field differences were not taken into account (substitute $a_i = 0$ and consequently $\sigma_a = 0$) then

$$\sigma_q^2 = \frac{4\,q^2\,\sin^2\beta + 2}{4\,(T + d_o)^2\sin^2\beta},$$ as was found already in section ..10... .

The corrected σ_q depends on γ, σ_a, σ_d and q itself, but also on ψ = wind direction. The σ_a represents the dependence on the true field value. Again σ_a must be taken from table 6.

A numerical example representing a practical case will illustrate the influence of the field difference on σ_q.

Suppose H_o = 17.8 mm; $\gamma = 3^o$; $\psi = 9.6^o$; q = 0.976. Without field differences, and only in consequence of measuring errors $\sigma_q = 0.004$, but taking also into account the field differences $\sigma_q = 0.015$. In the first case we would conclude that q differs too much from 1, the hypothetical mean value, that is more than could be explained only by chance, but if σ_q is calculated in the right way, then 1 - q = 0.024 is smaller than twice $\sigma_q = 0.015$.

Conclusion: By applying the extended value of σ_q "all" q values prove to differ from 1 less than 2 σ_q; in other words: We may assume that q varies only by chance.

APPENDIX B

The mean horizontal velocity and the "effective height" of the drops in the rain.
The water content (liquid water per m^3 air) over the raingage during the rain.

1. The pluviogram enabled us to measure the total durations of each of the 38 rainfalls, 10 of which have been mentioned in table 5. The wind speed was recorded at heights of 0.40 m (level of raingage), 10 m and 20 m. These registrations allowed to compute the corresponding mean wind speed during the rainfall periods; they may be called $W_{0.4}$, W_{10} and W_{20} m/sec. The exponential relation $\hat{r} = a\ I^b$, mentioned in 8, gave for given value of the mean intensity I (derived from the pluviogram) the corresponding $\hat{r}$ - value ($\hat{r}$- median value of the drop radius) and this value furnished the corresponding terminal fall (vertical) speed V_v by means of the empirical Gunn-Kunzer relation (see fig. 9).

2. Now $W_h/tg\,\gamma$ (with W_h = wind speed at the height of h meters) represents an estimate of the true vertical speed V_v of the drops over the raingage and also W_h represent an estimate of the true horizontal speed $V_h = V_v tg\,\gamma$ of the drops. However, which value of h (called the "effective height") should be used? Is there a "best" value of h? The corresponding values of W_h, $tg\,\gamma$ and V_v are interdependent in a rather complicated manner. The value of V_v is determined by the mean intensity (see section 1) and the Gunn-Kinzer relation. However, since it is possible to use at least four empirical $\hat{r}$ - I -relations (see chapter 8) it is possible to distinguish four values of V_v, called V_v (S) for the sake of simplicity, where S = B.E. (Best; England), B.G. (Best; Germany), L.P. (Laws and Parsons; Washington) and M.P. (Marshal and Palmer; Ottawa). We decided to choose 5 values of h : 0.4 1 2 10 and 20 m. In this way 8 x 5 = 40 figures could be drawn. In each figure 38 points have been plotted. Consequently 20 figures with V_v(S) against $W_h/tg\,\gamma$ and 20 figures with $V_v(S).tg\,\gamma$ against W_h.

For each of the collections of paired values we computed the correlationcoefficient between V_v (S) and $W_h/tg\,\gamma$ or between V_v(S) $tg\,\gamma$ and W_h. Next we computed to best linear (least square) relations

$$V_v = a\ (W_h/tg\,\gamma) + b \quad \text{and}$$

$$V_v\ tg\,\gamma = c\ W_h + d.$$

Finally we prefered to select from these 40 regression lines only that one which represents the largest correlation (strongest linear relation). Since each figure is based on only 38 points this is a statistically difficult question. In this way the regression between V_v (B.E) tg γ and W_2 proved to be preferable. The straight line is

$$\underline{V_v \text{ tg } \gamma = 0.96 \, W_2 + 0.20} \qquad V_v \text{ and } W_2 \text{ in m/sec.}$$

The correlation coefficient is 0.73 (highly significant, because the 95% level is 0.32). So the "effective height" proved to be on an average 2 m. This is in a rather good agreement with the theoretical results in table 3, when account is taken of the fact that in nearly all rains which we studied the median drop radius was below 1 mm.

The general mean values are $\overline{V_v \text{tg} \gamma}$ = 2.3 m/sec. and $\overline{W_2}$ = 2.2 m/sec. The 38 points scatter around the straight line mentioned with a standard deviation $1.46 \sqrt{1-0.73^2}$ = 1.1 m/sec. We stress the meaning of this value by giving an example. Suppose the mean wind speed is 5 m/sec during the rainfall, measured at a height of 2 m. Then the above mentioned linear relation gives for the mean horizontal speed of the raindrops near to the gage a value of almost 5 m/sec, <u>at least on an average</u>. There is a probability 0.95 that this mean horizontal speed is situated between 5.0 - 2 x 1.1 = 2.8 and 5.0 + 2 x 1.1 = 7.2 m/sec. Although the correlation coefficient is highly positive (0.73) it is not sufficiently large to decrease the standard deviation 1.46 m/sec of the V_v.tg γ-values appreciably by means of the linear regression (a diminution of 1.46 to 1.10 m/sec) It is to be hoped that this standard deviation will decrease much more if we also consider the value of the mean intensity I. For this purpose we should try to compute for specified ranges of I the linear regressions between V_v.tg γ and W_2. However, these calculations require a much larger material than is available now, but it is intended to make such computations in the near future.

3. The inequality of the correlation coefficients $r[V_v(S); W_h/\text{tg}\gamma]$ and $r[V_v(S)\text{tg}\gamma ; W_h]$ is caused by the fact that tg γ is far from constant, but in strong linear relation to the value of W_h itself. To show this we give here only the numerical linear regression tg γ = 0.26 W_2 - 0.15, which corresponds to a correlation coefficient $r[\text{tg}\gamma ; W_2]$ = 0.85 (highly statistically significant) and a scattering standard deviation around the least square straight regression line of 0.16 m/sec. The mean of all values tgγ is 0.42, giving $\hat{\gamma} = 22\frac{1}{2}^\circ$,

whereas $\bar{\gamma} = 19.8^{\circ}$. However, $\bar{\gamma} = \frac{1}{n}\sum_{1}^{n} \gamma_i \neq \text{arc tg}\left[\frac{1}{n}\sum_{1}^{n} \text{tg}\,\gamma_i\right]$

Lacy in England [4] found $\text{tg}\,\gamma = 0.32\ W - 0.04$. However, whereas he used monthly totals of rainfall (25 months and consequently 25 points in the $\text{tg}\,\gamma$ - W figure) and measured W at gage level, we used individual rains (or total falls in specified parts of the day). Consequently it is not easy to compare the results. Results as to individual rains seem more important in practice than as to monthly totals.

4. Next we refer to formula (18) for $db_v = H_o/OD$. As H_o represents the weight of rainfall water (in grams) caught by the gage through an horizontal orifice of O cm^2 in D seconds: $d = \frac{1}{60}\ I/V_v$ kg/m^3

d = water content of the air = quantity of water in kg (liters) per m^3 air over the gage

I = mean intensity of rainfall in mm/min = (water height in mm/min)/(rainfall duration in min)

V_v = vertical speed of the drops in m/sec

The results of section 2 showed that it is preferable to substitute $W_2/\text{tg}\,\gamma$ for V_v, so that $d = \frac{1}{60}\ I\,\text{tg}\,\gamma\ /W_2$.

In this way it is possible to compute for each of the 38 rains the value of d with the help of the values of the mean intensity I (derived from the pluviogram), the value of γ (measured by means of the four equally tilted gages) and the value of W_2 (mean wind speed at 2 m height, calculated as the mean value of $W_{0.4}$ and W_{10} because, if $W = a \lg z$, then $W_2 = \frac{1}{2}(W_{0.4} + W_{10})$. Next we plotted the 38 pairs d, I in a double logarithmic diagram and found a correlation coefficient 0.87 and the exponential relation

(43) $$\boxed{d = 0.0021\ I^{0.84}}$$

d in kg/m^3; I in mm/min.

It agrees surprisingly well with the relation $d = 0.0022\ I^{0.85}$ found by Best for English rainfalls [8].

Table 9 shows some numerical results. For mean intensity I (given by the registration) the value of the water content d is deduced from (43), whereas the relation $\hat{r} = 1.73\ I^{0.227}$ (Best; English rainfalls; median value of radius r in mm) gives $\hat{r}$. Consequently the drop volume $t = \frac{4}{3}\pi(\hat{r})^3$ mm^3 is known. Then the total number of drop per m^3 air becomes $N = 10^6 d/t = 97.I^{0.16}$

TABLE 9

I mm/min	d cm^3/m^3	$\hat{r}$ mm	N dr/m^3	$\bar{D}$ cm	qualification
0.001	0.0063	0.36	32	31	very light drizzle
0.01	0.046	0.61	49	27	
0.1	0.304	1.03	66	25	
1.0	2.1	1.72	98	22	very heavy

Stress is laid on some facts

1) If the mean intensity I is increased by a factor 1000 (from 0.001 to 1.0 mm/min, that is from very light drizzle rains to very heavy rains) the water content increases by a factor 334, the median drop radius increases by a factor 5, and the total number of drops by a factor 3, while the mean mutual drop distance decreases by a factor $\sqrt[3]{3} = 1.4$.

2) Even for the very rare, extremely intense rains with a mean intensity of 1 mm/min (occurring in the Netherlands on an average once per year) the total volume of liquid water (all raindrops together) in the air close to the gage is only about 2 cm^3 per m^3 air.

