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THE ACCURACY OF MEASURING AREAL PRECIPITATION WITH A RAIN GAUGE NETWORK

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1. SYNOPSIS

The main purpose of this technical meeting of the Committee for Hydrological Research TNO, is to report on the use of radar for precipitation measurements. In practice, areal precipitation is at the moment mainly determined by rain-gauge measurements at individual points. The aim of this paper is to discuss the accuracy which this method may yield.

Firstly, the instrumental errors of a rain-gauge proper are described. Secondly, the spatial variability of a precipitation field is discussed. Thirdly, a theoretical approach of the problem is given and, lastly, some results are shown of experiments carried out with a relatively dense network of rain-gauges.

2. INTRODUCTION

For many hydrological applications, information on the precipitation depth averaged over a certain area, S, and fallen in a certain time interval, T, is desirable. In practice, this average is mostly determined with rain-gauge measurements taken at a finite number of individual points.

It is perhaps interesting to relate that this method was first described many centuries ago; Biswas (1971) reports that the Chinese scientist Chiu Chiu Shao, who lived about 1200 A.D., discussed it in his book "Shu Shu Chiu Chang".

The accuracy of the method depends on several factors. The instrumental accuracy is, of course, important; the spatial statistical structure of a precipitation field, depending on the climatological characteristics of the area of interest, plays a prominent part, while further the quantities S, T, the number of rain-gauges, N, the spatial distribution of the gauges, and the method of estimation itself may affect the accuracy of the method.

3. INSTRUMENTAL ERRORS OF A RAIN GAUGE

General remarks

A rain gauge is an instrument that has been used for thousands of years (Biswas, 1971). It usually has the form of a collector above a funnel leading into a receiver.

In the Netherlands, the so-called "national standard rain gauge" is installed with its rim 40 cm above ground level, while since 1962 its orifice has an area of 200 cm^2. (Before 1962, its area was 400 cm^2 and before 1946 its height 1.50 m).

In general, the catch of this rain gauge differs from the true precipitation depth because of instrumental and observational errors. Some of these are discussed in the following sections.

Systematic errors

A rain gauge is affected by a number of systematic errors. For instance: evaporation and wetting losses; rain can splash in and out, and wind also acts on the gauge. Although these errors depend on such factors as the nature of the gauge, the meteorological conditions and characteristics of the site, Kurtyka (1953) (see also Rodda (1971)) has made the following approximations (see Table 1) of the magnitude of the most important systematic measuring faults:

Table 1

Evaporation	Adhesion	Colour	Inclination	Splash	Wind and exposure
−1.0%	−0.5%	−0.5%	−0.5%	+1%	−5 to −80%

It will be seen from this table that, except for the error caused by splash, the effects are all negative, while the error due to wind and exposure appears to be the most important. Therefore, this error will now be discussed in more detail.

The process of wind acting on a rain gauge is rather complicated. Qualitatively it can be described as follows:
Wind interacts with the gauge and the feature of the site to produce eddies and turbulence in such a way that, in the region immediately above the gauge, raindrops or snowflakes are accelerated. This results in an alteration of their trajectories by which a fraction of the drops or flakes, which otherwise would have reached the orifice without disturbance, is not caught by the gauge.

In Figure 1, taken from Green and Helliwell (1972), streamlines are drawn as measured above a rain gauge installed in a steady wind flow of about 3 m/s produced in a wind tunnel. It is immediately noticeable that the effect of the rain-gauge is to cause a convergence of streamlines from the windward edge of the gauge. The alteration of raindrop trajectories is due to this convergence. Obviously, the trajectories of small raindrops are

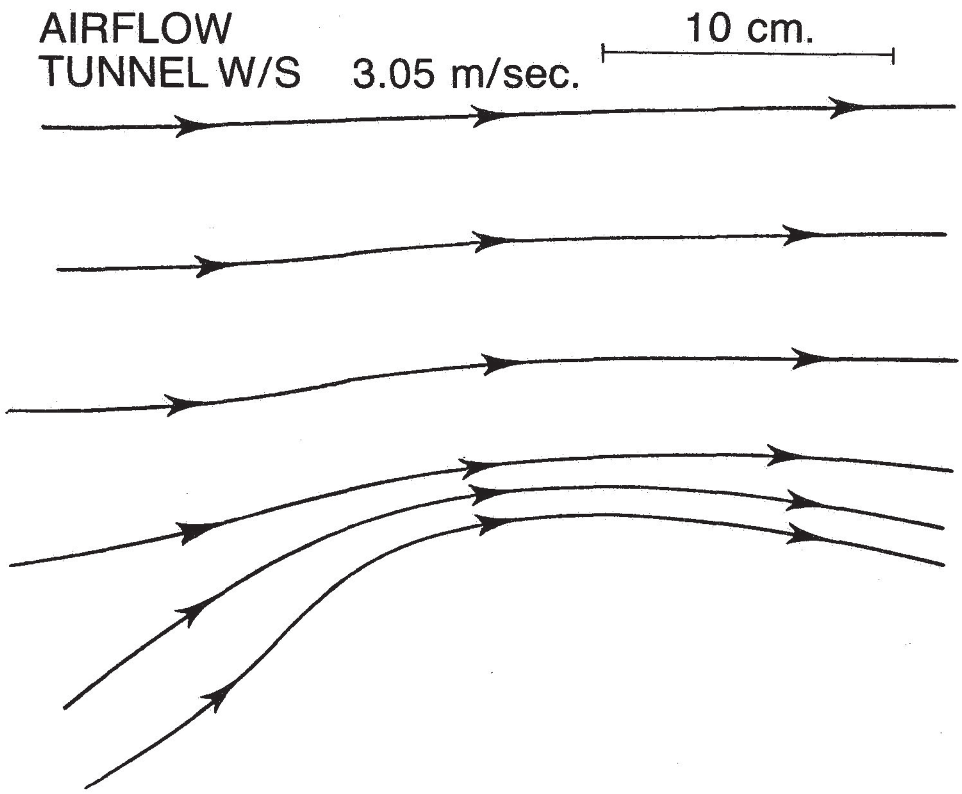


Fig. 1. Streamlines above the Mk 2 rain-gauge funnel with a wind speed of 3.05 m/s, taken from Green et. al. (1972).

altered more than those of bigger ones, while a much greater proportion of snowflakes is being deviated because of their larger surface area and their lower fall-velocities.

The results shown in Figure 1 are valid for steady wind flows. In natural field conditions, the flow will be far from steady: it will be more gusty. Furthermore, the wind flow is influenced by obstructions, such as bushes, in the vicinity of the rain-gauge. All these factors together make that a quantitative description of the process is hardly possible.

In practice, the order of magnitude of the error due to the wind, as far as liquid precipitation is concerned, is obtained by comparing the catch of the rain gauge of interest with that of a so-called "pit gauge". This is a rain gauge installed in such a way that its rim is at ground level. Usually an anti-splash grid is constructed to protect the pit gauge against splash.

It is generally accepted that a pit gauge is not affected by wind. In the Netherlands, a number of comparisons between the national standard rain gauge and a pit gauge was carried out (see e.g. Braak (1945), Dey (1968), Colenbrander and Stol (1970)).

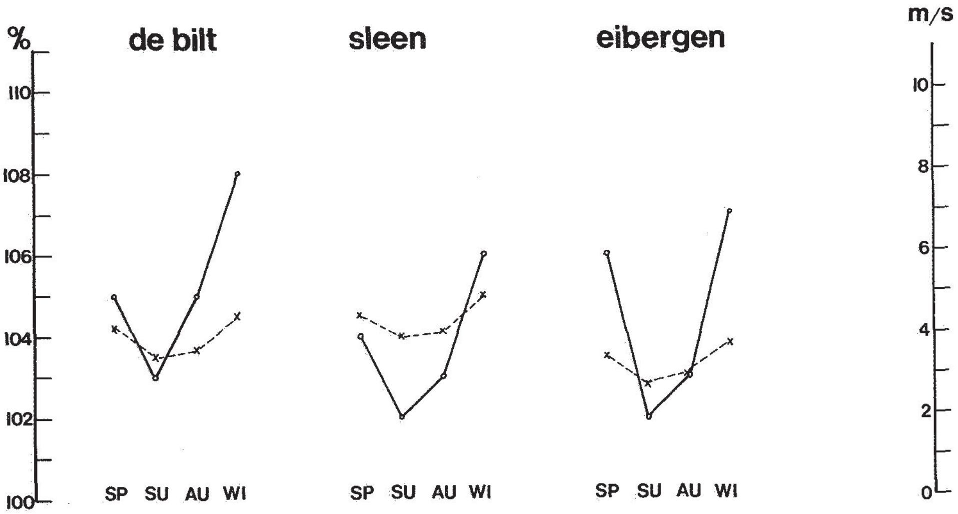


Fig. 2. Seasonal fluctuation of the difference in precipitation depth between the standard rain gauge, h_s, and an international reference pit gauge, h_p (o-o-o), and the seasonal fluctuation of the mean wind speed (x-x-x); $h_p = 100\%$.

In Figure 2, the result of such comparisons made in 1972–1975 within the framework of an international comparison programme, organized by the World Meteorological Organization through its Commission for Instrumentation and Methods of Observation, are given for the Dutch stations De Bilt, Eibergen (Hupsel) and Noord Sleen. It is seen that in winter the pit gauge catches 6–8 per cent and in summer about 3 per cent more than the national standard gauge.

4. RANDOM ERRORS OF A RAIN GAUGE

Random errors due to reading the measuring glass

In the Netherlands, the measuring glass is read to 0.1 mm. The average daily rainfall amount is about 4 mm, if only wet days are taken into account. So the random error due to reading the measuring glass is about 2.5 per cent of the average daily precipitation depth. As about 50 per cent of the days are dry, the random error of monthly totals due to this effect can be estimated with an accuracy better then 1 per cent.

Errors due to the "random character of rain"

Let us imagine a cloud which procedures raindrops, so that the dropsize distribution and the rainfall intensity do not vary with time. During fixed time intervals, T, we take samples of the rainfall depth, h. If N_i is the number of raindrops caught during the i^{th} sample, we can write:

$$h_i = \frac{\pi}{6} \frac{N_i(\widetilde{D^3})_i}{0} \qquad (1)$$

where $(\tilde{D}^3)_i$ is the average value of the third power of the diameter of the raindrops caught during the i^{th} sample, and 0 the area of the rain gauge. Doing so, it will be observed that in general the values of h_1, h_2, h_3, ... are not equal. This variation is caused by random variation in N and $\tilde{D}^3$, due to sampling effects. Appendix I shows, that the relative value of the random error caused by this effect is given by:

$$\frac{\sigma(h)}{\bar{h}} \approx \sqrt{\frac{\tilde{D}^6}{N(\tilde{D}^3)^2}} \qquad (2)$$

It should be noted that N is proportional to 0, so $\sigma(h)/\bar{h} \sim \sqrt{1/0}$.