LITERATURE

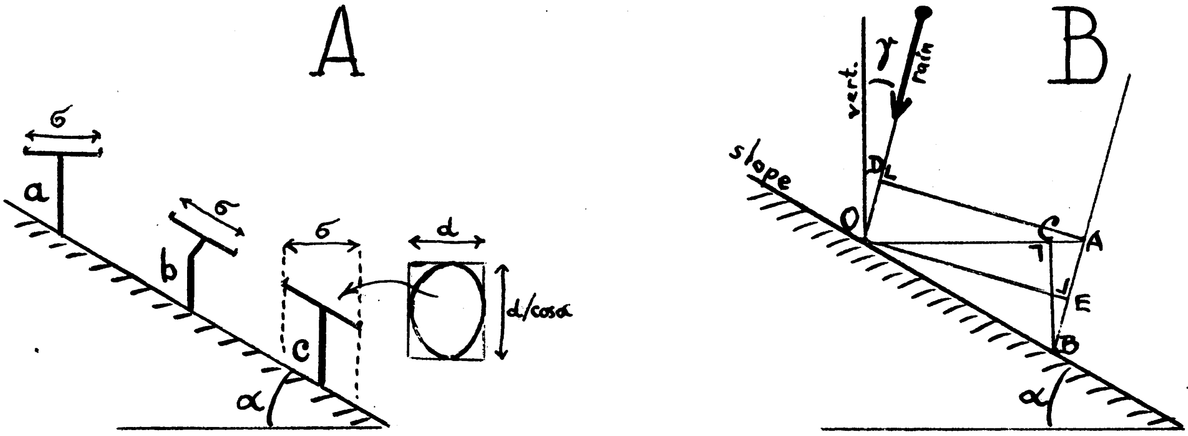
(not exhaustive)

1. Serra, L.	Interprétation des mesures pluviométriques.	U.G.G.I. 1951; Ass.Int. d'Hydr. Scientifique; Tome III, 9.
2. Serra, L.	La mesure correcte des précipitations; pluviomètre horizontal et pluviomètres inclinés.	Mémoires et traveaux de la Société Hydroélectrique de France (novembre 1952), 46.
3. Pers, M.R.	Etude théorique sur la variation des précipitations avec l'altitude.	La Météor. 10, 468, 1934.
4. Lacy, R.E.	Observations with a directionel rain gage.	Quart. J. Roy Met. Soc. 77, 233, 1951.
5. Pers, U.R.	Relations entre les données pluviométriques et les precipitations totales recueillis par un bassin. Introduction à l'étude théorique de la stéréopluviometrie.	La Météor. 8, 101, 1932.
6. Pers, M.R.	Note complémentaire sur la stéréopluviometrie.	La Météor. 8, 106, 1932.
7. Pers, M.R.	De l'influence du relief sur les précipitations en haute-montagne. Premiers résultats expérimentaux de stéréopluviométrie.	La Météor. 10, 473, 1934.
8. Best, A.C.	The size distribution of rain drops.	Quart. J. Roy Met. Soc. 76, 10, 1950.
9. Laws, J.O.	Measurements of the fall velocity of water drops and rain drops.	Trans. Amer. Geoph. Union 22, 709, 1941.
10. Marshall, J.S. and Palmer, W.Mc.K.	The distributions of rain drops with size.	J. Met. 5, 165, 1948.
11. Gunn, B. and Kinzer, G.T.	Falling speeds of drops.	J. of Meteor. 6, 243, 1949.

Wiesner, J.	Tropfengrözse.	Sitz.Ber.Akad.Wiss. Wien. Abt. I, 104, 1397, 1895.
Lenard, P.	Über Regen.	Met. Z. 21, 249, 1904.
Mache, H.	Über die Geschwindigkeit und Grösze der Regentropfen.	Met. Z. 39, 278, 1904.
Bentley, W.A.	Studies of raindrops and raindrop phenomena.	Mon. Weather Rev. 450, 1904.

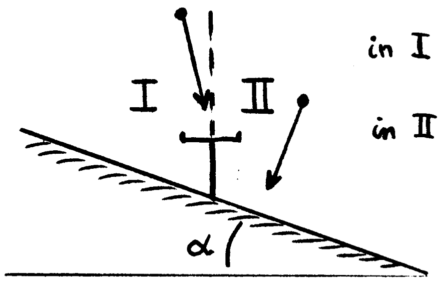
Defant, A.	Gesetzmäszigkeiten in der Verteilung der verschiedenen Tropfengröszen bei Regenfällen.	Sitzungsber. der Kais. Akad. der Wiss. in Wien, 114, 585, 1905
Becker, A.	Zur Messung der Tropfengröszen bei Regenfällen nach der Absorptionsmethode.	Met. Z. 24, 247, 1907.
Schmidt, W.	Zur Erklärung der gesetzmäszigen Verteilung der Tropfengröszen bei Regenfällen.	Met. Z. 25, 496, 1908.
Schmidt, W.	Eine unmittelbare Bestimmung der Fallgeschwindigkeit von Regentropfen.	Sitzungsber. der Math. Naturw.Klasse der Kais. Akad. der Wiss. 118, Abt. IIa, 1909.
Knudsen, M. and Weber, S.	Résistance de l'air au mouvement lent de petites boules.	Ann. der Phys. 36, 981, 1911.
Liznar, J.	Die Fallgeschwindigkeit der Regentropfen.	Met. Z. 31, 339, 1914.
Köhler, H.	Eine quantische Verteilung von Materie in der Atmosphäre.	Met. Z. 39, 263, 1922.
Köhler, H.	Über Tropfengröszen.	Met. Z. 42, 463, 1925.
Niederdorfer, E.	Messung der Gröszen von Regentropfen.	Met. Z. 49, 1, 1932.
Bastanoff, M.S.	Sur les pluviomètres.	La Météor. 8, 94, 1932.
Niederdorfer, E.	Über die Häufigkeitsverteilung der Regentropfengröszen.	Met. Z. 53, 33, 1936.
Lütschg, O.	Der Kugelniederschlagsmesser Haas-Lütsch.	Gerlands Beitr. zur Geophysik 50, 423, 1937.
Hann-Süring.	Lehrbuch der Meteorologie I.	1939.
Seeliger, R.	Untersuchungen über Regentropfen.	Dissertation, Greifswald, 1939.
Langmuir, J. and Blodgett, K.A.	A mathematical investigation of waterdrop trajectoires.	AAF Techn. Rep. no. 5418, 66 pp. Washington D.C. 1946.
Anderson, L.J.	Drop size measurements in orographic rains.	Bul. Amer. Mat. Soc. 29, 362, 1948.
Spilhaus, H.F.	Raindrop, size, shaps and falling speed.	J. of Meteor. 5, 108, 161, 1948.
Bergeron, T.	The problem of artificial control of rainfall on the globe. 1. General effects of ice-nuclei in clouds.	Tellus 1, 32, 1949.
Kopp, W.	Niederschlagsanalysen mit dem Tropfenschreiber.	Ber. Deut. Wetterd. U.S.-Zone no. 12. 1 Tagung Met. Ges., Bad Kissingen, 1949.

Green, R.L.	Evaluation of air resistance to freely falling drops of water.	J. Amer. Soc. Agric. 33, 286, 1952.
Blanchard, D.C.	A simple recording technique for determining raindrop size and time of occurrence of rainshowers.	J. Météor. 10, 457, 1953. Trans.Amer.Geoph.Union 34, 534, 1953.
Rigby, E.C., Marshall, J.S. and Hitschfield, W.	The development of the size distribution of raindrops during their fall.	J. of Meteor. 11, 362, 1954.
Hamilton, E.L.	Rainfall sampling on rugged terrains.	Techn. Bull. no. 1096 U.S. dep. of Agric., Washington D.C. 1954.
Weickman, H.	A nomogram for the calculation of collision efficiences.	Art. Stimulation of Rain; 1° . Congres Phys. of Clouds and Prec. particles, Massachusets 1955.
East, T.W.R.	Precipitation of convective clouds.	"
Blanchard, D.C.	Discussion of raindrop distributions made during the Project Shower, Hawaii 1954.	"
Gunn, K.L.S. and Marshall, J.S.	The effect of wind sheer on falling precipitation.	J. Met. 12, 339, 1955.
Diem, M.	Messung der Gröszen der Regentropfen in natürlichen Regen und bei künstlicher Beregnung.	Beitr. zur natürkündlichen Forschung in Südwestdeutschland 15, 84, 1956. Ann. Hydrol.mar.Meteor. 70, 142, 1942.
Taylor, G.J.	Surveys in mechanics.	1956.
Ed. Byers, H.R.	Thunderstorm Electrcity.	1956 pg. 96.
Mason, B.J.	The physics of clouds.	1957.
Blanchard, D.C. and Spencer, A.T.	Raindrop measurements during Project Shower.	Tellus 9, 541, 1957.
Noth, H.	Vertikale Luftbewegung und Niederschlag.	Met. Runds. 10, 50, 1957.
Weickman, H. and Smith, A.T.	Artificcal Stimulation of rain.	1° Congr. Phys. of Clouds and Prec. Particles. Massachusets 1957.
* Weiss, L.L. and Wilson, W.T.	Precipitation gage shields (with a very extensive list of literature).	U.G.G.I. Assemblée Générale de Toronto, Tome 1, p. 462, 1957. Comptes Rendus et Rapports.
Roulleau, J. et Tronchon, R.	Précipitations.	Météorologie Générale Tome II, Paris, 1958.
Lamp, R.	Das Tropfenspektrum in Niederschlägen und die Radarreflectivität (with extensive list of literature)	Beitr. zur Phys. der Atm. 30, 233, 1958.



orifice a : circular σ cm²
" b : circular σ cm²
" c : elliptical $\sigma/\cos\alpha$ cm²