During 1968 and the first few months of 1969, raindrop-size measurements were carried out at De Bilt, using a filter-paper technique (Wessels, 1972). These data (one purpose to collect them was to determine the relationship between rainfall rate and radar reflectivity) were used to get an impression of the order of magnitude of $\sigma(h)/\bar{h}$. It should be remarked that the measurements could only be carried out in working time. Table 2 shows some results for "daily" rainfall amounts and for two values of 0 (20 and 200 cm^2).

Table 2

		0 = 20 cm^2	0 = 200 cm^2	
date	$\bar{h}$ mm	$\sigma(h)/\bar{h}.100$	$\sigma(h)/\bar{h}.100$	N(0=20 cm^2)
5-1-68	0.16	6.7	2.1	1355
9-2-68	0.60	4.3	1.4	4651
14-2-68	0.06	4.9	1.5	3402
21-2-68	0.48	2.5	0.8	5898
21-3-68	1.9	2.4	0.8	11216
2-4-68	1.3	2.5	0.8	13820
4-4-68	2.7	2.8	0.9	12462
23-4-68	3.0	1.5	0.5	23892
29-4-68	0.2	5.2	1.6	1727
7-5-68	1.6	3.4	1.1	8705
13-5-68	0.3	6.5	2.1	2903
6-6-68	4.0	1.7	0.5	34815
17-6-68	4.3	2.9	0.9	18364
10-7-68	8.3	1.5	0.5	39856
26-9-68	6.3	1.0	0.3	50025
20-2-69	1.4	3.9	1.2	6916
13-3-69	11.2	1.0	0.3	48142

Table 3 shows some examples of short time intervals:

Table 3

		$0 = 20$ cm^2	$0 = 200$ cm^2	
ΔT min	$\bar{h}$ mm	$\sigma(h)/\bar{h}.100$	$\sigma(h)/\bar{h}.100$	N($0 = 20$ cm^2)
5	0.427	7.3	2.3	1271
5	0.096	8.8	2.4	1691
1	0.177	13.0	4.1	463
1	0.1732	14.6	4.6	512
1	0.329	8.9	2.8	799
1	0.052	21.0	6.7	175
1	0.241	17.6	5.6	1099
5	0.245	10.3	3.3	422

From these tables it can be concluded that the error due to random character of rain is negligible for rain gauges having an area of 200 cm^2.

In the literature (see for instance Kalma et al. 1969), rain gauges with smaller areas are described. It is seen from Tables 2 and 3 that for these types of gauges the values of $\sigma(h)/\bar{h}$ are not negligible.

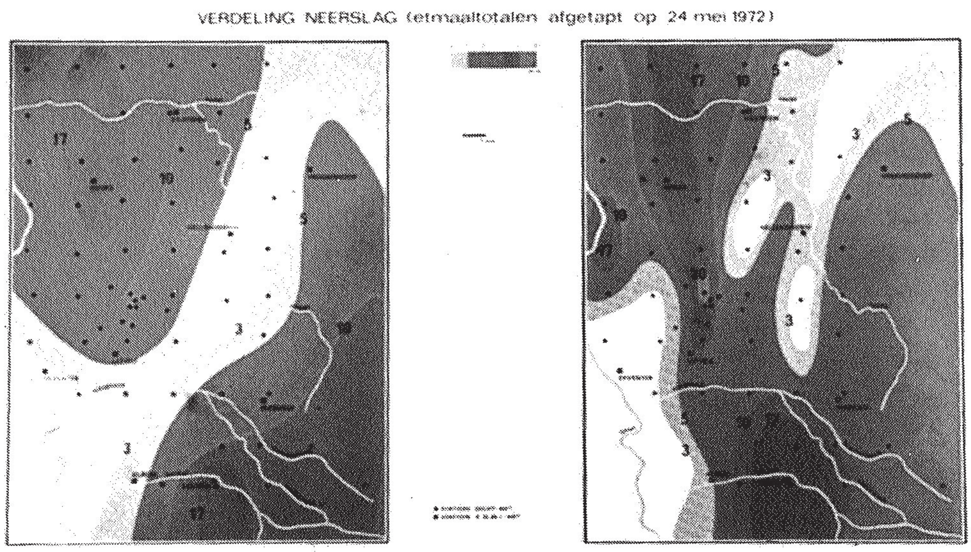


Fig. 3. Spatial distribution of daily precipitation amounts, as measured in Salland on May 24, 1972, with the national rain gauge network (Fig. 3a) and a dense network (Fig. 3b).

5. SPATIAL VARIABILITY WITHIN A PRECIPITATION FIELD

From our own experiences we know that the spatial distribution of precipitation can be highly irregular. The meteorological processes responsible for this variability were reviewed this morning by Schmidt (1976).

To illustrate the possible differences over a short distance, the spatial distribution of the daily precipitation amounts is shown in Figure 3, as measured on May 24, 1972, with a dense network installed in Salland, a district in the eastern part of the Netherlands, in 1970–1972. This was done within the framework of a hydrological research programme (Santing et al., 1974).

In this figure the same precipitation field is drawn as "seen" by the (less denser) national rain gauge network of the Royal Netherlands Meteorological Institute.

This example shows that, over distances of 1 kilometre, differences of 20 mm/day can occur.

Generally, these differences within areas such as Salland are not systematic. For most of the practical applications, areas in the Netherlands of, say, 1000 km^2 can largely be considered to be climatologically homogeneous and isotrope.

It will be pointed out in the next section that a convenient quantity to describe the statistical spatial structure of a precipitation field is the correlation coefficient, ρ, defined by:

$$\rho_{ik} = \frac{\overline{h_i h_k} - \overline{h}_i \overline{h}_k}{\sigma_i \sigma_k} \tag{3}$$

in which the bars indicate an average over a long time series; h_i and h_k are the precipitation depth measured at the i^{th} and k^{th} station, while σ_i and σ_k are the standard deviations (of time series) of these stations.

If the distance between the two stations is denoted by r_{ik}, it is to be expected that ρ_{ik} decreases with increasing r_{ik}, if a homogeneous and isotrope area is considered.

It has been found (see e.g. Stol (1972) and De Bruin, (1975) that the "correlation function", $\rho(r)$, can be (more or less) satisfactorily described by:

$$\rho(r) = \rho_0 \, e^{-r/r_0} \tag{4}$$

which relationship can be written for $r << r_0$ in the form:

$$\rho(r) = \rho_0 - r/r_0 \tag{5}$$

Stol fitted the daily observations of a 10-year period from 9 selected stations situated in the eastern part of the Netherlands to equation (4). He did so for each month. The ("smoothed") results are given in Figure 4 (according to Stol, 1967), while the values of ρ_0 and r_0, as found by Stol, are listed in Table 4. It should be noted that he only took those days when at least 0.5 mm had been recorded at each station.

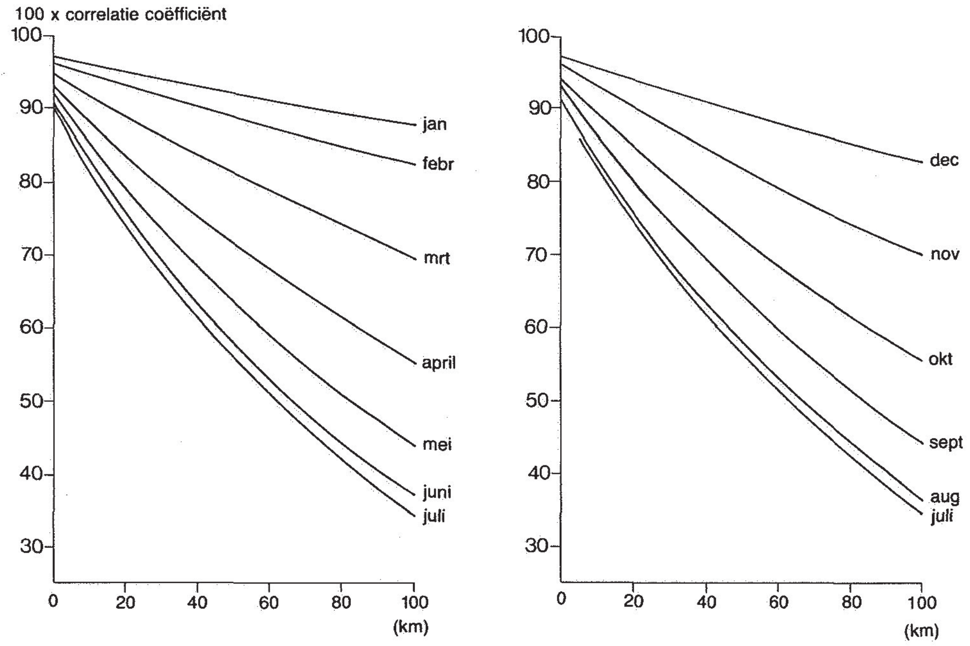


Fig. 4. The correlation coefficient, ρ, as a function of the distance, r, for each month (daily totals), taken from Stol (1967).

Table 4

month	ρ_0	r_0 km
1	.97	962
2	.96	630
3	.95	320
4	.94	190
5	.93	135
6	.91	111
7	.91	104
8	.92	110
9	.93	133
10	.95	186
11	.96	309
12	.97	609
Year	.94	188

Ten days total from 36 stations in the same area and of the period 1966–1972, were processed by De Bruin (1975). He concluded that for r, up to about 30 km, equation (5) can be used. However, he found a big scatter plotting ρ against r (a scatter which, after all, is also present in the data of Stol). De Bruin pointed out that this scatter is partly due to random observational errors, because these lower the value of ρ and, therefore, increase its variability.

Table 5 shows the values of ρ_0 and r_0 from (5), as found by De Bruin for 14 selected stations. Furthermore, the corresponding values of all 36 stations are listed.

Table 5

period	14 selected stations		all 36 stations	
	ρ_0	r_0 km	ρ_0	r_0 km
Nov.–March	.99	1000	.95	1000
Sept., Oct., Apr., May	.99	270	.98	270
July–Aug.	.99	190	.99	190

It is seen from Tables 4 and 5 that a seasonal effect exists: in winter, r_0 has a maximum and in summer a minumum value. This is so because in winter mainly frontal (that is large-scale) precipitation occurs, whereas in summer also convective storms (small-scale precipitation) are likely to occur.

Table 6 lists the values of ρ_0 and r_0 from equation (5) for 5-day periods and different "seasons", as found in Salland by means of the records of the dense raingauge network mentioned before. The values found in this network for 1 and 10-days periods were similar to those listed in Tables 4 and 5.

Table 6

period	selected stations	
	ρ_0	r_0 km
"year"	.97	230
"summer"	.97	150

6. ACCURACY OF THE ESTIMATES OF AREAL PRECIPITATION

Estimate with one rain gauge

A homogeneous, isotrope and square area, S, with edge ℓ is considered. In the centre. P, of this area, a rain gauge is installed.