fig.1



in I $f_1 = 1 + \mathrm{tg}\,\alpha\,\mathrm{tg}\,\gamma$

in II $f_1 = 1 - \mathrm{tg}\,\alpha\,\mathrm{tg}\,\gamma$

fig.2

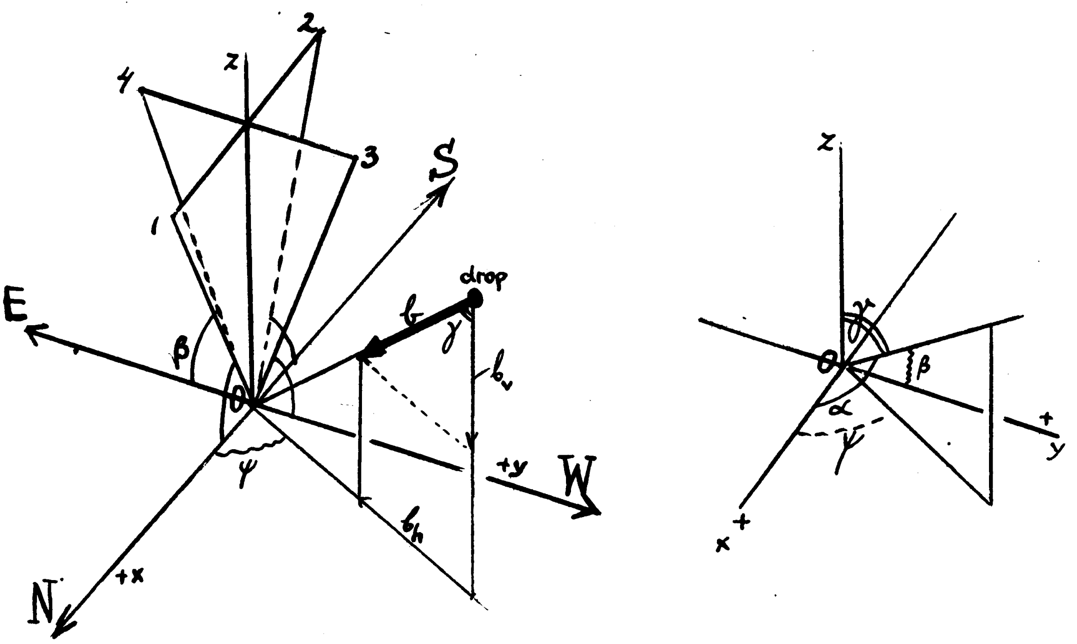
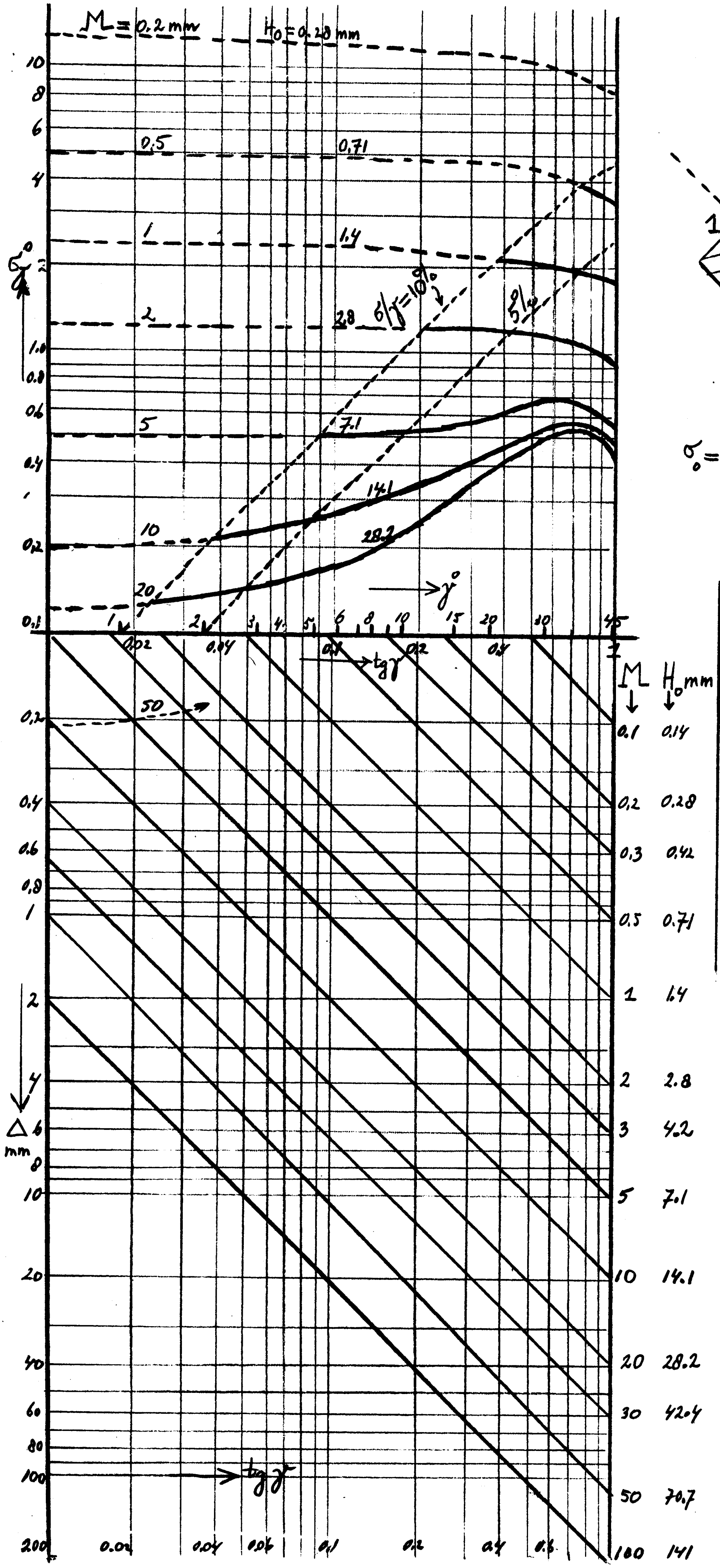
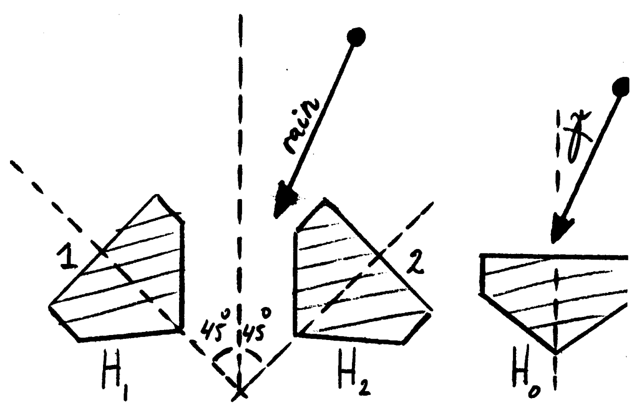


fig.3





$0 \leq \gamma < 45^\circ$

$\sigma_0 = 0.06$ mm ; $\sigma_\alpha = 0.6^\circ$

text 4.1.1.

$$\mathrm{tg}\,\gamma = \Delta / 2M\,\mathrm{tg}\,45^\circ = \frac{\Delta}{2M}$$

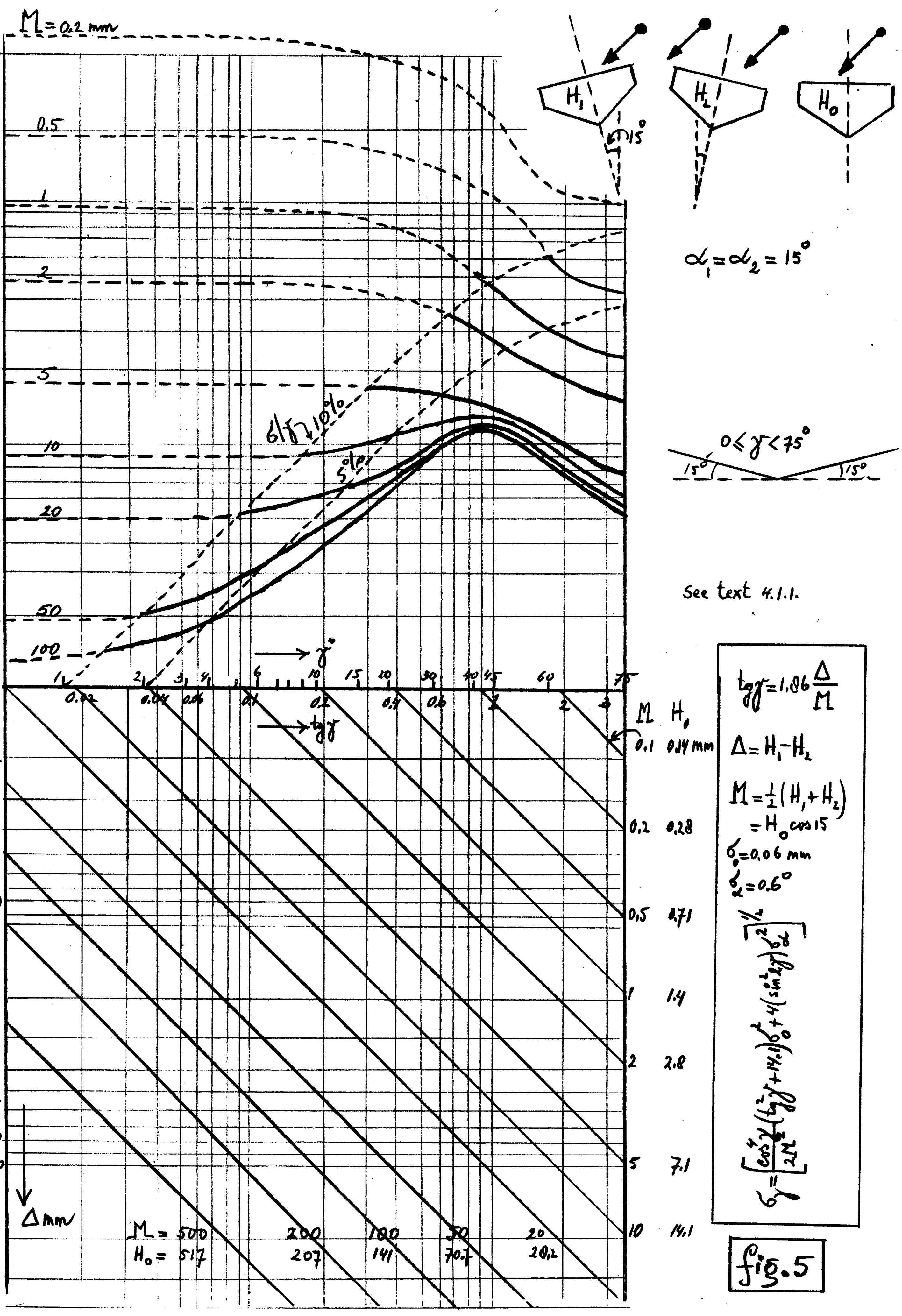
$$\Delta = H_1 - H_2$$

$$M = \tfrac{1}{2}(H_1 + H_2) = H_0 \cos 45^\circ$$

$$\sigma_\gamma = \left[\frac{\cos^2\gamma}{2M^2}\sigma_0^2 + (\sin^2 2\gamma)\,\sigma_\alpha^2\right]^{1/2}$$

always $H_0 > \frac{1}{2}\Delta$

fig.4



M = 0.2 mm
0.5
1
2
5
10
20
50
100
σ_γ^0
σ/γ 10%
5%
→ γ^0
→ tg γ
Δ mm
M = 500 200 100 50 20
H_0 = 517 207 141 70.7 20.2
M H_0
0.1 0.14 mm
0.2 0.28
0.5 0.71
1 1.4
2 2.8
5 7.1
10 14.1
H_1
H_2
H_0
15°
$\alpha_1 = \alpha_2 = 15°$
$0 \leq \gamma < 75°$
See text 4.1.1.
$tg\gamma = 1.86 \frac{\Delta}{M}$
$\Delta = H_1 - H_2$
$M = \frac{1}{2}(H_1 + H_2) = H_0 \cos 15$
$\sigma_0 = 0.06$ mm
$\sigma_\alpha = 0.6°$
$\sigma_\gamma = \left[\frac{\cos^4\gamma}{2M^2}(tg^2\gamma + 14.1)\sigma_0^2 + 4(\sin^2 2\gamma)\sigma_\alpha^2\right]^{1/2}$

fig.5

In both cases $\sigma_0 = 0{,}06$ mm, $\sigma_\alpha = 0^\circ$

text 4.1.1.