The precipitation depth averaged over the area is denoted by h_G, while that measured at P is called h_0. As the area is supposed to be homogeneous, an estimate of h_G is:

$$\hat{h}_G = h_0 \tag{6}$$

Appendix II shows that the standard error of this estimate, E, defined by $E^2 = \overline{(h_G - \hat{h}_G)^2}$, can be written as:

$$E^2 = \sigma^2 \left[(1 - \rho_0) + 0.23 \frac{\sqrt{S}}{r_0} \right] \tag{7}$$

where σ is the standard deviation of a long time series taken at an arbitrary point in S.

This relationship is valid, if equations (4) or (5) are supposed to be true.

Introducing $Z = E/\bar{h}$ and $C_v = \sigma/\bar{h}$, we obtain:

$$Z = C_v \sqrt{(1 - \rho_0) + 0.23 \frac{\sqrt{S}}{r_0}} \tag{8}$$

Table 7 shows some characteristic values of C_v for different time intervals.

Table 7

T days	$C_v = \sigma/h$
1	1.4
5	1.1
10	0.9
30	0.5

Accuracy obtained with N evenly distributed rain gauges

Next, the case is considered that N rain gauges are evenly distributed over area S. The recorded precipitation depths at these stations are denoted by $h_1, h_2, ..., h_N$.

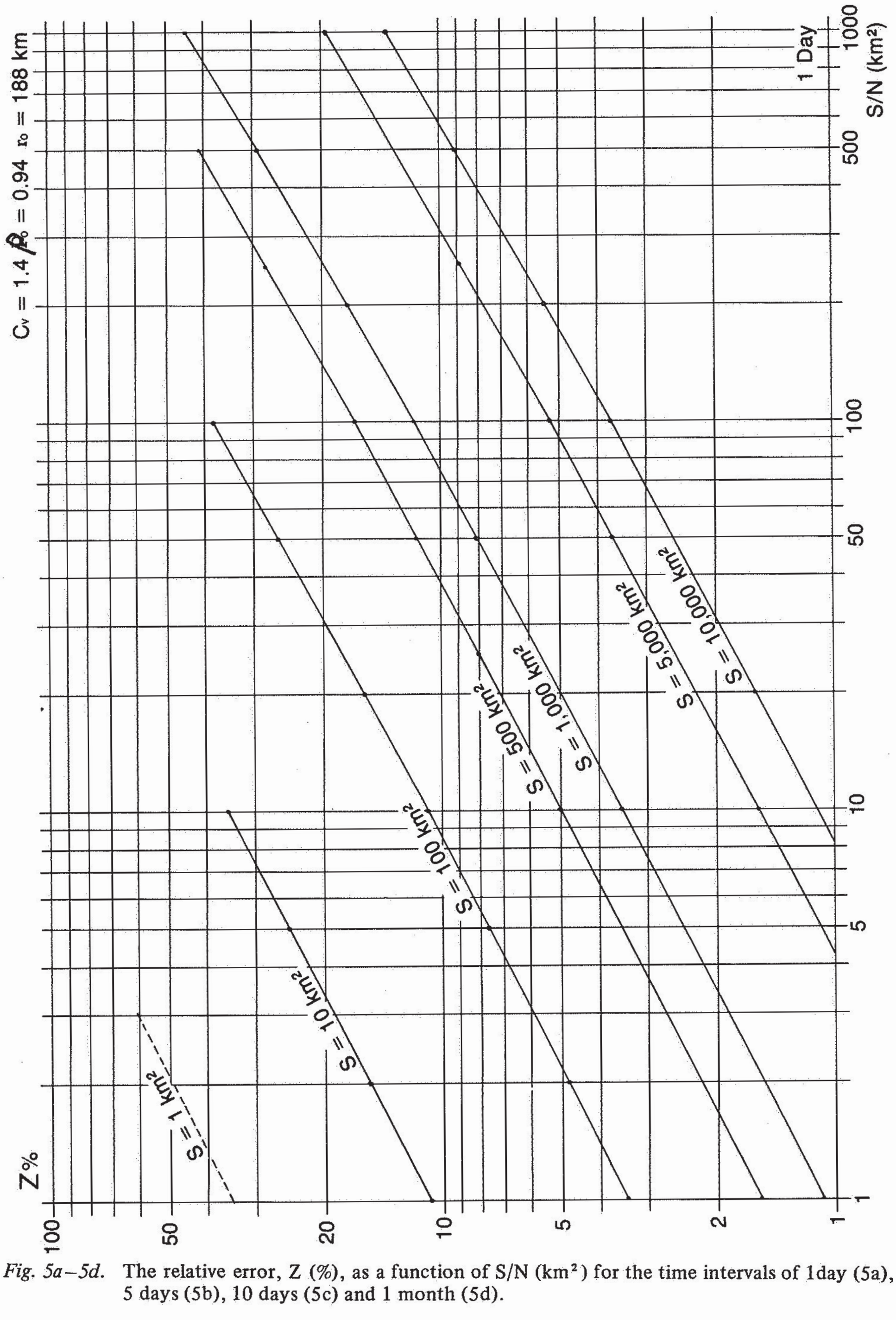


Fig. 5a–5d. The relative error, Z (%), as a function of S/N (km^2) for the time intervals of 1day (5a), 5 days (5b), 10 days (5c) and 1 month (5d).

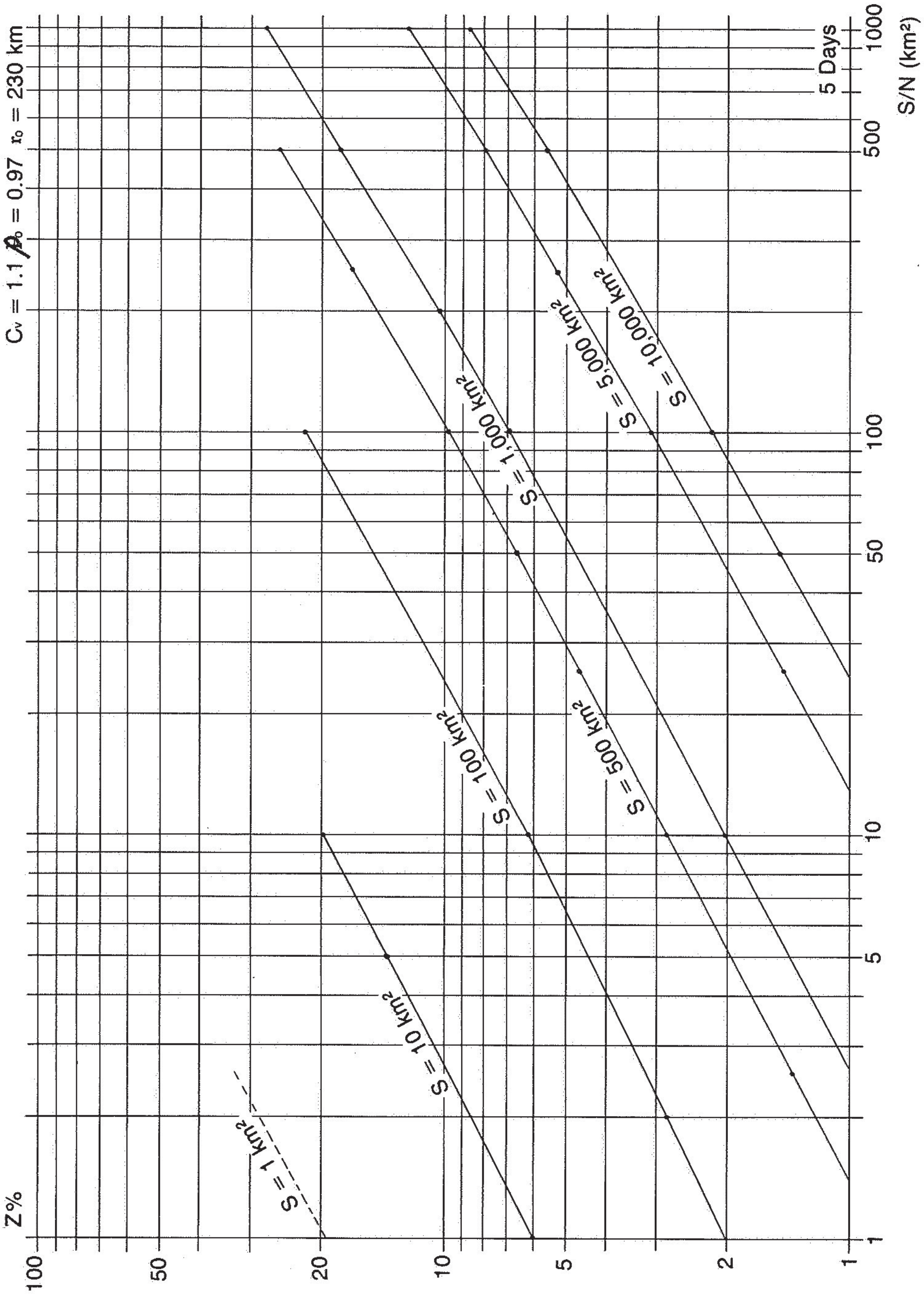
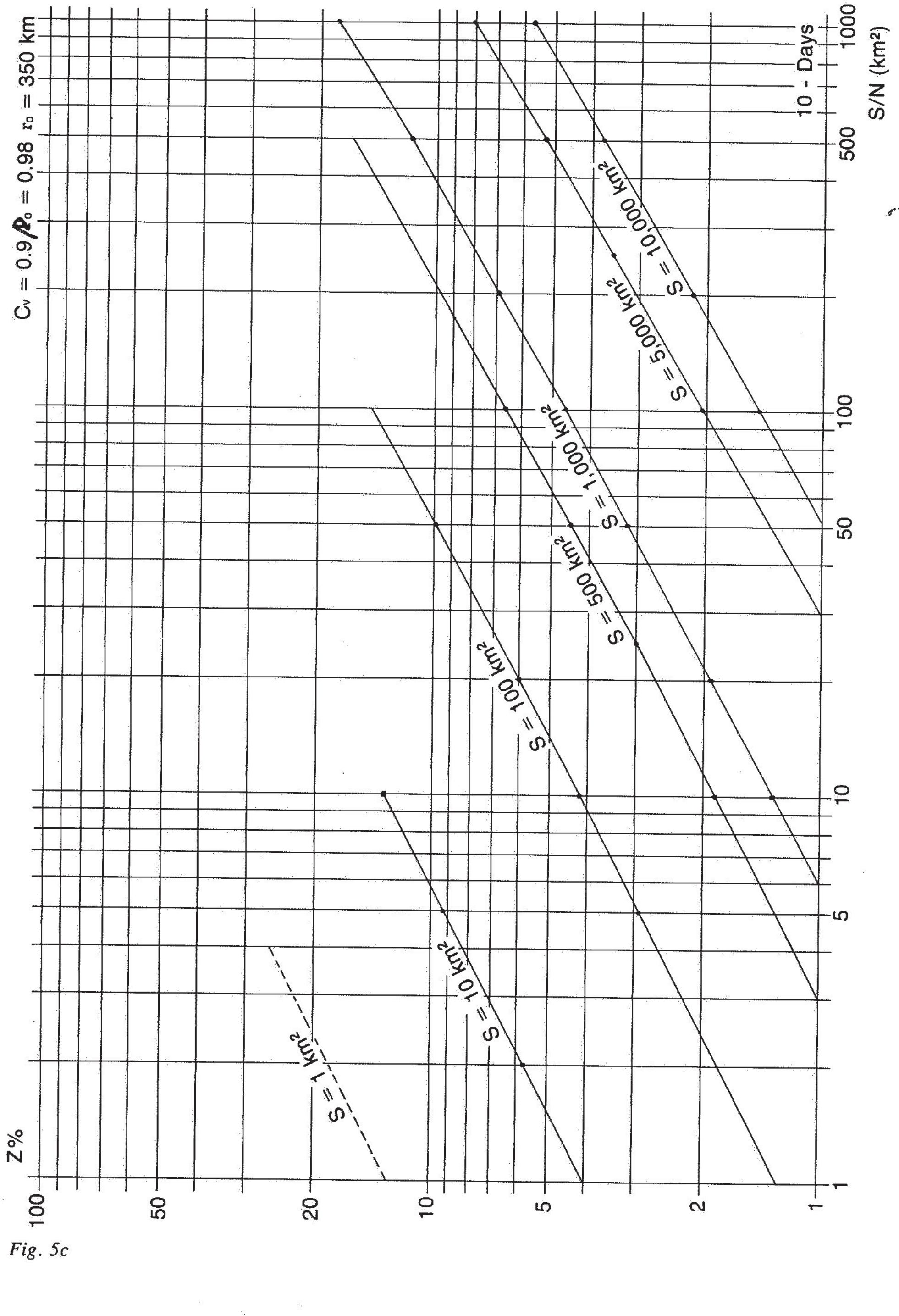
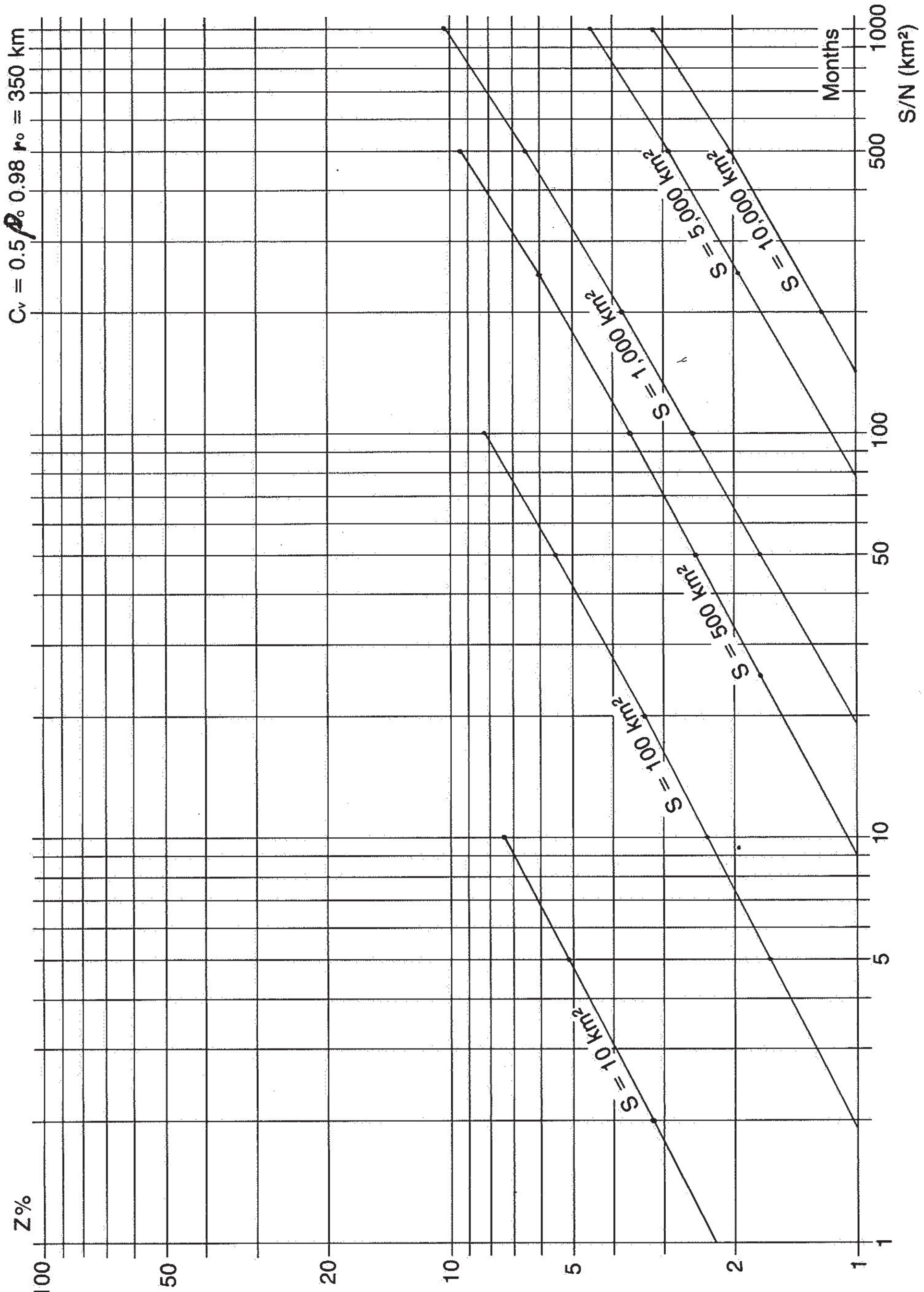

Z%
S/N (km²)
5 Days
S = 1 km²
S = 10 km²
S = 100 km²
S = 500 km²
S = 1,000 km²
S = 5,000 km²
S = 10,000 km²
100
50
20
10
5
2
1
1000
500
100
50
10
5
1

Fig. 5b



$C_v = 0.9$ $\rho_0 = 0.98$ $r_0 = 350$ km
Z%
10 - Days
S/N (km²)
S = 10,000 km²
S = 5,000 km²
S = 1,000 km²
S = 500 km²
S = 100 km²
S = 10 km²
S = 1 km²
100
50
20
10
5
2
1
1000
500
100
50
10
5
1

Fig. 5c



$C_v = 0.5$ ρ_o 0.98 $r_o = 350$ km
Z%
Months
S/N (km²)
S = 10 km²
S = 100 km²
S = 500 km²
S = 1,000 km²
S = 5,000 km²
S = 10,000 km²
100
50
20
10
5
2
1
1
5
10
50
100
500
1000

Fig. 5d

An estimate of h_G is now given by:

$$\hat{h}_G = \frac{1}{N} \sum_{i=1}^{N} h_i \tag{9}$$

while the relative error Z can be written as

$$Z = C_v \sqrt{\frac{1}{N}\left[(1-\rho_0) + \frac{0.23}{r_0}\sqrt{\frac{S}{N}}\right]} \tag{10}$$

(See Appendix II, and also Kagan, 1972).

Figures 5a, b, c and d present Z drawn as a function of S/N for various values of S and for different time intervals.

Some remarks on equation (10)

The reliability of equation (10) depends on the quantities ρ_0 and r_0, found with the empirical correlation function given by (4) and (5). But, because of the rather big scatter observed when correlation coefficient ρ is plotted against distance r, the accuracy of these quantities (notably that of ρ_0) is not very high. This is one of the main problems of this "theoretical" approach.

Quantity ρ_0 has been introduced as a result of the random errors of observation (see Appendix II). Although ρ_0 can be determined only with a relatively low accuracy, it is interesting to see what happens when the value of ρ_0 is altered.

In Figure 6, Z is drawn as a function of S/N for 1000 km^2 and 10-day totals, while the following values of ρ_0 were chosen: 0.96, 0.99 and 0.9975, corresponding with $\Delta = 0.20$, 0.10 and 0.05 respectively (Δ is the random error of observation divided by σ). Suppose we have a rain-gauge network with S/N = 100 km^2 and $\Delta = 0.20$. It is then seen from Figure 6 that in a network with S/N $\approx$ 230 km^2 and $\Delta = 0.10$, or S/N $\approx$ 320 km^2 and $\Delta = 0.05$, the same value of Z is obtained. This example is given to illustrate that under certain circumstances it can be more economical to decrease the number of rain gauges, if at the same time the random measuring error is lowered (for instance, by means of automation of the network).

7. COMPARISONS WITH A DENSE NETWORK IN SALLAND AND IN ZUID-HOLLAND

It is in itself a problem to test the reliability of equation (10). The only thing one can do is compare the records of a dense and a less dense network of rain-gauges with one another, assuming that the densest network gives the true areal precipitation depth.

In this manner, the data of a network of rain gauges installed in Salland during 1970–1972 were used.