$tg\gamma = \frac{\Delta}{0.52 H_0}$; $\Delta = H_1 - H_2$, $M = \frac{1}{2}(H_1 + H_2)$ $0 \leq \gamma < 75^\circ$ $\sigma_\gamma = \frac{\cos\gamma}{H_0}\sigma_0\sqrt{1+6.5\cos^2\gamma}$

$tg\gamma = \frac{\Delta}{1{,}41 H_0}$ $0 \leq \gamma < 45^\circ$ $\sigma_\gamma = \frac{\cos\gamma}{H_0}\sigma_0$

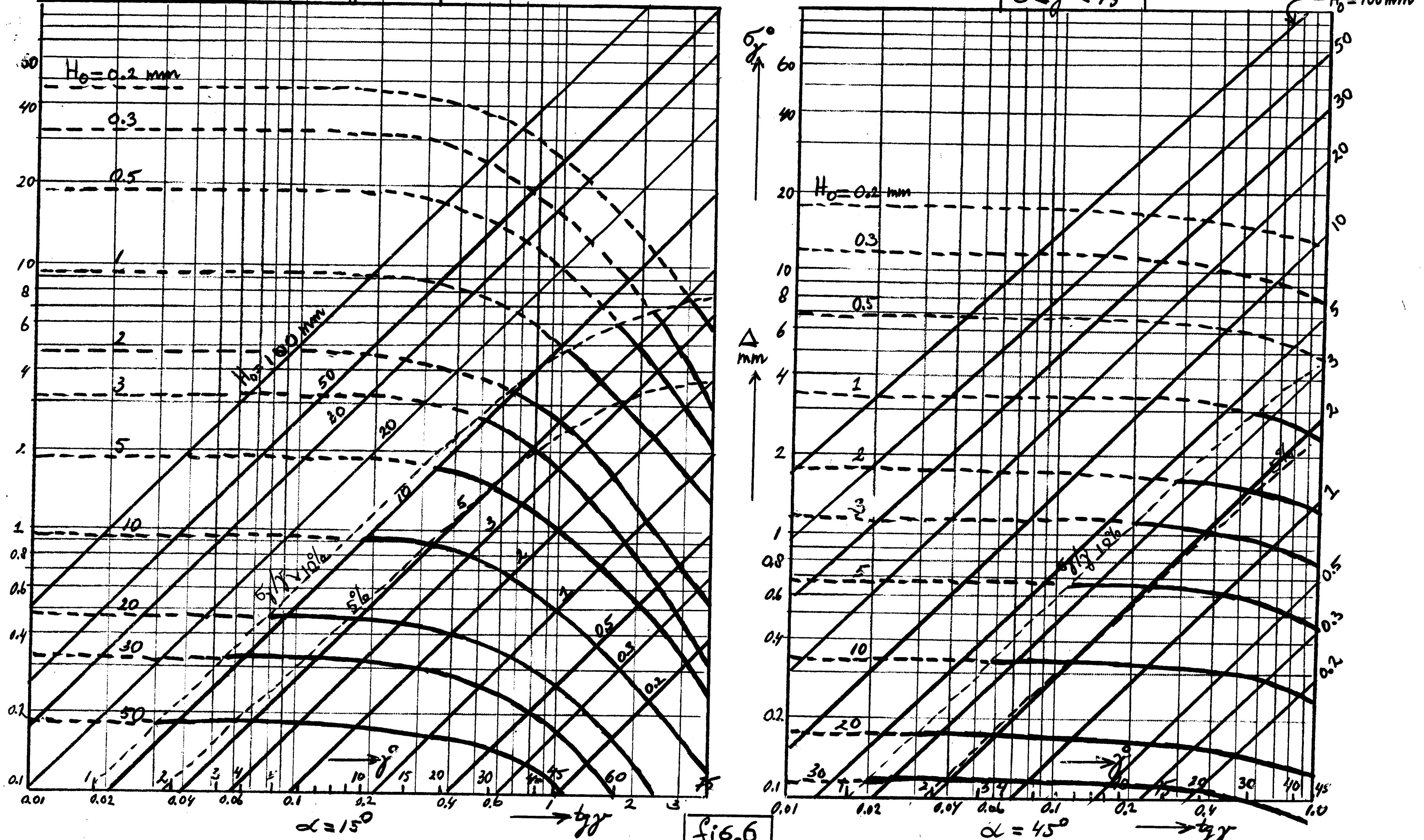
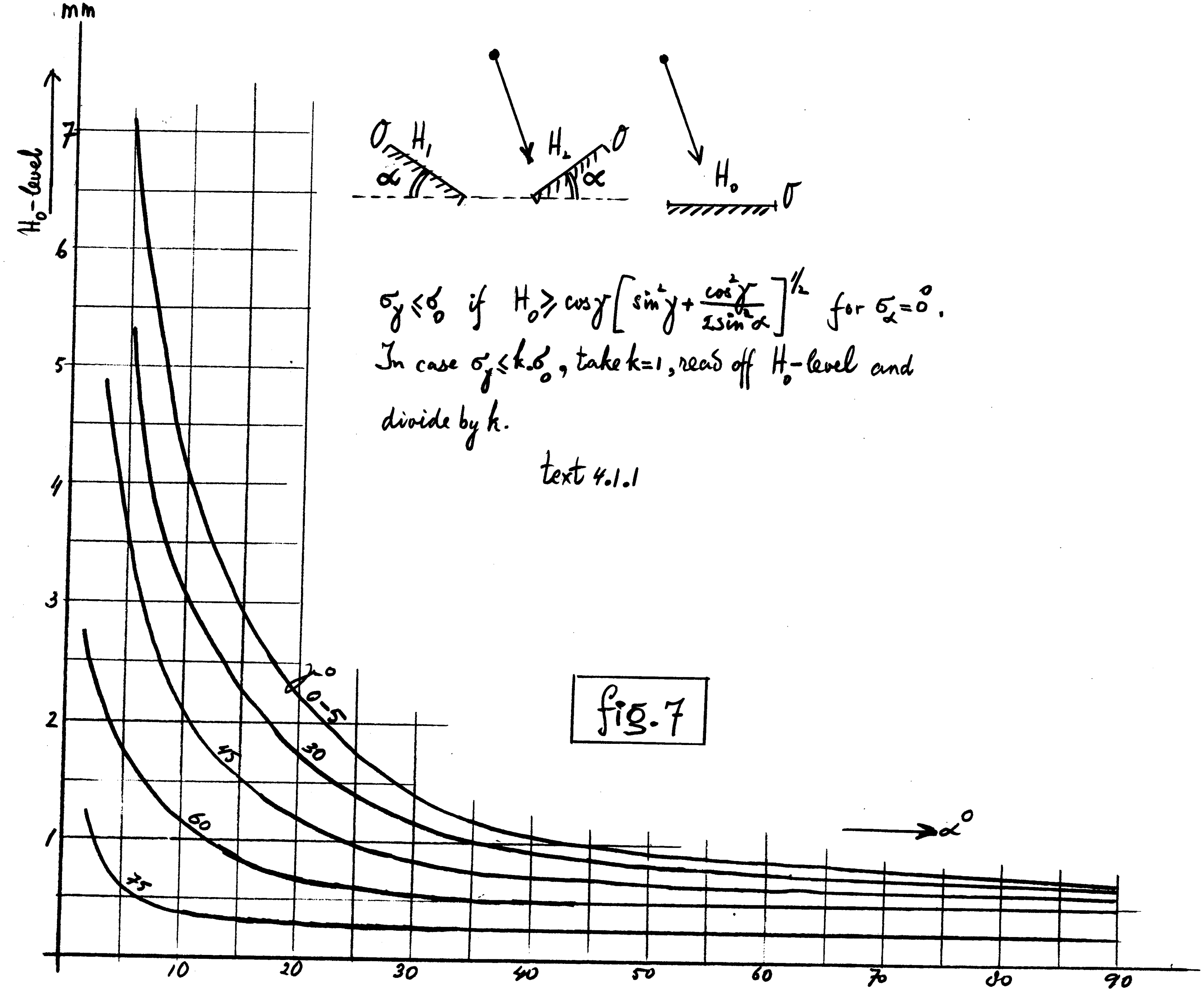


fig.6

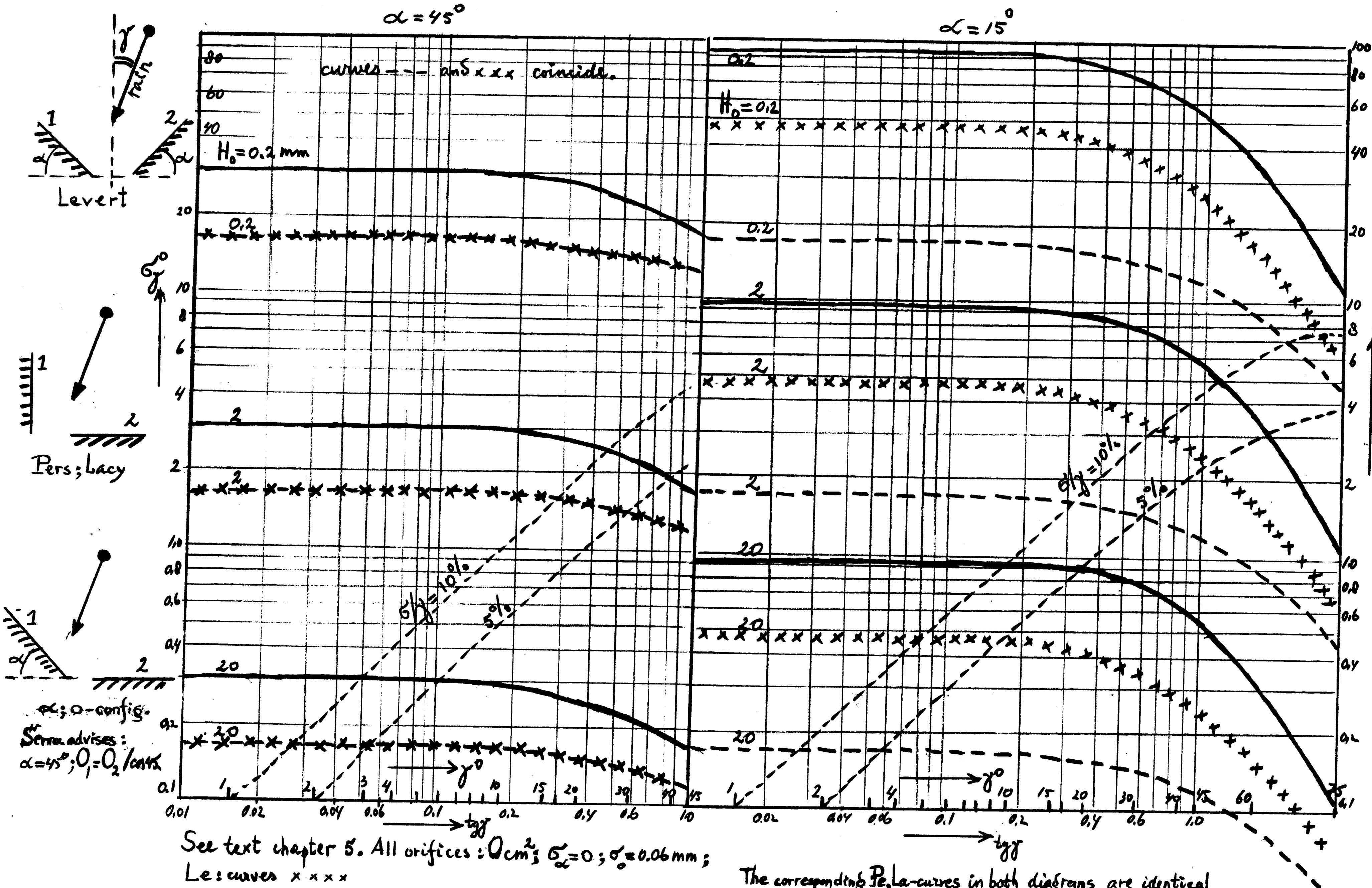


$\sigma_\gamma \leq \sigma_0$ if $H_0 \geq \cos\gamma\left[\sin^2\gamma + \frac{\cos^2\gamma}{2\sin^2\alpha}\right]^{1/2}$ for $\sigma_\alpha = 0^0$.