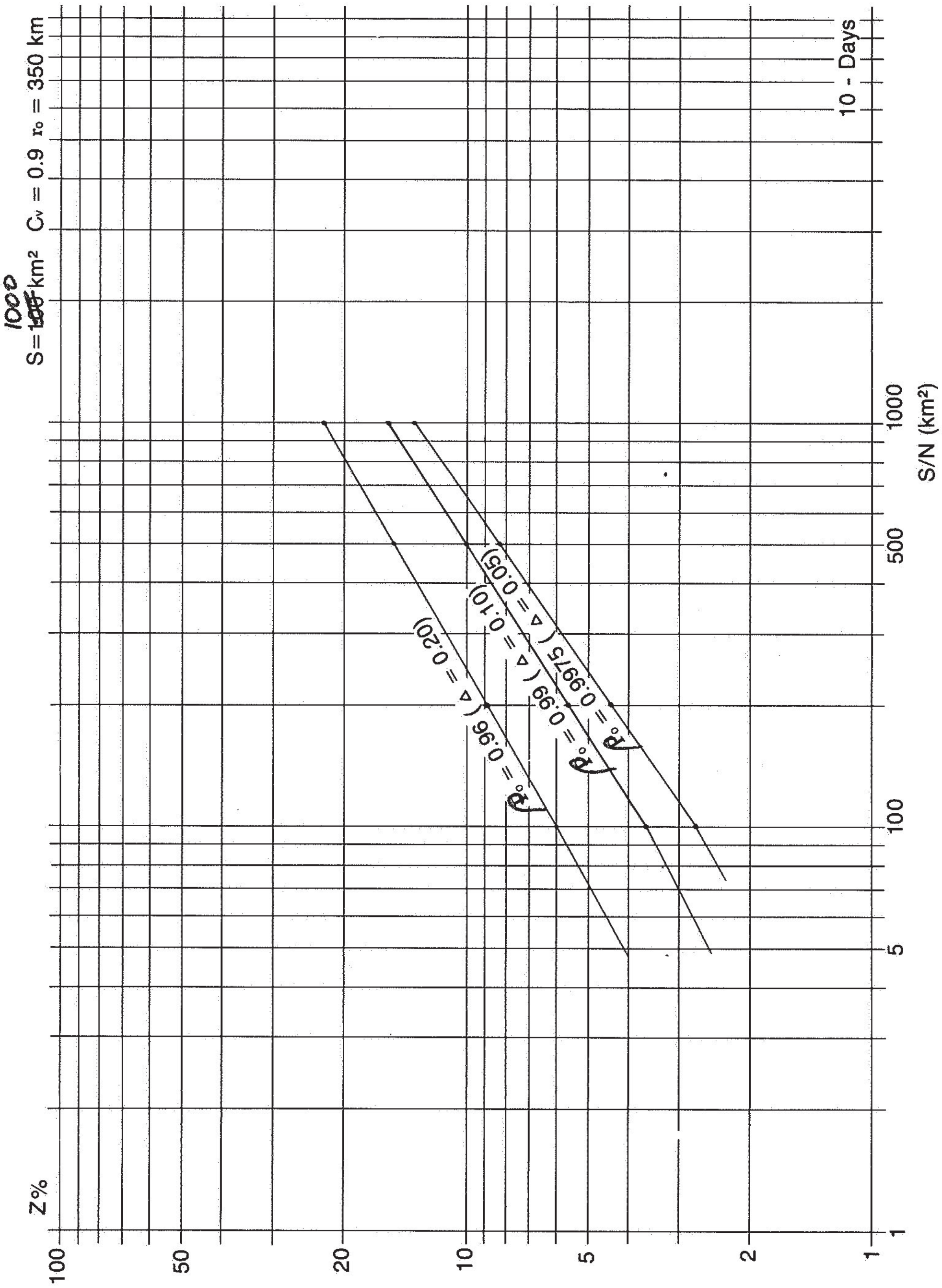


Fig. 6. The relative error, Z (%), as a function of S/N (km²) for different values of ρ_0 (10-days period and S = 1000 km²).

Within an area of about 1100 km^2, rain gauges were erected at 65 points. These are plotted in Figure 7. It is seen that in principle the gauges are evenly distributed over the area. They form a 5 x 5 km grid. In the centre of the area (in the neighbourhood of Heeten), the network had a special density.

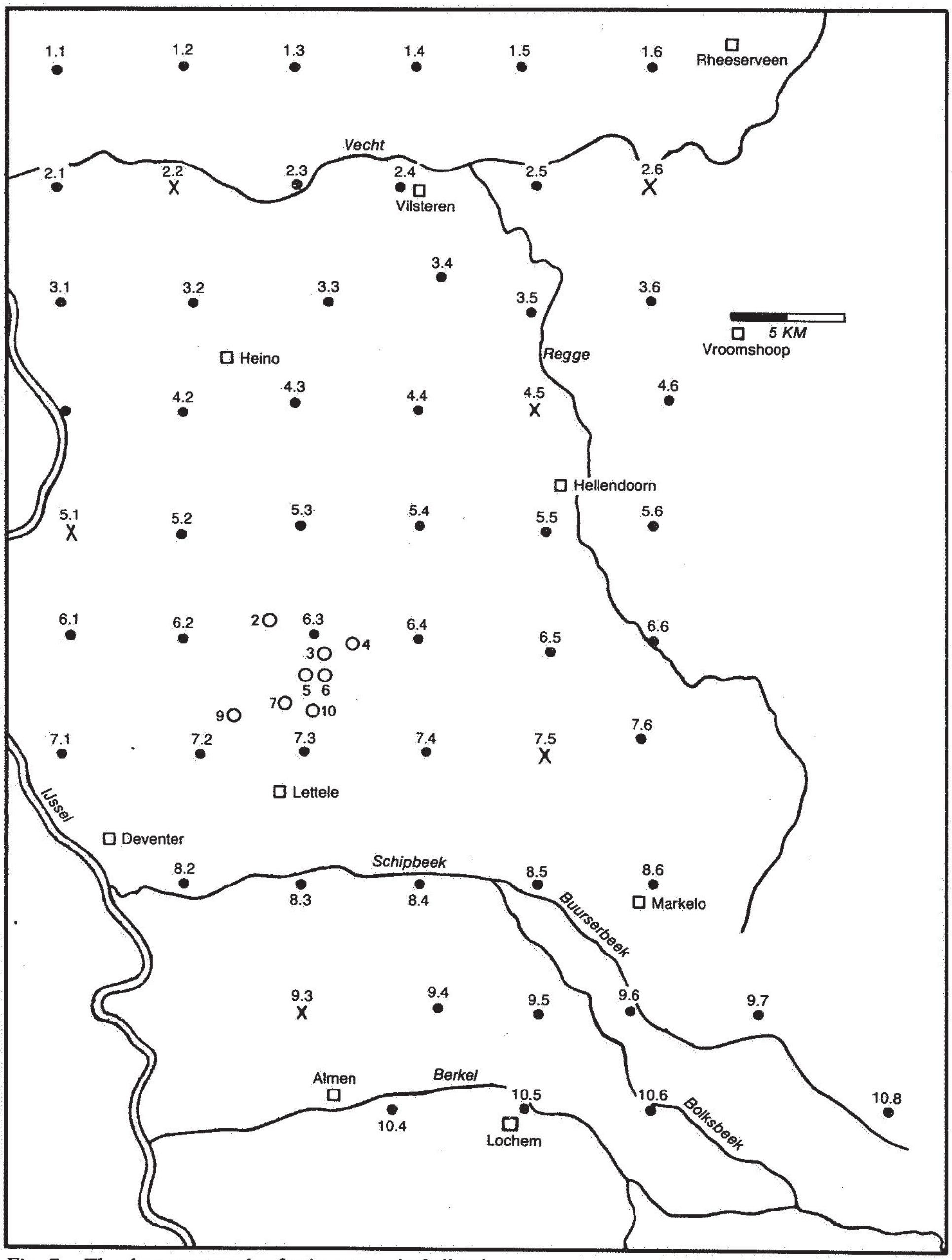


Fig. 7. The dense network of rain gauges in Salland.

Here the distances between two stations were up to about 750 m. In order to avoid the systematic measuring error due to wind, pit gauges were used. For budgetary and organizing reasons, a special, cheap type was developed; see Figure 8.

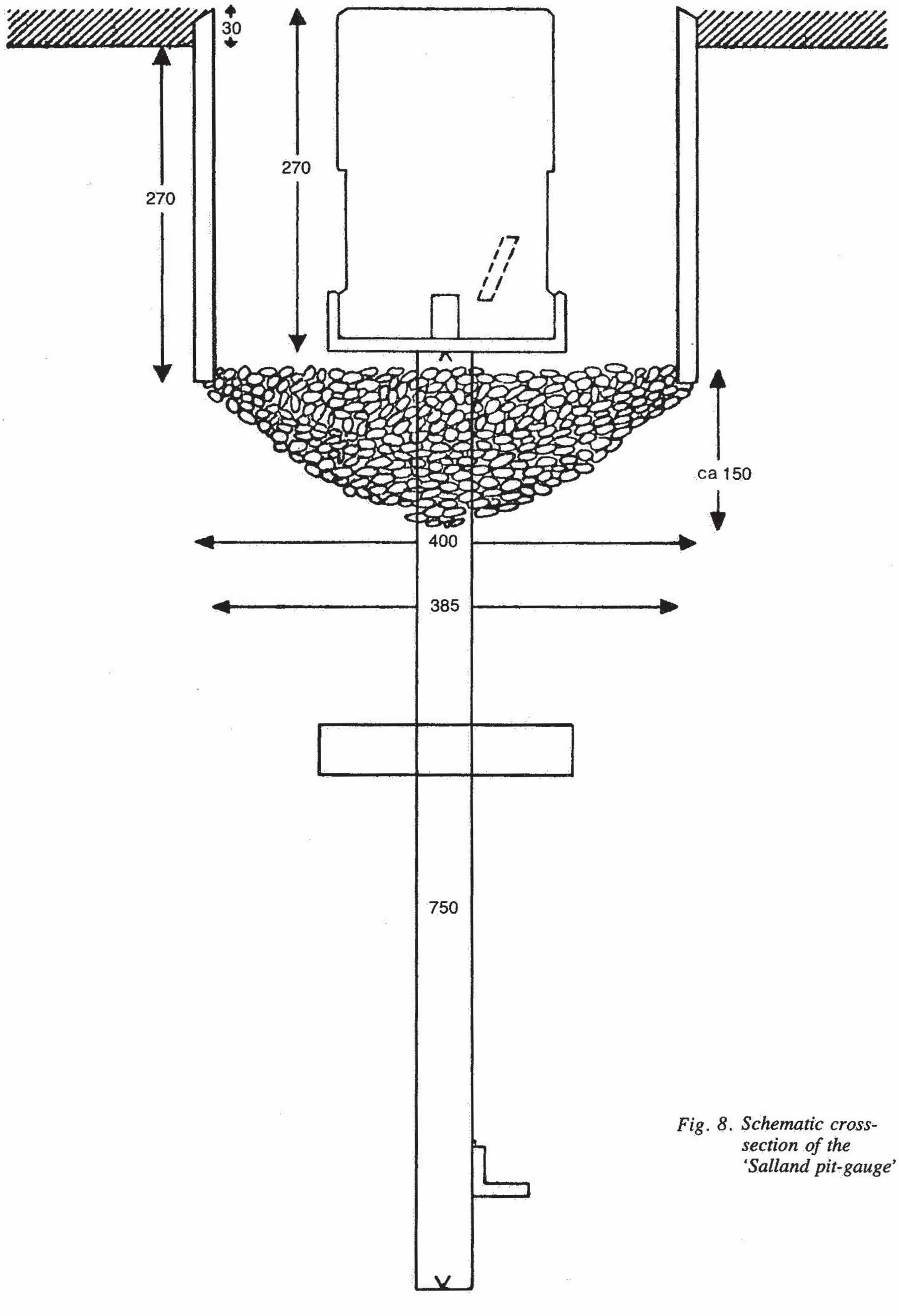


Fig. 8. Schematic cross-section of the 'Salland pit-gauge'

Comparisons between this type of rain gauge and an international standard pit gauge, mentioned in one of the previous sections, were carried out at De Bilt in 1972–1975.

Two "international" pit gauges were installed next to three "Salland" pit gauges.