In case $\sigma_\gamma \leq k.\sigma_0$, take $k=1$, read off H_0-level and divide by k.

text 4.1.1

fig.7



See text chapter 5. All orifices: $O\ cm^2$; $\sigma_\alpha=0$; $\sigma_o=0.06$ mm;

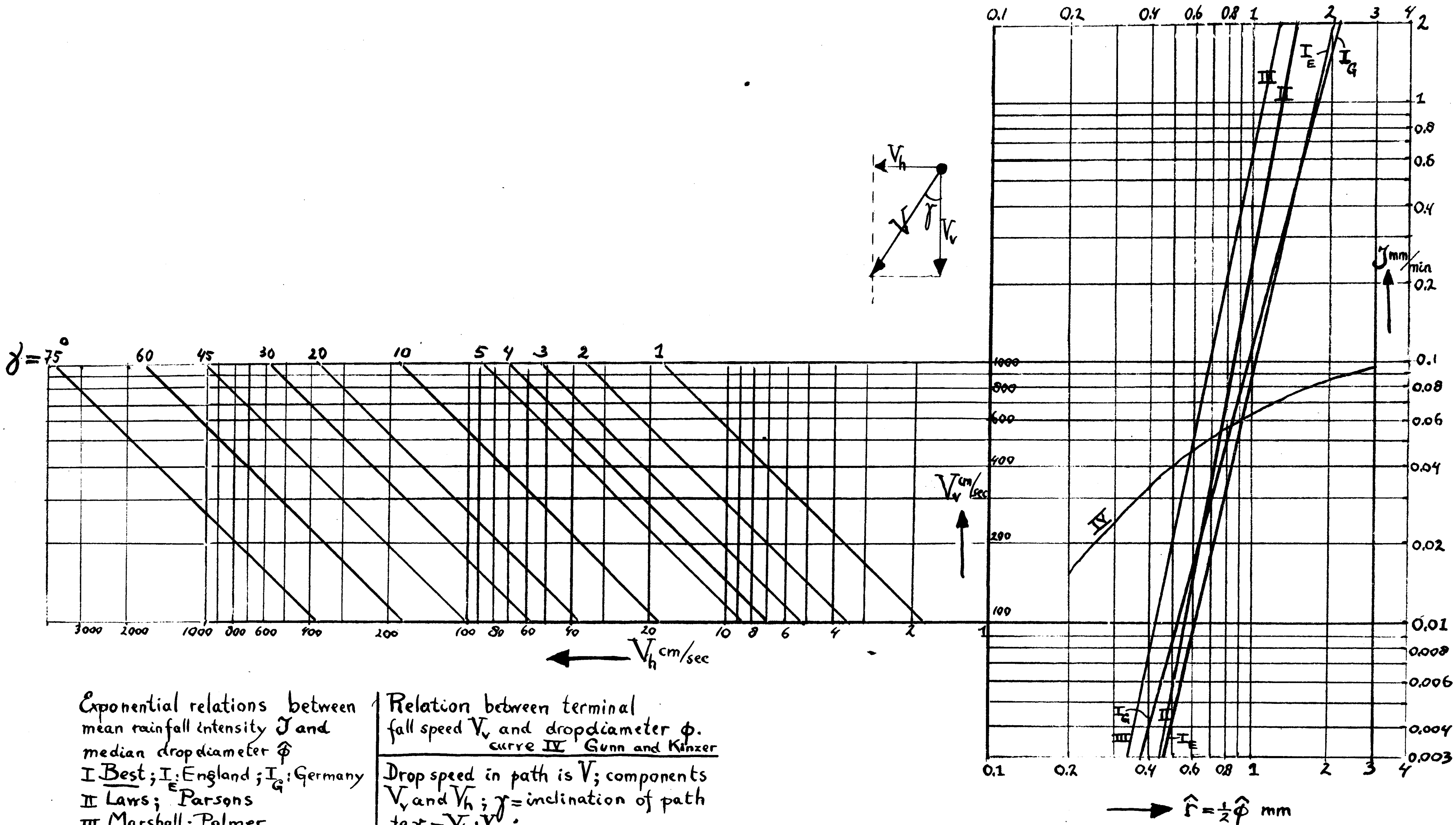
Le: curves x x x x

Pe, La: - - - -

Serra ———

The corresponding Pe, La-curves in both diagrams are identical

Fig. 8



Exponential relations between mean rainfall intensity J and median dropdiameter $\hat{\phi}$

I Best; I_E: England; I_G: Germany

II Laws; Parsons

III Marshall; Palmer.

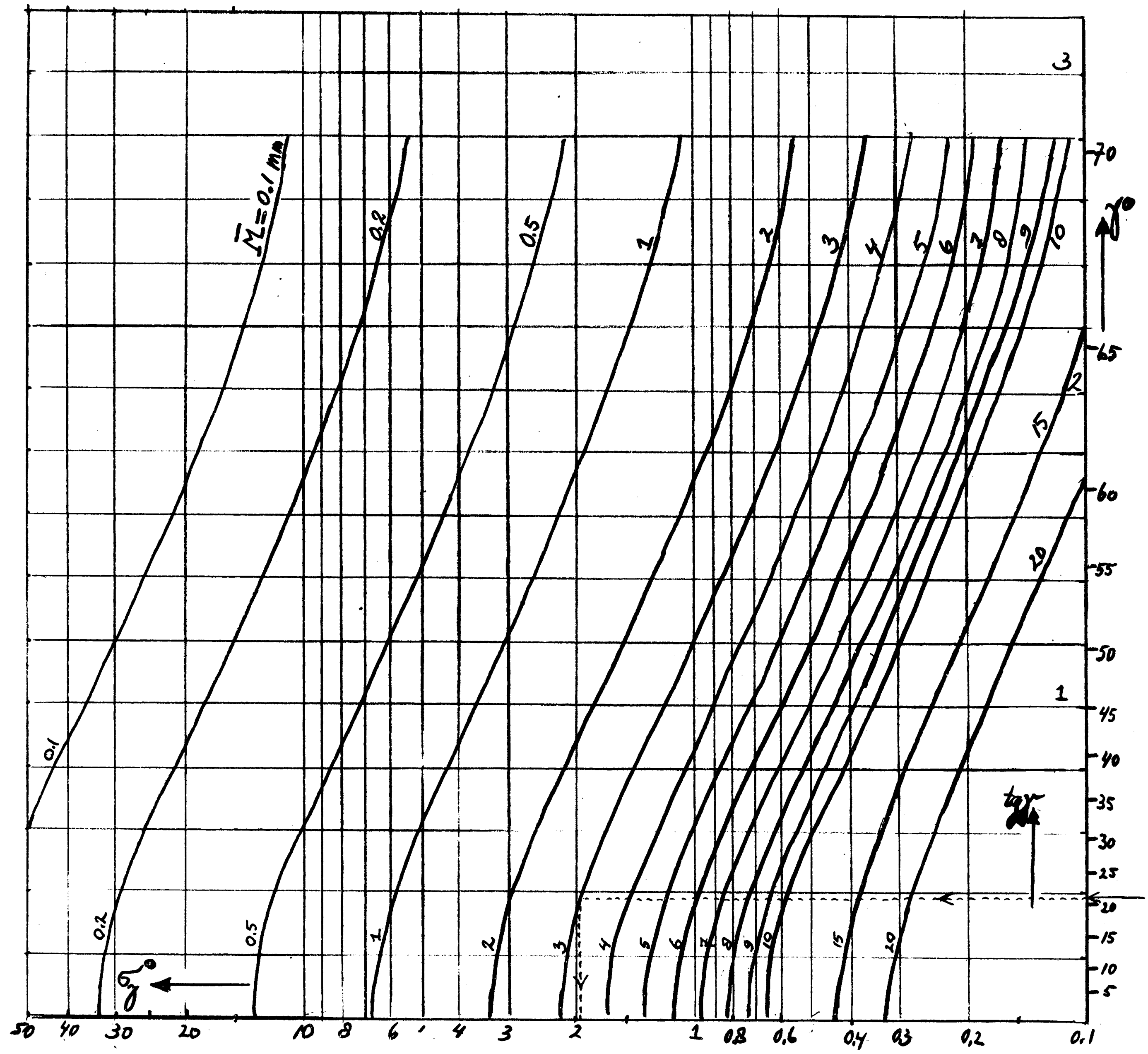
Relation between terminal fall speed V_v and dropdiameter ϕ.

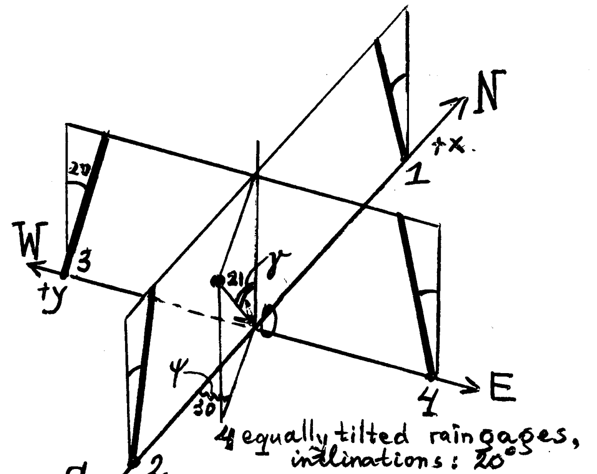
curve IV Gunn and Kinzer

Drop speed in path is V; components V_v and V_h; γ = inclination of path

$tg\,\gamma = V_h : V_v$;

fig.9





EXPLANATION

Let be $\Delta_1 = H_1 - H_2 = 0.7$ mm; $\Delta_2 = H_3 - H_4 = 0.4$ mm;
$\bar{M} = \frac{1}{4}[H_1 + H_2 + H_3 + H_4] = 3.0$ mm.