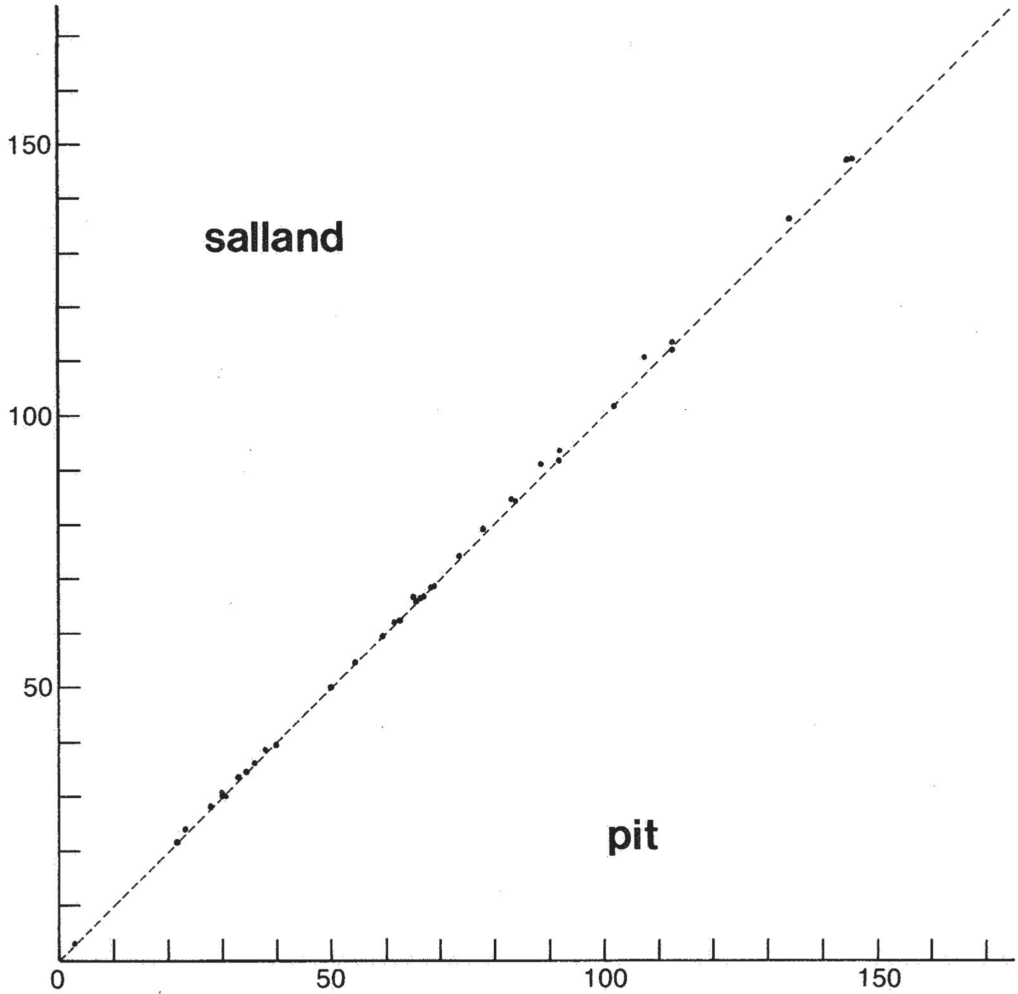
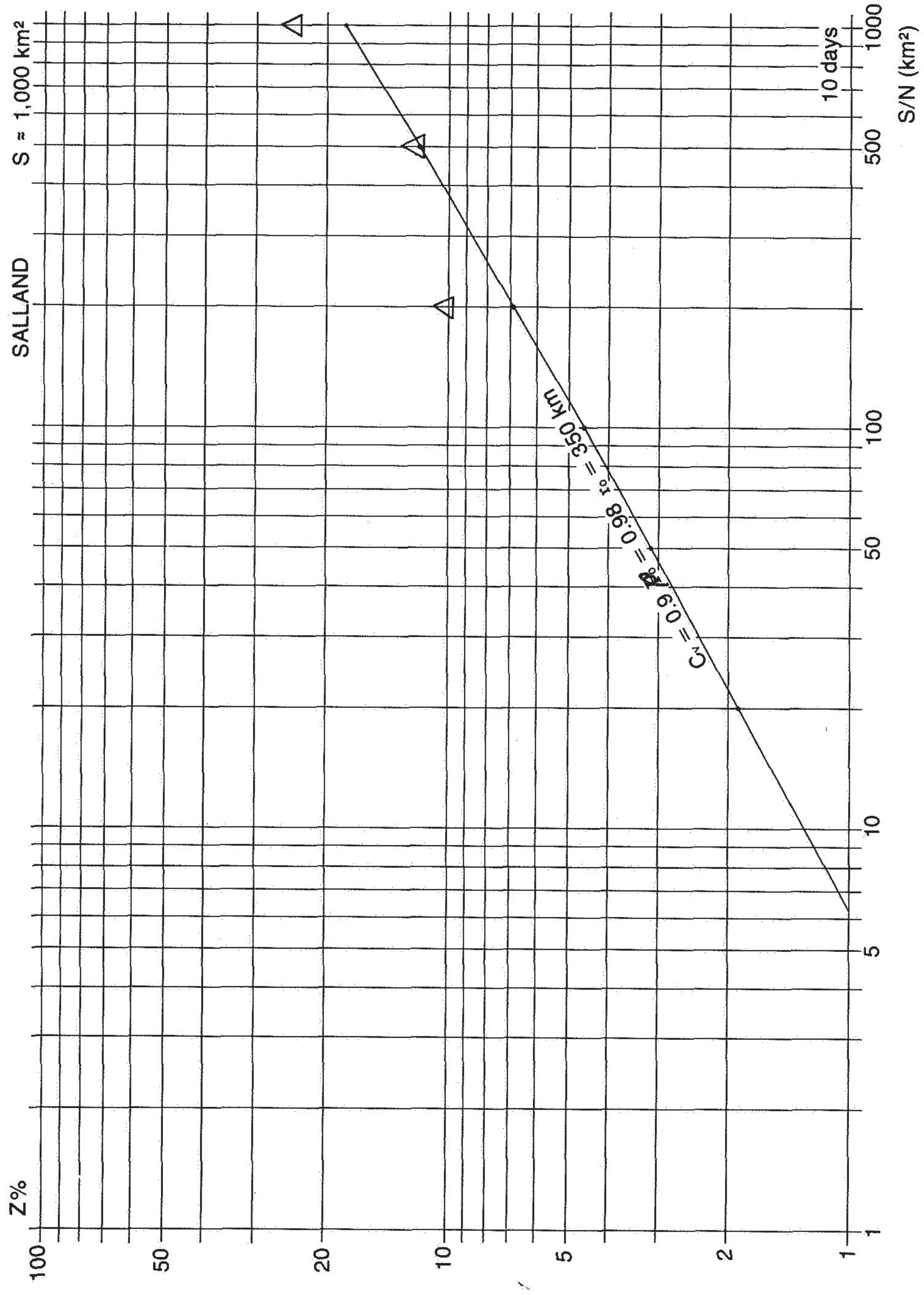


Fig. 9. Comparison between the monthly precipitation depth measured at De Bilt with the "Salland pit-gauge" and the international reference pit-gauge (mm).

In Figure 9, the means of the monthly totals measured with the three "Salland" gauges are plotted against those of the two "international" gauges. It is seen that there are hardly any differences between the two types of pit gauges. (De Bruin, 1973).

Unfortunately, it appeared that the daily records of about 30% of the stations were inadequate (Yperlaan, 1974), so only the data of 42 stations could be used for testing equation (10) as far as daily totals were considered. For 10-day totals all 65 stations were



Figs. 10–12. Comparison between theoretical and experimental values of Z (%) as found, with a dense network of rain gauges in Salland for:
an area of ≈ 1000 km² and a time interval of 10 days (Fig. 10);
an area of ≈ 1000 km² and 1 day (Fig. 11);
and ≈ 1 and ≈ 100 km² and 1 day (Fig. 12).

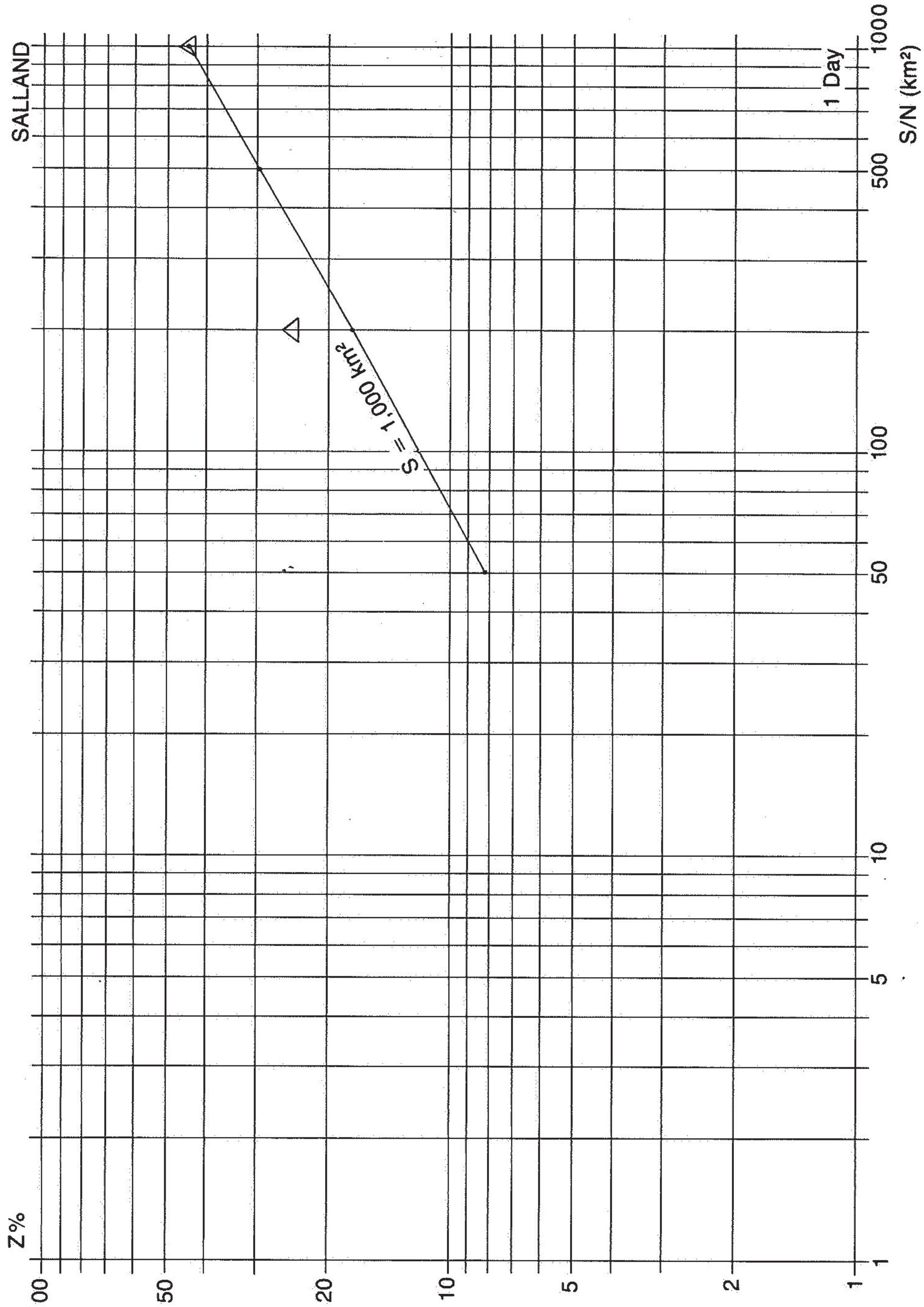

SALLAND
Z%
1 Day
S/N (km²)
S = 1,000 km²
100
50
20
10
5
2
1
1000
500
100
50
10
5
1