Then in fig. 10b the point Δ_1, Δ_2 is situated on the straight line $\psi \cong 30°$ (between SSE and SE) and on the circle $\sigma_\psi \cong 6°$. This circle cuts the Δ_1-axis in 0.81. The vertical through 0.81 cuts the straight line $\bar{M} = 3.0$ in a point with ordinate $tg\gamma = 0.38$ and $\gamma \cong 21°$. In fig. 10a the horizontal through 0.38 cuts the curve $\bar{M} = 3.0$ in a point with $\sigma_\gamma \cong 2°$

fig. 10a

fig.10b

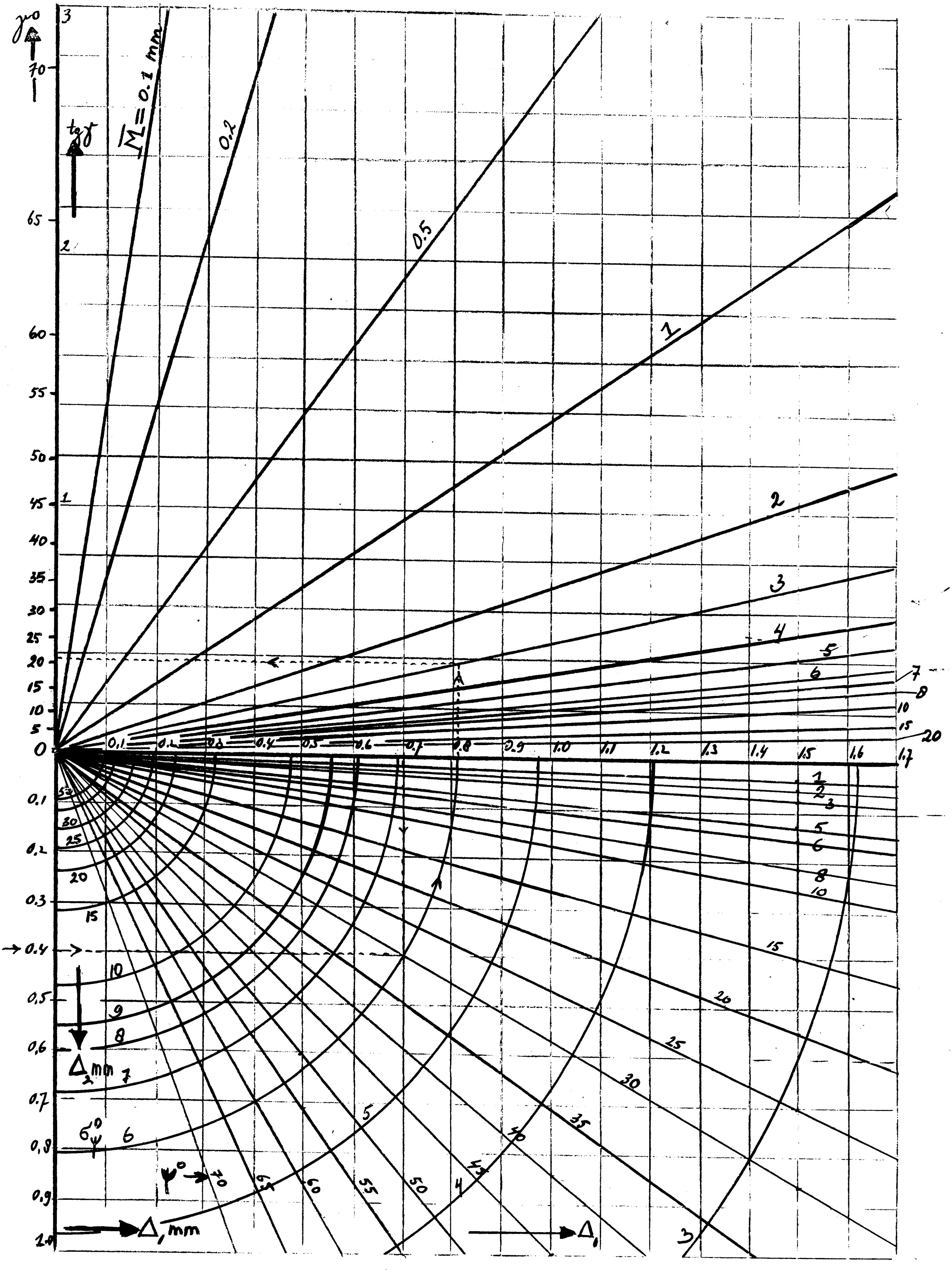
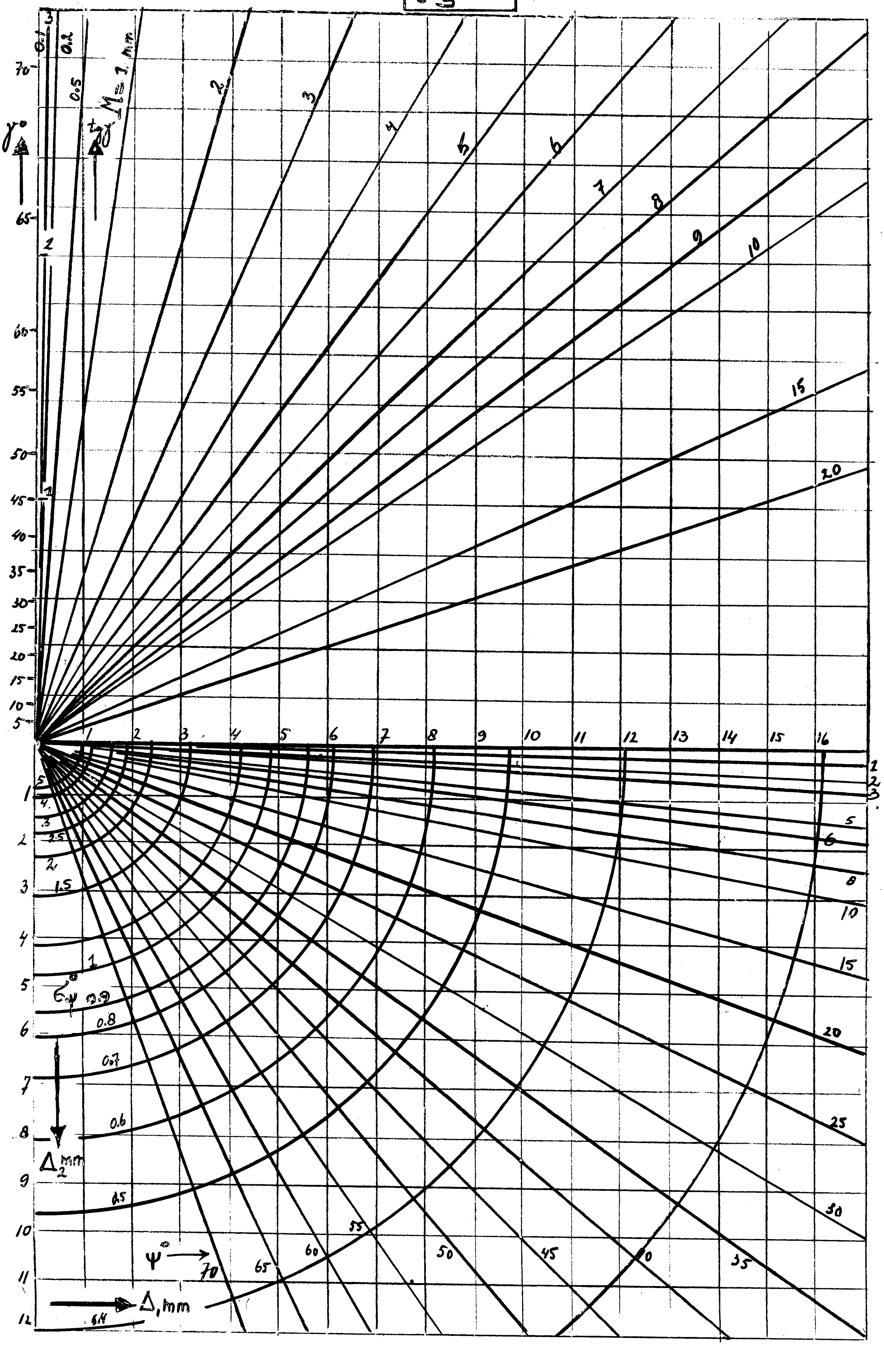


fig.10c



γ°
tg γ
M = 1 mm
0.1
0.2
0.5
2
3
4
5
6
7
8
9
10
15
20
70
65
60
55
50
45
40
35
30
25
20
15
10
5
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
ε_ψ°
0.9
0.8
0.7
0.6
0.5
0.4
1.5
2.5
Δ_2 mm
ψ°
70
65
60
55
50
45
40
35
30
25
Δ_1 mm

THE REPRESENTATIVITY OF A POINT MEASUREMENT OF THE RAINFALL.