Fig. 11

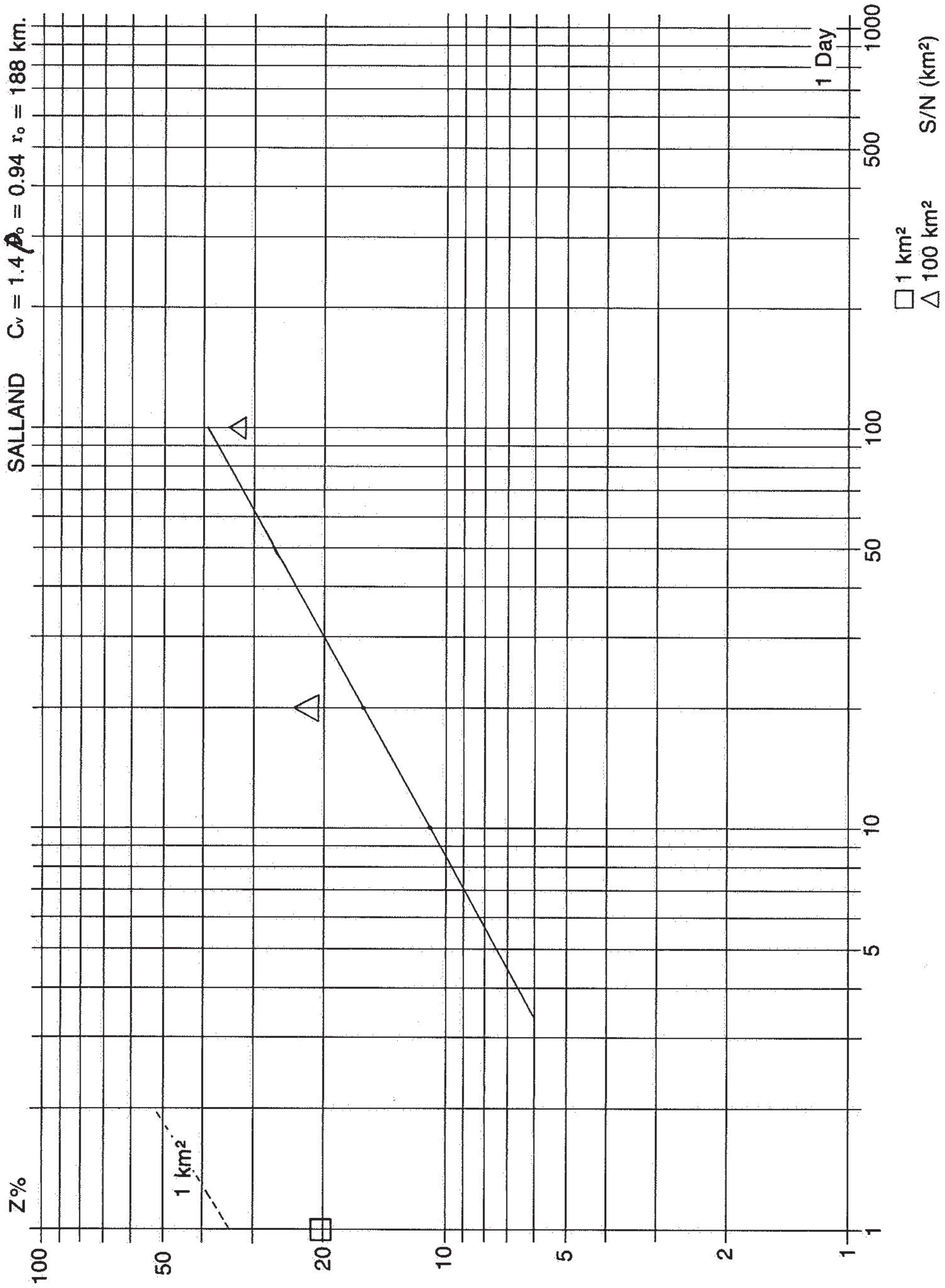

SALLAND Cv = 1.4 ρo = 0.94 ro = 188 km.
Z%
100
50
20
10
5
2
1
1 km²
1 Day
1
5
10
50
100
500
1000
S/N (km²)
□ 1 km²
△ 100 km²

Fig. 12

used. The arithmetic means of the records of these data were taken as the true areal precipitation depth.

The areal precipitation depths of 72 10-day periods were estimated with 1, 2 and 5 stations of the national rain gauge network, respectively, each covering about 1000, 500 and 200 km^2, respectively. As the five stations were not evenly distributed over the area, the "Thiessen average" was taken as an estimate. These estimates were compared with the "true" precipitation depth obtained with the dense network. The respective values of Z were calculated; they are plotted in Figure 10. In this figure, also the theoretical line given by equation (10) is drawn. It is seen that there is fairly good agreement between experimental and theoretical values of Z. It can be shown that (10) underestimates Z, if the stations are not evenly distributed. This explains the rather big deviation of the experimental Z-value for $S/N = 200\ km^2$.

The same procedure was followed for daily totals. The results for 1000, 100 and 1 km^2 are given in Figures 11 and 12. Again, there is agreement although with a tendency that (10) underestimates Z.

Unfortunately, the procedure under consideration only gives a few examples for testing equation (10); it is, therefore, dangerous to draw definite conclusions.

Finally, similar experiments are being considered, carried out by Kruizinga and Yperlaan, 1976 with the records of 35 stations situated in the province of Zuid Holland, in the western part of the Netherlands. The total area was about 4000 km^2.

The mean daily precipitation depth over this area was estimated with the daily records of 4 stations, and the average over a sub-area of about 400 km^2 was estimated with the measurements at 1, 3 and 4 points, respectively. In the case of four points, the stations were situated outside the area.

Kruizinga and Yperlaan did not use quantity Z as a measure of error of estimate. They took the mean of the absolute value of the differences between the estimate and the true areal rainfall depth. So, in the first instance, their results are not comparable with equation (10). However, experiments with the records of the "Salland network" reveal that the "mean absolute difference" error is about 0.5 times the "root mean square" error, from which Z is derived. This is partly due to the sensitiveness of Z to outliers.

Using this conversion factor, we plotted the results of Kruizinga and Yperlaan in Figure 13, where also the theoretical lines for 400 and 4000 km^2

It should be noted that Kruizinga and Yperlaan described the error of estimates as a function of the "true" areal rainfall depth and the four seasons, while dry days were also concerned. The values given in Figure 13 are recalculated for rainy days. No distinction is made between the rainfall depth and the seasons.

Again, it can be concluded that equation (10) fits fairly well to the experimental values of Z. However, we should bear in mind that the experimental values of Z for this example were obtained in a rather crude way.

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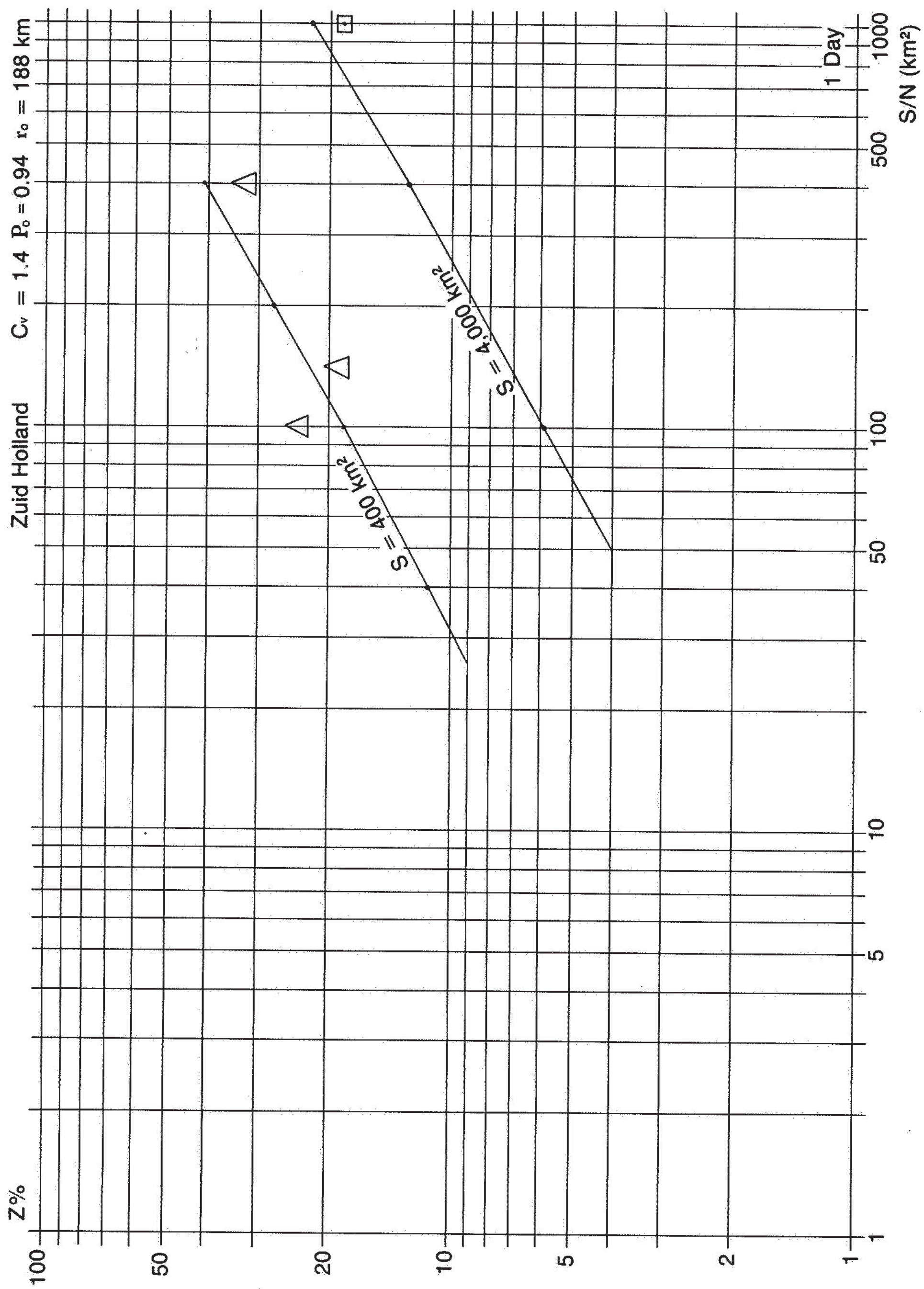

Fig. 13. Comparison between theoretical and the experimental values of Z (%) as found by Kruizinga and Yperlaan in Zuid-Holland ($\approx$ 400 and $\approx$ 4000 km^2, 1 day).