in a field of 4000 m^2

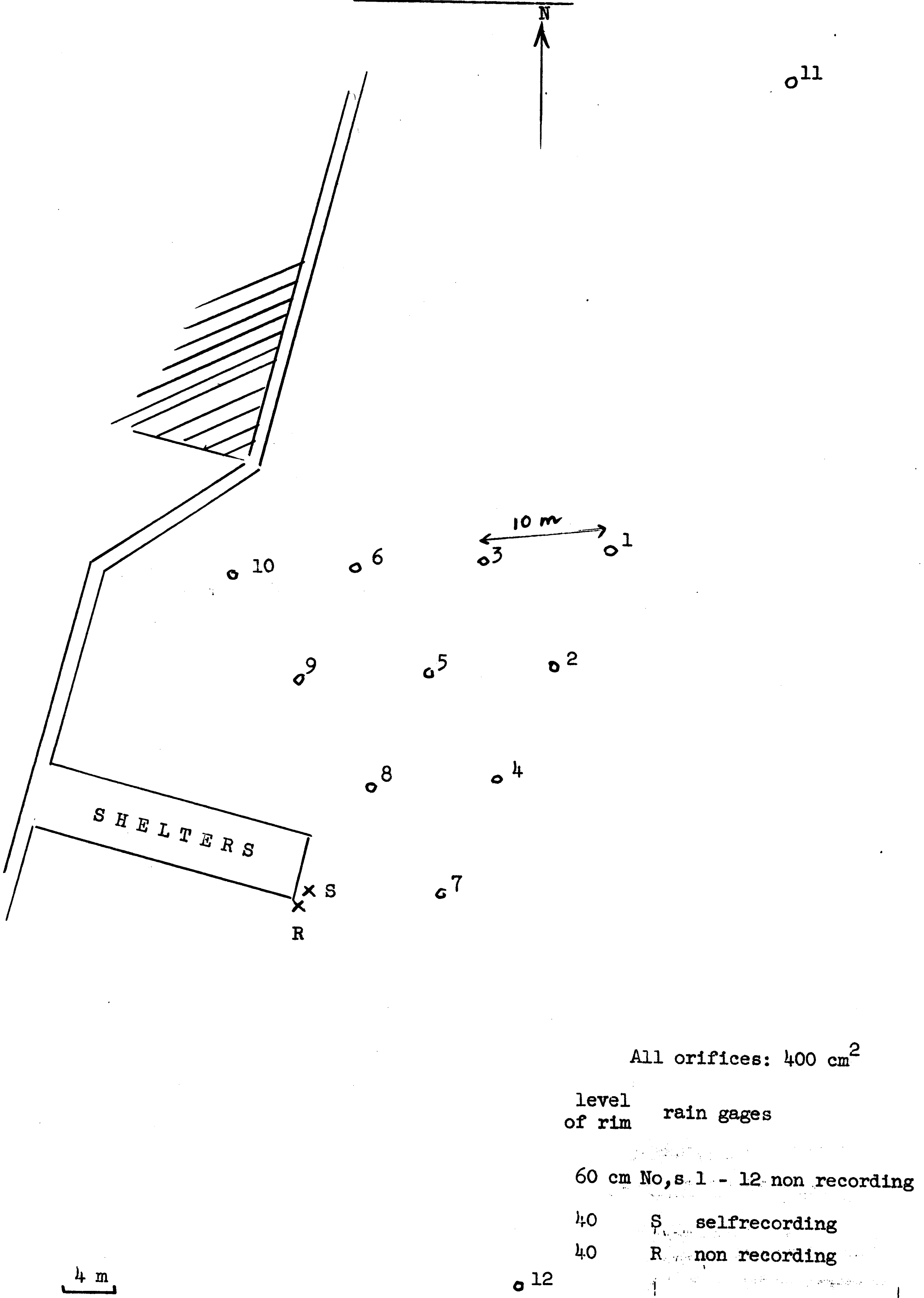


FIG. 11

Experiments Aug.,Sept. 1956

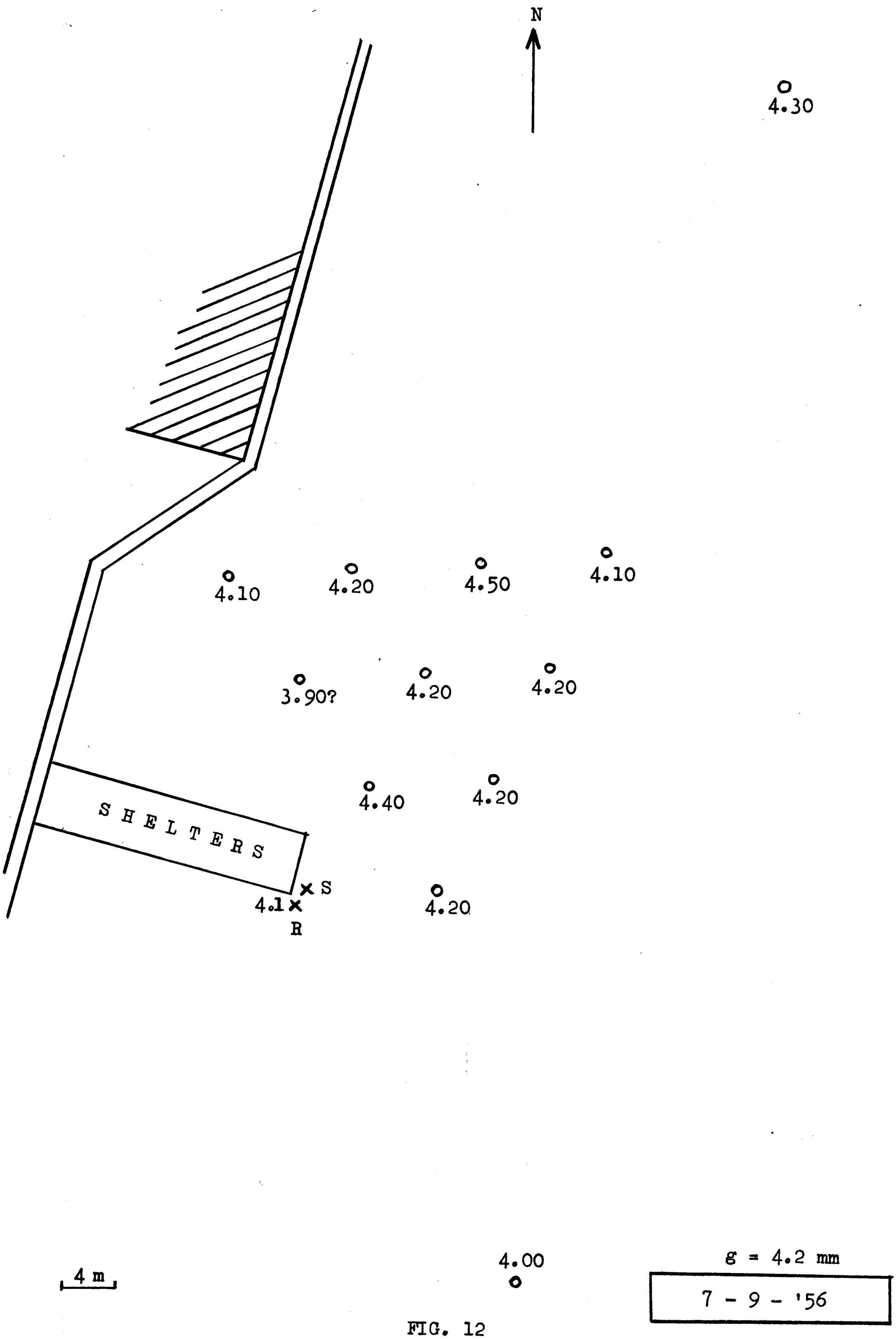

N
4.30
4.10
4.20
4.50
4.10
3.90?
4.20
4.20
4.40
4.20
SHELTERS
S
4.1
R
4.20
4 m
4.00
g = 4.2 mm
7 - 9 - '56

FIG. 12

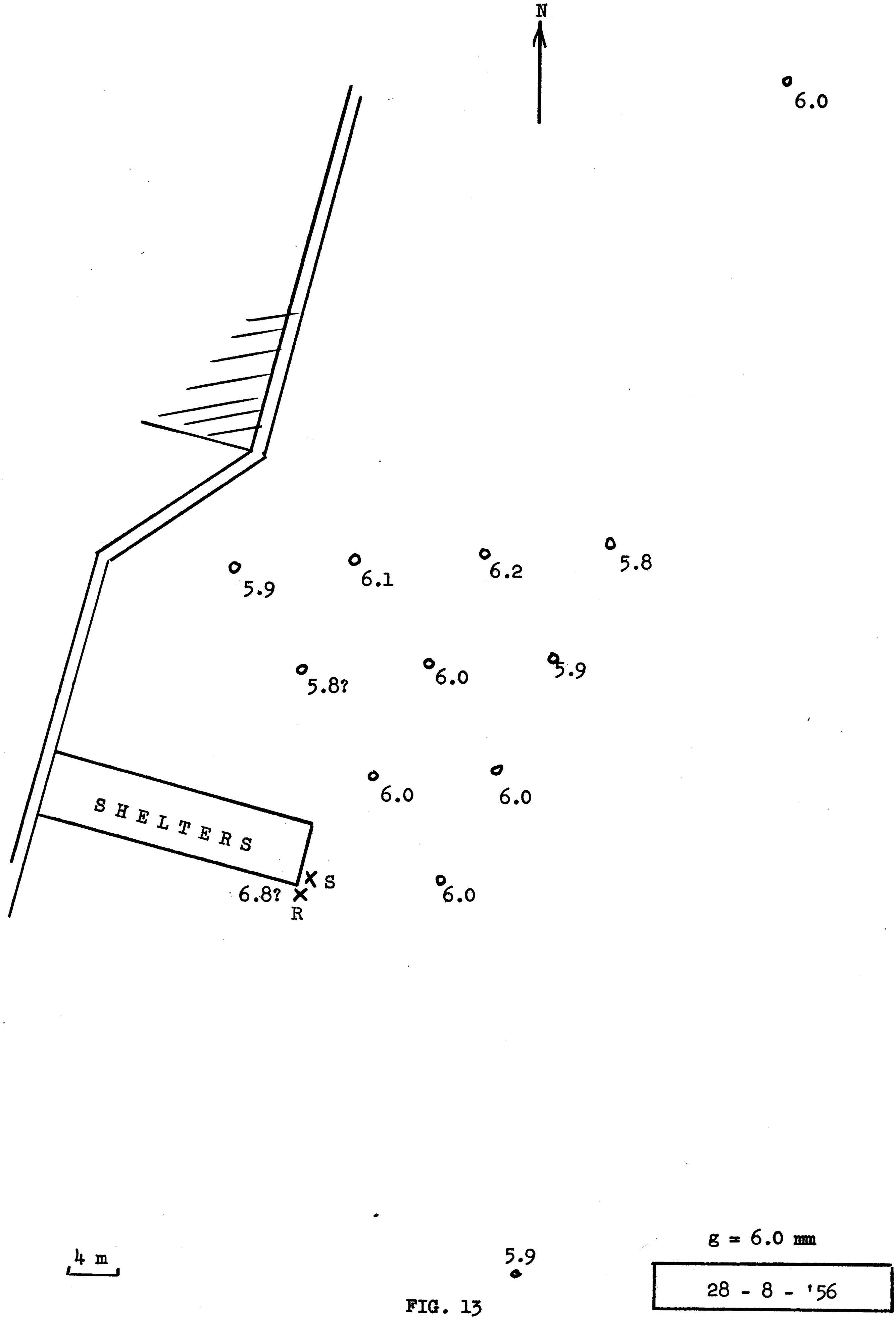


FIG. 13

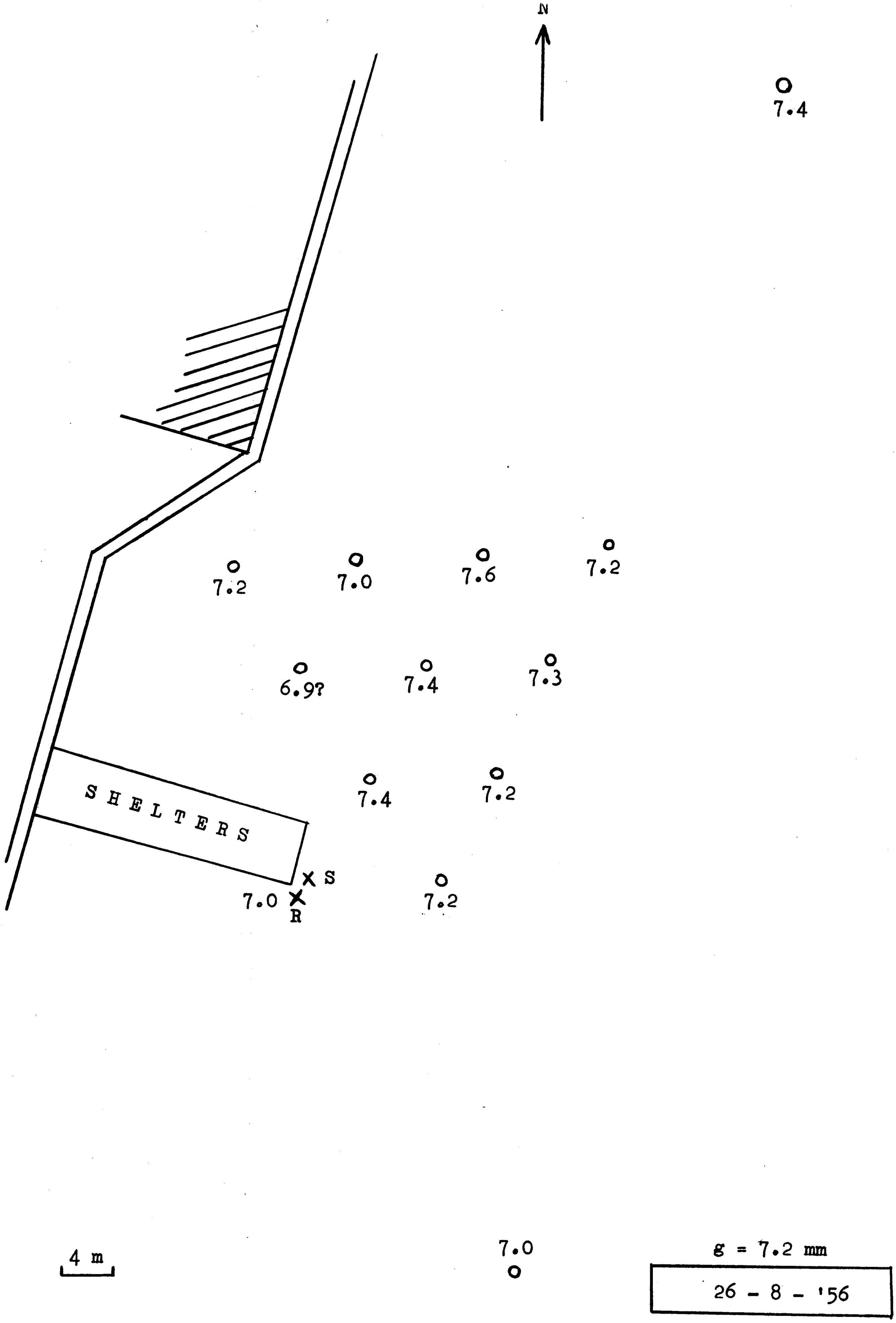


FIG. 14

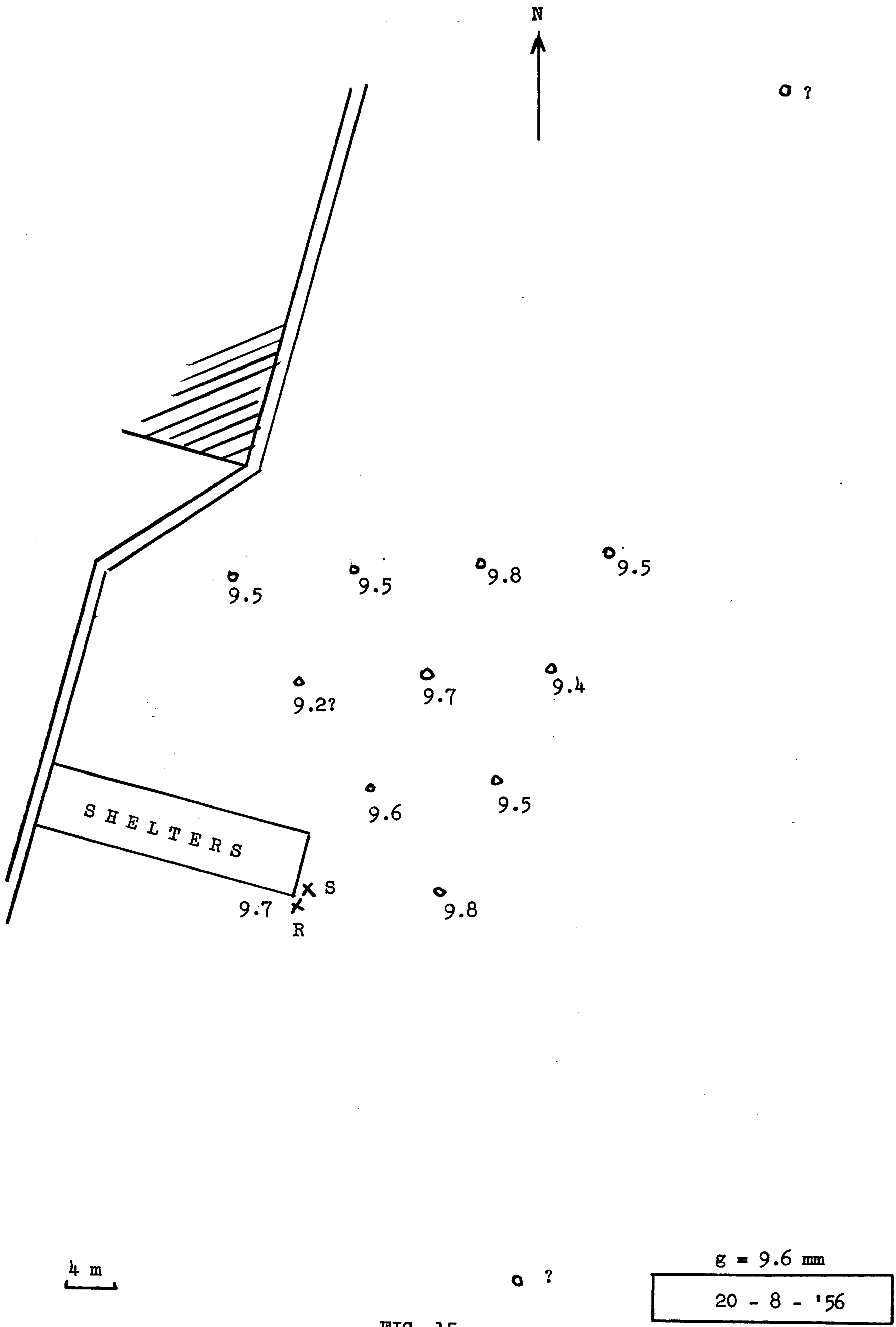


FIG. 15

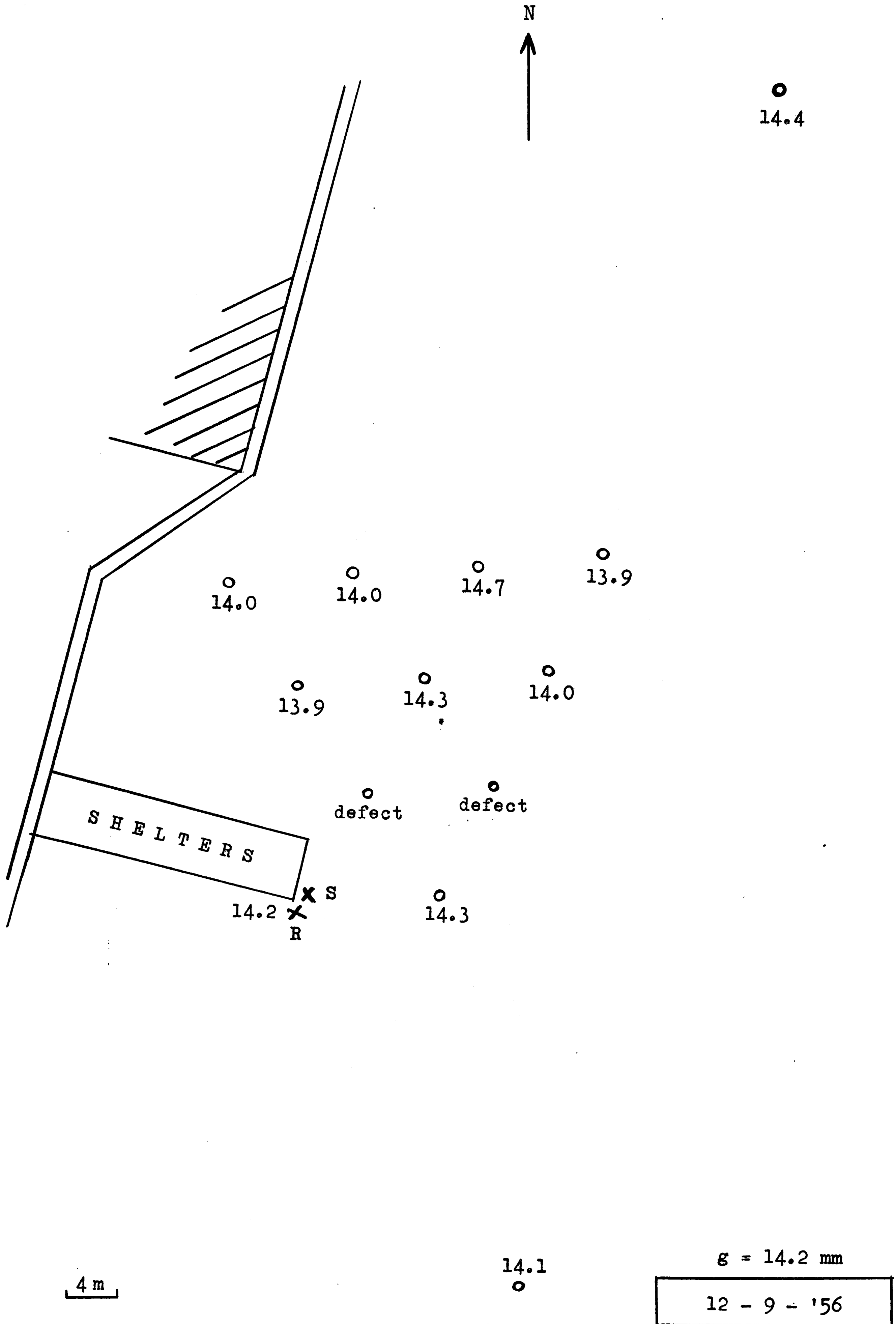

N
14.4
14.0
14.0
14.7
13.9
13.9
14.3
14.0
defect
defect
SHELTERS
S
14.2
R
14.3
14.1
4 m
g = 14.2 mm
12 - 9 - '56

FIG. 16