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APPENDIX I

ERRORS DUE TO THE "RANDOM CHARACTER OF RAIN"

When the precipitation depth, h, is measured with a rain gauge, a number of raindrops, N, with different diameters is caught during a certain time, T.

If we were able to repeat the experiment, it would be expected that we would not find the same number of drops and the same dropsize distribution. This indefiniteness in both the number of drops and the size distribution springs from the random nature of the rain-producing process.

We can write:

$$h = \frac{\pi}{6} \frac{N \widetilde{D}^3}{0} \tag{I.1}$$

where $\widetilde{D}^3$ is the average value of the third power of the dropdiameters, and O is the area of the orifice of the rain gauge. If we could repeat the measurement of h many times, we would get the average value of h, N and $\widetilde{D}^3$, denoted by respectively $\overline{h}$, $\overline{N}$ and $\overline{\widetilde{D}^3}$.

Our problem is to find the random fluctuations of h, N and $\widetilde{D}^3$ round their means.

A measure for these functions is the standard deviation σ defined by:

$$\sigma^2(x) = \overline{(x - \overline{x})^2} = \overline{x^2} - \overline{x}^2$$

First, we remark that it is reasonable to assume that the random fluctuations of N and $\widetilde{D}^3$ are independent, so we can write for small σ (N) and σ ($\widetilde{D}^3$):

$$\frac{\sigma(h)}{\overline{h}} = \sqrt{\left(\frac{\sigma(N)}{\overline{N}}\right)^2 + \left(\frac{\sigma(\widetilde{D}^3)}{\overline{\widetilde{D}^3}}\right)^2} \tag{I.2}$$

Further, we note that the standard deviation of the mean of P samples of a stochastic variable is equal to the standard deviation of the variable itself, divided by $\sqrt{P}$. So we can estimate $\sigma(\widetilde{D}^3)$ with

$$\sigma^2(\widetilde{D}^3) = \frac{\sigma^2(D^3)}{\overline{N}} \approx \frac{\overline{\widetilde{D}^6} - (\overline{\widetilde{D}^3})^2}{\overline{N}}$$

This results into:

$$\left(\frac{\sigma(\widetilde{D}^3)}{\overline{\widetilde{D}^3}}\right)^2 = \frac{1}{\overline{N}} \frac{\overline{\widetilde{D}^6}}{(\overline{\widetilde{D}^3})^2} - \frac{1}{\overline{N}} \tag{I.3}$$

The problem to determine σ (N) remains. We, therefore, divide the time interval, T, into M equal time intervals, where $M \gg \overline{N}$. If it is assumed that two raindrops never reach the orifice simultaneously, which makes it possible to choose M in such a manner, that never more than one drop is caught during an interval; the chance that during any interval a raindrop is caught is $\bar{N}/M$. The chance that during any given interval no drop is caught is $(1-\bar{N}/M)$. It can then be proved that the probability that during N intervals a raindrop is caught can be written as:

$$P(N) = \frac{M!}{N!(M-N)!}\left(\frac{\bar{N}}{M}\right)^N\left(1-\frac{\bar{N}}{M}\right)^{M-N} \qquad (I.4)$$

For large N, this equation transforms into:

$$P(N) = \frac{\bar{N}^N e^{-\bar{N}}}{N!} \qquad (I.5)$$

This is the Poisson distribution, for which

$$\sigma(N) = \sqrt{\bar{N}} \qquad (I.6)$$

From (I.2), (I.3) and (I.6) we obtain:

$$\frac{\sigma(h)}{\bar{h}} = \sqrt{\frac{1}{\bar{N}}\frac{\overline{\widetilde{D}^6}}{(\widetilde{D}^3)^2}} \qquad (I.7)$$

If we have only 1 sample $\frac{\sigma(h)}{\bar{h}}$ is of course estimated with $\sqrt{\frac{1}{N}\frac{\widetilde{D}^6}{(\widetilde{D}^3)^2}}$

APPENDIX II

DERIVATION OF EQUATIONS (7) AND (10)

It is assumed that the region of interest is square-shaped and has an area S, while at its centre, P, a rain gauge is installed. Let h_0 be the precipitation depth measured at P during a certain time interval, T. The mean precipitation depth over S is denoted by h_G.

Further, it is assumed that the area is climatologically homogeneous and isotrope.

Because of these assumptions, a reasonable estimate of h_G will be:

$$\hat{h}_G = h_0 \qquad (II.1)$$

Now let us imagine that S is covered by a dense and regular network of ideal rain gauges. The precipitation depths obtained at these stations are denoted by $h_1^*, h_2^* ..., h_N^*$.

We can write:

$$h_G = \frac{1}{N} \sum_{i=1}^{N} h_i^*$$

In fact:

$$h_G = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} h_i^* \tag{II.2}$$

A measure of accuracy of the estimate given by equation (II.1) is the quantity defined by

$$E^2 = \overline{(h_G - \hat{h}_G)^2} \tag{II.3}$$

The bar denotes an average over a large series of h_G and $\hat{h}_G$.

From equations (II.1), (II.2) and (II.3), we obtain:

$$E^2 = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left(\overline{h_i^* h_j^*} - \overline{h_i^* h_0} - \overline{h_j^* h_0} + \overline{h_0^2} \right) \tag{II.4}$$

The measured precipitation depth, h_0, can be written as

$$h_0 = h_0^* + \delta \tag{II.5}$$

where h_0^* is the true precipitation depth at P, and δ is a random measuring error with $\overline{\delta} = 0$. (For the sake of simplicity it is assumed that h_0 has been corrected for systematic errors such as those due to wind).

Further, it is reasonable to assume that:

$$\overline{h_i^* \delta} = 0 \quad (i = 0, 1, 2 \ldots N) \tag{II.6}$$

The Quantity $\overline{\delta^2}$ is the standard error of observations at P.

From equations (II.4), (II.5) and (II.6) we get:

$$E^2 = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left(\overline{h_i^* h_j^*} - \overline{h_i^* h_0^*} - \overline{h_i^* h_0^*} + \overline{h_0^{*2}} + \overline{\delta^2} \right) \tag{II.7}$$

As the homogeneity of S has been assumed, we can write:

$$\overline{h_0^*} = \overline{h_i^*} \equiv x \quad \text{and} \quad \left(\overline{h_0^*}\right)^2 - \overline{h_0^{*2}} = \left(\overline{h_i^*}\right)^2 - \overline{h_i^{*2}} \equiv \sigma^2 \tag{II.8}$$

$(i = 1, \ldots N)$.

Thus the correlation coefficient

$$\rho_{ij}^* = \frac{\overline{h_i^* h_j^*} - \overline{h_i^*}\,\overline{h_j^*}}{\sigma_i \sigma_j} \tag{II.9}$$

is equal to

$$\rho^*_{ij} = \frac{\overline{h^*_i h^*_j} - x^2}{\sigma^2} \tag{II.10}$$

So we obtain:

$$E^2 = \lim_{N\to\infty} \frac{\sigma^2}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left(\rho^*_{ij} - \rho^*_{i0} - \rho^*_{j0} + 1 + \Delta^2\right) \tag{II.11}$$

where

$$\Delta^2 = \frac{\overline{\delta^2}}{\sigma^2} \tag{II.12}$$

As we have seen, the correlation coefficient can be described by equation (4):

$$\rho\,(r) = \rho_0\, e^{-r/r_0}$$

It is noted that this relationship has been found for the correlation coefficients computed with the series of measured precipitation depths, while ρ^*_{ij} stands for the correlation between two series of true precipitation.

If, analogously to (II.5), we write:

$$h_i = h^*_i + \delta_i, \quad h_j = h^*_j + \delta_j \tag{II.13}$$

then it can easily be shown that:

$$\rho_{ij} = \rho^*_{ij} \frac{1}{\Delta^2 + 1} \quad (i \neq j)$$

If it is assumed that:

$$\overline{\delta_i \delta_j} = 0,\ \overline{\delta_i} = \overline{\delta_j} = 0,\ \overline{\delta_i h^*_j} = 0,\ \overline{\delta^2_i} = \overline{\delta^2_j} = \overline{\delta^2},\ \text{while}\ \overline{\delta^2}/\sigma^2 = \Delta^2$$

the "true" correlation function, ρ^*_{ij}, will reach the value 1 for $r \to 0$.*) Thus, combining equations (4) and (II.13), we find:

$$\rho_0 = \frac{1}{\Delta^2 + 1} \tag{II.14}$$

and

$$\rho^*_{ij} = e^{-r_{ij}/r_0} \tag{II.15}$$

If $\Delta \lesssim 0.25$, then $\frac{1}{\Delta^2 + 1} \approx 1 - \Delta^2$, so $\Delta^2 \approx 1 - \rho_0$ (II.16)

*) assuming that ρ_0 is independent of r_{ij}.

Substituting (II.15) and (II.16) into (II.11) we get:

$$E^2 = \sigma^2 \left[(1-\rho_0) + \lim_{N \to \infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{i=1}^{N} e^{-r_{ij}/r_0} - e^{-r_{io}/r_0} - e^{-r_{jo}/r_0} + 1 \right] \tag{II.17}$$

With the aid of a computer, the term

$$\lim_{N \to \infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left(e^{-r_{ij}/r_0} - e^{-r_{io}/r_0} - e^{-r_{jo}/r_0} + 1 \right)$$

was determined for a square area S (see Kagan, 1972).

In this way

$$E^2 = \sigma^2 \left[(1-\rho_0) + \frac{0.23}{r_0} \sqrt{S} \right] \tag{II.18}$$

is obtained.

Next, we consider the situation that n rain gauges are evenly distributed over S. The precipitation depths recorded at these points are denoted by $h_1, h_2, \ldots, h_n$.

Each rain gauge covers a subarea S/n.

The areal precipitation depths of this subarea is now estimated with the precipitation depth recorded at the station belonging to it.

That means that:

$$\hat{h}_G = \frac{1}{n} \sum_{i=1}^{n} h_i \tag{II.19}$$

The standard error of estimation for the i^{th} subarea is given by (II.18):

$$E_i = \sqrt{\sigma^2 \left[(1-\rho_0) + \frac{0.23}{r_0} \sqrt{\frac{S}{n}} \right]} \tag{II.20}$$

Assuming that the errors of estimate for the various subareas are independent, we can write for the standard error of the estimate given by (II.19):

$$E = \sqrt{\frac{1}{n} E_i^2} = \sqrt{\frac{1}{n} \sigma^2 \left[(1-\rho_0) + \frac{0.23}{r_0} \sqrt{\frac{S}{n}} \right]} \tag{II.21}$$